PATH INTEGRAL APPROACH TO MULTIDIMENSIONAL QUANTUM TUNNELLING

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ABSTRACT

Path integral formulation of the coupled channel problem in the case of multidimensional quantum tunneling is presented and two-time influence functionals are introduced. The two-time influence functionals are calculated explicitly for the three simplest cases : Harmonic oscillators linearly or quadratically coupled to the translational motion and a system with finite number of equidistant energy levels linearly coupled to the translational motion. The effects of these couplings on the transmission probability are studied for two limiting cases, adiabatic case and when the internal system has a degenerate energy spectrum. The condition for the transmission probability to show a resonant structure is discussed and exemplified. Finally, we study the properties of the dissipation factor in the adiabatic limit, and study its correlation with the friction coefficient in the classically accessible region.

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I. INTRODUCTION

Until recently the cross sections for fusion of two heavy ions have been analyzed in terms of a simple model, where one starts with a local, one-dimensional real potential barrier formed by the nuclear and Coulomb interactions and assumes that the absorption into the fusion channel takes place at the region inside the barrier after the system is transmitted through. As a further approximation one replaces this barrier by a parabola with the same height, location and curvature at the top, and varies these three parameters to fit the cross sections. The systematics of fusion cross-sections have been studied in this way in Ref. 1. However, a number of recent experiments have shown that the fusion cross-sections for intermediate-mass systems below the Coulomb barrier are much larger than those expected from such a simple picture[2–8]. The inadequacy of this model for intermediate and heavy mass systems has been explicitly demonstrated by inverting the experimental data to obtain directly the effective one-dimensional barrier.[9]

The analysis of Ref. (9) and other theoretical work[10–23] where the enhancement has been attributed to the coupling of the relative motion to other degrees of freedom motivated this study of the multidimensional dynamics of the barrier transmission process. Quantum tunnelling in a multidimensional dissipative system is relevant not only to fusion and deep-inelastic reactions between heavy ions, but also to a number of other physical phenomena. For example, dissipative quantum tunnelling is an important mechanism for a super-conducting quantum interference device (SQUID) at low temperatures.[24]

The natural language to study multidimensional barrier penetration is the coupled channels formalism[25]. In the next section we present the path integral approach to the coupled channels problem and introduce the two-time influence functional. Path integral approach is especially useful for a semi-classical analysis of our problem, which is appropriate for sufficiently massive heavy-ions. A WKB-like approximation in a similar spirit has been introduced earlier.[26] In Section III, we explicitly calculate the two-time influence functionals for three particular systems : a harmonic oscillator linearly or quadratically coupled to the translational motion, and a spin system with $(2j+1)$ levels linearly coupled to the translational motion.

Section IV includes an analysis of the transmission probability in those cases when these three internal systems has a degenerate spectrum. We show that for all the systems considered the transmission probability is given as an integral with appropriate weights over probabilities for transmission across frozen one-dimensional barriers as shown in Ref.15 for the particular case of a linear coupling to a harmonic oscillator. In Section V we study the conditions for the transmission probability to show resonant behavior.

In Section VI we investigate the transmission probability for these particular systems in the adiabatic limit. We show that the total transmission probability is given as a product of the probability for transmission across a one-dimensional effective adiabatic barrier and a dissipation factor which depends on the coupling Hamiltonian. In Section VII we consider a model where the translational motion is linearly coupled to a damped harmonic oscillator; study the correlation between friction coefficient and the dissipation factor, and compare our results with previous studies. Finally Section VIII includes a summary and a discussion of our results.

II. PATH INTEGRAL APPROACH TO THE COUPLED CHANNELS PROBLEM: TWO-TIME INFLUENCE FUNCTIONAL

The Hamiltonian of the system we want to study is

$$H = -\frac{\hbar^2}{2\mu}\frac{\partial^2}{\partial R^2} + V(R) + H_0(q) + H_{\rm int}(q,R), \tag{2.1}$$

where R is a translational degree of freedom and $H_0(q)$ is the Hamiltonian of the internal system coupled to this degree of freedom. Although our motivation is to study the effects of coupling to internal degrees of freedom on the transmission through a potential barrier in heavy-ion collisions, for the purpose of discussion in this and next sections R will be taken as *any* translational degree of freedom and $V(R)$ to be *any* potential which decreases sufficiently rapidly for large $|R|$. Eventually, in order to investigate the subbarrier fusion of two heavy ions we will specify R as the separation of their centers of mass and $V(R)$ as the barrier formed by the nuclear and Coulomb forces. Nevertheless, the formalism developed here will apply in most cases.

We assume that the eigenvalues and eigenstates of the internal Hamiltonian $H_0(q)$ are known and $H_{\rm int}(q,R)$ vanishes except when $R_f \leq R \leq R_i$. Hence for $R > R_i$ and $R < R_f$, the internal system $H_0(q)$ has well-defined initial and final states with energies ϵ_i and ϵ_f, characterized by the two sets of quantum numbers $\{n_i\}$ and $\{n_f\}$, respectively. In the cases where we calculate only an inclusive probability and sum over all final states, we can relax the condition above and let $H_{\rm int}(q,R)$ be non-zero for $R < R_f$.

We want to calculate first the amplitude for transition from an initial state characterized by R_i and n_i to the final state characterized by R_f and n_f at a given energy. Starting from the expression which relates Green's function and the T-operator[27]

$$G^+(E) = G_0^+(E) + G_0^+(E)T(E)G_0^+(E), \tag{2.2}$$

where $G_0^+(E)$ and $G^+(E)$ are the unperturbed and total Green's functions respectively, one can identify the S-matrix elements as [28]

$$\begin{aligned} S_{n_f,n_i}(E) = -\lim_{\substack{R_i\to\infty \\ R_f\to-\infty}} \left(\frac{P_iP_f}{\mu^2}\right)^{\frac{1}{2}} \exp\left[\frac{i}{\hbar}(P_fR_f - P_iR_i)\right] \\ \times (R_f, n_f|G^+(E)|R_i, n_i), \end{aligned} \tag{2.3}$$

where

$$P_i = P(R_i) \equiv [2\mu(E - \epsilon_i - V(R_i))]^{\frac{1}{2}}. \tag{2.4}$$

The G-matrix elements can be written as

$$(R_f, n_f|G^+(E)|R_i, n_i) = \int_0^\infty dT\, e^{+\frac{iET}{\hbar}} K(R_f, n_f, T; R_i, n_i, 0), \tag{2.5}$$

where

$$\begin{aligned} K(R_f, n_f, T; R_i, n_i, 0) = \int_{-\infty}^{+\infty} dq_i dq_f (R_f, n_f|R_f, q_f) \\ \times (R_f, q_f|\hat{U}(T,0)|R_i, q_i)(R_i, q_i|R_i, n_i). \end{aligned} \tag{2.6}$$

The middle term in the integral (2.6) can be expressed as the path integral

$$(R_f, q_f|\hat{U}(T,0)|R_i, q_i) = \int \mathcal{D}[R(t)]\mathcal{D}[q(t)] e^{\frac{i}{\hbar}S_{\rm tot}(q,R;T)}, \tag{2.7}$$

where the integral is over all paths which satisfy the boundary conditions $q_f = q(T)$, $q_i = q(0)$, $R_f = R(T)$ and $R_i = R(0)$. In Eq. (2.7) the total action is a sum of three parts:

$$\begin{aligned} S_{\rm tot}(q,R;T) &= \int_0^T dt[\mathcal{L}_t(R) + \mathcal{L}_0(q) + \mathcal{L}_{\rm int}(q,R)] \\ &= S_t(R,T) + S_0(q,T) + S_{\rm int}(q,R;T), \end{aligned} \tag{2.8}$$

where

$$L_t(R) = \frac{1}{2}\mu\dot{R}^2 - V(R). \tag{2.9}$$

In order to calculate the path integrals in Eq. (2.7) we first pick a particular path $R(t)$, do all integrals over all paths $q(t)$ keeping $R(t)$ fixed, and repeat this procedure for all paths $R(t)$:

$$\langle R_f, q_f|\hat{U}(T,0)|R_i, q_i\rangle = \int \mathcal{D}[R(t)]e^{\frac{i}{\hbar}S_t(R,T)} \int \mathcal{D}[q(t)]e^{\frac{i}{\hbar}[S_0(q,T)+S_{\rm int}(q,R;T)]}. \tag{2.10}$$

The second path integral in Eq. (2.10) is then the propagator for the internal system to propagate from the initial position $q = q_i$ at time $t = 0$ to the final position $q = q_f$ at time $t = T$ under the influence of an external time-dependent interaction $H_{\rm int}(q,R)$ which vanishes outside the period $0 \leq t \leq T$. Inserting Eq. (2.10) into Eq. (2.6) we get

$$K(R_f, n_f, T; R_i, n_i, 0) = \int \mathcal{D}[R(t)]e^{\frac{i}{\hbar}S_t(R,T)} \mathcal{W}_{n_f,n_i}[R(t); T, 0], \tag{2.11}$$

where we denoted the transition amplitude for the internal system as

$$\mathcal{W}_{n_f,n_i}[R(t); T, 0] = \langle n_f|\hat{U}(R(t); T, 0)|n_i\rangle, \tag{2.12}$$

where $\hat{U}$ satisfies the differential equation

$$i\hbar\frac{\partial\hat{U}}{\partial t} = [H_0 + H_{\rm int}(q, R(t))]\hat{U}, \tag{2.13}$$

subject to the boundary condition $\hat{U}(t = 0) = 1$.

A quantity of interest in heavy-ion physics is the inclusive transmission probability, $P(E)$, i.e. the total probability that the internal system emerges in any final state. We have

$$P(E) = \sum_{n_f=0}^{\infty} |S_{n_f,n_i}(E)|^2; \tag{2.14}$$

substituting Eqs. (2.3), (2.5) and (2.11) into (2.14) we get

$$\begin{aligned} P(E) = \lim_{\substack{R_i\to\infty \\ R_f\to-\infty}} \left(\frac{P_i P_f}{\mu^2}\right) \int_0^\infty dT e^{\frac{i}{\hbar}ET} \int_0^\infty d\tilde{T} e^{-\frac{i}{\hbar}E\tilde{T}} \\ \times \int \mathcal{D}[R(t)]\mathcal{D}[\tilde{R}(\tilde{t})]e^{\frac{i}{\hbar}[S_t(R,T)-S_t(\tilde{R},\tilde{T})]} \rho_M(\tilde{R}(\tilde{t}), \tilde{T}; R(t), T), \end{aligned} \tag{2.15}$$

where we defined the *two-time influence functional*, ρ_M, as

$$\rho_M(\tilde{R}(\tilde{t}), \tilde{T}; R(t), T) = \sum_{n_f} \mathcal{W}^*_{n_f,n_i}[\tilde{R}(\tilde{t}); \tilde{T}, 0]\mathcal{W}_{n_f,n_i}[R(t); T, 0]. \tag{2.16}$$

In writing Eqs. (2.15) and (2.16), we assumed that energy dissipated to the internal system is small as compared to the total energy, and took P_f outside the summation over final states. Note that one can repeat the same steps as above when $H_{\rm int}(q,R) \neq 0$ for $R < R_f$. In such cases the sum over all final states n_f should be replaced by an integral over all final coordinates q_f.

Influence functionals were originally introduced by Feynman[29] to provide a convenient description of systems interacting with their environment. However, Feynman is interested in calculating the transition probability *during a given time interval*, rather than the transition probability *for a given energy*, $|G(E)|^2$. Hence in his definition there is only one time variable and both $\tilde{R}$ and R are the functions of the same variable. Nuclear physics applications of the influence functionals defined in this way have been extensively investigated[30,31]. Since the expression defined by Eq. (2.10) is a generalization to the case when there are two independent time variables of Feynman's original expression, we call it the two-time influence functional. Of course, the two-time influence functional calculated for any internal system can be reduced to the ordinary influence functional by setting $T = \tilde{T}$. Consequently, the two-time influence functionals we calculate in the next section can also be used as ordinary influence functionals in other contexts.

We note that Eq. (2.15) is an *exact* expression for the inclusive transmission probability. As long as no semi-classical (i.e. stationary phase or saddle point) approximations are invoked, all time and path integrals are exact, hence both T and $\tilde{T}$ are real. When such approximations are done for the time integrals, both T and $\tilde{T}$ will remain real for classically allowed regions, but they will become pure imaginary for classically forbidden regions. For the latter cases the real time influence functional in Eq. (2.15) would be replaced by the imaginary-time influence functional[17]. We defer the discussion of the imaginary-time influence functionals to a forthcoming publication. However, if we specify $V(R)$ to be a potential barrier and can somehow calculate Eq. (2.15) exactly, without invoking any such approximations, we should keep both T and $\tilde{T}$ real (see Section IV).

If the interaction term $H_{\rm int}(q,R)$ is zero at all times, the influence functional is identially 1, hence the internal and translational degrees of freedom decouple. If we further take $V(R)$ as a potential barrier, then Eq. (2.15) gives the transmission probability of a single particle:

$$P_0(E) = \lim_{\substack{R_i\to\infty \\ R_f\to-\infty}} \left(\frac{P_i P_f}{\mu^2}\right) \int_0^\infty dT d\tilde{T} e^{\frac{i}{\hbar}E(T-\tilde{T})} \int \mathcal{D}[R(t)]\mathcal{D}[\tilde{R}(\tilde{t})] \times e^{\frac{i}{\hbar}[S_t(R,T)-S_t(\tilde{R},\tilde{T})]} . \tag{2.17}$$

Especially when the potential barrier has the same topological structure as a quadratic function, Eq. (2.17) can be calculated[32] in a uniform approximation with a proper treatment of the multiple reflections under the barrier to obtain the usual WKB expression for penetrability:

$$P_0(E,V(R)) = \left[1 + \exp\!\left(2\int_{r_1}^{r_2} \sqrt{\frac{2\mu}{\hbar^2}(V(r)-E)}\right)\right]^{-1} . \tag{2.18}$$

Eq. (2.18) is valid uniformly from below to the above of the barrier.

III. EXACT CALCULATION OF THE TWO-TIME INFLUENCE FUNCTIONAL FOR PARTICULAR SYSTEMS:

Using Eq. (2.12) and the completeness of final states, the two-time influence functional, Eq. (2.16) can be rewritten as

$$\rho_M(\tilde{R}(\tilde{t}),\tilde{T};R(t),T) = \langle n_i|\hat{U}^\dagger(\tilde{R}(\tilde{t});\tilde{T},0)\hat{U}(R(t));T,0)|n_i\rangle, \tag{3.1}$$

where the operator $\hat{U}$ satisfies the equation

$$i\hbar\frac{\partial \hat{U}}{\partial t} = [\hat{H}_0 + \hat{H}_{\rm int}(q,R(t))]\hat{U}, \tag{3.2a}$$

with the initial condition

$$\hat{U}(R(t);0,0) = 1. \tag{3.2b}$$

We wish to catalogue all those systems for which the two-time influence functionals can be calculated exactly.

