ATS/Note/2012-076 TECH

2012-10-02

christian.scheuerlein@cern.ch

EDMS no: 1244158

Statistical analysis of LHC main interconnection splices room temperature resistance (R-8) results

S. Heck, C. Scheuerlein / TE-MSC

1 Introduction

During the 2008/2009 shutdown the so-called R-8/R-16 room temperature resistance test has been introduced for the quality control of the LHC main interconnection splices [1]. It has been found that at present two groups of LHC main interconnection splices can be distinguished, so-called "old" splices produced during LHC installation, and so-called "new" splices produced during 2009 [2]. 2009 production splices are considered as the state-of-the art, which is reflected by a much smaller R-8 distribution as compared to that of splices produced during first LHC installation.

Statistical analysis of R-16 data has been reported in [3]. The main goal of the previous analysis was to estimate the maximum R-8 excess resistances of the existing LHC main splices for the estimation of the LHC safe energy. For this purpose the "new" splices results could be neglected and the statistical analysis concentrated on R-16 results of "old" splices.

Here we present the statistical analysis of the R-8 results of "new" splices and compare the "new" and "old" R-8 data sets. The main goal of the analysis of "new" splice R-8 results is to estimate the uncertainty of the R-8 measurements and to provide fit functions that can be compared with future splice production R-8 results in order to monitor a potential degradation in the splice manufacturing process. The R-8 results of 2009 production are compared with R-8 of splices produced during LHC installation and the possible reasons for the different R-8 distributions are discussed. The number of "old" splices to be repaired because of a too high R-8 excess resistance (R-8_{excess}) before consolidation is estimated for different R-8_{excess} threshold values.

All R-8 measurements were performed with a Digital Low Resistance Ohm Meter (Avo DUCTER DLRO 10) [4] with a voltage tap distance of 8 cm (R-8), a test current of 10 A and a resolution of 0.1 μΩ. The accuracy of the DLRO 10 stated by the manufacturer is ±0.2 μΩ. The inhomogeneous current distribution due to the point like current injection close to the voltage taps causes a systematic error in the R-8 results of approximately +10%. The influence of the splice temperature on R-8 is neglected. The estimated temperature variation in the LHC tunnel is between 14 °C to 20 °C. This causes a maximum uncertainty in the R-8 results of 0.21 uΩ and 0.12 uΩ for quadrupole and dipole splices, respectively.

For binning and curve fitting the statistical analysis software EasyFit Professional Version 5.5 from MathWave Technologies [5] has been used.

2 Statistical analysis of "new" splices R-8 data sets

2.1 Kolmogorov-Smirnov test of "new" quadrupole splices R-8 data set

Assuming that the R-8 distribution obtained for "new" splices would be caused entirely by random uncertainties like resistance measurement errors, voltage tap distance errors, temperature uncertainties and random variations of the busbar stabiliser cross sections a nearly normal distribution of the R-8 results would be expected.

A Kolmogorov-Smirnov (KS) test [6] has been performed in order to verify if the "new" splices R-8 data sets follow a normal distribution. As a first step of the KS test the data to be investigated is standardized (z-transformed) to a standard normal distribution and is plotted as a cumulative distribution function (CDF). The CDF of "new" quadrupole splices (M1, M2) with $\sigma^2 = 1$ and $\mu = 9.51$ is shown in Figure 1 (red squares). The blue curve is a step curve called empirical distribution function (EDF), in which the cumulative probability increases in R-8 steps of 1/n, with n being the number of measurement values (here n=336).

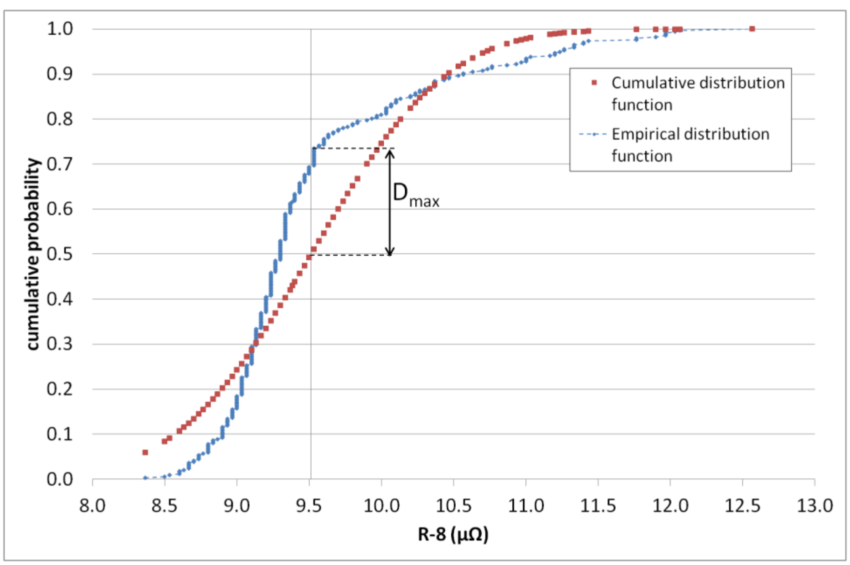

Figure 1: Comparison of the EDF and CDF calculated from the R-8 data set of "new" quadrupole splices (n=336, μ =9.51 μΩ, σ=0.74 μΩ). D_{max}=0.22.

The larger the distance (D) between EDF and CDF, the less likely it is that the investigated data set is normal distributed. If at any point the absolute difference between EDF and CDF (D_{max}) is larger than D_α the hypothesis of a normal distribution is rejected with a selected significance level α. For a data set with n>40 values, the D_α value is calculated as shown in Equation 1:

Equation 1:

$$D_\alpha = \frac{\sqrt{\boldsymbol{ln}(\frac{2}{\alpha})}}{\sqrt{2\boldsymbol{n}}}$$

In case of the entire "new" quadrupole splices R-8 data set D_{max} is 0.22 and $D_{0.3\%}$ is 0.098. Since $D_{max}>D_{0.3\%}$ one can therefore conclude with a confidence level of 99.7% (3σ) that this R-8 data set is not following a normal distribution, which is obvious in the histogram shown in Figure 2.

In a next step it has been verified if a normal distribution is obtained when excluding high R-8 results from the data set. For this purpose it is assumed that the R-8 distribution is symmetric with μ =9.23 μΩ, and that R-8 values >10.1 μΩ (empty symbols in Figure 2) are caused by splice imperfections and can be excluded from the data set when estimating the random measurement uncertainties in the R-8 data set.

