Bayesian Global Fit of CMSSM and EWSB

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The BayesFITS Group (A. Fowlie, A. Kalinowski, M. Kazana, S. Tsai,...)

Outline

- TH vs EXPT
- Bayesian analysis of the CMSSM
- likelihood
- (preliminary) results
- summary

A conjecture

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SUSY cannot be experimentally ruled out

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...or abandoned

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  inclusive, least model-dep.

- "razor"

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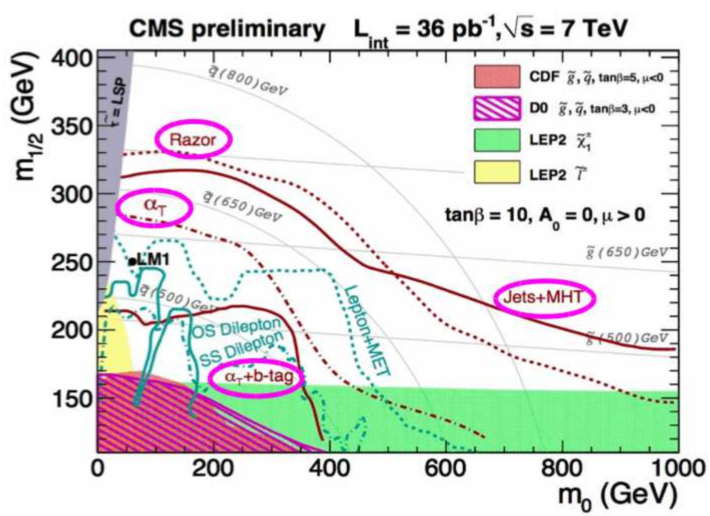

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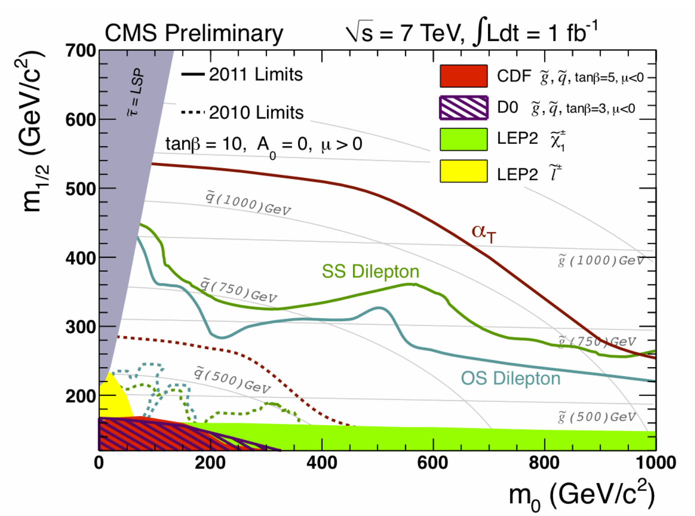

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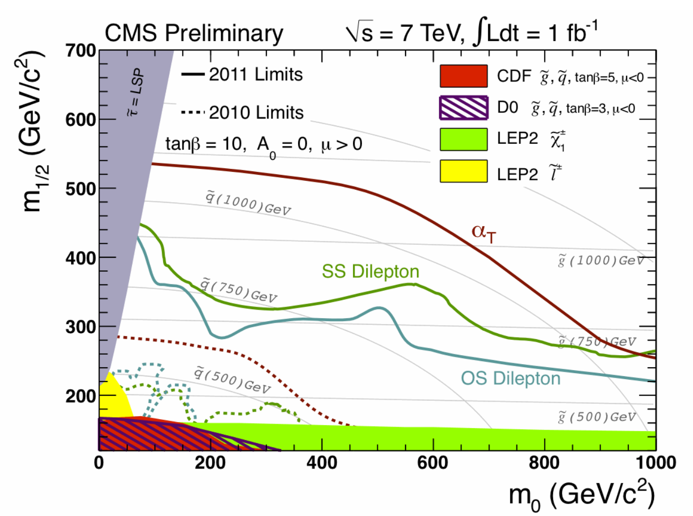
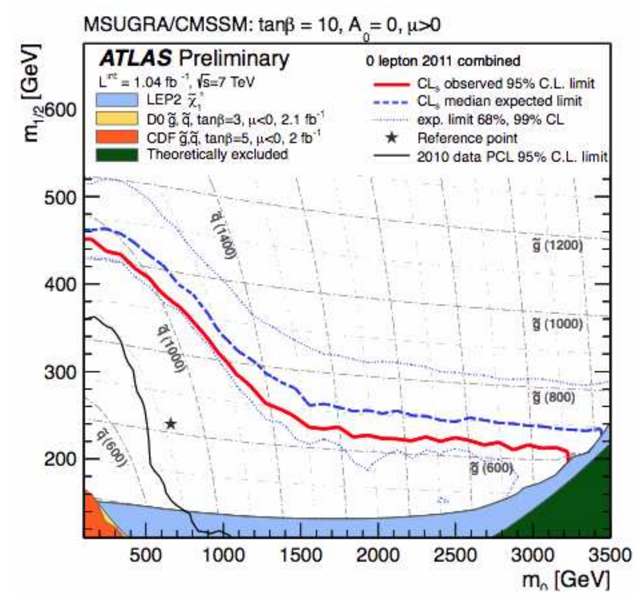