Exact solutions of Eq. (3.1) can be obtained when $\hat{H}_0 + \hat{H}_{\text{int}}$ can be expressed a a linear combination of the *generators* of a given group. For such cases $\hat{U}$ defined by Eq. (3.2) will be an *element* of this group[33]. Consequently for these systems the two-time influence functional, given by Eq. (3.1) is a particular diagonal matrix element of the group in the appropriate representation. Furthermore, one can add various Casimir operators of the group to $\hat{H}_0 + \hat{H}_{\text{int}}$ and still calculate Eq. (3.1) exactly.

In this section, we calculate Eq. (3.1) for three simplest cases, where we take the interaction term to be separable, i.e. $H_{\text{int}}(q,R) = f(R)g(q)$. These systems are an harmonic oscillator, linearly or quadratically coupled to a translational coordinate and a system with finite number of equidistant energy levels. Of course using groups other than those studied in this section, it is possible to formulate Hamiltonians which could model more complicated phenomena, but here we restrict ourselves to the simplest cases to illustrate our method.

a) Harmonic oscillator linearly coupled to the translational motion :

In this case the internal system is a harmonic oscillator

$$\hat{H}_0(q) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial q^2} + \frac{1}{2}m\omega_0^2 q^2, \tag{3.3}$$

and the interaction term is taken to be

$$\hat{H}_{\text{int}}(q,R) = f(R)q, \tag{3.4}$$

where $f(R)$ is a real function of R. Introducing the creation and annihilation operators a and $a^\dagger$ for the harmonic oscillator we can write the above Hamiltonian as

$$\hat{H}_0 + \hat{H}_{\text{int}} = (a^\dagger a + \frac{1}{2})\hbar\omega_0 + \alpha_0 f(R)(a + a^\dagger), \tag{3.5}$$

where α_0 is the amplitude of the zero point motion of the harmonic oscillator and is given by

$$\alpha_0 = \sqrt{\frac{\hbar}{2m\omega_0}}. \tag{3.6}$$

The Hamiltonian (3.5) is now written as a linear combination of the generators of the Weyl group [33]: $a^\dagger a$, a, $a^\dagger$ and I. The resulting two-time influence functional, assuming that the oscillator is initially in the ground state, is

$$\begin{aligned}\rho_M(\tilde{R}(\tilde{t}),\tilde{T};R(t),T) = \mathrm{e}^{-\frac{i}{2}\omega_0(T-\tilde{T})}\exp\Bigg\{-\frac{\alpha_0^2}{\hbar^2}\Bigg[\int_0^T dt\int_0^t ds\, f(R(t))f(R(s))e^{-i\omega_0(t-s)} \\ + \int_0^{\tilde{T}} dt\int_0^t ds\, f(\tilde{R}(t))f(\tilde{R}(s))e^{i\omega_0(t-s)} \\ -\mathrm{e}^{i\omega_0(\tilde{T}-T)}\int_0^T dt\, f(R(t))e^{i\omega_0 t}\int_0^{\tilde{T}} ds\, f(\tilde{R}(s))e^{-i\omega_0 s}\Bigg]\Bigg\}.\end{aligned} \tag{3.7}$$

For $T = \tilde{T}$, this is the influence functional originally calculated by Feynman[29].

b) Harmonic oscillator quadratically coupled to the translational motion :

We again take the internal system to be a harmonic oscillator given by Eq. (3.3) and the interaction term as

$$H_{\rm int}(q,R) = h(R)q^2, \tag{3.8}$$

where $h(R)$ is another, arbitrary real function of R which vanishes as $|R|\to\infty$. In terms of the creation and annihilation operators we then have

$$\hat{H}_0 + \hat{H}_{\rm int} = (a^\dagger a + \frac{1}{2})\hbar\omega_0 + \alpha_0^2 h(R)(a^\dagger a^\dagger + aa + 2a^\dagger a + 1). \tag{3.9}$$

Introducing the three generators of the non-compact group $SU(1,1)$ as[33]

$$K_+ = \frac{1}{2}a^\dagger a^\dagger, \tag{3.10a}$$

$$K_- = \frac{1}{2}aa, \tag{3.10b}$$

$$K_0 = \frac{1}{2}(a^\dagger a + \frac{1}{2}), \tag{3.10c}$$

Eq. (3.9) takes the form

$$\hat{H}_0 + \hat{H}_{\rm int} = 2\hbar\omega_0 K_0 + 2\alpha_0^2 h(R)[K_+ + K_- + 2K_0]. \tag{3.11}$$

The solution of Eq. (3.2) is given as

$$\hat{U} = \exp(-i\phi K_0)\exp(\varsigma K_+)\exp[K_0\log(1-|\varsigma|^2)]\exp(-\varsigma^* K_-), \tag{3.12}$$

where

$$\varsigma(R(t);t) = z(R(t);t)e^{i\phi(R(t);t)}, \tag{3.13}$$

and z and ϕ satisfy the following equations :

$$i\frac{dz}{dt} = 2\omega_0 z + \frac{h(R(t))}{m\omega_0}(z+1)^2, \tag{3.14}$$

and

$$\frac{d\phi}{dt} = 2\omega_0 + \frac{2h(R(t))}{m\omega_0} + \frac{h(R(t))}{m\omega_0}(z+z^*), \tag{3.15}$$

respectively with the initial conditions $z(t=0)=0$ and $\phi(t=0)=0$. Inserting Eq. (3.12) into Eq. (3.1), and assuming again that the oscillator is initially in the ground state, we obtain the two-time influence functional as

$$\rho_M(\tilde{R}(\tilde{t}),\tilde{T};R(t),T) = \frac{e^{\frac{i}{4}[\phi(\tilde{R})-\phi(R)]}(1-|z(\tilde{R})|^2)^{\frac{1}{4}}(1-|z(R)|^2)^{\frac{1}{4}}}{[1-z^*(\tilde{R})z(R)]^{\frac{1}{2}}}, \tag{3.16}$$

where $\phi(\tilde{R})[\phi(R)]$ and $z(\tilde{R})[z(R)]$ are solutions of Eqs. (3.15) and (3.14) respectively when $t = \tilde{T}[T]$.

The quantum tunneling problem for the Hamiltonian (3.9), where the function $h(R)$ is taken to be a Gaussian, has been previously discussed as a model for nuclear fission[34].

c) Linear coupling to a system with finite number of equidistant energy levels:

We consider a simple system with equidistant $(2j+1)$-energy levels:

$$H_o = \omega_0 J_z, \tag{3.17}$$

coupled to a translational degree of freedom as

$$H_{\rm int} = 2\beta(R) J_x, \tag{3.18}$$

where J_x and J_z are the SU(2) generators in the $(2j+1)$-dimensional representation and $\beta(R)$ is a real function of R. One can think of this system, for example, as a two-level Lipkin-Meshov-Glick system[35] with no self-interactions, where transitions between two levels are caused only by the coupling to the external degree of freedom. Of course Eqs. (3.17) and (3.18) can be used to model any system with equidistant energy levels whatever the underlying dynamics might be. The solution of Eq. (3.2) is

$$\hat{U} = \exp\left(-\frac{i\varphi}{\hbar}J_z\right)\exp\left(\frac{\tau J_+}{\hbar}\right)\exp\left[\frac{J_z}{\hbar}\log(1+|\tau|^2)\right]\exp\left(-\frac{\tau^* J_-}{\hbar}\right), \tag{3.19}$$

where

$$\tau(R(t);t) = y(R(t);t)e^{i\varphi(R(t);t)}, \tag{3.20}$$

and y and φ satisfy the equations

$$i\frac{dy}{dt} = \omega_0 y + \beta(R(t))(1-y^2), \tag{3.21}$$

and

$$\frac{d\varphi}{dt} = \omega_0 - \beta(R(t))(y+y^*), \tag{3.22}$$

respectively with the initial conditions $y(t=0)=0$ and $\varphi(t=0)=0$. Again assuming that the system described by H_0 is initially in the ground state, we get the two-time influence functional as

$$\rho_M(\tilde{R}(\tilde{t}),\tilde{T};R(t),T) = \frac{e^{-ij[\varphi(\tilde{R})-\varphi(R)]}[1+y^*(\tilde{R})y(R)]^{2j}}{[1+|y(\tilde{R})|^2]^j[1+|y(R)|^2]^j}, \tag{3.23}$$

where $\varphi(\tilde{R})[\varphi(R)]$ and $y(\tilde{R})[y(R)]$ are solutions of Eqs. (3.22) and (3.21) respectively when $t=\tilde{T}[T]$.

The reader would observe the similarities between Eq. (3.16) and Eq. (3.23). This is because the relevant group representations can be obtained by a proper analytic continuation of SU(1,1) to SU(2)[36] and the appropriate representations of SU(1,1) for transitions from the ground state of the oscillator are the discrete series[33] with the quantum number $k=\frac{1}{4}$.

A two level version of the model defined by Eqs. (3.17) and (3.18) has been discussed previously by one of us[37] to investigate barrier-top resonances, a topic which we will discuss in Section V.

IV. DEGENERATE SPECTRUM RESULTS

Our aim is to calculate Eq. (2.16) with the expressions we derived in section III for the two-time influence functional. We might choose to calculate it by invoking saddle point or uniform approximations, or exactly, maybe numerically. The former is more convenient whenever we have an explicit expression for the influence functional, such as Eq. (3.7). On the contrary, some influence functionals we derived are expressed in terms of solutions of given Riccati-type differential equations, which, in general, cannot be solved analytically. For numerical investigatons, this should not pose any problem. However, when we want to do analytical approximations for the integrals in Eq. (2.15), we also need analytical, but approximate solutions of these differential equations.

There is yet a limit in which we can obtain exact expressions for the penetrability. This is the limit where the internal system has a degenerate energy spectrum. To achieve this limit, for the harmonic oscillator we let $\omega_0 \rightarrow 0$ and the mass parameter of the oscillator, $m \rightarrow \infty$, keeping $\alpha_0^2 \sim 1/m\omega_0$ fixed. For the system with a finite number of energy levels we simply let $\omega_0 \rightarrow 0$. We will now study these cases separately:

a) Harmonic oscillator linearly coupled to a translational motion :

For the limit discussed above, Eq. (3.7) takes the form

$$\rho_M(\tilde{R}(s), \tilde{T}; R(t), T) = \exp\left\{-\frac{\alpha_0^2}{2\hbar^2}\left[\int_0^T dt f(R(t)) - \int_0^{\tilde{T}} ds f(\tilde{R}(s))\right]^2\right\}. \tag{4.1}$$

We can linearize the exponent using the formula

$$\int_{-\infty}^{+\infty} dx e^{-(ax^2+bx)} = \sqrt{\frac{\pi}{a}} e^{\frac{b^2}{4a}}. \tag{4.2}$$

The result reads,

$$\rho_M(\tilde{R}(s), \tilde{T}, R(t), T) = \frac{1}{\alpha_0\sqrt{2\pi}} \int_{-\infty}^{+\infty} d\alpha e^{-\frac{1}{2}\left(\frac{\alpha}{\alpha_0}\right)^2} e^{-\frac{i\alpha}{\hbar}\left[\int_0^T dt f(R(t)) - \int_0^{\tilde{T}} ds f(\tilde{R}(s))\right]}. \tag{4.3}$$

Finally inserting Eq. (4.3) into Eq. (3.15) we get the inclusive transmission probability as

$$\begin{aligned} P(E) = \frac{1}{\alpha_0\sqrt{2\pi}} \int_{-\infty}^{+\infty} d\alpha e^{-\frac{1}{2}\left(\frac{\alpha}{\alpha_0}\right)^2} \Bigg\{ \lim_{\substack{R_i \to \infty \\ R_f \to -\infty}} \left(\frac{P_i P_f}{\mu^2}\right) \\ \times \left| \int_0^{\infty} dT e^{\frac{i}{\hbar}ET} \int \mathcal{D}[R(t)] e^{\frac{i}{\hbar}\int_0^T dt\left[\frac{1}{2}\mu\dot{R}^2 - V(R) - \alpha f(R)\right]} \right|^2 \end{aligned} \tag{4.4}$$

Note that the expression in the curly brackets in Eq. (4.4) is the exact expression for the transmission probability of a single particle of mass μ across the potential barrier $V(R) + \alpha f(R)$, as given in Eq. (2.17). Consequently, we can write Eq. (4.4) in the form[15]

$$P(E) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dx e^{-\frac{x^2}{2}} P_0(E, V(R) + x\alpha_0 f(R)). \tag{4.5}$$

Eq. (4.5) is an average with the probability distribution of an oscillator ground-state representing zero-point fluctuations[10].

b) Harmonic oscillator quadratically coupled to a translational motion :

In the limit $\omega_0 \rightarrow 0$, $m \rightarrow \infty$ with $m\omega_0 =$ fixed, the differential equations (3.14) and (3.15) can be solved analytically, yielding

$$\rho_M(\tilde{R}(s), \tilde{T}; R(t), T) = \left\{1 + \frac{2i\alpha_0^2}{\hbar}\left[\int_0^T h(R(t))dt - \int_0^{\tilde{T}} h(\tilde{R}(s))ds\right]\right\}^{-\frac{1}{2}}. \tag{4.6}$$

Again using Eq. (4.2), we can write Eq. (4.6) as

$$\rho_M(\tilde{R}(s),\tilde{T};R(t),T)=\frac{1}{\sqrt{\pi}}\int_{-\infty}^{+\infty}dx e^{-x^2}\exp\left\{-\frac{2i\alpha_0^2x^2}{\hbar}\left[\int_0^T h(R(t))dt-\int_0^{\tilde{T}}h(\tilde{R}(s))ds\right]\right\}. \qquad (4.7)$$

Inserting Eq. (4.7) into Eq. (2.15) and following the same steps as above we get for the inclusive transmission probability

$$P(E)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}dx e^{-\frac{x^2}{2}}P_0(E,V(R)+x^2\alpha_0^2h(R)). \qquad (4.8)$$

This is again the zero point motion formula of Ref. 10.

c) Harmonic oscillator both linearly and quadratically coupled to a translational motion : We can also easily study the system

$$\hat{H}_0+\hat{H}_{\rm int}=-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial q^2}+\frac{1}{2}m\omega_0^2q^2+f(R)q+h(R)q^2, \qquad (4.9)$$

in the limit $\omega_0\to 0$, $m\to\infty$, with $m\omega_0$ = fixed. The corresponding solution of Eq. (3.2) is given as

$$\hat{U}=\exp\left\{-\frac{i}{\hbar}\hat{M}\int_0^T\alpha_0 f(R(t))dt-\frac{2i}{\hbar}\hat{N}\int_0^T\alpha_0^2h(R(t))dt\right\}, \qquad (4.10)$$

with

$$\hat{M}=a+a^\dagger,$$

$$\hat{N}=K_++K_-+2K_0.$$

Inserting Eq. (4.10) into Eq. (3.1) we obtain the two-time influence functional as

$$\rho_M(\tilde{R}(s),\tilde{T};R(t),T)=\left[1+\frac{2i\alpha_0^2}{\hbar}\left(\int_0^T h(R(t))dt-\int_0^{\tilde{T}}h(\tilde{R}(s))ds\right)\right]^{-\frac{1}{2}}$$
$$\times\exp\left\{-\frac{\alpha_0^2}{2\hbar^2}\left[\int_0^T f(R(t))dt-\int_0^{\tilde{T}}f(\tilde{R}(s))ds\right]\Big/\left[1+\frac{2i\alpha_0^2}{\hbar}\left(\int_0^T h(R(t)dt-\int_0^{\tilde{T}}h(\tilde{R}(s))ds\right)\right]\right\} \qquad (4.11)$$

Using Eq. (4.2), we can write Eq. (4.11) as

$$\rho_M(\tilde{R}(s),\tilde{T};R(t),T)=\frac{1}{\sqrt{\pi}}\int_{-\infty}^{+\infty}dx e^{-x^2}\exp\left\{-\frac{2ix^2\alpha_0^2}{\hbar}\left[\int_0^T h(R(t))dt-\int_0^{\tilde{T}}h(\tilde{R}(s))ds\right]\right\}$$
$$\times\exp\left\{\frac{-i\sqrt{2}\alpha_0x}{\hbar}\left[\int_0^T f(R(t))dt-\int_0^{\tilde{T}}f(\tilde{R}(s))ds\right]\right\}. \qquad (4.12)$$

Finally substituting Eq. (4.12) into Eq. (2.15) and following the same steps as in subsection IV.a, one obtains the inclusive transmission probability as

$$P(E) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dx e^{-\frac{x^2}{2}} P_0(E, V(R) + x\alpha_0 f(R) + x^2 \alpha_0^2 h(R)). \tag{4.13}$$

Note that Eq. (4.13) reduces to Eqs. (4.5) and (4.8) in the limits $f(R) \to 0$ and $h(R) \to 0$ respectively, as it should.