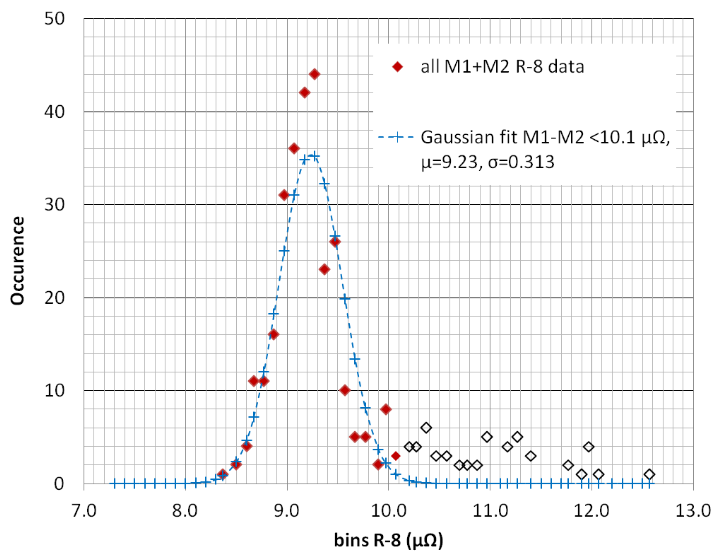

Figure 2: Histogram of "new" quadrupole R-8 results. The Gaussian fit considers only R-8 results <10.1 μΩ. The empty symbols represent the R-8 data that has not been considered because it is assumed to be caused by splice imperfections. Bin size is 0.1 μΩ.

When R-8 values >10.1 μΩ are excluded, the D_{max} between EDF and CDF is reduced to 0.081 (see Figure 3), which is equal to $D_{5\%}$=0.081 (for n=280). This indicates that a normal distribution is approached when considering R-8 smaller 10.1 μΩ only, but the hypothesis of a normal distribution is still rejected with a probability of 95 %.

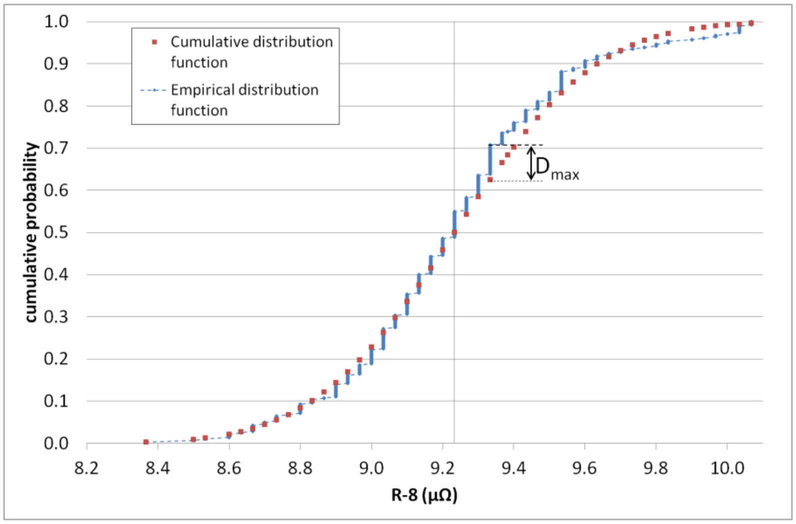

Figure 3: Comparison of the EDF and CDF calculated from the R-8 data set of quadrupole splices produced during 2009 considering R-8 values <10.1 µΩ only (n=280, µ =9.23 µΩ, σ=0.31 µΩ). D_{max} is with 0.081 equal to $D_{5\%}$.

2.2 Kolmogorov-Smirnov test of "new" dipole splices R-8 data set

The KS test was also performed on "new" dipole (M3) splices considering all R-8 data, which is presented in Figure 4.

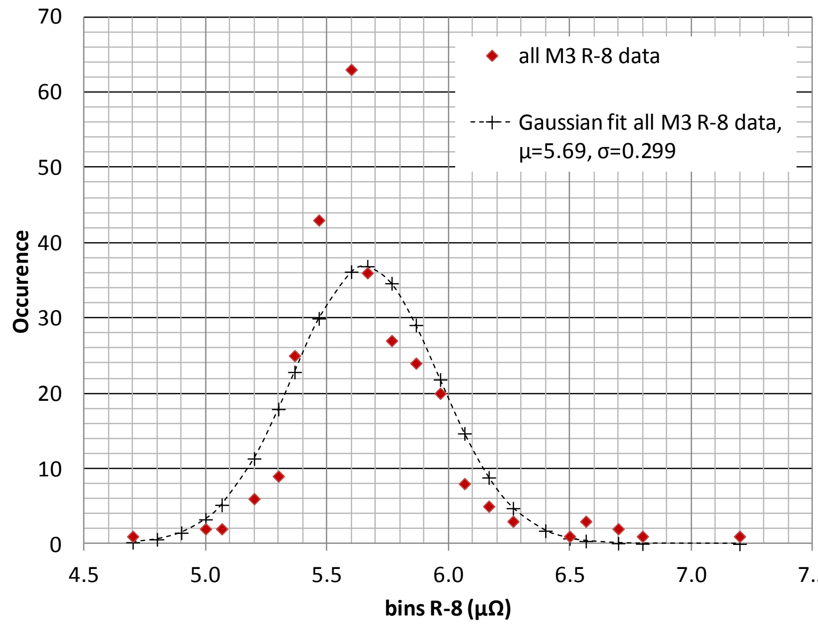

Figure 4: Histogram of all R-8 results for "new" M3 splices and Gaussian fit. Bin size is 0.1 µΩ.

The EDF and CDF for "new" dipole splices are shown in Figure 5. D_{max} (0.11) is larger than $D_{5\%}$ (0.080), and equal to the $D_{0.3\%}$ (0.11), and it is concluded that with a confidence level of 95% this R-8 data set is not following a normal distribution.

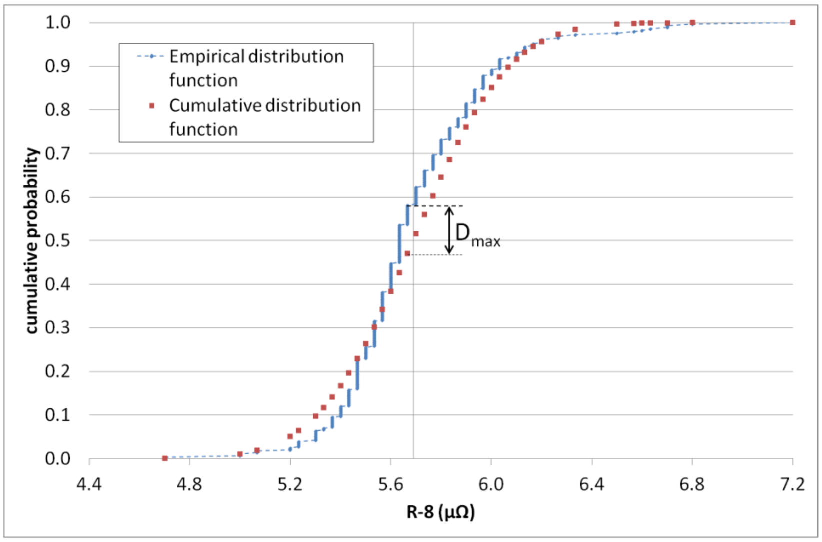

Figure 5: Comparison of the EDF and CDF calculated from R-8 data measured on "new" dipole splices (n=286, μ =5.69 μΩ, σ=0.29 μΩ) D_{max}=0.11.