ATLAS:

- jets + missing transverse momentum
- ...



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- rigid step-function approach (e.g., 95%)
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Frequentist: “probability is the number of times the event occurs over the total number of trials, in the limit of an infinite series of equiprobable repetitions”

Bayesian: “probability is a measure of the degree of belief about a proposition”

Bayesian Analysis of the CMSSM

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- CMSSM parameters $\theta = m_{1/2}, m_0, A_0, \tan\beta$
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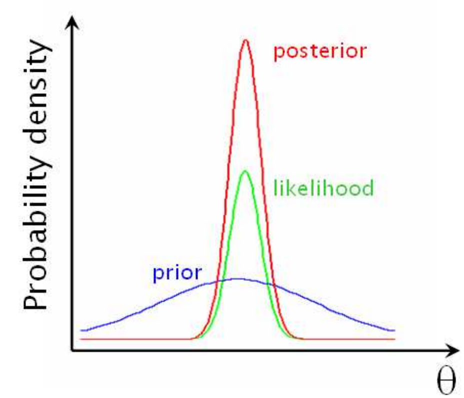

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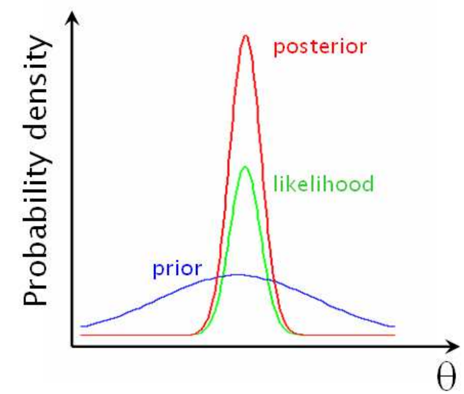
- Bayes' theorem: posterior pdf

$$p(\theta, \psi | d) = \frac{p(d|\xi)\pi(\theta,\psi)}{p(d)}$$

- $p(d|\xi) = \mathcal{L}$: likelihood
- $\pi(\theta, \psi)$: prior pdf
- $p(d)$: evidence (normalization factor)



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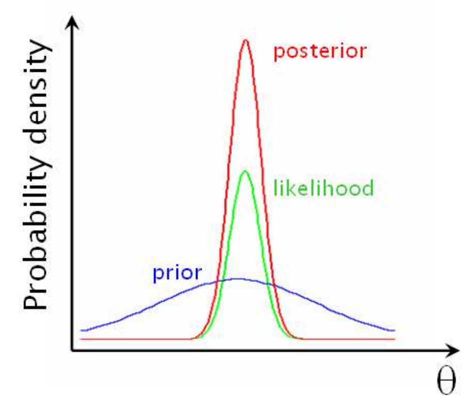

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- usually marginalize over SM (nuisance) parameters ψ $\Rightarrow$ $p(\theta|d)$

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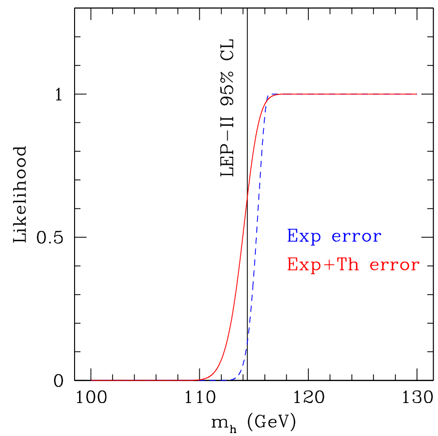
- for several uncorrelated observables (assumed Gaussian):

$$\mathcal{L} = \exp\left[-\sum_i \frac{\chi_i^2}{2}\right]$$

Limits: eg. light Higgs in the CMSSM

LEP: $m_h > 114.4\,\mathbf{GeV}$ (95% CL) - if SM-like

- include both experimental and theoretical error: $\sigma \to s = \sqrt{\sigma^2 + \tau^2}$



- apply similar way to LHC exclusion limits

compute expected nr of events, compare with data

compute cross sections, efficiency, apply cuts...