We can also study a system where internal system is a harmonic oscillator and $\hat{H}_{\rm int}(q, R) = V_{\rm int}(q, R)$ is an arbitrary function of q and R. Expanding $V_{\rm int}(q, R)$ in a Taylor series around $q = 0$

$$V_{\rm int}(q, R) = V_{\rm int}(q = 0, R) + \left.\frac{\partial V_{\rm int}}{\partial q}\right|_{q=0} q + \frac{1}{2!} \left.\frac{\partial^2 V_{\rm int}}{\partial q^2}\right|_{q=0} q^2 + .. \tag{4.14}$$

and following similar steps leading to Eq. (4.13) we obtain

$$P(E) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dx e^{-\frac{x^2}{2}} P_0(E, V(R) + V_{\rm int}(\alpha_0 x, R)). \tag{4.15}$$

A model where $V_{\rm int}$ is taken to be a surface-surface interaction between heavy ions, has been studied by Esbensen, Wu and Bertsch[16].

d) Linear coupling to a system with finite number of equidistant levels:

In the limit $\omega_0 \to 0$, the differential equations (3.21) and (3.22) can be solved analytically yielding

$$\begin{aligned} \rho_M(\tilde{R}(s), \tilde{T}; R(t), T) &= \cos^{2j}\left[\int_0^T \beta(R(t))dt - \int_0^{\tilde{T}} \beta(\tilde{R}(s))ds\right] \\ &= \frac{1}{4^j} \sum_{k=0}^{2j} \frac{(2j)!}{k!(2j-k)!} \exp\left\{-\frac{2i(j-k)}{\hbar}\left[\int_0^T \beta(R(t))dt - \int_0^{\tilde{T}} \beta(\tilde{R}(s))ds\right]\right\} \end{aligned} \tag{4.16}$$

Consequently, substitution of Eq. (4.16) into Eq. (2.15) gives

$$P(E) = \frac{1}{4^j} \sum_{k=0}^{2j} \frac{(2j)!}{k!(2j-k)!} P_0(E, V(R) + 2(j-k)\hbar\beta(R)). \tag{4.17}$$

(4.17) is a direct generalization of the two level system[37]. In Appendix A, we derive Eq.(4.17) by a direct diagonalization method of the Hamiltonian.

Eqs. (4.5), (4.8), (4.13) and (4.17) are exact results for the inclusive transmission probability as long as $P_0(E, V_{\rm eff}(R))$ is calculated exactly. One may also choose to calculate P_0 in the uniform approximation, using Eq. (2.18).

Our results show that in the degenerate spectrum limit, the potential barrier is renormalized by an amount determined by the coupling form factor and auxiliary variables (the continuous variable x in Eq. (4.15) and the discrete variable k in Eq. (4.17). In order to get the final result for the inclusive transmission probability, we have to multiply the transmission probability from the effective, renormalized potential with a weight function, $e^{-\frac{x^2}{2}}$ or $\binom{2j}{k}$, representing the probability of having a particular renormalization, i.e. a particular value of the auxiliary variable. We have also shown that in this particular limit it is appropriate to average over probabilities rather than amplitudes, hence Esbensen's zero-point motion prescription[10] becomes exact. Eq. (4.5) was originally obtained by Jacobs and Smilansky[15]. Our results, Eqs. (4.8), (4.13), (4.15) and (4.17), confirm and generalize their conclusions. We also note that in this limit, there is no energy dissipation, since the internal spectrum is degenerate.

In Figures 1-3, we show how the inclusive transmission probability changes as a function of the strength of the coupling form factor for linear and quadratic coupling to a harmonic oscillator with a degenerate spectrum. We have taken both the potential barrier and coupling form factor to be of the Eckart type, so the results shown are exact. The barrier plotted in Fig. 1 is given by $V(R) = V_0/Cosh^2 bR$ with $V_0 = 10MeV$ and $b = 1fm^{-1}$. The coupling form factors in Figs. 2 and 3 are given by $f(R) = f_0/Cosh^2 bR$ and $h(R) = h_0/Cosh^2 bR$ respectively with the coupling strengths shown. We observe that, in the harmonic oscillator case for a linear coupling, irrespective of the sign of the coupling strength, the penetrability decreases above the barrier as compared to the decoupled case, but increases below the barrier. This confirms the previous results[10,12]. For a quadratic coupling, however, the penetrability decreases for positive coupling strength, but increases for negative coupling strength *at all energies*. The opposite limit $\omega_0 \to \infty$ gives similar results[17] (See Section VI). We therefore expect that the situation will be similar for finite ω_0. Consequently, one has to be careful in studying models of the kind as in Eq. (4.14) where the sign of $\frac{\partial^2 V}{\partial q^2}$ might make a considerable difference.

In Fig.4 we plot the transmission probability when the internal system is the spin system with a degenerate spectrum defined by Eqs. (3.17) and (3.18). The coupling form factor is taken to be $\beta(R) = \beta_0/Cosh^2 bR$ with $\hbar\beta_0 = 5MeV$. Transmission probability shows a trend similar to the case of linear coupling to a harmonic oscillator. Furthermore, the decrease above the barrier and the increase below the barrier are more pronounced as the number of levels increases.

For general values of ω_0, one needs either numerical analysis, or further approximations such as the perturbative solutions of the Riccati equations with respect to ω_0. These topics will be treated in forthcoming articles.

Let us assume that the cross-section for the fusion of two heavy-ions at a center-of-mass energy E is given by a sum over all partial waves :

$$\sigma = \sum_{l=0}^{\infty} \sigma_l, \tag{4.18}$$

with

$$\sigma_l = \frac{\pi\hbar^2}{2\mu E}(2l+1)P^{(l)}(E, V(R)), \tag{4.19}$$

where $P^{(l)}(E)$ is the transmission probability for the l-th partial wave. For the case of an internal oscillator degree of freedom with a degenerate spectrum coupled to the center-of-mass motion, using Eq.(4.15), Eq.(4.19) can be rewritten as

$$\sigma_l = \frac{\hbar^2}{2\mu E}\sqrt{\frac{\pi}{2}}(2l+1)\int_{-\infty}^{+\infty} dx e^{-\frac{x^2}{2}} P_0^{(l)}(E, V_{\rm eff}^{(l)}(R,x)), \tag{4.20}$$

where

$$V_{\rm eff}^{(l)}(R,x) = V(R) + \frac{\hbar^2}{2\mu R^2}\frac{l(l+1)}{} + V_{\rm int}(\alpha_0 x, R). \tag{4.21}$$

Let us further assume that the series expansion Eq.(4.14) can be truncated after the first order term in q. Since linear coupling decreases transmission for energies above the barrier, but increases it for energies below the barrier, if E is larger than the s-wave barrier height, σ_l given in Eq.(4.20) will be depleted for lower partial waves, but enhanced for higher partial waves, as shown in the lower portion of Fig. 5. If E is below the s-wave barrier height, σ_l will be enhanced for all values of l, as shown in the upper portion of Fig. 5.

One might be able to study this effect of the linear coupling on the angular momentum distribution of the fusion cross section by examining the gamma-ray multiplicities in the decay of the compound nucleus[37].

V. RESONANCES IN THE INCLUSIVE TRANSMISSION PROBABILITY

The coupling functions $\beta(R)$ in Eq. (3.18) and $f(R)$ and $h(R)$ in Eq. (4.9) can be general as far as they asymptotically vanish. If these functions have bound-state solutions separately, then the transmission probability and consequently fusion cross-section would show resonant behaviour. As a particular model we will consider the degenerate spin system studied in the previous section where we take

$$\beta(R) = \lambda\delta(R - R_B), \tag{5.1}$$

where R_B is the barrier top position for the potential $V(R)$.

For the degenerate spectrum case we have to calculate transmission probabilities for the potential $V(R) + 2(j-k)\lambda\delta(R - R_B)$ as shown in Eq. (4.17). Assuming that the potential $V(R)$ is parabolic, the transmission amplitude can be calculated by the method of comparison function[38] to be

$$A = t\frac{1}{1+\nu f(\mathcal{E},\mathcal{E})}, \tag{5.2}$$

where t is the bare transmission amplitude through the potential barrier $V(R)$ and is given by

$$t = \frac{1}{\sqrt{1+e^{2\pi\mathcal{E}}}}e^{i\arg\Gamma(\frac{1}{2}+i\mathcal{E})}. \tag{5.3}$$

$\mathcal{E}$ in the above equations is the action integral between two turning points of the potential barrier, i.e.

$$\mathcal{E} = \frac{i}{\pi}\int_{R_1}^{R_2}\sqrt{\frac{2\mu}{\hbar^2}(E - V(R))}dR. \tag{5.4}$$

The quantities ν and f are given by

$$\nu = \lambda(j-k)\,\hbar^{-\frac{3}{2}}\left(\frac{\mu}{\Omega}\right)^{\frac{1}{2}}, \tag{5.5}$$

where

$$\Omega = \left(\frac{\left|\frac{d^2V(R)}{dr^2}\right|}{\mu}\right)^{\frac{1}{2}}, \tag{5.6}$$

and

$$f(\mathcal{E},\mathcal{E}) = e^{\frac{i\pi}{4}}\frac{\Gamma(\frac{1}{4}+i\frac{\mathcal{E}}{2})}{\Gamma(\frac{3}{4}+i\frac{\mathcal{E}}{2})}. \tag{5.7}$$

Whenever $\lambda(j-k)$ is negative, the transmission probability will show a resonance peak at the energies where the condition

$$1 = -\nu Re f(\mathcal{E},\mathcal{E}) \tag{5.8}$$

is satisfied. We call this a barrier-top resonance. A plot of the real and imaginary parts of $f(\mathcal{E},\mathcal{E})$ is given in Fig. 6. Note that $\mathcal{E}$ decreases as E increases. Hence the figure indicates that the width of the resonance gets larger as the resonance energy increases. Since there is one-bound state in the one-dimensional delta function potential well, there will be one barrier-top resonance for each negative value of $\lambda(j-k)$. However, some of these values might not satisfy the condition given in Eq. (5.8), especially at higher energies. Therefore, since

$0 \leq k \leq 2j$, in the transmission probability we expect at most $\left[\frac{2j+1}{2}\right]$ resonance peaks for a given value of λ, [ ] being the Gauss symbol. In fig. 7 we plot the penetration probability for several different values of j.

VI. TRANSMISSION PROBABILITY IN THE ADIABATIC LIMIT

In this section, we study the transmission probability in the limit where the energy quantum of the internal motion, ω_0, is very large. This is the adiabatic case, i.e. the internal system remains in the initial state. Hence the two-time influence functional will be well approximated by

$$\rho_M(\tilde{R}(\tilde{t}), \tilde{T}; R(t), T) \approx \mathcal{W}^*_{n_i,n_i}[\tilde{R}(\tilde{t}); \tilde{T}, 0]\mathcal{W}_{n_i,n_i}[R(t); T, 0]. \tag{6.1}$$

Note that $\mathcal{W}_{n_i,n_i}$ is the unique matrix element of the Green's function which appears in studying microscopic foundation of the optical potential by means of the path integral method[39]. Therefore, Eq.(6.1) means to approximate the adiabatic influence functional by the influence functional for elastic scattering. In the following, we successively deal with those three cases which have been considered in Sects. III and IV.

a) Linear coupling to a harmonic oscillator :

The corresponding effective Hamiltonian for the internal motion is given by Eq.(3.5), where R is a given time-dependent c-number path $R(t)$. The Green's function $\hat{U}$ is then given by

$$\hat{U}(R(t); t, 0) = e^{-\frac{i}{\hbar}\hat{H}_0 t} e^{i\Phi(\omega_0)} \hat{D}(I(\omega_0; t)), \tag{6.2}$$

where

$$\Phi(\omega_0) = \frac{\alpha_0^2}{\hbar^2}\int_0^t dt_1 \int_0^{t_1} dt_2 f(R(t_1)) f(R(t_2)) sin\omega_0(t_1 - t_2), \tag{6.3}$$

$$\hat{D}(z) = e^{za^\dagger - z^* a}, \tag{6.4}$$

and

$$I(\omega_0; t) = -\frac{i\alpha_0}{\hbar}\int_0^t f(R(t_1)) e^{i\omega_0 t_1} dt_1. \tag{6.5}$$

The operator $\hat{D}(z)$ is the displacement operator, which generates the coherent state $|z\rangle_c$ defined by[40],

$$a|z\rangle_c = z|z\rangle_c. \tag{6.6}$$

Assuming that the oscillator degree of freedom is initially in the ground state, i.e. $n_i = 0$, Eqs.(6.2) through (6.5) yield

$$\mathcal{W}_{00} = exp\left[-\frac{i}{\hbar}\int_0^T W(R(t); t) dt\right], \tag{6.7}$$

where

$$W(R(t); t) = \frac{1}{2}\hbar\omega_0 - i\frac{\alpha_0^2}{\hbar} f(R(t)) \int_0^t f(R(t_1)) e^{-i\omega_0(t-t_1)} dt_1. \tag{6.8}$$