Since the R-8 distribution shown in the histogram in Figure 4 is not normal distributed it can be concluded that the spread in R-8 results is not only due to random measurement errors, but it is also influenced by splice imperfections. It is therefore assumed that the random uncertainties are somewhat smaller than the variance of the "new" splices R-8 results, and that the uncertainty of the R-8 measurements performed in 2009 is smaller than ±0.3 μΩ (±σ).

2.3 Bin size of the R-8 histograms

The bin sizes of the following R-8 histograms have been automatically determined as the best bin size by the software EasyFit Professional Version 5.5. A comparison of R-8-histograms with manually selected bin size of 0.1 μΩ (for "new" splices and excess resistances <5 μΩ) and 1 μΩ (for the entire data set of "old" splices) is presented in the appendix of this note.

2.4 Curve fitting

The EasyFit software is able to fit 60 different mathematical functions to a given data set. For each data set the software makes a ranking of the 60 fit functions based on a KS and an Anderson Darling goodness of fit test.

2.4.1 Probability density distribution of "new" dipole splices R-8 results

The goodness of fit ranking for different functions is shown in Figure 6. According to the KS test the best function to fit the R-8 results of "new" dipole splices is the Johnson SU function, while according to the Anderson Darling test the best fit is achieved with a 4 Parameter (4P) Burr function.

#	Distribution	Kolmogorov Smirnov		Anderson Darling	
		Statistic	Rank	Statistic	Rank
30	Johnson SU	0.1149	1	2.6708	3
21	Gen. Extreme Value	0.11528	2	3.307	7
3	Burr (4P)	0.12268	3	2.4753	1
2	Burr	0.12273	4	2.4771	2

Goodness of Fit - Summary

Figure 6: Kolmogorov Smirnov and Anderson Darling goodness of fit ranking of "new" dipole splice R-8 data set.

The 3 parameter (3P) Burr function (see Equation 2) can fit well the "new" R-8 data sets analysed in this note, and only slight differences in the goodness of fit were found between the 3P and 4P Burr functions. In the following the simpler 3P Burr function is preferred over the 4P Burr function.

$$f(x) = \frac{\alpha k \left(\frac{x}{\beta}\right)^{(\alpha-1)}}{\beta\left(1+\left(\frac{x}{\beta}\right)^{\alpha}\right)^{k+1}}$$

Equation 2:

In Figure 7 the R-8 histogram for "new" dipole splices is shown with the 3P Burr probability density function.

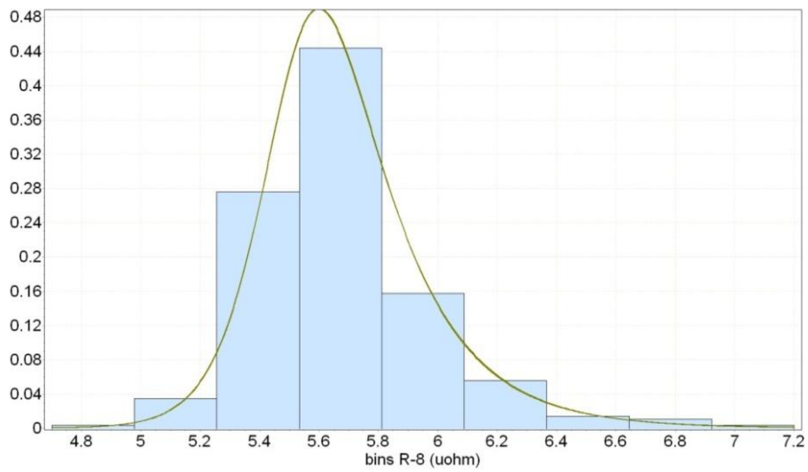

Figure 7: R-8 histogram of "new" dipole splices (bin size= 0.27 μΩ, n=286) and 3P Burr probability density function f(x).

The corresponding 3P Burr fit parameters and the values determined for mode, skewness and kurtosis of the fit are summarized in Table 1.

Table 1: Summary of 3P Burr fit parameters and fit characterizing values mode (number of most frequent value), skewness and kurtosis of "new" dipole R-8 results (arithmetic mean =5.69 μΩ). The skewness and kurtosis of a normal density distribution are 0 and 3, respectively.

3P Burr fit parameters	k=0.509	α=50.93	β=5.53
3P Burr fit characterization	mode=5.60 μΩ	skewness=1.12	kurtosis=3.59

2.4.2 Probability density distribution of "new" quadrupole splices R-8 results

The histogram of the entire "new" quadrupole R-8 data set, and for the "new" quadrupole R-8<10.1 μΩ data with the 3P Burr fits are shown in Figure 8 and Figure 9, respectively.

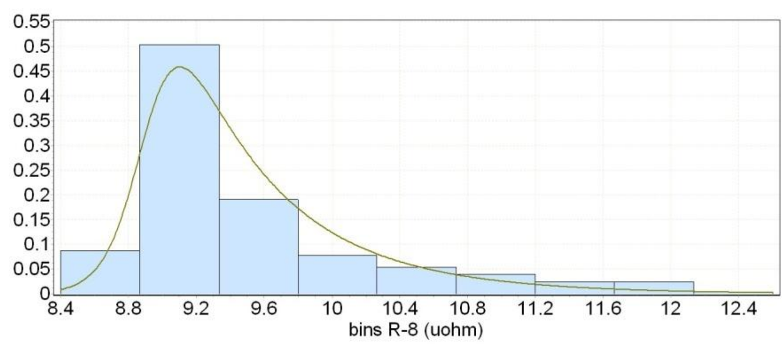

Figure 8: R-8 histogram of "new" quadrupole splices (bin size=0.46 μΩ, n=336) and 3P Burr probability density function f(x).