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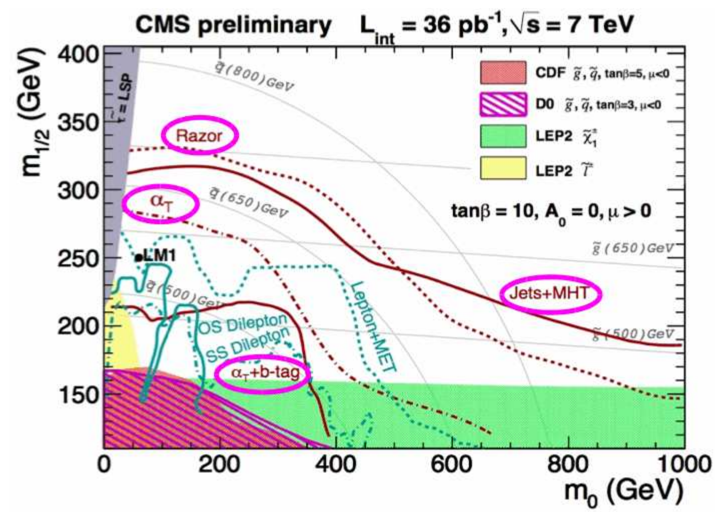
consider CMS limit from α_T at 1 fb^{-1}

analogous tests for α_T and razor at 35 pb^{-1} equally encouraging



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$$s = \epsilon \cdot \sigma \cdot \int L$$

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simulate detector efficiency with Pythia

σ with Herwig++

use SoftSusy, SUSY-HIT

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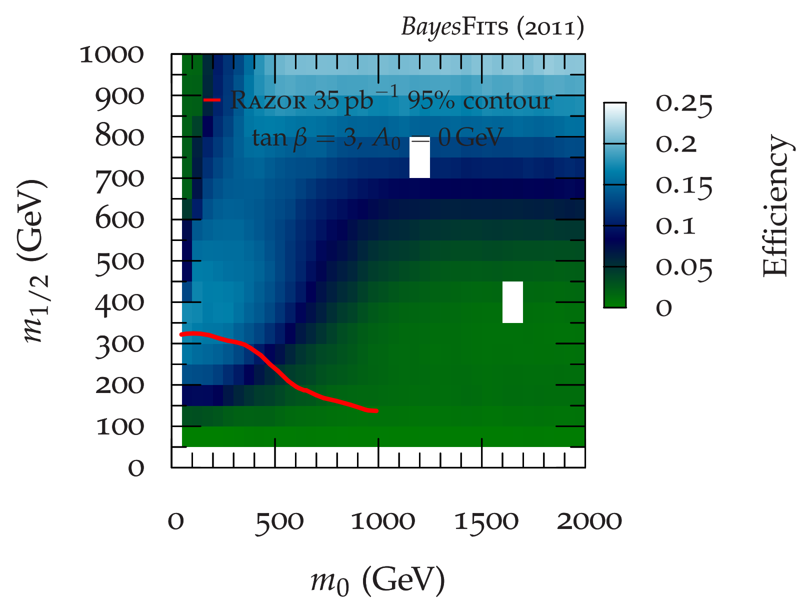

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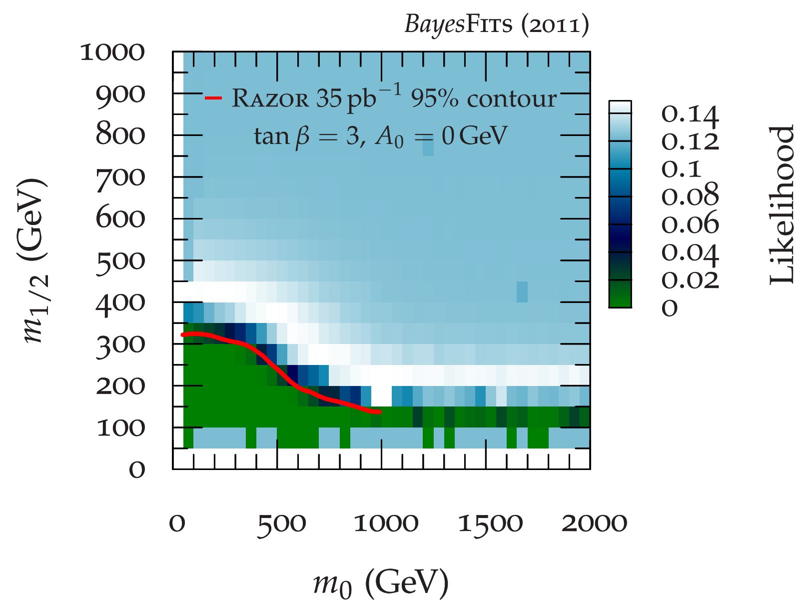

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$\Rightarrow$ very good agreement

encouraging!