On the other hand Eqs.(2.15),(6.1) and (6.7) lead to

$$P(E) = \lim_{\substack{R_i \to \infty \\ R_f \to -\infty}} \left(\frac{P_i P_f}{\mu^2}\right) \left| \int_0^\infty dT e^{\frac{i}{\hbar}ET} \int \mathcal{D}[R(t)] e^{\frac{i}{\hbar}S_{eff}(R,T)} \right|^2, \tag{6.9}$$

where

$$S_{eff}(R, T) = S_t(R, T) + \delta S_t(R, T), \tag{6.10}$$

with

$$\delta S_t(R,T) = -\int_0^T W(R(t);t)dt. \tag{6.11}$$

Integrating the time integral in Eq.(6.8) by parts repeatedly we can write the influence potential as

$$\begin{aligned} W(R(t);t) = \frac{1}{2}\hbar\omega_0 - \frac{\alpha_0^2}{\hbar\omega_0} f(R(t)) \sum_{n=0}^{\infty} A_n(t) \\ + \frac{\alpha_0^2}{\hbar\omega_0} f(R(t)) e^{-i\omega_0 t} \sum_{n=0}^{\infty} A_n(0), \end{aligned} \tag{6.12}$$

where

$$A_n(t) = \left(\frac{i}{\omega_0}\right)^n \frac{d^n f(R(t))}{dt^n}. \tag{6.13}$$

Eqs.(6.12) and (6.13) give the influence potential exactly for all possible coupling form factors. Assuming $f(R_i) = f(R(t=0)) = 0$ as we did before, we can drop the last term in Eq.(6.12). Furthermore, since in the adiabatic limit ω_0 is large enough, we can truncate the series in Eq.(6.12) after the second term and obtain the adiabatic influence functional as

$$W(R(t);t) = \frac{1}{2}\hbar\omega_0 - \frac{\alpha_0^2}{\hbar\omega_0}[f(R)]^2 - \frac{i\alpha_0^2}{\hbar\omega_0^2} f(R)\frac{df}{dR}\frac{dR}{dt} + O(\frac{1}{\omega_0^3}). \tag{6.14}$$

Hence the additional term in the action, δS of Eq.(6.11), due to adiabatic coupling is given by

$$\delta S \approx -\frac{1}{2}\hbar\omega_0 T + \frac{i\alpha_0^2}{2\hbar\omega_0^2}\Delta f^2 + \frac{\alpha_0^2}{\hbar\omega_0}\int_0^T dt[f(R)]^2, \tag{6.15}$$

where

$$\Delta f^2 = f^2(R(T)) - f^2(R(0)) = f^2(R_f) - f^2(R_i) = f^2(R_f). \tag{6.16}$$

Finally inserting Eqs.(6.15) and (6.16) into Eqs.(6.9) and (6.10), we get the following expression for the transmission probability :

$$\begin{aligned} P(E) = \lim_{\substack{R_i\to\infty \\ R_f\to-\infty}} \left(\frac{P_i P_f}{\mu^2}\right) \Bigg| \int_0^\infty dT e^{\frac{i}{\hbar}(E-\frac{1}{2}\hbar\omega_0)T} e^{\left[-\left(\frac{\alpha_0}{\hbar\omega_0}f(R_f)\right)^2\right]} \\ \int D[R(t)] \exp\frac{i}{\hbar}\int_0^T dt\left[\frac{1}{2}\mu\dot{R}^2 - V(R) + \frac{\alpha_0^2}{\hbar\omega_0}f^2(R)\right] \Bigg|^2 \end{aligned} \tag{6.17}$$

Since while doing the time and path integrals we keep R_f fixed, we can take the term including R_f in the above expression outside all the integrals. The result reads

$$P(E) = e^{\left[-\left(\frac{\alpha_0}{\hbar\omega_0}f(R_f)\right)^2\right]} \left\{ \lim_{\substack{R_i\to\infty \\ R_f\to-\infty}} \left(\frac{P_i P_f}{\mu^2}\right) \left| \int_0^\infty dT e^{\frac{i}{\hbar}(E-\frac{1}{2}\hbar\omega_0)T} \int D[R(t)] e^{\frac{i}{\hbar}\int_0^T dt\left[\frac{1}{2}\mu\dot{R}^2 - V(R) + \frac{\alpha_0^2}{\hbar\omega_0}f^2(R)\right]} \right|^2 \right\}, \tag{6.18}$$

where $f(R_f)$ is the limiting value of $f(R)$ as $R_f \to -\infty$. In those cases where $f(R)$ does not have an asymptotic value , R_f should be chosen on the basis of appropriate physical considerations. Most of the time it is sufficient to take R_f to be the point where the tunneling process is completed. In any case, $f(R_f)$ in the expression above should be computed at the same point as the one used to calculate the path integral, for example, by the uniform approximation as in Ref. 32. Note that the expression in curly brackets in Eq.(6.18) is the exact expression for the transmission probability of a single particle of mass μ and energy $E - \frac{1}{2}\hbar\omega_0$ across the one-dimensional adiabatic potential barrier

$$V_{ad}(R) = V(R) - \frac{\alpha_0^2}{\hbar\omega_0} f^2(R). \tag{6.19}$$

Eq.(6.18) is valid at all energies, both below and above the barrier. If it cannot be calculated exactly, one can employ the uniform approximation as given in Eq.(2.18). The final result for the adiabatic transmission probability for the linear coupling to an oscillator degree of freedom reads as

$$P(E) \approx P_{ad}(E).P_D, \tag{6.20}$$

where

$$P_{ad}(E) = P_0\Big(E - \frac{1}{2}\hbar\omega_0, V(R) - \frac{\alpha_0^2}{\hbar\omega_0} f^2(R)\Big), \tag{6.21}$$

and

$$P_D = \exp\left[-\Big(\frac{\alpha_0}{\hbar\omega_0} f(R_f)\Big)^2\right]. \tag{6.22}$$

For energies well below the barrier Eqs.(6.20), (6.21), and (6.22) agree with the result obtained in Ref. 26 based on an extended WKB approximation, and in Ref. 17 by studying the imaginary time propagator. In contrast to Refs. 17 and 26, however, the results given here are valid at *all* energies from below to the above of the barrier. Furthermore, we have explicitly used the fact that the coupling form factor is of finite range. In heavy-ion collisions, $f^2(R_f)$ is considerably larger than $f^2(R_i)$ for a realistic coupling Hamiltonian[23], which justifies our assumption. The factor P_D corresponds to the dissipation factor in Ref. 17. In fact, as mentioned above, $P_D < 1$ for heavy-ion collisions. However, the dissipation effect is much smaller than the potential renormalization effect in the adiabatic limit[17].

b) Quadratic coupling to a harmonic oscillator :

In this case, the Green's function for the internal motion is given by Eq. (3.12). Again assuming that the oscillator remains in the ground state, the matrix element $\mathcal{W}_{00}$ of Eq.(2.12) is given by

$$\mathcal{W}_{00}[R(t); T, 0] = \exp\left[-\frac{i}{4}(\phi + i\rho)\right], \tag{6.23}$$

where

$$\rho = \log(1 - |z|^2). \tag{6.24}$$

The quantity $(\phi + i\rho)$ can be related to the frequency of the harmonic oscillator and the coupling form factor by using Eqs.(3.14) and (3.15). The result reads,

$$\phi + i\rho = 2\omega_0 T + \frac{4\alpha_0^2}{\hbar}\int_0^T h(R(t))[z(R(t); t) + 1]dt. \tag{6.25}$$

The matrix element $\mathcal{W}_{00}$ thus takes the standart form expressed as in Eq. (6.7), where the influence potential is given by

$$W(R(t); t) = \frac{1}{2}\hbar\omega_0 + \alpha_0^2 h(R(t))[z(R(t); t) + 1]. \tag{6.26}$$

Since we are considering the adiabatic limit, we determine the functional $z(R(t);t)$ by solving Eq.(3.14) up to the second order with respect to the strength of the coupling form factor divided by ω_0. Substituting this result into Eq.(6.26) one obtains the influence potential as

$$
\begin{aligned}
W(R(t);t) = \frac{1}{2}\hbar\omega_0 + \alpha_0^2 h(R)\left[1 - \frac{\alpha_0^2}{\hbar\omega_0}h(R) + \frac{2\alpha_0^4}{(\hbar\omega_0)^2}h^2(R)\right] \\
- \frac{i\alpha_0^4}{2\hbar\omega_0^2}h(R)\frac{dh}{dR}\frac{dR}{dt} + O(\frac{1}{\omega_0^3}).
\end{aligned}
\tag{6.27}
$$

Substituting Eq.(6.27) into Eqs. (6.9), (6.10), and (6.11) and following the same steps as in the previous subsection, one obtains the adiabatic transmission probability for a quadratic coupling to an oscillator degree of freedom as

$$P(E) \approx P_{ad}(E).P_D, \tag{6.28}$$

where

$$P_{ad}(E) = P_0(E - \frac{1}{2}\hbar\omega_0, V_{ad}(R)), \tag{6.29}$$

$$P_D = \exp\left[-\frac{1}{2}\left(\frac{\alpha_0^2}{\hbar\omega_0}h(R_f)\right)^2\right] \tag{6.30}$$

and

$$V_{ad}(R) = V(R) + \alpha_0^2 h(R)\left[1 - \frac{\alpha_0^2}{\hbar\omega_0}h(R) + \frac{2\alpha_0^4}{(\hbar\omega_0)^2}h^2(R)\right], \tag{6.31}$$

where $h(R_f)$ is the limiting value of $h(R)$ as $R_f \to -\infty$.

The adiabatic potential given by Eq.(6.31) agrees with that obtained in Ref. 17, where one uses the imaginary time propagator. The present work is superior to Ref. 17 in two respects. The dissipation factor has been explicitly obtained. Also, the present result is true for all energies, since one can calculate Eq.(6.29) in a uniform approximation. Note that, as we have already pointed out in Sect. IV, the potential renormalization depends crucially on the sign of the coupling form factor in the adiabatic limit as well as in the degenerate limit.

c) Linear coupling to a spin system :

In this case we want to calculate

$$\mathcal{W}_{00} = \langle j, -j|\hat{U}|j, -j\rangle. \tag{6.32}$$

Using Eq.(3.19) we obtain

$$\mathcal{W}_{00}[R(t);T,0] = e^{ij(\varphi + i\chi)}, \tag{6.33}$$

where

$$\chi = \log(1 + |r|^2). \tag{6.34}$$

The quantity $(\varphi + i\chi)$ can be related to the level spacing and the coupling form factor by using Eqs.(3.21) and (3.22). The result reads,

$$\varphi + i\chi = \int_0^T [\omega_0 - 2\beta(R(t))y(R(t);t)]dt. \tag{6.35}$$

The matrix element $\mathcal{W}_{00}$ thus takes the standart form in this problem as well, where the influence potential is given by

$$W(R(t);t) = -j\hbar\omega_0 + 2j\hbar\beta(R(t))y(R(t);t). \tag{6.36}$$

Similarly to the previous subsection, we determine the functional $y(R(t);t)$ by solving Eq.(3.21) up to the lowest order with respect to the strength of the coupling form factor. This gives us

$$y(R(t);t) = -i\int_0^t dt_1 \beta(R(t_1))e^{-i\omega_0(t-t_1)}. \tag{6.37}$$

Integrating Eq.(6.37) by parts twice and keeping terms up to second order in $\frac{1}{\omega_0}$, we obtain the influence potential as

$$W = -j\hbar\omega_0 + 2j\hbar\beta(R(t))\left[-\frac{1}{\omega_0}\beta(R(t)) - \frac{i}{\omega_0^2}\frac{d\beta}{dR}\frac{dR}{dt}\right]. \tag{6.38}$$

Finally the adiabatic transmission probability for the linear coupling to a spin system becomes

$$P(E) \approx P_{\rm ad}(E).P_D, \tag{6.39}$$

where

$$P_{\rm ad}(E) = P_0(E + j\hbar\omega_0, V_{\rm ad}(R)), \tag{6.40}$$

$$P_D = \exp\left[-2j\left(\frac{\beta(R_f)}{\omega_0}\right)^2\right]. \tag{6.41}$$

and

$$V_{\rm ad}(R) = V(R) - \frac{2j\hbar}{\omega_0}\beta^2(R), \tag{6.42}$$

where $\beta(R_f)$ is the limiting value of $\beta(R)$ as $R_f \to -\infty$. Note that the potential renormalization gets larger and the dissipative factor gets smaller as the number of levels increases.

VII. CORRELATION TO A FRICTION COEFFICIENT

Caldeira and Leggett[41] have considered a quantum system which can tunnel out of a metastable state and whose interaction with the environment is adequately described in the classically accessible region by a phenomenological friction coefficient η. They have assumed that the environment is represented as an aggregation of harmonic oscillators and that the coupling Hamiltonian is linear with respect to the coordinates of both the quantum system and the harmonic oscillators. They have thus argued that the tunneling probability is multiplied by a dissipation factor, which is related to η such that

$$P_D = e^{-A\eta(\Delta R)^2/\hbar}, \tag{7.1}$$

where ΔR is the distance under the barrier and A is a numerical factor which is generally of order of unity.

In order to see if the present approach leads to a similar result in the situations appropriate to heavy-ion collisions, we consider a model, where the translational motion directly couples to a collective harmonic oscillator which further couples with many other non-collective harmonic oscillators (see Appendix B). This model will mimic the nuclear response when a giant resonance state is excited in heavy-ion collisions[42]. Furthermore, the average trajectory of the translational motion in the classically accessible region obeys a Markovian equation of the type assumed in Ref. 40, i.e.

$$\mu\ddot{R} = -\frac{dV}{dR} - \eta\dot{R} + F^{(\rm ind)}(R), \tag{7.2}$$

if the coupling between the collective and the non-collective harmonic oscillators is strong enough (see Eq. (B.25)). In Eq. (7.2), $F^{(\rm ind)}(R)$ is the conservative force induced by the coupling. In a similar way to the previous section, we consider the adiabatic limit, where the dissipation effect can be easily separated from the effect of the potential renormalization. Accordingly we employ the approximation given by Eq. (6.1). The resulting transmission probability is given as slight generalization of the formulae in Section VI.a (the derivation is sketched in Appendix B). One obtains

$$P(E) \simeq \lim_{\substack{R_i \to \infty \\ R_f \to -\infty}} \left(\frac{P_i P_f}{\mu^2}\right) \left| \int_0^\infty dT e^{\frac{i}{\hbar}(E - \sum_j \frac{1}{2}\hbar\omega_j)T} \int D[R(t)] e^{\frac{i}{\hbar} S_{eff}^{(\Gamma)}(R,T)} \right|^2, \tag{7.3}$$

where

$$S_{\rm eff}^{(\Gamma)}(R,T) = S_t(R,T) + \delta S_t^{(\Gamma)}(R,T), \tag{7.4}$$

$$\delta S_t^{\Gamma}(R,T) = \int_0^T dt \Bigg\{ \frac{\alpha_0^2}{\hbar} f(R(t)) \left[f(R(t)) \frac{\omega_0}{\omega_0^2 + (\frac{\Gamma}{2})^2} - \frac{df}{dR}\frac{dR}{dt} \frac{\omega_0 \Gamma}{[\omega_0^2 + (\frac{\Gamma}{2})^2]^2} \right]$$
$$+ i \frac{\alpha_0^2}{\hbar} f(R(t)) \left[f(R(t)) \frac{\frac{\Gamma}{2}}{\omega_0^2 + (\frac{\Gamma}{2})^2} + \frac{df}{dR}\frac{dR}{dt} \frac{\omega_0^2 - (\frac{\Gamma}{2})^2}{[\omega_0^2 + (\frac{\Gamma}{2})^2]^2} \right] \Bigg\}. \tag{7.5}$$

In these equations Γ is the width of the strength distribution of the collective vibrational state (see Eqs. (B.11) and (B.12)). We have ignored the terms proportional to $e^{-(\frac{\Gamma}{2}+i\omega_0)t}$ (see Eq. (B.21)). Such terms will be negligible if Γ is sufficiently large in the classically allowed region, or if ω_0 is sufficiently large in the classically forbidden region. Also, we have left out the terms involving the second and higher order derivatives of $f(R)$ and $R(t)$ with respect to R and t, respectively. Note that those terms include second and higher order inverse powers of $\omega_0^2 + (\frac{\Gamma}{2})^2$ which can be neglected in the adiabatic case.

