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ADDENDUM TO

A PHENOMENOLOGICAL PROFILE OF THE HIGGS BOSON

John Ellis, Mary K. Gaillard *) and D.V. Nanopoulos +)
CERN - Geneva

ABSTRACT

This addendum discusses further considerations on the Higgs boson mass, including a lower limit due to S. Weinberg. It also contains a calculation of Higgs production in neutrino collisions, and some remarks on the model dependence of Higgs phenomenology.

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*) And Laboratoire de Physique Théorique et Particules Elémentaires, associé au C.N.R.S., Orsay.

+) Address after 1 January 1976 : Ecole Normale Supérieure, Paris.

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Since writing our paper we have learnt of some more considerations [55)-57)] about the mass of the Higgs boson. Also, we have been encouraged [58)] to calculate its production in neutrino collisions. We also make here some further remarks about the model dependence of our previous results.

In two papers [55),56)], K. Sato and H. Sato have given astrophysical arguments against very light Higgs bosons. They argue that present understanding of the cosmic background radiation excludes $0.1 \text{ eV} < m_H < 100 \text{ eV}$ [55)], and that stellar evolution would be drastically affected if $m_H < 0.1 \times m_e$ [56)].

Most recently, S. Weinberg has derived [57)] an approximate lower bound on m_H from an analysis of Coleman and E. Weinberg [59)]. These authors pointed out that a simple Higgs potential

$$V_0(H) = \mu^2 H^2 + \lambda H^4 \quad (\mu^2 < 0, \lambda > 0) \tag{A.1}$$

acquires radiative corrections in perturbation theory. The one-loop graphs of Fig. 20 yield

$$V_1(H) = \mu^2 H^2 + B H^4 \ln\left(H^2/M^2\right) \tag{A.2}$$

where M is a mass parameter chosen to absorb all H^4 terms, and

$$B = \frac{1}{64\pi^2 v^4}\left[3\sum_{V=W,Z} M_V^4 - \sum_f M_f^4\right] \tag{A.3}$$

where $v^2 = 1/\sqrt{2}\, G_F$ as before [*)]. Then by requiring that the value $H = v$ be a global minimum of the potential (A.2), Weinberg [57)] showed that if $m_{W,Z} \gg m_f$

$$m_H^2 \geqslant \frac{3\sqrt{2}\, G_F}{16\pi^2}\left(2m_W^4 + m_Z^4\right) = \frac{3\alpha^2\left(2+\sec^4\theta_W\right)}{16\sqrt{2}\, G_F \sin^4\theta_W} \tag{A.4}$$

[*)] The potential is actually gauge dependent, the original calculations of Coleman and E. Weinberg [59)] being performed in the Landau gauge so that no ghost loops appear in Fig. 20. However, the conclusions of physical interest are gauge independent to all orders in perturbation theory [60)]. There is also a Higgs contribution to (A.3) which is negligible for the comparatively light Higgs bosons we are interested in.

which is $O(4.9)$ GeV if $\theta_W \sim 35^\circ$ as currently favoured by experiment. Higher order loops introduce $H^4 [\ln(H^2/M^2)]^n$ terms into the effective potential, but the limit (A.4) is not greatly affected thereby. Is there any other way of evading the lower limit (A.4) ?

One might entertain the idea of relaxing the condition that $V(v)$ be a global minimum, and just demand that it be a local minimum, with the global minimum at the origin $H = 0$ *). According to standard arguments 31), the minimum at $H = v$ would not then be stable, but the theory with no spontaneous breakdown would be probably so singular as to be undefined 61). The only way for the local minimum at $H = v$ to be relevant might be if, as suggested in Section 2.4 for other reasons, the world Lagrangian has another spontaneous breakdown which cannot occur independently of the one relevant to the weak and electromagnetic interactions.

The bound (A.4) is significantly altered if there is at least one fundamental fermion field with mass $m_f \approx m_W$. There are no theoretical arguments for or against such an object. Experiment has so far revealed only small fermion masses, but our sampling techniques are clearly biased in favour of low masses. It would be an act of bravado to suggest that the discovery of a Higgs boson with a mass appreciably below the limit (A.4) would be an argument for the existence of ultra-high mass fermions.

We now turn to Higgs boson production in neutrino collisions. Three relevant Feynman diagrams are shown in Fig. 21. The Higgs boson may be emitted either from the muon line (Fig. 21a), or the hadronic system (Fig. 21b) or from the virtual exchanged W boson **) (Fig. 21c). The first two graphs will give a cross-section rising linearly with E_ν like the total neutrino cross-section, and we expect them to contribute a cross-section ratio resembling that in purely hadronic processes :

$$\frac{\sigma(\nu + N \to \mu^- + H + X)\big|_{a+b}}{\sigma(\nu + N \to \mu^- + X)} \lesssim 10^{-7} \qquad (A.5)$$

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*) Note that in deriving the bound (A.4) it is not necessary that $\mu^2 < 0$. If $\mu^2 > 0$ could be excluded, then the bound on m_H would be improved by a factor $\sqrt{2}$.