Impact on CMSSM parameters

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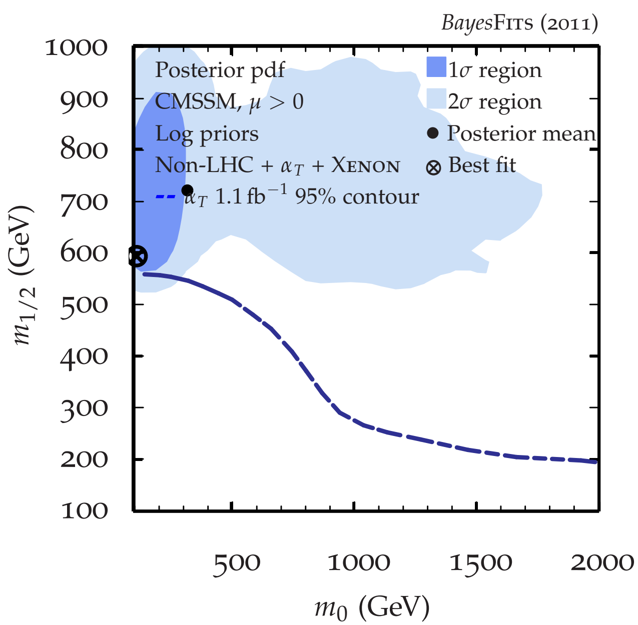
scans and stat analysis done with SuperBayeS

apply: "Non-LHC"+ α_T (1 fb^{-1}) + DM Xenon-100

"Non-LHC": $\Omega_\chi h^2$, $b \to s\gamma$, $\mathrm{BR}(\bar{B}_s \to \mu^+\mu^-)$, $(g-2)_\mu$, LEP,...

posterior pdf



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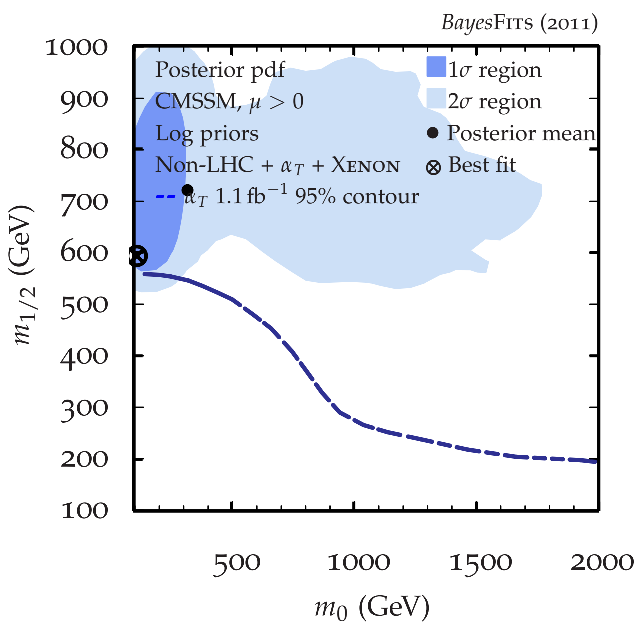
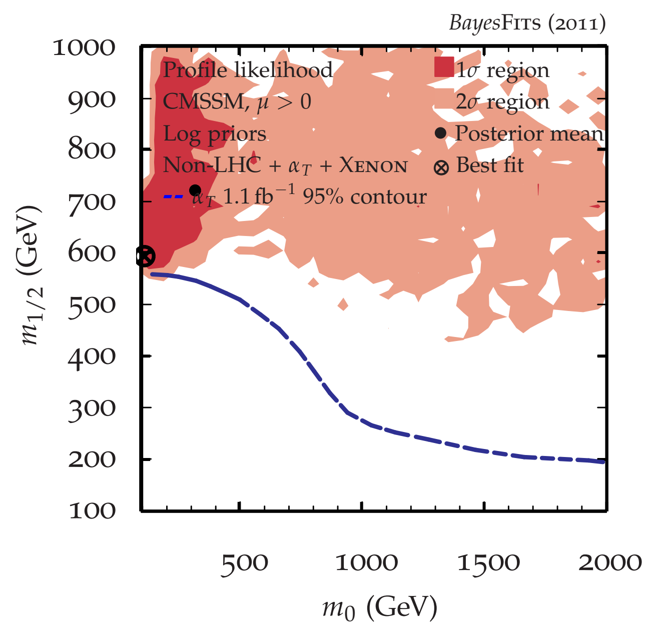