Following the same steps as those leading to Eqs. (6.20),(6.21) and (6.22), we obtain the transmission probability

$$P(E) \approx P_{\rm ad}(E).P_D, \tag{7.6}$$

where

$$P_{\rm ad}(E) = P_0(E - \sum_j \frac{1}{2}\hbar\tilde{\omega}_j, V_{\rm ad}(R)), \tag{7.7}$$

with

$$V_{\rm ad}(R) = V(R) - \frac{\alpha_0^2}{\hbar}[f(R)]^2 \frac{\omega_0 + i\frac{\Gamma}{2}}{\omega_0^2 + (\frac{\Gamma}{2})^2}, \tag{7.8}$$

and

$$P_D = \exp\left[-\left(\frac{\alpha_0}{\hbar}\right)^2 \Delta f^2 \frac{\omega_0^2 - (\frac{\Gamma}{2})^2}{[\omega_0^2 + (\frac{\Gamma}{2})^2]^2} \right], \tag{7.9}$$

where the quantity Δf^2 is given by Eq.(6.16). The result given by Eqs. (7.6) through (7.9) coincides with the result given by Eqs.(6.20) through (6.22) if $\Gamma \ll \omega_0$. Likewise Eqs.(6.20),(6.21) and (6.22), Eqs. (7.6) through (7.9) are valid for all energies from above to the below the barrier and for coupling form factors of finite range. Note that the adiabatic potential is complex this time.

The terms which are proportional to Γ in Eq. (7.5) would yield a dissipation factor that looks like Eq.(7.1), because Γ is proportional to η (see Eq. (B.29)). Note that the second term in the curly brackets in Eq. (7.5) contributes only a phase, proportional to Δf^2, to the path integral. The third term gives the imaginary (absorptive) part of the complex potential. However, for energies well below the barrier maximum, the effect of the absorptive part is negligable, as can easily be seen by calculating Eq. (2.17) within saddle point approximation using the imaginary time prescription. Hence for this energy region, coupling to internal degrees of freedom does not introduce a dissipation factor of the form asserted in Ref. 41. Instead, the "dissipation effect" is given by Eq.(7.9), which is a generalization of the result in Ref.17 for a general, but finite range, form factor. Note, however, that the terms proportional to Γ in Eq.(7.5) reduce the transmission probability in the classically accessible region. Clearly, the magnitude of this effect is related to the strength of the friction coefficient η. Furthermore, we note that the strength distribution of the internal oscillators used in our model is markedly different from that of Ref. 41 (cf. Eqs. (B.11) and (B.13)).

VIII. SUMMARY AND CONCLUSIONS

We have applied the path integral method to elucidate the effects of internal degrees of freedom on the transmission probability of a translational degree of freedom across a potential barrier. In this paper, we have dealt only with the inclusive transmission probability, where the effects of coupling to the internal degrees of freedom are taken into account through the two-time influence functional. Three cases have been explicitly considered : translational motion linearly or quadratically coupled to a harmonic oscillator, or linearly coupled to an equally spaced spin system. The two-time influence functional has been determined for these cases, although for the latter two cases one still needs to solve a Riccati-type differential equation to express the answer directly in terms of the coupling form factor.

The result becomes extremely simple in the case of degenerate spectrum, i.e. when the level spacing is zero. Clearly, in this case there is no dissipation effect, and the internal degrees of freedom only renormalize the potential barrier. When the internal degree of freedom is a harmonic oscillator, then the inclusive transmission probability is given by the so-called zero point motion formula[10]. On physical grounds this is expected, because for zero oscillator quanta it takes infinite time for the amplitude of the oscillator to change. Therefore, one should first calculate the transmission probability for an effective barrier corresponding to a given amplitude of the harmonic oscillator, and then calculate the final inclusive transmission probability by averaging with a weight proportional to the probability of the oscillator to be in the ground state. When the internal degree of freedom is a spin system with $(2j+1)$ levels, the inclusive transmission probability is given as an appropriate average of the probabilities of transmission through $(2j+1)$ one-dimensional barriers.

The exact results mentioned above reveal several interesting aspects of the effects of the internal degrees of freedom on the transmission probability. The effects of the linear coupling to an oscillator do not change by inverting the sign of the coupling form factor, whereas the effects of the quadratic coupling crucially depends on the sign of the coupling form factor. For the linear coupling the transmission probability is enhanced at sub-barrier energies, but is hindered at above-barrier energies. Contrarily, for quadratic coupling the transmission probability is enhanced throughout all energy region by a negative form factor, but is always hindered by a positive form factor. As a result a linear coupling enhances the high angular momentum component of the fusion cross-section. This explains the recent discovery in Ref.43 that the coupled channel calculations, which include the effects of surface vibrations, have larger high angular momentum component in the fusion cross section than those with a "bare" potential. In the case of a linear coupling to a spin system, half of the $(2j+1)$ effective potentials are higher (or thicker) than the bare potential, and the other half are lower (or thinner). This is a natural extension of the situation encountered in the degenerate two level model[38].

In order to study the effects of a well localized coupling form factor, we have studied a parabolic potential barrier coupled to a spin system with a delta function form factor at the top of the barrier. The sub-barrier transmission probability was strongly enhanced by the coupling. Moreover, this model led to at most $(2j+1)/2$ resonance peaks in the excitation function. This behavior has been interpreted in terms of the barrier-top resonances[38] based on semiclassical theory. The fact that the experimentally measured heavy-ion fusion excitation functions do not show such a resonant behavior indicates that the coupling form factors are rather non-localized.

We also investigated the effects of the adiabatic coupling, i.e. when the energy quantum of the internal system is very large. In this case we have approximated the influence functional by the influence functional for elastic scattering. This led to an expression of the transmission probability in terms of a single time integral. We have shown that the transmission probability can be written as a product of two factors for *all* energies: 1. A dissipation factor which depends on the asymptotic value of the form factor, 2. Another factor, which is the probability for transmission across a one-dimensional effective barrier (adiabatic potential barrier). When the internal system is a harmonic oscillator, the adiabatic potential barrier agrees with those obtained in Refs. 17 and 26 by using different methods, although our expressions are valid above and near the top of the barrier as well, unlike previous results. Furthermore, the dissipation factor for the quadratic coupling to a harmonic oscillator is not derived in these references. We have also studied the adiabatic coupling to a spin system. We found that both the potential renormalization and the dissipation factor depends on the number of levels.

In the last section, we studied the transmission probability in a model, where the translational motion is linearly coupled to a damped harmonic oscillator. In particular, we considered the case when the frequency and the width of the oscillator are very large. Unlike the argument in Ref. 41, we obtained a "dissipation factor", which has only a weak connection to the friction coefficient in the classically allowed region. We should, however, note that our way of introducing the friction coefficient and the strength distribution is very different from that in Ref.41. Also, our coupling Hamiltonian is of finite range as contrast to that of infinite range in Ref. 41. Further investigations are definitely needed to understand how the correlation between the hindrance factor of the tunneling probability and the friction coefficient depends on the properties of the internal system and the coupling form factor.

Finally we want to remark that the potential renormalization in the adiabatic limit, given by Eq. (6.19), is minus the counter Hamiltonian introduced in Ref. 24 to avoid double counting, i.e. to cancel out those potential renormalization effects which are already included in the "bare" potential, when the bare potential is phenomenologically determined. For heavy-ion reactions such a phenomenological potential is the real part of the optical potential, which is determined by fitting the data for elactic scattering[22]. The present study indicates that introduction of a counter Hamiltonian is adequate regarding the role of high-lying vibrational states. However, for low-lying vibrational states this is not the case. Potential renormalization due to such states should still be considered even if one takes the real part of the phenomenological optical potential as the "bare" potential. In this respect, we note that the potential renormalization due to coupling to low-lying vibrational states more strongly enhances the sub-barrier fusion cross section than that due to coupling to high-lying vibrational states[23]. The problem of double counting is hence of practical importance in heavy-ion collisions, and will be discussed in a seperate publication.

APPENDIX A: Diagonalization method to obtain the influence functional for a degenerate spin system

We consider the internal system which has been treated in Sect. IIIc, in the limit of degenerate spectrum. In this limit, the Green's function is determined by

$$i\hbar\frac{\partial}{\partial t}\hat{U}(t,t_0)=2\beta(t)\hat{J}_x\hat{U}(t,t_0), \tag{A.1}$$

with the boundary condition

$$\hat{U}(t_0,t_0)=\hat{1}. \tag{A.2}$$

Operating the unitary operator $e^{-i\hat{J}_y\theta}$, θ being $-\frac{\pi}{2}$, on Eq. (A1), we obtain

$$i\hbar\frac{\partial}{\partial t}\left[e^{-i\hat{J}_y\theta}\hat{U}\right] = 2\beta(t)\hat{J}_z[e^{-i\hat{J}_y\theta}\hat{U}]. \tag{A.3}$$

Therefore, the matrix element of the transformed Green's function in the $(2j+1)$ representation can be easily obtained as

$$\begin{aligned}\langle j,m\left|e^{-\hat{J}_y\theta}\hat{U}(t,t_0)\right|j,-j\rangle &= e^{\frac{1}{i\hbar}\int_{t_0}^{t}2\beta(t_1)mdt_1}\langle j,m\left|e^{-i\hat{J}_y\theta}\hat{U}(t_0,t_0)\right|j,-j\rangle \\ &= e^{\frac{1}{i\hbar}\int_{t_0}^{t}2m\beta(t_1)dt}\langle j,m\left|e^{-\hat{J}_y\theta}\right|j,-j\rangle\end{aligned} \tag{A.4}$$

where we have used eq. (A.2). The second factor in the right hand side of eq. (A.4) is nothing but $d^j_{m-j}(\theta)$,which is well known. We thus obtain

$$\langle j,m\left|e^{-i\hat{J}_y\theta}\hat{U}(t,t_0)\right|j,-j\rangle = \begin{pmatrix}2j\\ j+m\end{pmatrix}^{\frac{1}{2}}\left(\frac{1}{\sqrt{2}}\right)^{2j}e^{-2im\int_{t_0}^{t}\beta(t_1)dt_1/\hbar}. \tag{A.5}$$

We now express the two-time influence functional as

$$\rho_M(\tilde{R}(\tilde{t}),\tilde{T};R(t),T) = \sum_m (m|e^{-i\hat{J}_y\theta}\hat{U}(\tilde{R}(\tilde{t});\tilde{T},0)|-j)^*(m|e^{-i\hat{J}_y\theta}\hat{U}(R(t);T,0)|-j). \tag{A.6}$$

Eq. (A.5) then leads to

$$\begin{aligned}\rho_M(\tilde{R}(\tilde{t}),\tilde{T};R(t),T) &= \sum_m\begin{pmatrix}2j\\ j+m\end{pmatrix}(\frac{1}{2})^{2j}e^{-2im[\int_0^T\beta(R(t))dt/\hbar-\int_0^{\tilde{T}}\beta(\tilde{R}(\tilde{t}))d\tilde{t}/\hbar]} \\ &= \cos^{2j}\left[\int_0^T\beta(R(t))dt-\int_0^{\tilde{T}}\beta(\tilde{R}(s))ds\right].\end{aligned} \tag{A.7}$$

This is the formula eq. (5.16), which we have derived before in Section IV.d.

APPENDIX B : Influence functional for the case of a linear coupling to a damped harmonic oscillator

In this appendix we explain the model which we used in Section VII and derive the corresponding influence functional.

a) Model Hamiltonian

We take the Hamiltonian for the internal system to be

$$\hat{H}_0 = \hbar\omega_0(a_0^\dagger a_0+\frac{1}{2})+\sum_{i=1}^{m}\hbar\omega_i(b_i^\dagger b_i+\frac{1}{2})+\hbar\kappa\sum_i^m(a_0^\dagger b_i+a_0b_i^\dagger), \tag{B.1}$$

and

$$\hat{H}_{\rm int} = \alpha_0 f(R)(a_0^\dagger+a_0), \tag{B.2}$$

where $(a_0^\dagger, a_0)$ and $(b_i^\dagger, b_i)$ are the (creation, annihilation) operators of the collective and the i-th non-collective harmonic oscillators, respectively. The unperturbed Hamiltonian $\hat{H}_0$ can be diagonalized by introducing the normal modes $\tilde{a}_j^\dagger, \tilde{a}_j$) as,

$$\hat{H}_0 = \sum_{j=1}^{m+1} \hbar \tilde{\omega}_j (\tilde{a}_j^\dagger \tilde{a}_j + \frac{1}{2}), \tag{B.3}$$

$$\tilde{a}_j = \chi_{j1} a_0 + \sum_{i=1}^{m} \chi_{j,i+1} b_i, \tag{B.4}$$

where

$$\chi_{j1} = \left[1 + \sum_{i=1}^{m} \frac{\kappa^2}{(\tilde{\omega}_j - \omega_i)^2} \right]^{-\frac{1}{2}}, \tag{B.5}$$

$$\chi_{j,i+1} = \frac{\kappa}{\tilde{\omega}_j - \omega_i} \chi_{j1}, \tag{B.6}$$

and

$$\tilde{\omega}_j - \omega_0 = \kappa^2 \sum_{i=1}^{m} \frac{1}{\tilde{\omega}_j - \omega_i}. \tag{B.7}$$

By inverting Eq.(B.4), the interaction Hamiltonian is given as

$$\hat{H}_{\rm int} = \alpha_0 \sum_{j=1}^{m+1} \chi_{j1} f(R) (\tilde{a}_j^\dagger + \tilde{a}_j). \tag{B.8}$$

We thus transformed the original problem to another one where the relative motion is linearly coupled to many independent oscillators with a coupling strength which is related to the strength distribution of the original collective vibrational state.