**) This diagram has been mentioned as a possible source of Higgs bosons by Ross and Veltman 18).

On the other hand, the diagram of Fig. 21c can give a contribution rising as E_ν^2. We assume scaling for the deep inelastic structure functions as in the quark parton model with negligible antiquark distributions, we ignore m_H, and assume $m_H^2 \ll m_N E_\nu \ll m_W^2$. Introducing the ν, N, μ^-, W and H momenta as in Fig. 21c, and defining

$$\left.\begin{aligned} \nu &\equiv 2q\cdot p_N \quad , \nu' \equiv 2q'\cdot p_N \ , \quad x \equiv -q^2/\nu \\ y &\equiv \frac{\nu}{2mE_\nu} \quad , \quad y' \equiv \frac{\nu'}{2mE_\nu} \quad ; \quad x' \equiv -q'^2/\nu' \end{aligned}\right\} \tag{A.6}$$

then we have the standard result

$$\frac{d^2\sigma(\nu+N\to\mu^-+X)}{dxdy} = \frac{G_F^2 m_N E_\nu}{\pi} F_2(x) \tag{A.7}$$

for the total cross-section with no H production, and

$$\frac{d^4\sigma(\nu+N\to\mu^-+H+X)|_c}{dxdydx'dy'} = \frac{G_F^3 m_N^2 E_\nu^2}{\sqrt{2}\,\pi^3} F_2(x') \times$$

$$\times\left[1 - y + y' - \frac{x}{x'} - \frac{y'}{y} + \frac{2y'x}{yx'} + \frac{xy}{x'} - \frac{xy'}{x'}\right] \tag{A.8}$$

for the inclusive H production from Fig. 21c. The Higgs boson would emerge at an angle Θ_H to the neutrino beam, where $\Theta_H \sim \sqrt{1/E_\nu}$ is largest when $y \approx 0$ or $y \sim y'$. Integrating (A.8) with the kinematical restrictions

$$x' > x \quad , \quad y > y'$$

we find

$$\frac{\sigma(\nu+N\to\mu^-+H+X)|_c}{\sigma(\nu+N\to\mu^-+X)} \simeq \frac{1}{6\sqrt{2}\,\pi^2} G_F m_N E_\nu \langle\xi\rangle \tag{A.9}$$

where

$$\langle \xi \rangle \equiv \frac{\int_0^1 \xi F_2(\xi)\,d\xi}{\int_0^1 F_2(\xi)\,d\xi} \approx 0{\cdot}2 \quad \text{experimentally}$$

Putting numbers into (A.9), we get

$$\left.\frac{\sigma(\nu+N\to\mu^-+H+X)}{\sigma(\nu+N\to\mu^-+X)}\right|_c \approx 3\times10^{-8}\,\frac{E_\nu}{m_N} \tag{A.10}$$

in the range $m_H^2 \ll m_N E_\nu \ll m_W^2$. Thus the diagram of Fig. 21c dominates over the other diagrams (A.5) if $E_\nu \approx O(100\ \text{GeV})$, but the Higgs boson production rate is not enormous.

We should add some remarks about the sensitivity of some results of our paper to specifics of the simplest Weinberg-Salam model we discussed. Generally our production estimates might apply to any semi-weakly coupled particle of similar mass. Possible differences occur when we consider couplings to the electron, which is $\alpha\ m_e$ and hence very small in the simplest Weinberg-Salam model. If the He^+e^- coupling were characteristic of other masses and hence much larger, as in some gauge theories, then our estimates of $\Gamma(H\to e^+e^-)$ and $\Gamma(H\to\gamma\gamma)$ would be altered, decreasing the lifetimes and changing the decay branching ratios of low mass Higgs bosons. Also, direct production in e^+e^- collisions 49) might no longer be negligible. Another sensitive area is the assumption in the model that the Higgs quark couplings $H\bar{u}u$ and $H\bar{d}d$ are essentially equal. If they were unequal, giving the Higgs couplings a large $I_3 = 1$ component $\alpha\ u_3$ in the $(3,\bar{3})+(\bar{3},3)$ representation of $SU(3)\times SU(3)$, then the estimates in Section 3.3.1 on the decays $\eta\to\pi^0+H$, $\Sigma^0\to\Lambda+H$ could be modified, the branching ratios becoming much larger 61). We should also observe that in a model with physical charged Higgs bosons $H^\pm$ the production rates in related reactions should be similar to those for the neutral Higgs boson discussed here. On the other hand, the leptonic decay modes of $H^\pm$ into final states $e^\pm\nu$, $\mu^\pm\nu$ would be less distinctive than the e^+e^-, $\mu^+\mu^-$ decays of the neutral Higgs boson.