profile likelihood



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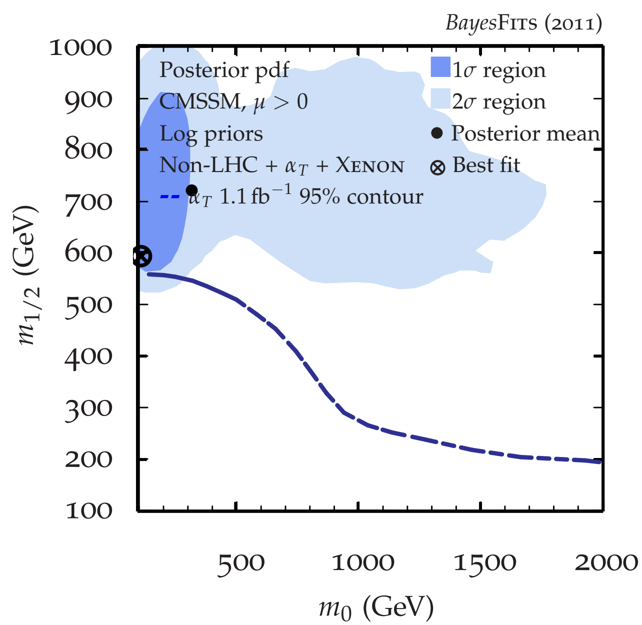

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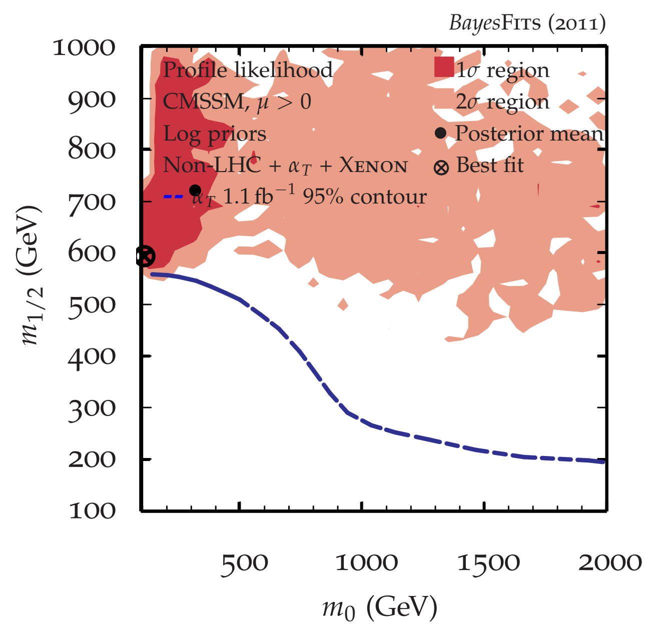

- large $m_{1/2}$ with $m_0 \lesssim m_{1/2}$ favored
- large m_0 and small $m_{1/2}$ (FP, HB,...) strongly disfavored by Xenon-100 limit

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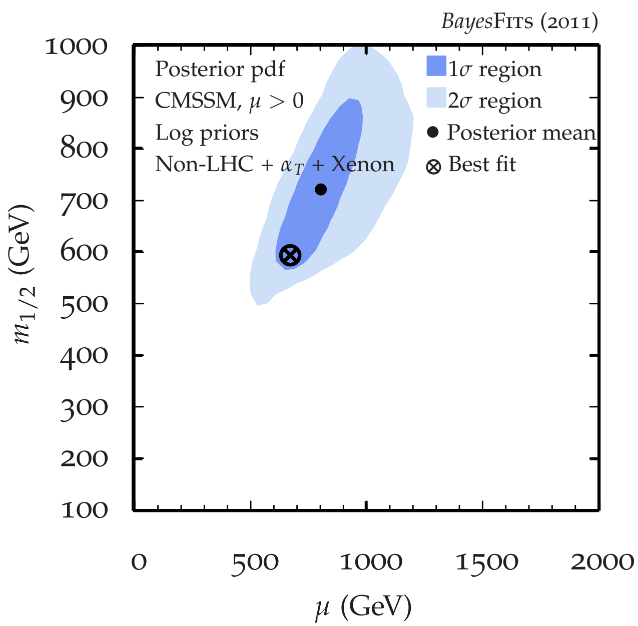