In the following, we consider a particular case, where the original non-collective vibrational states are distributed with equal energy spacing from $-\infty$ to $+\infty$, i.e. we assume[42]

$$\omega_i = i\Delta (i = 0, \pm 1, \pm 2, ...). \tag{B.9}$$

By futher assuming

$$\frac{\pi\kappa}{\Delta} \gg 1, \tag{B.10}$$

we obtain the following expression for the strength distribution of the collective harmonic oscillator state[42]

$$J(\tilde{\omega}_j) = \frac{\chi_{j1}^2}{\Delta} \approx \frac{1}{2\pi} \frac{\Gamma}{(\tilde{\omega}_j - \omega_0)^2 + (\frac{\Gamma}{2})^2}, \tag{B.11}$$

where

$$\Gamma = \frac{2\pi\kappa^2}{\Delta}. \tag{B.12}$$

Compare Eq.(B.11) with the strength distribution assumed in Ref.(41), which reads

$$J(\omega) = \eta\omega. \tag{B.13}$$

b) The Green's function for the internal motion and the influence functional

Eqs. (B.3) and (B.8) lead to

$$\hat{U}(R(t); t, 0) = \prod_j \hat{U}_j(R(t); t, 0) = e^{-i\sum_j \tilde{\omega}_j (\tilde{a}_j^\dagger \tilde{a}_j + \frac{1}{2})t} e^{i\sum_j \Phi_j} \prod_j \tilde{D}_j(I_j), \tag{B.14}$$

where

$$\Phi_j = \chi_{j1}^2 \frac{\alpha_0^2}{\hbar^2} \int_0^t dt_1 \int_0^{t_1} dt_2 f(R(t_1)) f(R(t_2)) \sin\tilde{\omega}_j(t_1 - t_2) = \chi_{j1}^2 \Phi(\tilde{\omega}_j), \tag{B.15}$$

$$\tilde{D}_j(I_j) = exp\Big[I_j \tilde{a}^\dagger - I_j^* \tilde{a}_j\Big], \tag{B.16}$$

and

$$I_j = -\chi_{j1} \frac{i\alpha}{\hbar} \int_0^t f(R(t_1)) e^{i\tilde{\omega}_j t_1} dt_1 = \chi_{j1} I(\tilde{\omega}_j; t). \tag{B.17}$$

It is then straightforward to calculate the matrix element $W_{n_f n_i}$. It takes the standart form Eq. (6.7), where the influence potential is given by an obvious extension of Eq. (6.8) as

$$W(R(t);t) = \frac{1}{2}\sum_j \hbar\,\tilde{\omega}_j - i\frac{\alpha_0^2}{\hbar}\sum_j \chi_{j1}^2 f(R(t)) \int_0^t f(R(t_1)) e^{-i\tilde{\omega}_j(t-t_1)} dt_1. \tag{B.18}$$

Introducing the distribution function we now replace the sum over j by an integral over $\tilde{\omega}_j$. This leads to

$$\sum_j \chi_{j1}^2 e^{-i\tilde{\omega}_j(t-t_1)} \approx \int_{-\infty}^{+\infty} d\tilde{\omega}_j J(\tilde{\omega}_j) e^{-i\tilde{\omega}_j(t-t_1)} = e^{-i(\omega_0 - i\frac{\Gamma}{2})(t-t_1)}. \tag{B.19}$$

The influence potential thus becomes

$$W(R(t);t) = \frac{1}{2}\sum_j \hbar\,\tilde{\omega}_j - i\frac{\alpha_0^2}{\hbar} f(R(t)) \int_0^t dt_1 f(R(t_1)) e^{-\frac{\Gamma}{2}(t-t_1)} e^{-i\omega_0(t-t_1)}. \tag{B.20}$$

In the adiabatic limit ω_0 is taken to be very large. In deriving the strength distribution above we have also assumed that Γ is large, cf Eq. (B.10). Hence for both classically allowed and classically forbidden regions (real and imaginary time), due to the exponentials in Eq. (B.20), the dominant contribution to the t_1 integral will come from those values of t_1 which are very close to t. Therefore expanding $f(R(t_1))$ around $f(R(t))$ we obtain

$$W(R(t);t) = \frac{1}{2}\sum_j \hbar\,\tilde{\omega}_j - \frac{\alpha_0^2}{\hbar} f(R(t)) \Big[f(R(t)) \frac{\omega_0 + i\frac{\Gamma}{2}}{\omega_0^2 + (\frac{\Gamma}{2})^2} \Big(1 - e^{-(\frac{\Gamma}{2} + i\omega_0)t}\Big)$$

$$+ i\frac{df}{dR}\frac{dR}{dt}\frac{\omega_0^2 - (\frac{\Gamma}{2})^2 + i\omega_0\Gamma}{[\omega_0^2 + (\frac{\Gamma}{2})^2]^2}\Big(1 - e^{-(\frac{\Gamma}{2} + i\omega_0)t}\Big) + \frac{df}{dR}\frac{dR}{dt}\frac{\omega_0 + i\frac{\Gamma}{2}}{\omega_0^2 + (\frac{\Gamma}{2})^2} t e^{-(\frac{\Gamma}{2} + i\omega_0)t} + O_2\Big], \tag{B.21}$$

where O_2 is the aggregation of the terms involving second or higher order derivatives of $f(R)$ and $R(t)$ with respect to R and t, respectively. If we ignore O_2 and the terms proportional to $e^{-(\frac{\Gamma}{2} + i\omega_0)t}$, then Eq. (B.21) leads to Eq. (7.5). Note that an expansion with respect to the inverse powers of $\omega_0^2 + (\frac{\Gamma}{2})^2$, as done in Eq. (6.12), gives the same answer.

c) Induced force in the classically accessible region

In order to relate Γ to the friction coefficient η, let us remark that the force acting on the translational motion due to the coupling to the internal degrees of freedom is given by[44]

$$\mathcal{F}(t) = -\mathrm{Tr}\Big[\Big\{\frac{\partial}{\partial R} H_{\mathrm{int}}(\hat{q}, R(t))\Big\} \hat{\rho_{\mathrm{int}}}(t)\Big], \tag{B.22}$$

where $\hat{\rho_{int}}(t)$ is the density operator for the internal degrees of freedom. Therefore

$$\mathcal{F}(t) = -\langle 0|\hat{U}^{\dagger}(R(t);t,0)\left\{\frac{\partial}{\partial R}H_{int}(\hat{q},R(t))\right\}\hat{U}(R(t);t,0)|0\rangle. \tag{B.23}$$

Inserting Eq. (B.14), we obtain

$$\mathcal{F}(t) = \alpha_0^2\sum_{j=1}^{m+1}\chi_{j1}^2\frac{2}{\hbar}f'(R(t))\int_0^t dt_1 f(R(t_1))\sin\tilde{\omega}_j(t-t_1), \tag{B.24}$$

where f' (R) means the first order derivative with respect to R(t).

We now proceed in the same way as in the previous subsection, and obtain

$$\mathcal{F}(t) = \mathcal{F}^{ind}(t) - \eta(t)\dot{R}(t) + \ldots, \tag{B.25}$$

where

$$\mathcal{F}^{ind}(t) = \frac{2}{\hbar}\alpha_0^2 f'(R)f(R)\int_0^t dt_1 e^{-\frac{\Gamma}{2}(t-t_1)}\sin\omega_0(t-t_1), \tag{B.26}$$

and

$$\eta(t) = \frac{2}{\hbar}\alpha_0^2 f'(R)f'(R)\int_0^t dt_1 e^{-\frac{\Gamma}{2}(t-t_1)}(t-t_1)\sin\omega_0(t-t_1). \tag{B.27}$$

Especially when $\Gamma t \gg 1$,

$$\mathcal{F}^{ind}(t) \approx \frac{2}{\hbar}\alpha_0^2 f'(R(t))f(R(t))\frac{\omega_0}{\omega_0^2+(\frac{\Gamma}{2})^2}, \tag{B.28}$$

and

$$\eta(t) \approx \frac{2}{\hbar}\alpha_0^2 f'(R(t))f'(R(t))\frac{\Gamma\omega_0}{[\omega_0^2+(\frac{\Gamma}{2})^2]^2} \tag{B.29}$$

The truncation of the induced force by friction term as is shown in Eq.(B.25) will also be justified if Γ is sufficiently large.

Finally we wish to remark that the average value of the coordinate of the collective harmonic oscillator satisfies the equation of motion for a forced harmonic oscillator. Namely, if we define

$$\langle\hat{Q}\rangle_t = \mathrm{Tr}(\hat{Q}\hat{\rho}_{int}(t)), \tag{B.30}$$

with

$$\hat{Q} = \alpha_0(a_0^{\dagger}+a_0), \tag{B.31}$$

then

$$\frac{d^2}{dt^2}\langle\hat{Q}\rangle_t + \Gamma\frac{d}{dt}\langle\hat{Q}\rangle_t + \left\{\omega_0^2+\left(\frac{\Gamma}{2}\right)^2\right\}\langle\hat{Q}\rangle_t = -\frac{2\omega_0}{\hbar}\alpha_0^2 f(R(t)). \tag{B.32}$$

The quantity Γ therefore represents the damping width of the collective harmonic oscillator due to the coupling to the surrounding non-collective harmonic oscillators.

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FIGURE CAPTIONS

Figure 1. The potential barrier $V(R)$, used in the examples shown in Figs. 1 through 5 and Fig.7.

Figure 2. The effect on the inclusive transmission probability of the linear coupling of the potential in Fig. 1 to a harmonic oscillator with a degenerate spectrum. The coupling form factors $\alpha_0 f(R)$ with different strengths and signs are plotted in the upper panel. Lower panel shows the corresponding changes in the inclusive transmission probability.

Figure 3. The effect on the inclusive transmission probability of the quadratic coupling of the potential in Fig. 1 to a harmonic oscillator with a degenerate spectrum. Coupling form factors $\alpha_0^2 h(R)$ with different strengths and signs are plotted in the upper panel. Lower panel shows the corresponding changes in the transmission probability.

Figure 4. The effect on the inclusive transmission probability of the linear coupling of the potential in Fig. 1 to a spin system with a degenerate spectrum for different values of the number of levels,$2J+1$. The coupling form factor is taken to be $\hbar\beta(R) = \beta_0/Cosh^2 bR$, with $\beta_0 = 5MeV$ and $b = 1fm^{-1}$.

Figure 5. The effect of the linear coupling on the angular momentum distribution in the fusion process. To calculate partial fusion cross-section the potential barrier in Fig. 1 is used with the form factor $f(R)$ given in Section IV. Partial fusion cross-section σ_l is plotted versus partial wave number l for the case of no coupling (solid line), $\alpha f_0 = 1MeV$ (dashed line), and $\alpha f_0 = 2MeV$ (dash-dotted line). Upper panel corresponds to $E = 9.5MeV$ (below the barier), and the lower panel pertains to $E = 10.5MeV$ (above the barrier).

Figure 6. Real and imaginary parts of the function $f(\mathcal{E},\mathcal{E})$, defined in Eq.(5.7). Note that $Ref(\mathcal{E},\mathcal{E})$ and $Imf(\mathcal{E},\mathcal{E})$ are symmetric around $\mathcal{E} = 0$. Thick solid line is the real part and the thin solid line is the imaginary part of f.

Figure 7. Barrier-top resonances in the inclusive transmission probability for the case of linear coupling to a degenerate spin system with a delta function form factor $\lambda\delta(R - R_B)$, $\lambda = 4MeV$, for different values of the number of levels, $2J+1$. The dashed line is the bare transmission probability.

REFERENCES

1. L.C. Vaz, J.M. Alexander and G.R. Satchler, Phys. Rep. C69 (1981), 373.

2. M. Beckerman et al., Phys. Rev. Lett. 45 (1980), 1472; Phys. Rev. C23 (1981), 1581; C25 (1982), 837; C28 (1983), 1963; Phys. Rev. Lett. 50 (1983), 471.

3. M. Beckerman, Phys. Rep., to be published.

4. W. Reisdorf, et al., Phys. Rev. Lett. 49 (1982), 1811.

5. U. Jahnke, et al., Phys. Rev. Lett. 48 (1982), 17.

6. G.M. Berkowitz et al., Phys. Rev. C28 (1983), 667.

7. W.S. Freeman et al., Phys. Rev. Lett. 50 (1983), 1563.

8. R. Pengo et al., Nucl. Phys. A411 (1983), 255.

9. A.B. Balantekin, S.E. Koonin and J.W. Negele, Phys. Rev. C28 (1983), 1565.

10. H. Esbensen, Nucl. Phys. A352 (1981), 147.

11. S. Landowne and J.R. Nix, Nucl. Phys. A368 (1981), 352.

12. C.H. Dasso, S. Landowne and A. Winther, Nucl. Phys. A405 (1983), 381; A407 (1983), 221.

13. R.A. Broglia, C.H. Dasso, S. Landowne and A. Winther, Phys. Rev. C27 (1983), 2433.

14. R.G. Stockstad and E.E. Gross, Phys. Rev. C23 (1981), 281.

15. P.M. Jacobs and U. Smilansky, Phys. Lett. 127B (1983), 313.

16. H. Esbensen, J. Q. Wu and G.F. Bertsch, Nucl. Phys. A411 (1983), 275.

17. S.Y. Lee and N. Takigawa, Phys. Rev. C28 (1983), 1123.

18. L.C. Vaz, J.M. Alexander, M. Prakash and S.Y. Lee, Proc. Int. Conf. on Nuclear Physics with Heavy Ions, Stony Brook, N,Y., April 1983; Vol. VI Nucl. Sci. Research Conf. Ser. (Harwood Academic Publishers, Amsterdam).

19. H.J. Krappe, K. Mohring, M.C. Nemes and H. Rossner, Z. Phys. A314 (1983), 23.

20. H.R. Jaqaman, Phys. Rev. C29 (1984), 335.

21. M.J. Rhoades-Brown and V.E. Oberacker, Phys. Rev. Lett. 50 (1983), 1435.

22. M.J. Rhoades-Brown and P. Braun-Munzinger, Phys. Lett. 136B(1984), 19

23. N. Takigawa and G.F. Bertsch, Phys. Rev. C, in press.

24. A.O. Caldeira and A.J. Leggett, Ann. Phys. (NY) 149 (1983), 374 and references therein.

25. For a detailed discussion of coupled channels formalism see
G.R. Satchler, Direct Nuclear Reactions,(Oxford U. Press, Oxford,1983).

