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What is known about πK scattering lengths and the $K\pi$ atom lifetime?

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The DIRAC experiment aims to measure the (1s) lifetime or inverse decay width of the $K\pi$ atom. From this measurement, using the Deser-type formula $\Gamma \propto |\Delta a|^2$ (Δa: scattering length difference), data on basic πK scattering lengths can be extracted.

1 Introduction

The investigation of low-energy interaction between kaons and pions by means of $K\pi$ Coulomb bound states (atoms) enables an explicit probing of the 3-flavour structure of low-energy hadron scattering, an issue not covered in the $\pi\pi$ case. Measurements can be compared with predictions from $SU(3)_L \times SU(3)_R$ chiral perturbation theory (ChPT) and also with dispersion relation calculations based on Roy-Steiner equations.

In this note we present the status of the available theoretical and experimental information about low-energy πK scattering.

2 Atom lifetime and scattering lengths

For $\pi\pi$ atoms or pionium many authors [1–6] studied the relation between the atom lifetime or decay width and the corresponding scattering lengths. As a result of these investigations the following Deser-type formula for the decay rate $A_{2\pi}(\textit{ground state}) \rightarrow \pi^0\pi^0$ was derived by using low-energy QCD [7]:

$$\Gamma_{A_{2\pi}\rightarrow\pi^0\pi^0} = \frac{2}{9}\,\alpha^3\,p\,|a_0 - a_2|^2\,(1+\delta_\Gamma). \tag{1}$$

In Eq. (1) α is the fine structure constant, p the π^0 momentum in the pionium system, and a_0 and a_2 are the S-wave $\pi\pi$ scattering lengths in units of inverse charged pion mass m_π for isopin $I = 0$ and 2, respectively. The small term $\delta_\Gamma = (5.8 \pm 1.2)\cdot 10^{-2}$ [7] accounts for corrections of order α as well as for those due to the quark mass difference $m_u \neq m_d$.

In the framework of low-energy QCD, namely ChPT, the scattering length difference $|a_0 - a_2|$ has been calculated at the 2% level: $a_0 - a_2 = 0.265 \pm 0.004$ [8]. Inserting this value in (1) one gets in good approximation

$$\tau_{A_{2\pi}} \approx \Gamma^{-1}_{A_{2\pi}\rightarrow\pi^0\pi^0} = (2.9 \pm 0.1)fs. \tag{2}$$

In the case of the di-mesonic $K\pi$ atom [1)], the atom decays predominately by strong interaction into the neutral meson pair $K^0\pi^0$:

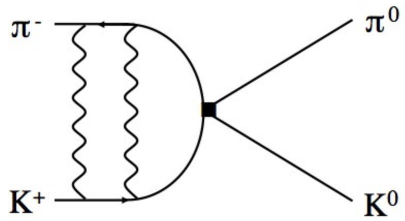

Figure 1: The dominant decay channel of the $K\pi$ atom ($A_{K\pi}$).

The decay width of the $K\pi$ atom in the ground state is given by the relation [3,9]:

$$\Gamma(A_{K\pi}) \simeq \Gamma(A_{K\pi} \to K^0\pi^0) = 8\ \alpha^3\ \mu_+^2\ p\ (a_0^-)^2\ (1+\delta_\Gamma). \tag{3}$$

In this formula, the S-wave isospin-odd πK scattering length $a_0^- = \frac{1}{3}(a_{1/2} - a_{3/2})$ (a_I for isospin I) is defined in pure QCD for (u,d) quark masses $m_u = m_d$. Further, α is the fine structure constant, μ_+ the reduced mass of the charged mesons π^+ and K^+ and p the outgoing K^0 or π^0 3-momentum in the $K\pi$ atom system. Finally, the term δ_Γ accounts for corrections, due to isospin breaking, at order α and quark mass difference $m_u - m_d$ [9]. Inserting $m_\pi a_0^- = 0.090 \pm 0.005$ [10] (dispersive analysis) and $\delta_\Gamma = 0.040 \pm 0.022$ [9] in Eq. (3) leads to a theoretical lifetime $\tau_{K\pi} = (3.7 \pm 0.4) \cdot 10^{-15}\ s$.

Hence, a measurement of $\tau_{K\pi} = \Gamma(A_{K\pi})^{-1}$, taking into account small corrections due to additional decay channels, provides a value for the scattering length a_0^- and so tests low-energy QCD including u, d as well as s quarks.

3 Predictions for scattering lengths

In the 1960s Weinberg started to investigate meson-meson and meson-nucleon scattering lengths in the mathematical framework of current algebra (CA). In his language he deduced for $\pi\pi$ scattering [11] $a_0 = \frac{7}{4}L$, $a_2 = -\frac{1}{2}L$ and so $a_0 - a_2 = \frac{9}{4}L$ with $L = \frac{m_\pi^2}{8\pi F_\pi^2} = 0.09078$. In the πK case he found for a_0^- the expression (tree level) [11]

$$m_\pi a_0^-(CA) = L(1 + \frac{m_\pi}{m_K})^{-1} = m_\pi \frac{\mu_+}{8\pi F_\pi^2} = 0.071. \tag{4}$$

Going one step further and including 1-loop ($1l$) effects (order p^4), $SU(3)$ ChPT provides – by treating the pion and the heavier strange kaon as "weakly interacting" Goldstone bosons – the following scattering length [12,13]:

$$m_\pi a_0^-(1l) = 0.0793 \pm 0.0006. \tag{5}$$

We notice that a_0^- at $O(p^4)$ is essentially parameter free and thus can be predicted to the high accuracy above.