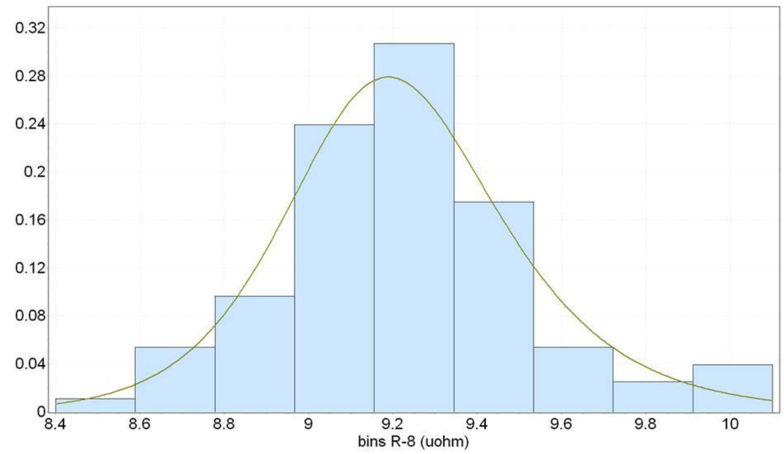

Figure 9: R-8 histogram of "new" quadrupole splices with R-8<10.1 μΩ (bin size=0.18 μΩ, n=280) and the 3P Burr probability density function f(x).

The 3P Burr fit parameters and the values determined for mode, skewness and kurtosis of the entire R-8 data of "new" quadrupole splices and for R-8 < 10.1 μΩ are compared in Table 2.

Table 2: Summary of fit parameters and fit characterizing values of R-8 results of "new" quadrupole splices. (a) entire R-8 data set (arithmetic mean =9.51 μΩ) and (b) only R-8 <10.1 μΩ (arithmetic mean =9.23 μΩ).

3P Burr fit parameters	a)	k=0.202	α=76.22	β=8.92
	b)	k=0.79	α=58.7	β=9.16
3P Burr fit characterization	a)	mode=9.10 μΩ	skewness=2.17	kurtosis=8.8
	b)	mode=9.19 μΩ	skewness 0.45	kurtosis=1.7

3) Comparison of "new" and "old" dipole splice R-8 results

3.1 Distribution of excess resistances in "old" dipole splices

A good R-8 overview is obtained when plotting the R-8_{excess} results ordered by the resistance values separately for "old" and "new" splices, and distinguishing between splices on the lyra and connection side. The influence of the improvements in the splice production introduced in 2009 on the R-8 results [1] is striking when comparing the R-8 distribution in Figure 10 (a) and Figure 10 (b). It can be seen that for "old" splices a large population has slightly varying excess resistance values below 4 μΩ and a strong change in the slope of the resistance increase is seen above 5 μΩ excess resistance.

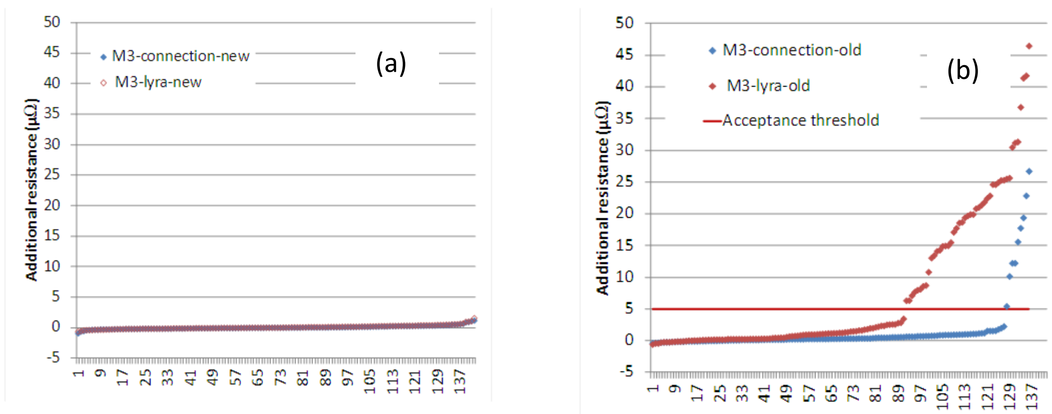

Figure 10: Distribution of excess resistance (defined as R-8 -5.6 μΩ) at the connection and lyra side of 2009 production (a) and first LHC installation (b) M3 splices, sorted ascending by resistance values.

The cumulative distribution of "old" M3 R-8_{excess} results is shown in Figure 11. Three excess resistance intervals are distinguished (R-8_{excess} <5 μΩ labeled I, 5 μΩ<R-8_{excess}<25 μΩ labeled II and R-8_{excess}>25 μΩ III).

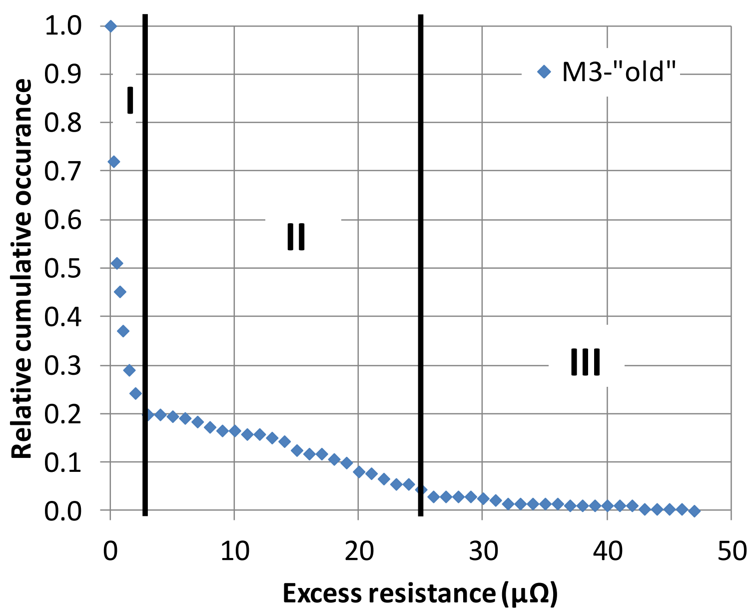

Figure 11: Cumulative distribution of "old" M3 R-8_{excess} values (n=272).

A plausible explanation for the first strong change of the slope of the cumulative distribution is that the R-8 results above 5 μΩ are all representative for splices with a complete electrical discontinuity between the busbar stabiliser and the splice profiles, where the R-8 resistance is entirely determined by the cable length that is not in contact with the busbar stabiliser (13 μΩ per cm insulated cable length).