Finally, since our paper was published, the authors of Ref. 23) have studied the e^+e^- invariant mass distribution in $K^+\to\pi^+e^+e^-$. With a mass resolution of 4 MeV, they are able to set a limit 62) (at 90% confidence level) :

$$\frac{\Gamma(K^+ \to \pi^+ X^0_{\hookrightarrow e^+e^-})}{\Gamma(K^+ \to \mathrm{all})} < 0.4 \times 10^{-7}$$

for 140 MeV $< M_{X^0} <$ 340 MeV. This level is still compatible with the estimate of Section 5, and therefore, unfortunately, inconclusive *).

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*) Recall that the Higgs boson is not expected to have a significant branching ratio into e^+e^- when $m_H > 2m_\mu$.

REFERENCES

55) K. Sato and H. Sato - Progr.Theor.Phys. 54, 912 (1975).

56) K. Sato and H. Sato - Progr.Theor.Phys. 54, 1564 (1975).

57) S. Weinberg - Harvard Preprint "Mass of the Higgs Boson" (1975).

58) D. Cline and C. Rubbia - Private communications.

59) S. Coleman and E. Weinberg - Phys.Rev. D7, 1888 (1973).

60) S. Weinberg - Phys.Rev. D7, 2887 (1973) ;
R. Jackiw - Phys.Rev. D9, 1686 (1974) ;
L. Dolan and R. Jackiw - Phys.Rev. D9, 2904 (1974) ;
J.S. Kang - Phys.Rev. D10, 3455 (1974) ;
W. Fischler and R. Brout - Phys.Rev. D11, 905 (1975),

62) N.K. Nielsen - Nuclear Phys. B101, 173 (1975) ;
J. Iliopoulos and N. Papanicolaou - Ecole Normale Supérieure Preprint PTENS 75/12 (1975).

61) M. Veltman - Private communications.

62) A.M. Diamant-Berger and R. Turlay - Private communication.

FIGURE CAPTIONS

Figure 20 Diagrams which contribute 59) to the effective Higgs potential in the Landau gauge and in the one-loop approximation. (Diagrams with an internal Higgs loop also contribute, but are irrelevant 57) in bounding the Higgs mass.)

Figure 21 Diagrams for Higgs production in neutrino-proton scattering via emission by
a) the muon line,
b) the hadronic system, and
c) the virtual W boson.

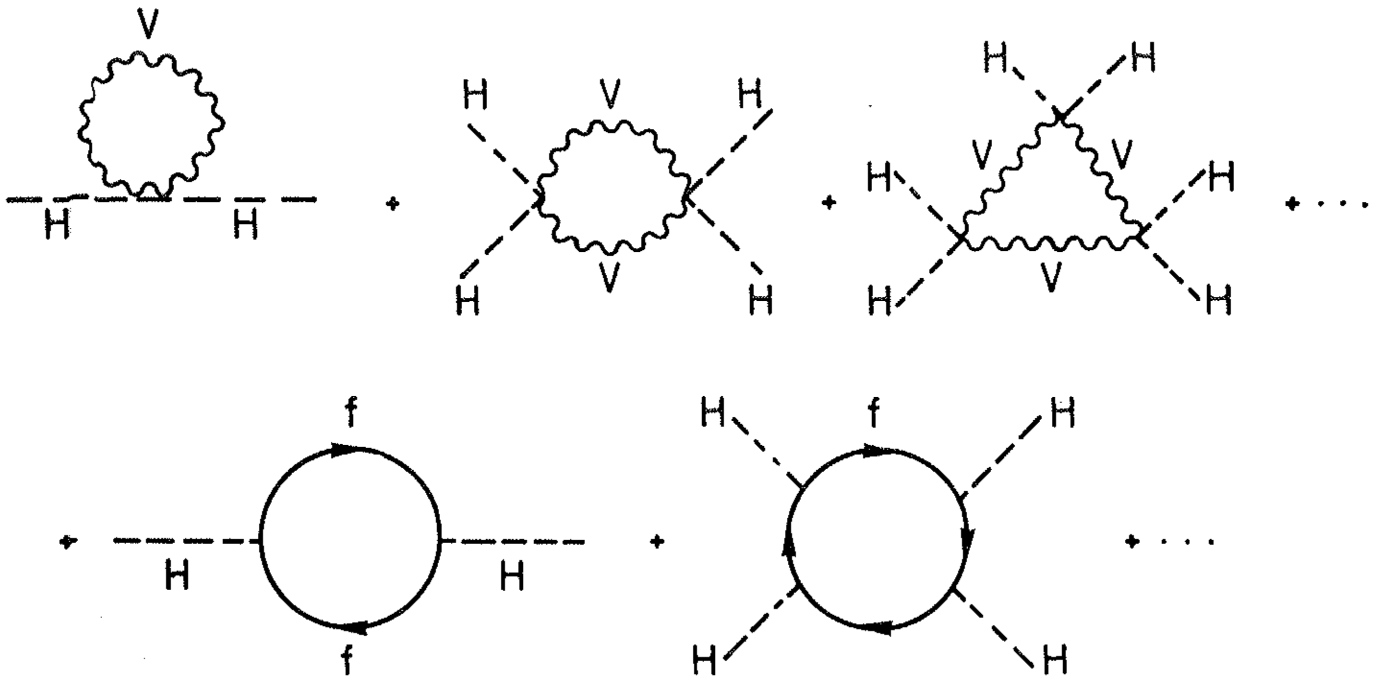
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FIG. 20