[1)] The $K\pi$ atom $A_{K\pi}$ is a Coulomb bound πK state with a Bohr radius of 248 fm and a ground state Coulomb binding energy of 2.9 keV.

Further progress could be achieved by calculating a_0^- at the 2-loop ($2l$) level or at order $O(p^6)$ [14]:

$$m_\pi a_0^-(2l) = 0.089. \tag{6}$$

In accordance with the results (4), (5) and (6), the $SU(3)$ chiral expansion of a_0^- can be written in the following way:

$$m_\pi a_0^- = m_\pi a_0^-(CA)(1 + \delta^{(2)} + \delta^{(4)} + \ldots) = m_\pi \frac{\mu_+}{8\pi F_\pi^2}(1 + 0.11 + 0.14 + \ldots). \tag{7}$$

It should be emphasized that the 1-loop contribution $\delta^{(2)}$ adds 11% to the current algebra value, whereas - surprisingly - the 2-loop contribution $\delta^{(4)}$ with 14% is even larger! According to a $SU(2)$ low-energy theorem with the s quark considered as heavy partner [15] one would expect, higher order corrections to the scattering lengths to be quite small.

At this point we present the latest outcome of the dispersive πK scattering length analysis from Roy-Steiner equations [10]:

$$m_\pi a_0^-(dis) = 0.090 \pm 0.005. \tag{8}$$

Let us graphically summarize the theoretical investigations above in Fig. 2:

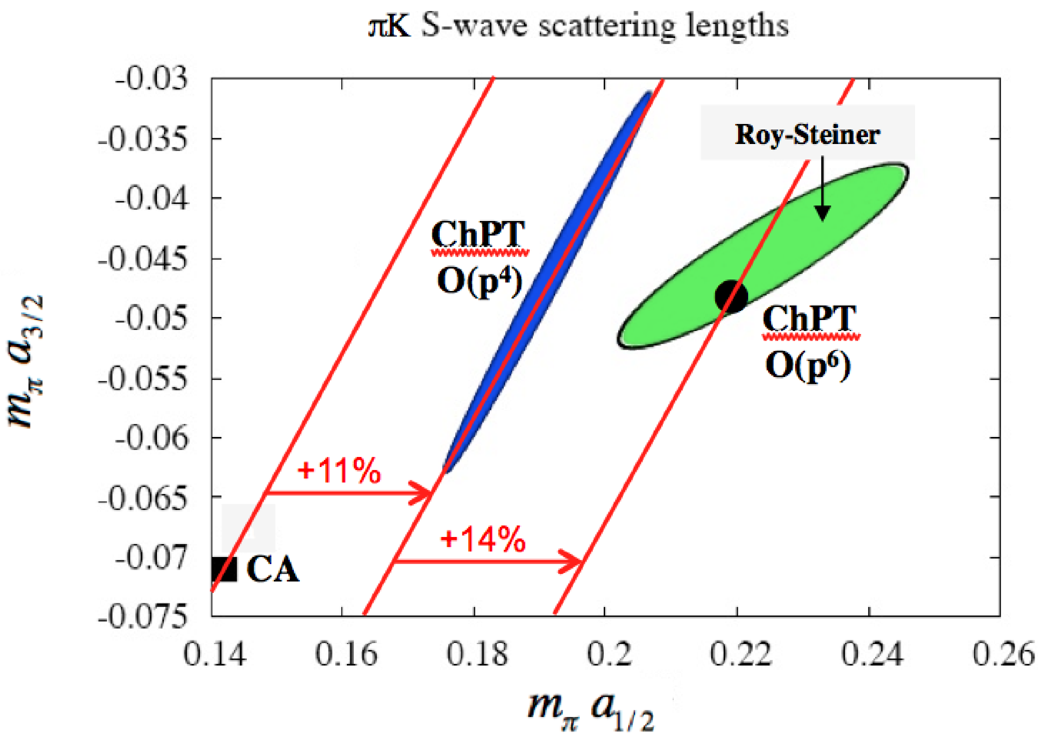

Figure 2: Predictions for S-wave πK scattering lengths $a_{1/2}$, $a_{3/2}$ and a_0^-. The three straight lines correspond to relation (7): $m_\pi(a_{1/2} - a_{3/2}) = m_\pi\ 3a_0^-\ (CA) \cdot \{1; 1.11; 1.25\}$, i.e. tree level, 1-loop and 2-loop calculations for a_0^-. The current algebra result CA, the standard error ellipse from ChPT at $O(p^4)$ and the result from ChPT at $O(p^6)$ are shown together with the standard error ellipse from solving the Roy-Steiner equations.

We notice that up to now experimental information about low-energy πK scattering parameters was only available indirectly, since this region of interest could not be investigated by experiments directly: πK phases, measured decades ago, were extrapolated by means of dispersion relations [16]. As already presented in Eq. (8), the same technique yielded quite precise results for πK scattering lengths [10].

4 Conclusion about low-energy πK interaction

The phenomenological dispersive analysis [10] based on the Roy-Steiner equations differs quite remarkably from the 1-loop prediction for a_0^-. Furthermore, an estimate at the 2-loop level leads to a large correction. Is there a convergence problem in ChPT for πK scattering?

To shed more light on this apparent discrepancy a first experimental study of low-energy πK interaction is needed. This can be done by investigating $K\pi$ atoms [4].

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