The 2nd change of slope in the cumulative excess resistance distribution at 25 μΩ excess resistance may be an indication that also for splices with an excess resistance >5 μΩ there are two failure modes. One failure mode for splices with excess resistance > 5 μΩ is underheating at the splice extremities, which causes an insulated cable length inside the splice profiles. A second failure mode is a transverse gap between busbar stabiliser and busbar profiles in combination with a certain insulated cable length inside the busbar stabiliser [1]. Radiographic examination revealed that most, if not all of M3 splices with very high excess resistance had unmolten solder foil at the splice extremities.

A possible explanation for the relatively high occurrence of very high R-8 resistances on the lyra side is the asymmetry of the tooling that has been used for the splice compression and inductive heating, which can cause a non-uniform temperature gradient across the splice during the soldering process.

Below the R-8_{excess} distribution in the three excess resistance intervals R-8_{excess} <5 μΩ, 5 μΩ<R-8excess<25 μΩ and R-8_{excess}>25 μΩ is described.

3.1.1 Comparison of the probability density function of "new" and "old" splices with an excess resistance up to 5 μΩ

The fits of the "new" and "old" R-8 data with an excess resistance <5 μΩ are compared in Figure 12 and Figure 13 for quadrupole and dipole splices, respectively. Both data sets have been fitted with a 3P Burr function.

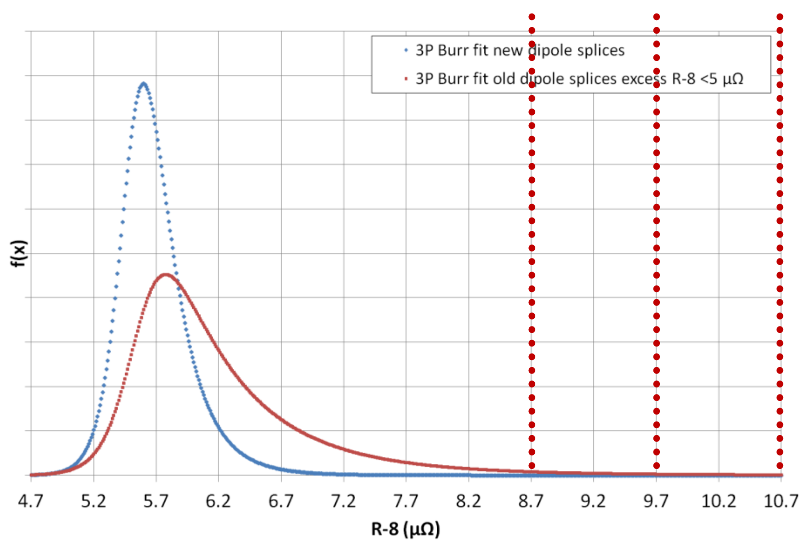

Figure 12: Comparison of the 3P Burr probability density distribution function for "new" and "old" R-8 dipole splice results with an excess resistance <5 μΩ (R-8<10.7 μΩ). R_{excess} values of 3 μΩ, 4 μΩ and 5 μΩ are indicated by the dotted vertical lines.

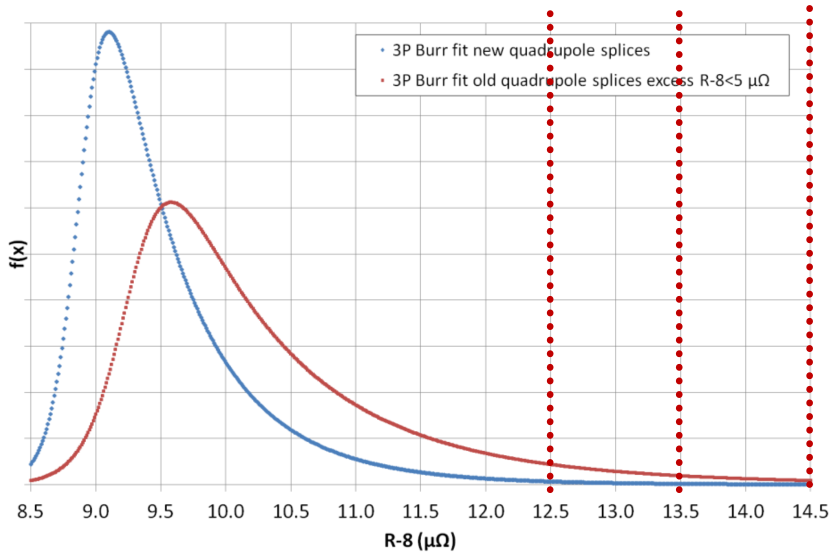

Figure 13: Comparison of the 3P Burr probability density distribution function for "new" and "old" R-8 quadrupole splice results with an excess resistance <5 μΩ (R-8<14.5 μΩ). R_{excess} values of 3 μΩ, 4 μΩ and 5 μΩ are indicated by the dotted vertical lines.

In Table 3 the 3P Burr fit parameters for the "new" and "old" dipole and quadrupole R-8 data sets with an excess resistance <5 μΩ are compared.

Table 3: 3P Burr fit parameters for R-8 data sets with R-8_{excess}<5 μΩ obtained for "new" and "old" dipole and quadrupole splices.

Parameter	k	α	β
"new" quadrupole fit	0.202	76.22	8.92
"old" quadrupole fit	0.18	54	9.3
"new" dipole fit	0.509	50.93	5.53
"old" dipole fit	0.24	40.1	5.59

The corresponding fit characterizing values are compared in Table 4.

Table 4: Fit characterizing values for "old" and "new" R-8 data sets.

	mode (μΩ)	skewness	kurtosis
Dipole "new"	5.60	1.12	3.59
Dipole "old"	5.77	2.48	12.9
Quadrupole "new"	9.10	2.17	8.80
Quadrupole "old"	9.58	2.62	13.8

Despite the fact that "old" splices of which R-8 was measured were not randomly selected, it is obvious from the results presented above that the splice assembly procedure and/or the process parameters used during LHC installation are not identical to those used during the 2008-2009 shutdown.

3.1.2 Influence of the R-8_{excess} acceptance threshold value on the number of "old" splices to be repaired during LS1

The baseline for the consolidation of the LHC main interconnection splices during the Long Shutdown 1 is to systematically repair "old" splices before application of shunts if R-8_{excess} exceeds a threshold value of 5 μΩ [2,7]. Below we estimate the number of splices that would need to be repaired for R-8_{excess} threshold values of 3 μΩ, 4 μΩ and 5 μΩ.

The number of splices to be repaired because of excessive R-8 values is estimated from the entire R-8 data set for "old" quadrupole splices with the following assumptions.