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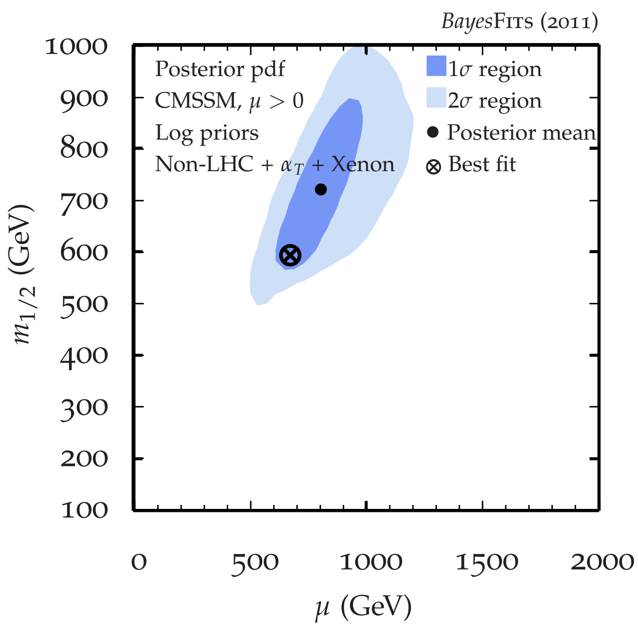

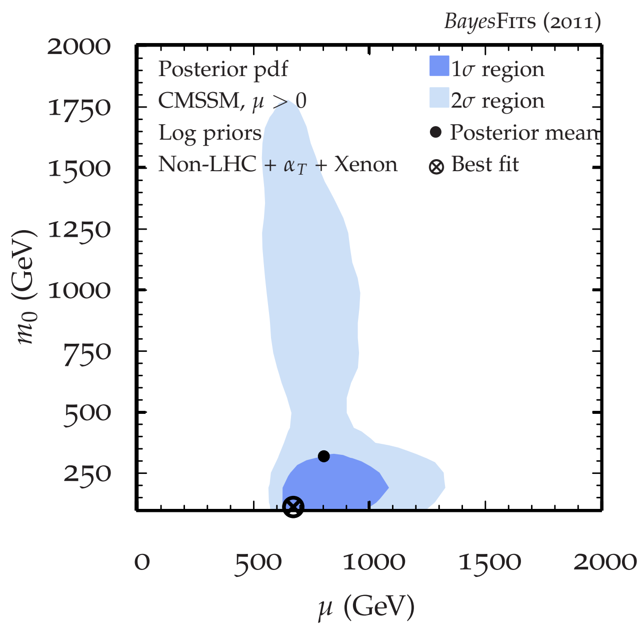

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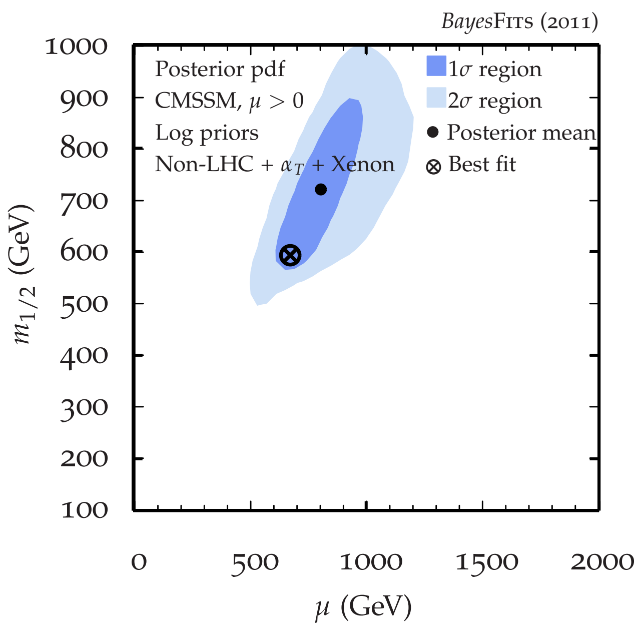