26. D.M. Brink, M.C. Nemes and D. Vautherin, Ann. Phys. (NY) 147, 171 (1983).

27. R.G. Newton, "Scattering Theory of Waves and Particles", pp. 177-197, McGraw-Hill, New York, 1966.

28. W.H.Miller, Adv. Chem. Phys. 35 (1974), 69.

29. R.P. Feynman and A.R. Hibbs, "Quantum Mechanics and Path Integrals", McGraw-Hill, New York, 1965; R.P. Feynman and F.L. Vernon, Jr., Ann. Phys. (NY) 24 (1963), 118.

30. K. Mohring and U. Smilansky, Nucl. Phys. A338 (1980), 227.

31. D.M. Brink, in "Progress in Particle and Nuclear Physics", edited by D. Wilkinson (Pergamon, Oxford, 1981), Vol. IV, p. 323.

32. D.M. Brink and U. Smilansky, Nucl. Phys. A405 (1983), 301.

33. R. Gilmore, "Lie Groups, Lie Algebras, and Some of Their Applications" (Wiley, New York, 1974);
B.G. Wybourne, "Classical Groups for Physicists" (Wiley, New York, 1974).

34. H. Hofmann, Nucl. Phys. A224 (1974), 116;
H. Massmann, K. Mohring, P. Ring, J. Chem. Phys. 66 (1977), 5368;
P. Ring, H. Massmann, and J.O. Rasmussen, Nucl. Phys. A296 (1978), 50.

35. H.J. Lipkin, N. Meshkov, and A.J. Glick, Nucl. Phys. 62 (1965), 188.

36. Y. Alhassid, F. Gursey and F. Iachello, Ann. Phys. (NY) 148 (1983), 346.

37. R. Vandenbosch, B.B. Back, S. Gil, A. Lazzarini and A. Ray, Phys. Rev. C28(1983), 1161.

38. N. Takigawa, to be published in the Proc. of the Seventh Oaxtepec Symposium on Nuclear Physics, Mexico, 1984 ; and N. Takigawa and G.F. Bertsch, in preparation .

39. D.M. Brink, Lecture Notes in Physics, 89(1978), p.340.

40. R.J. Glauber, Phys. Rev. 131(1963), 2766;
G.S. Agarwal and E. Wolf, Phys. Rev. D2(1970), 2161;2187;2206.

41. A.O. Caldeira and A.J. Leggett, Phys. Rev. Lett. 46(1981), 211

42. A. Bohr and B.R. Mottelson, Nuclear Structure, Vol. I (Benjamin, New York, 1969), p.302.

43. M.J. Rhoades-Brown and M. Prakash, SUNY,Stony Brook preprint (1984).

44. N. Takigawa, K. Niita, Y. Okuhara, and S. Yoshida, Nucl. Phys. A371(1981), 130.

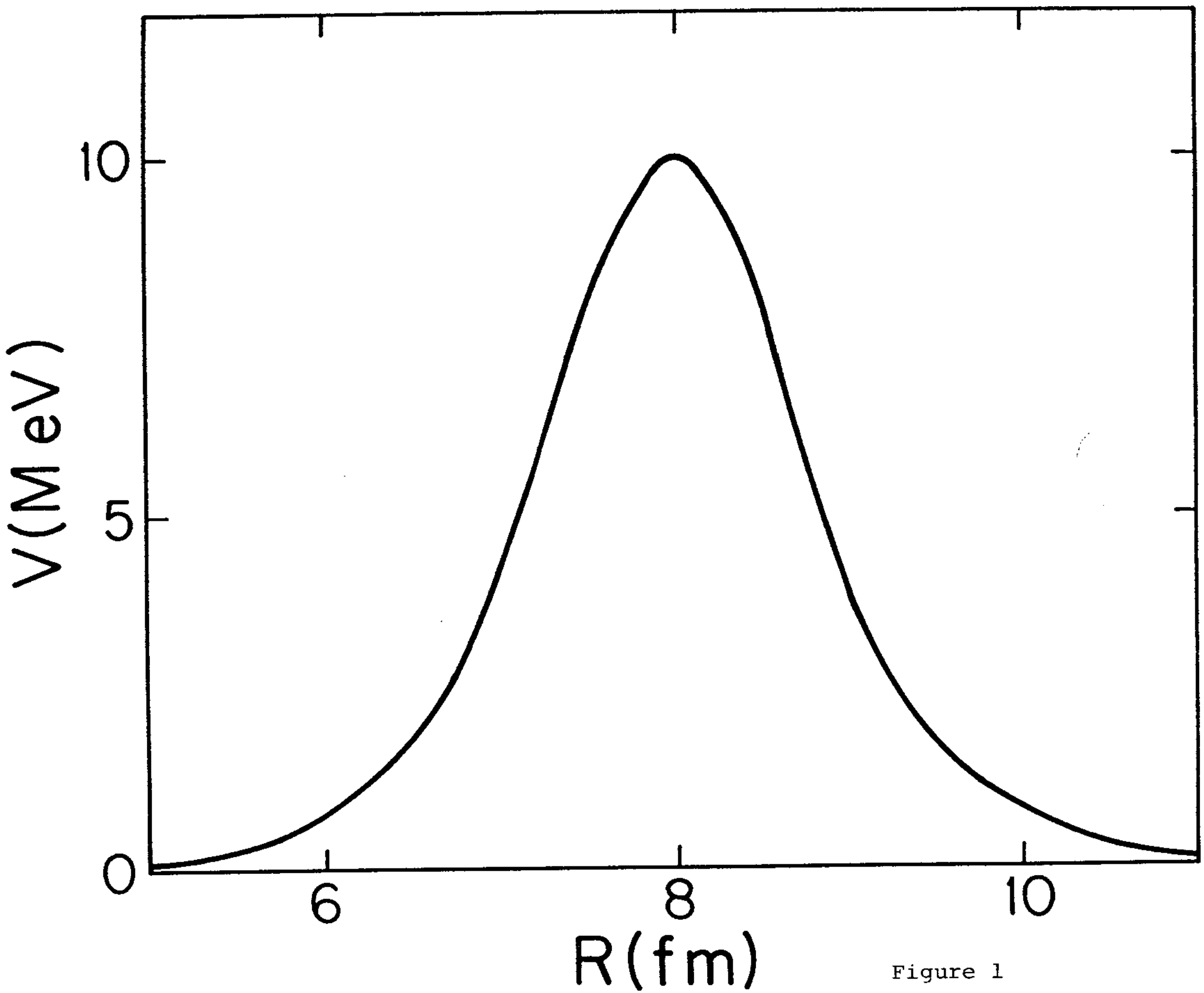
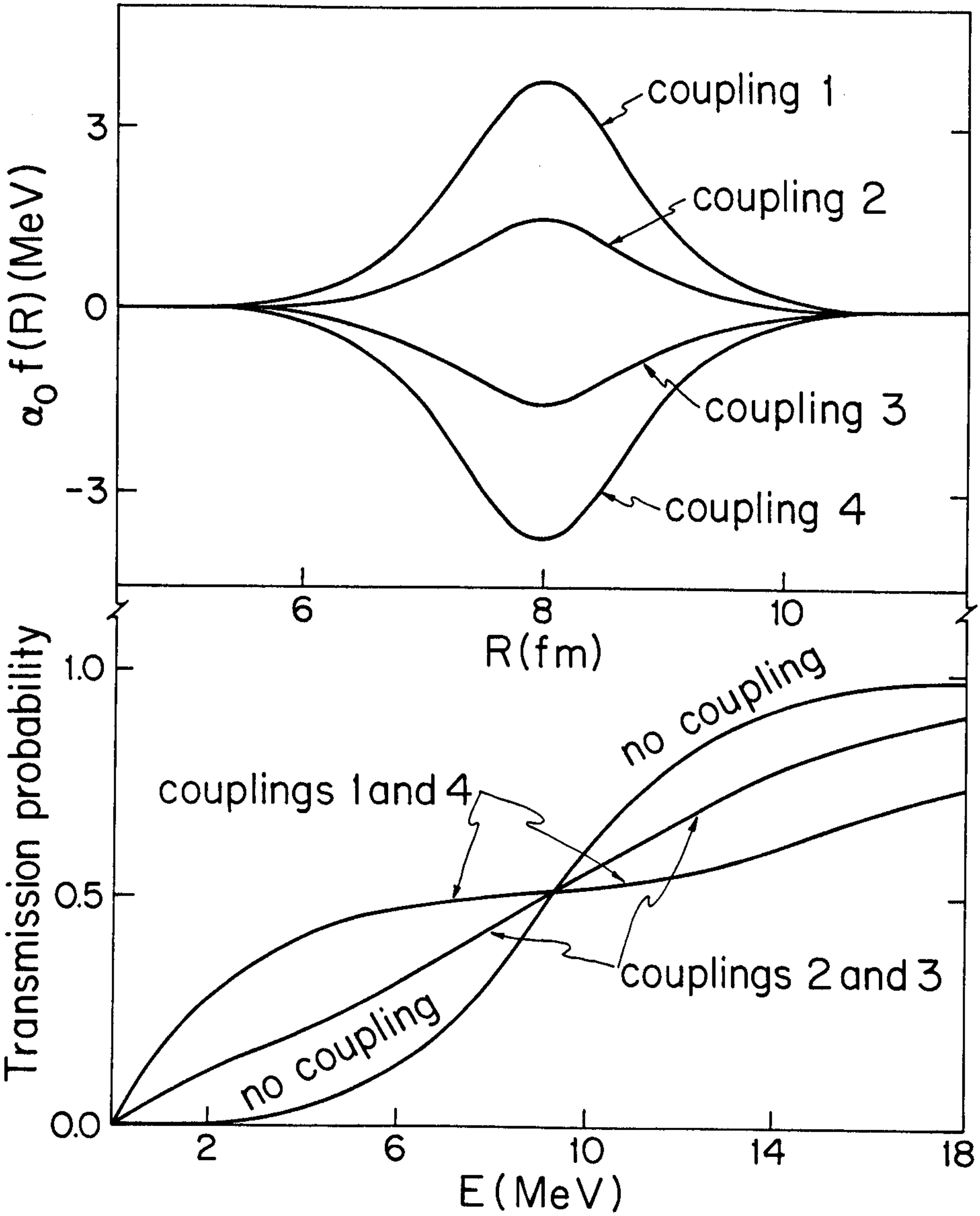


Figure 1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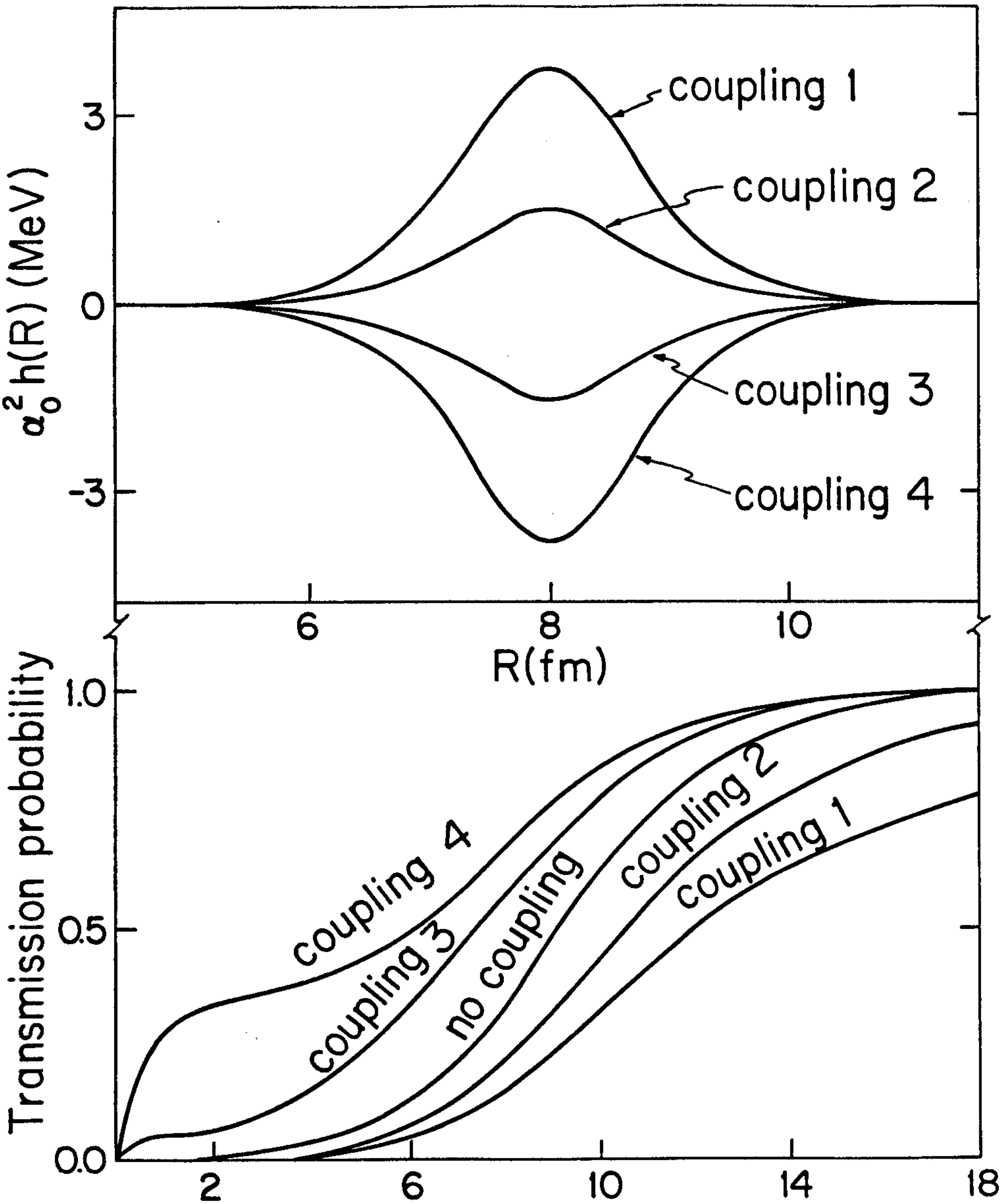
coupling 2
coupling 3
coupling 4
3
0
-3
$a_0 f(R)$ (MeV)
6
8
10
R(fm)
1.0
0.5
0.0
Transmission probability
no coupling
couplings 1 and 4
couplings 2 and 3
no coupling
2
6
10
14
18
E(MeV)

Figure 2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coupling 2
coupling 3
coupling 4
3
0
-3
$a_0^2 h(R)$ (MeV)
6
8
10
R(fm)
1.0
0.5
0.0
Transmission probability
coupling 4
coupling 3
no coupling
coupling 2
coupling 1
2
6
10
14
18
E(MeV)