- The R-8 distribution of "old" quadrupole splices measured in 2009 is representative for the entire LHC splice population (despite the fact that some high excess resistance quadrupole splices were found thanks to non invasive segment measurements).
- The R-8_{excess} distribution for "old" dipole and quadrupole splices is identical
- The excess resistance occurs always on one side of the splice
- The different R-8 distribution of "new" and "old" splices is neglected.

The estimated number of splices to be repaired is shown in Table 5 for the threshold values of 5 μΩ, 4 μΩ and 3 μΩ. As an example, 17 out of 202 (8.4%) R-8 results for "old" quadrupole splices gave an excess resistance >=5 μΩ, which corresponds to estimated 860 splices with R-8_{excess}>=5 μΩ in the LHC. It is further estimated that roughly 240 and 740 splices more need to be redone when the R-8 acceptance threshold value would be decreased from the present value of 5 μΩ to 4 μΩ and 3 μΩ, respectively. Other reasons to repair a splice, as for instance geometrical splices distortions that prevent the installation of shunts are not taken into account in these estimates.

Table 5: Estimated number of splices to be repaired due to too high R-8 as a function of the acceptance threshold value. *The assumed total number of splices in the LHC is 10170.

R-8_{excess} acceptance threshold (R-8)	Estimated number of splices to be redone*
5 μΩ (14.3 μΩ)	860 (8.4%)
4 μΩ (13.3 μΩ)	1100 (11%)
3 μΩ (12.3 μΩ)	1600 (16%)

3.1.3 Influence of the R-8_{excess} acceptance threshold value on the number of LS1 production splices to be repaired

The number of splices from LS1 production that has to be repaired because of a too high excess resistance is estimated based on the R-8 data for "new" quadrupole and dipole splices (see Table 6). At present the baseline is to repair LS1 production splices when R-8_{dipole}>=7.6 μΩ (R-8_{excess}>=2 μΩ) and R-8_{quad}>=12.3 μΩ (R-8_{excess} >=3 μΩ) [2].

Table 6: Estimated number of LS1 production splices to be repaired due to too high R-8 as a function of the acceptance threshold value. *The assumed number of M3 splices to be produced during LS1 is 500. **The assumed number of M1 and M2 splices to be produced during LS1 is 1000.

M3 R-8_{excess} acceptance threshold (R-8)	Estimated number of splices to be redone*
2 μΩ (7.6 μΩ)	0 (0%)
1 μΩ (6.6 μΩ)	21 (2.1%)
M1,M2 R-8_{excess} acceptance threshold (R-8)	Estimated number of splices to be redone**
3 μΩ (12.3 μΩ)	6 (0.3%)
2 μΩ (11.3 μΩ)	100 (5%)

The R-8 data indicates that the quality of "new" dipole splices exceeds that of "new" quadrupole splices. It can be seen that with the present acceptance criteria the number of LS1 production splices to be repaired can be neglected.

3.1.4 Excess resistance distribution for "old" dipole splices with an excess resistance >5 µΩ

As shown in Figure 11, for the "old" dipole R-8_{excess} data set two excess resistance intervals can be distinguished. The cumulated distribution of "old" M3 R-8_{excess} values in the range 5 µΩ<R-8_{excess}<25 µΩ is presented in Figure 14. The distribution of this data set can be described well by a 2nd order polynomial fit (cumulative occurrence=$0.034*R_{excess}^2-1.0788*R_{excess}+59.125$; $R^2=0.992$).

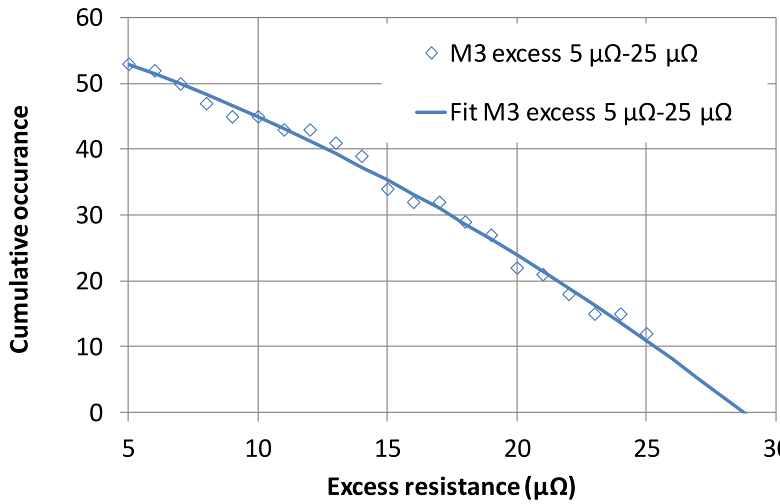

Figure 14: Cumulative distribution of "old" M3 R-8_{excess} values in the range 5 µΩ to 25 µΩ (n=42) and 2nd order polynomial fit.

The cumulated distribution of "old" M3 R-8_{excess} values >25 µΩ is presented in Figure 15. For dipole splices the maximum R-8_{excess} found in 5 out of 8 sectors in the LHC is 46 µΩ. Because of the limited number of only 8 data points a statistical analysis of M3 R-8_{excess} values >25 µΩ may not be very meaningful. However, assuming that in 5 sectors of the LHC all dipole splices with R-8_{excess} >25 µΩ have been detected by non-invasive resistance measurements [3], and that the R-8_{excess} distribution of all M3 LHC splices in the 3 LHC sectors is similar to what has been observed so far, Figure 15 suggests a maximum R-8_{excess} of roughly 50 µΩ as a crude estimate for the entire LHC dipole splice population, provided that there are no other splice failure modes as those described above.

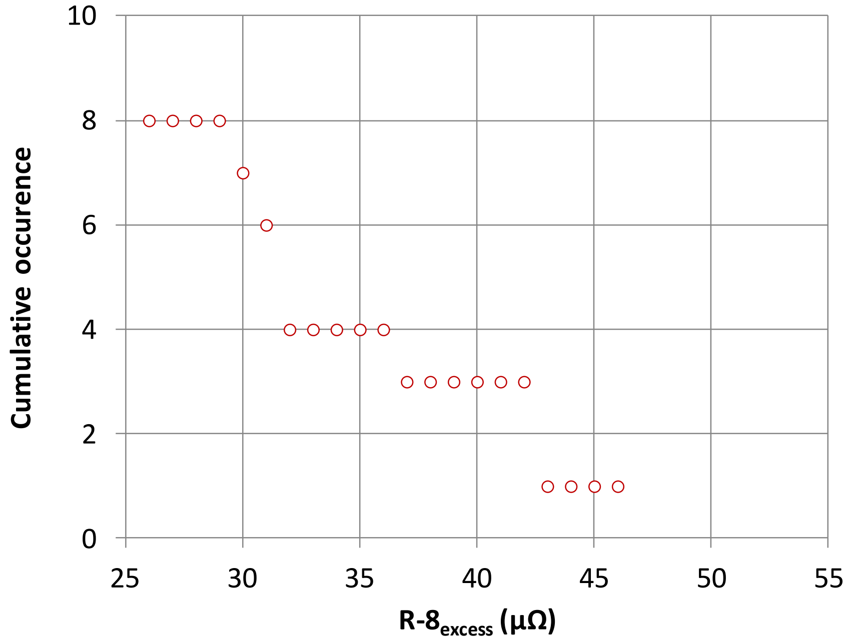

Figure 15: Cumulative distribution of "old" M3 R-8$_{excess}$ values with R-8$_{excess}$>25 μΩ.