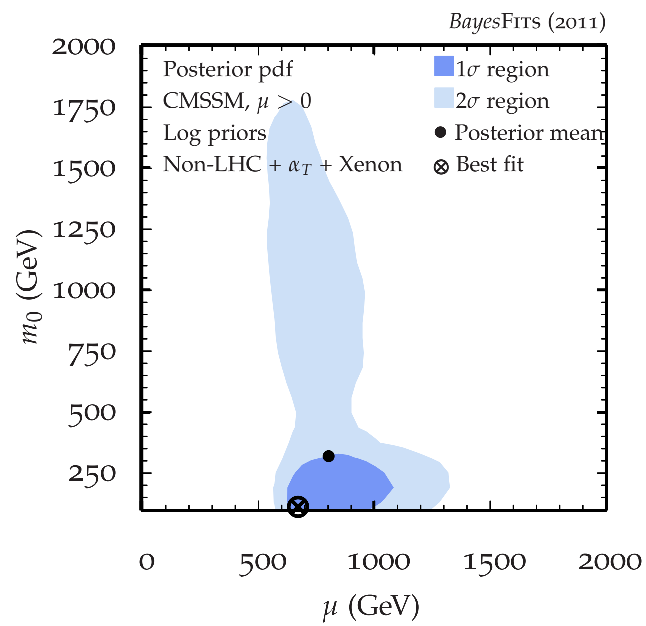

- pattern: $\mu \sim 1.25 m_{1/2}$
- similar pattern for profile likelihood

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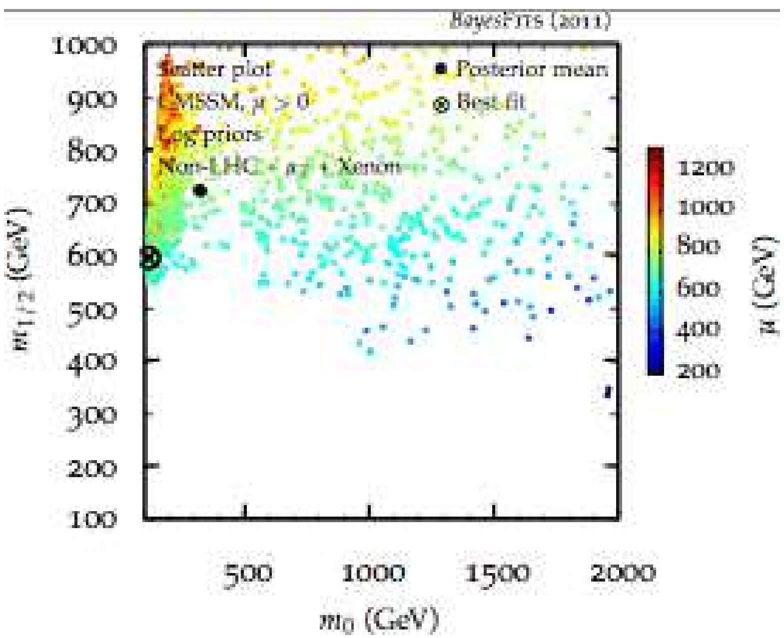
3D map of μ



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1D pdf for m_h

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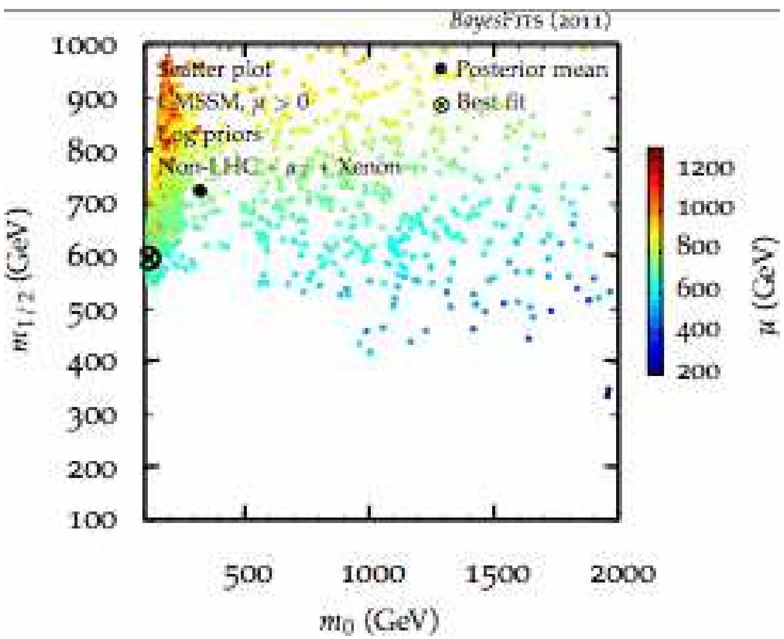