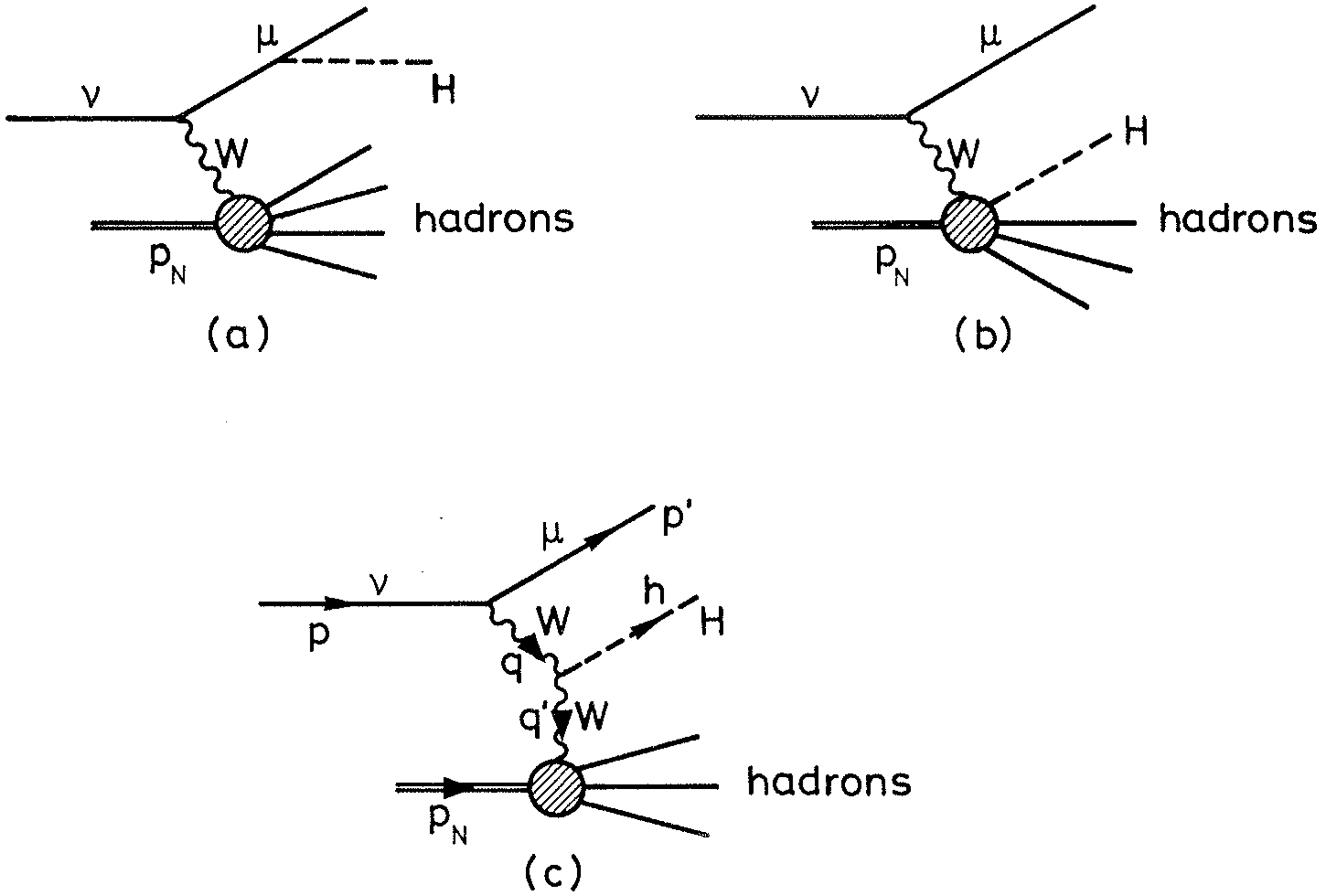
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FIG. 21