4 Discussion and Conclusion

A Kolmogorov-Smirnov test has been performed for the two data sets "new" dipole and "new" quadrupole splices, indicating that both data sets are not normal distributed. Thus, the variance of R-8 results is not only caused by random resistance measurement errors, random uncertainties in the distance between voltage taps, temperature variations and random variations of the splice Cu cross sections, but also by splice imperfections that systematically cause a R-8 increase. From the R-8 distribution measured for "new" splices it can be concluded that the precision of the R-8 measurements performed in 2009 is better than ±0.30 μΩ (± σ).

A reasonable fit of all R-8 data sets up to an excess resistance of 5 μΩ is obtained with a 3 parameter Burr function. The comparison of R-8 results for "new" (randomly selected) and "old" (biased selection [3]) splices shows that the R-8 distribution below 5 μΩ of both data sets differs strongly. The "old" splices distribution exhibits a larger skewness, kurtosis and a shifted mode to higher R-8 values, indicating that the splice assembly procedures and/or process parameters during LHC installation differed from those applied during the 2008/2009 shutdown.

For splices with >5 μΩ excess resistance it can be assumed that they have a complete discontinuity between busbar stabiliser and splice profiles and that they are mechanically not stable. Therefore, the splice resistance, which is in this case determined by the insulated cable length, can be estimated from the excess resistance. R-8 of such splices can change significantly when the splice moves slightly and therefore these splices need to be repaired before shunts can be applied.

The influence of the R-8 acceptance threshold value on the number of "old" splices that need to be repaired because of a too high room temperature excess resistance has been estimated from the entire "old" quadrupole R-8 data set. For the present baseline acceptance threshold of R_{excess}= 5 μΩ [2,7] it is estimated that roughly 860 splices need to be redone because of too high R-8 resistance. Decreasing the $R\text{-}8_{excess}$ acceptance criterion for “old” splices from 5 μΩ to 4 μΩ would require roughly 240 more splices to be repaired before consolidation. The assumptions made for these estimates are pessimistic and the number of splices to be repaired because of too high excess resistance may be somewhat smaller.

When applying the defined acceptance threshold values for “new” splices [2] it is estimated that in total 6 out of 1500 splices to be produced during LS1 need to be repaired because of too high excess resistance.

Acknowledgements

We are grateful to F. Bertinelli, L. Bottura, H. Neupert, E. Todesco and G. Willering for helpful discussions and suggestions.

Appendix: R-8 histograms of “new” and “old” dipole and quadrupole data sets

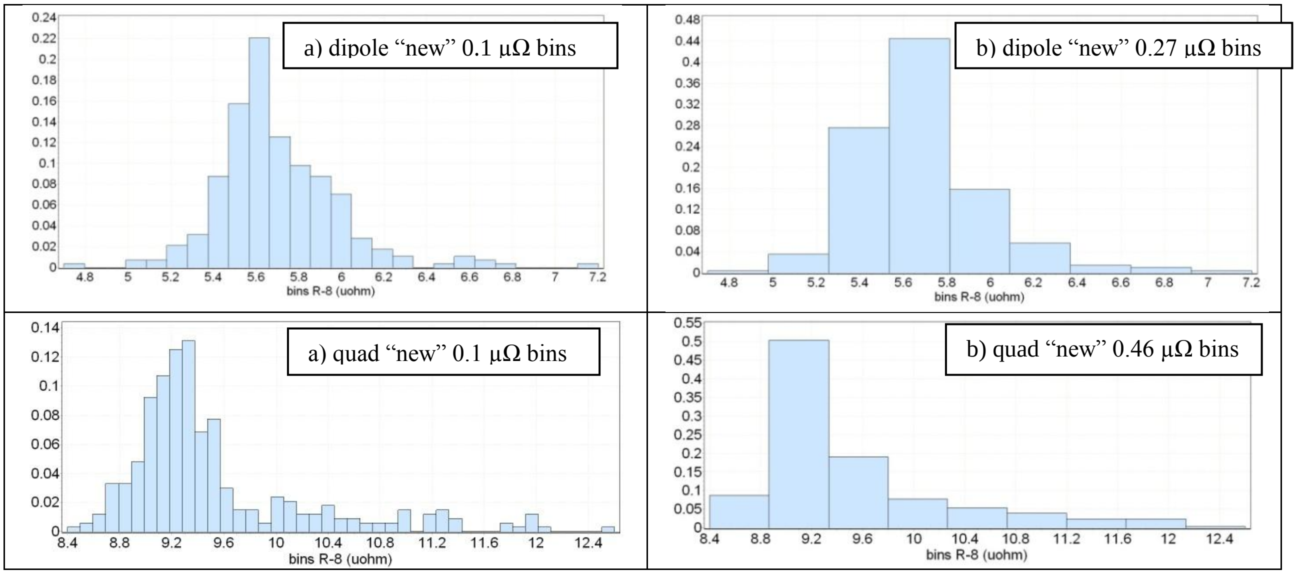

Figure 16: Comparison of R-8 histograms for “new” dipole and quadrupole splices with different bin size: (a) manually selected bin size 0.1 µΩ (b) automatically selected bin size.