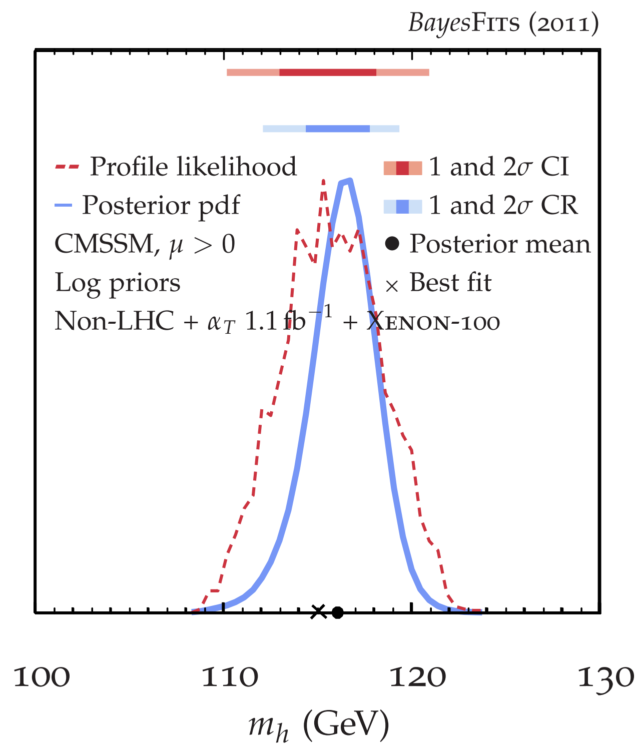

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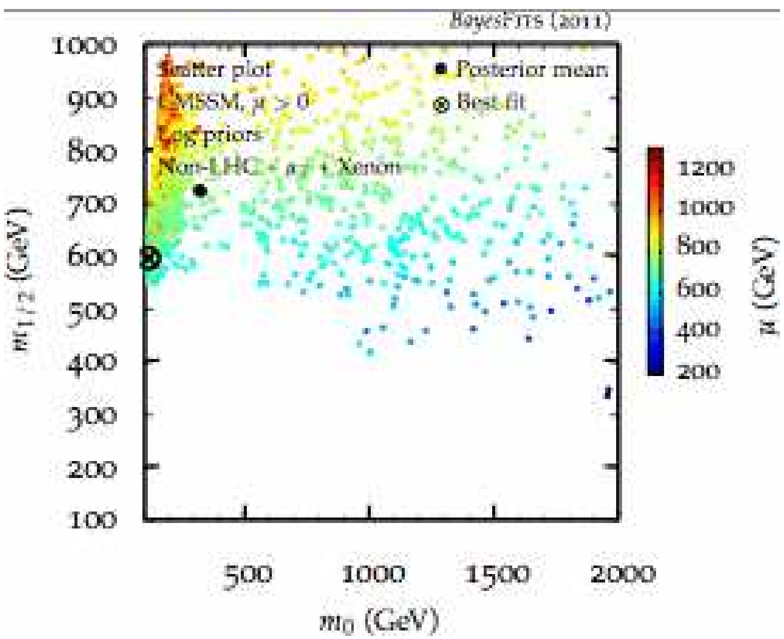

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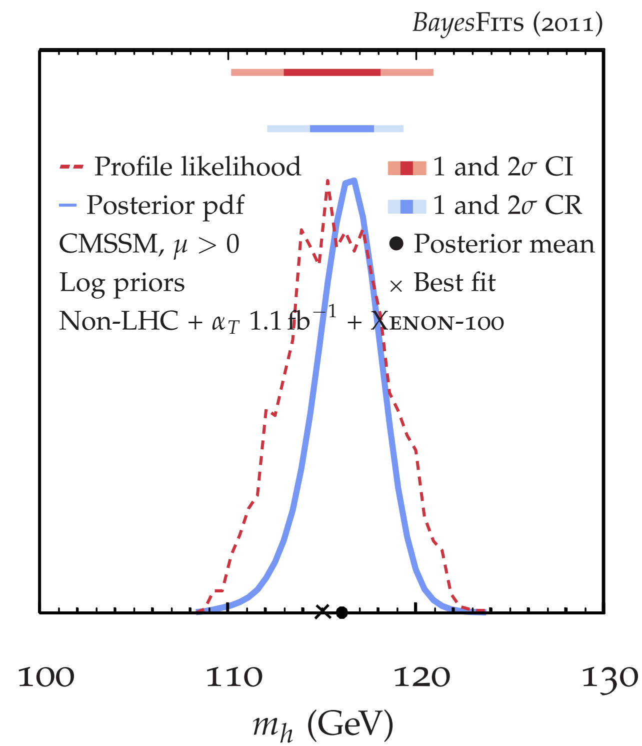

Summary

- EWSB: Bayesian global fits of the CMSSM favor

$$\mu \simeq 1.25 m_{1/2} \ \text{and} \ \text{small} \ m_0$$