Figure 3

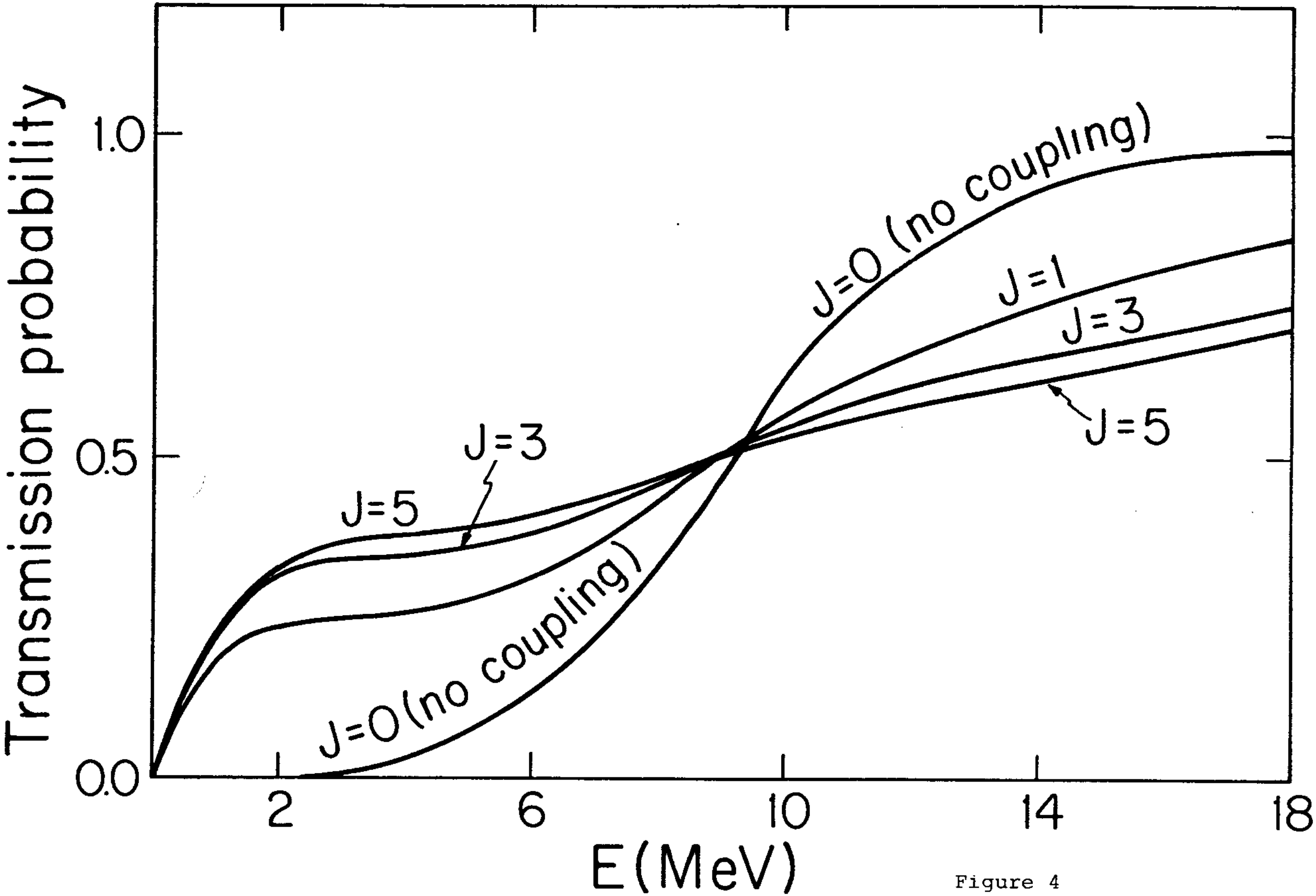
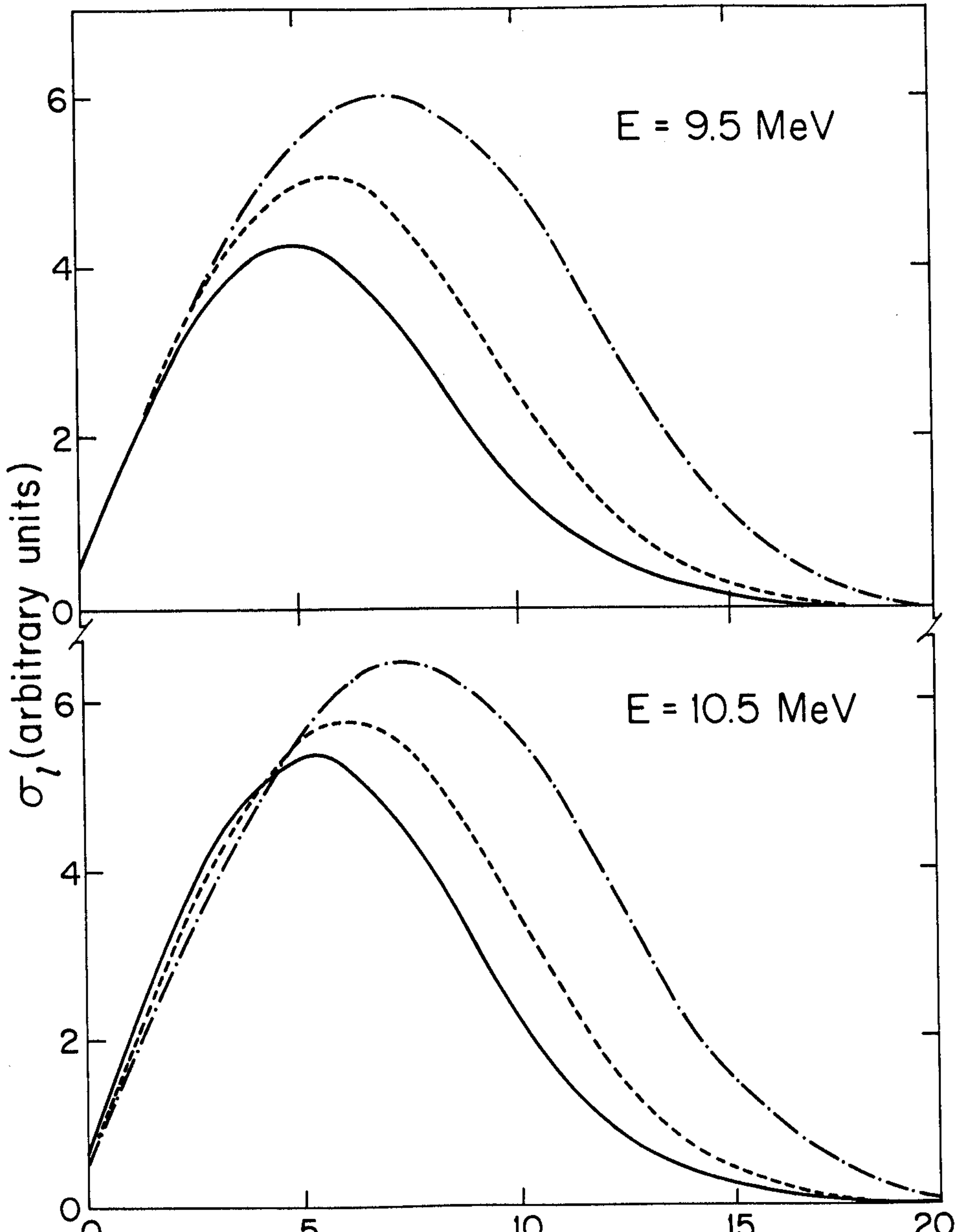

Transmission probability
1.0
0.5
0.0
J=0 (no coupling)
J=1
J=3
J=5
J=3
J=5
J=0(no coupling)
2
6
10
14
18
E(MeV)

Figure 4



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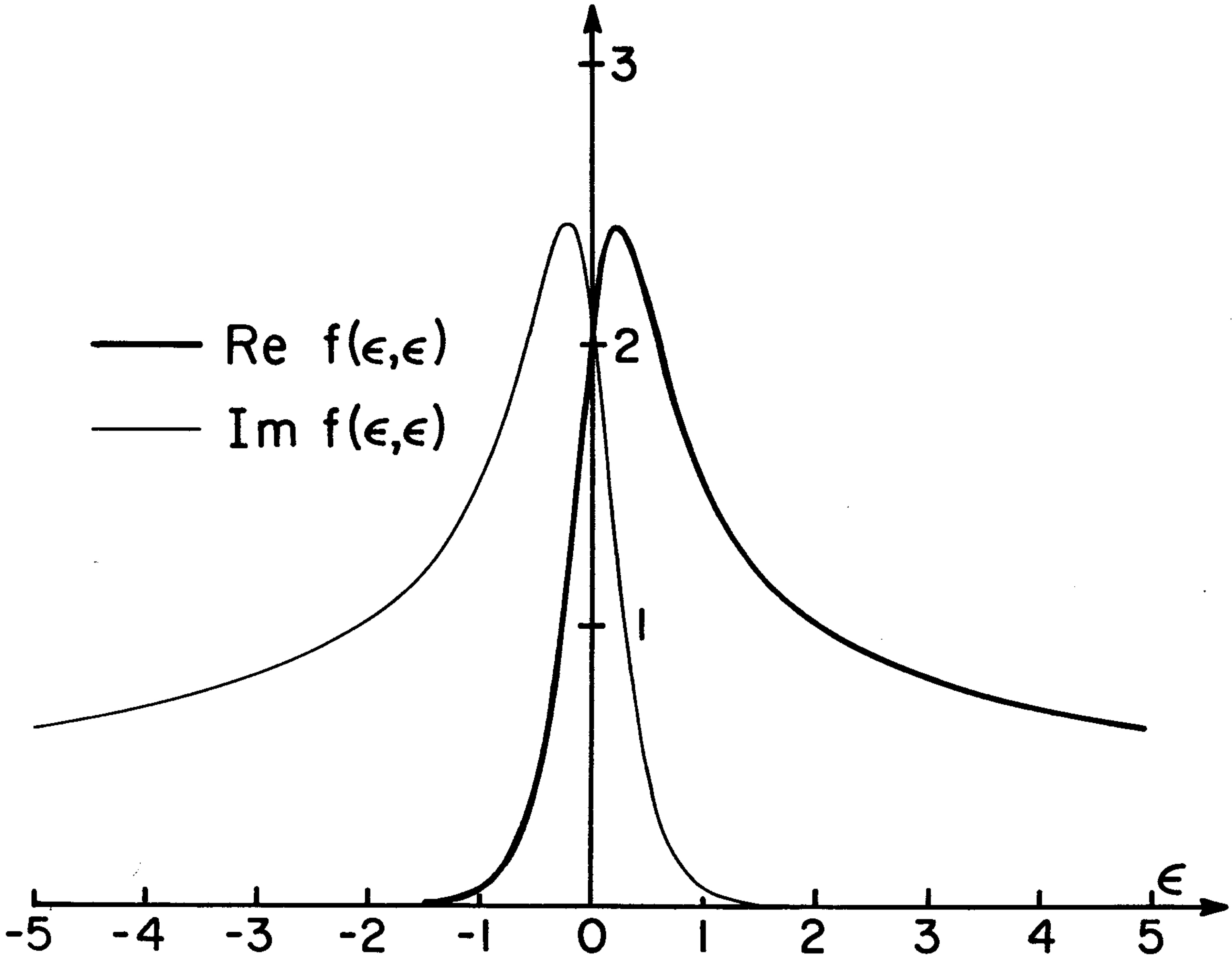

Figure 6

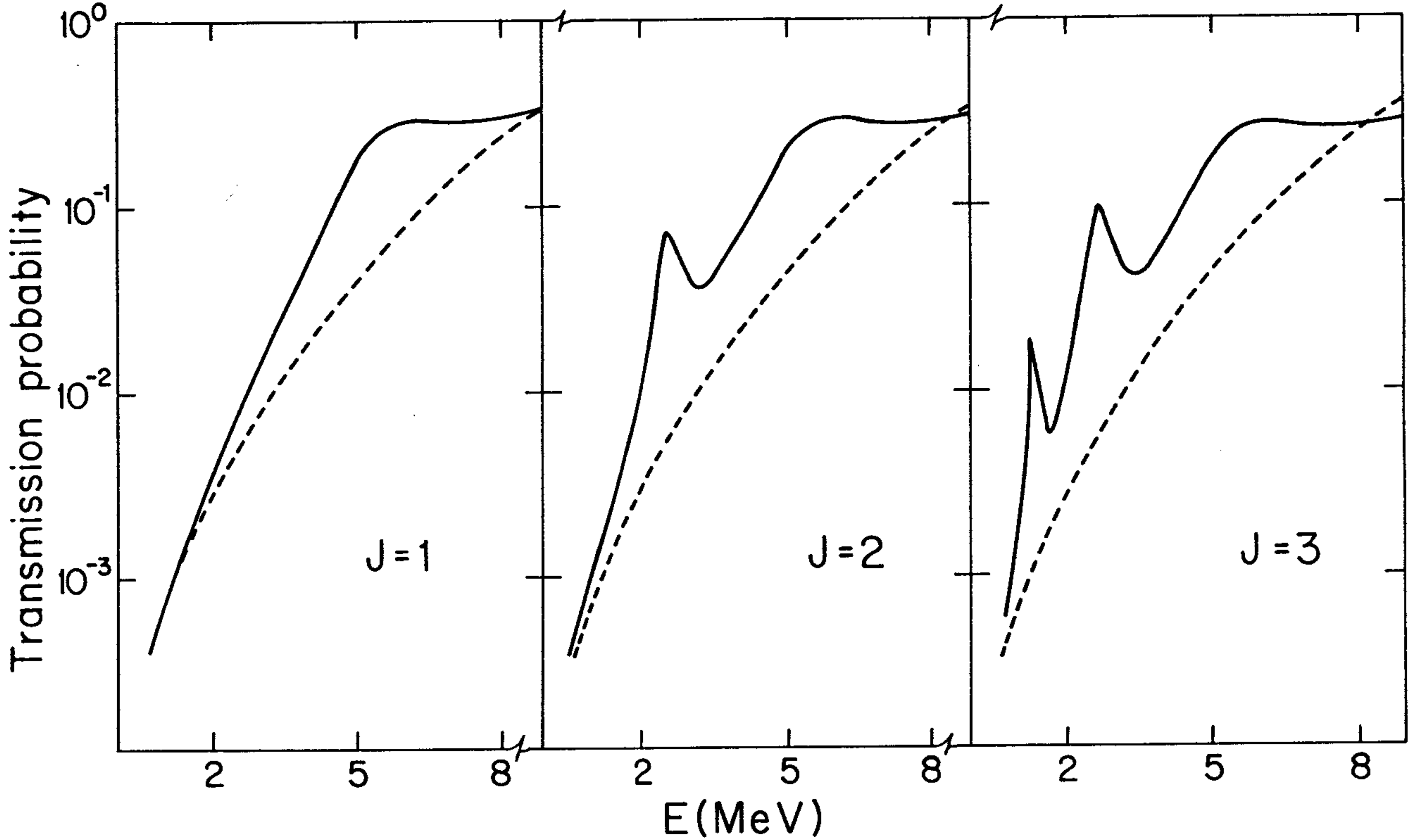

Figure 7