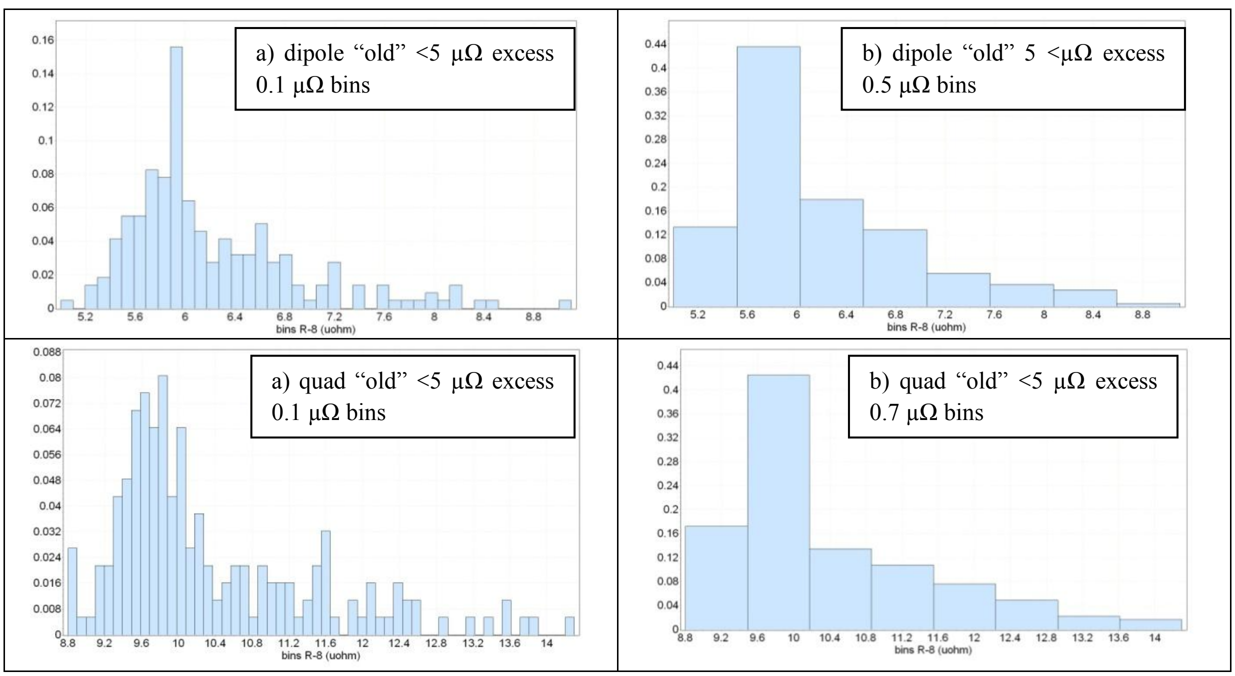

Figure 17: Comparison of R-8 histograms for “old” dipole and quadrupole splices with excess resistance <5 µΩ. (a) manually selected bin size 0.1 µΩ (b) automatically selected bin size.

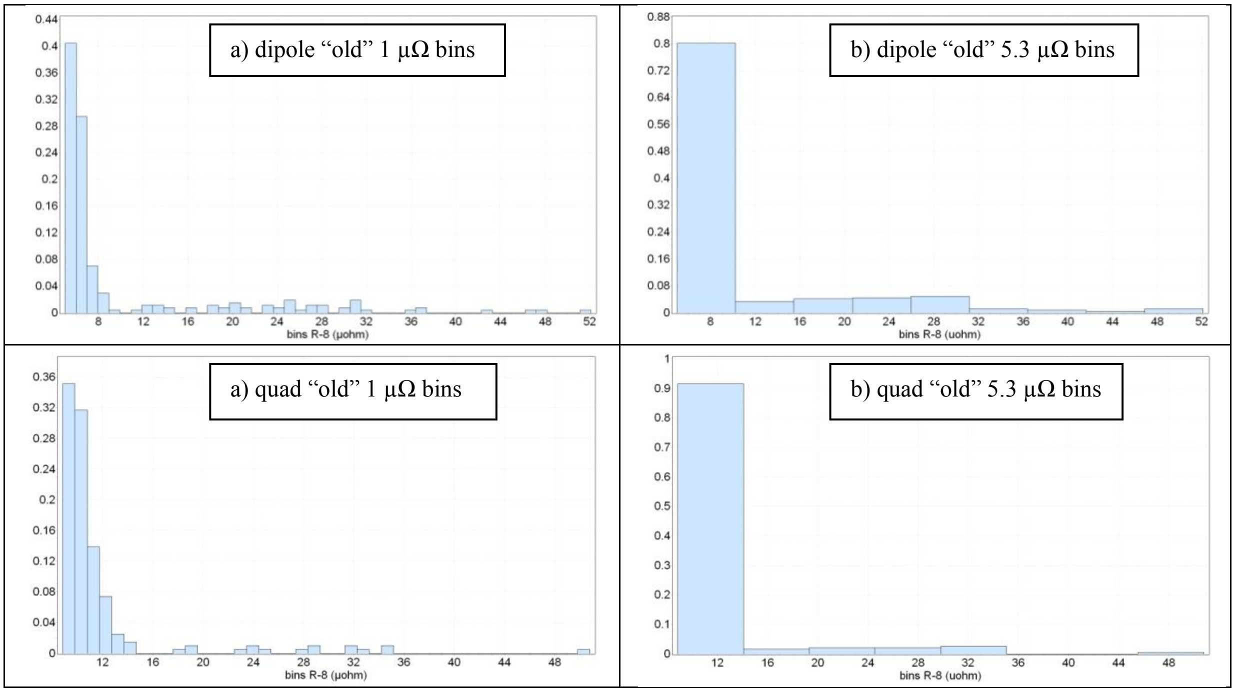

Figure 18: Comparison of R-8 histograms for “old” dipole and quadrupole splices with (a) manually selected bin size 1 μΩ (b) automatically selected bin size of 5.3 μΩ.

References

1 F. Bertinelli, L. Bottura, J.-M. Dalin, P. Fessia, R.H. Flora, S. Heck, H. Pfeffer, H. Prin, C. Scheuerlein, P. Thonet, J.-P. Tock, L. Williams, “*Production and Quality Assurance of Main Busbar Interconnection Splices during the LHC 2008-2009 Shutdown*”, IEEE Trans. Appl. Supercond., 22(3), (2011), 1786-1790

2 C. Scheuerlein, “*Quality control of the main interconnection splices before and after consolidation*”, presentation at the 2nd LHC splice review, 28.11.2011

3 M. Koratzinos, F. Bertinelli, R.H. Flora, H. Pfeffer, C. Scheuerlein, A. Siemko, J. Strait, “*Measurements of the Resistance of Main Circuit Busbar Segments at the LHC at Warm*”, CERN-ATS 2010-200, (2010)

4 http://www.megger.com/us/products/ProductDetails.php?ID=77&Description

5 http://www.mathwave.com/easyfit-distribution-fitting.html (06.08.2012)

6 I.M. Chakravarti, R.G. Laha, J. Roy, Handbook of Methods of Applied Statistics, Volume I, John Wiley and Sons, (1967), 392-394

7 J.Ph. Tock et al, “Consolidation of the LHC Superconducting Circuits: A Major Step towards 14 TeV Collisions”, CERN-ATS-2012-197