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CRITICAL AND SUPER-CRITICAL REGGEON FIELD THEORY *)

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A B S T R A C T

A path-integral formalism for the Reggeon field theory valid in all transverse dimensions is developed. In this formalism the strong coupling phase transition can be discussed through the introduction of an external source. It is then argued that the strong coupling transition occurs even in zero transverse dimensions but involves all higher-order couplings. The expanding disc and the analogous tunnelling effect in zero transverse dimensions are seen to be associated with a different weak coupling critical point. The conclusion is that the strong coupling critical Pomeron is the only known theoretically consistent (s and t channel unitary) description of rising total cross-sections.

The super-critical phase is shown to involve Reggeizing vector particles with associated singular Pomeron interactions and it is suggested that it be identified with the high-energy behaviour of a spontaneously broken non-Abelian gauge theory. Since the vector particles become massless and decouple from the Pomeron at the critical point it is conjectured that the critical Pomeron and hence rising total cross-sections are a direct consequence of local gauge invariance and the confinement of gluons.

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Reggeon Field Theory[1-3] (RFT) is best described both as summing Reggeon graphs derived from some underlying (field?) theory *and* as based on analyticity and unitarity, that is on multiparticle dispersion relations[4] leading to Reggeon unitarity[4,5]. Since unitarity in the physical world of four space-time dimensions is a very difficult property to achieve in any context, I am quite sure it should not be lightly neglected at high energy. In fact I shall argue that in order to obtain a complete understanding of the strong-coupling phase transition and consequent super-critical phase in RFT, unitarity is absolutely essential. In practice, this means that in looking for a formal sum of the Pomeron diagrams

 (1)

in which the bare Pomeron intercept α_0 is greater than the critical value α_{0c} we must be very careful not to violate t-channel unitarity in the form of Reggeon unitarity.

It is well known that when $\alpha_0 < \alpha_{0c}$ the total cross-section falls with energy (by definition) and that when $\alpha_0 = \alpha_{0c}$ we obtain the beautiful critical Pomeron[6,7]. The critical Pomeron not only produces factorizing rising total cross-sections but also gives detailed, *calculable* (with no parameters) scaling laws for all diffractive processes, i.e.

$$\sigma_T(s) \underset{s\to\infty}{\sim} [\ln s]^{\eta}\ , \quad \frac{d\sigma}{dt} \underset{s\to\infty}{\sim} [\ln s]^{2\eta} f(t[\ln s]^{\nu})\ , \quad \text{etc.} \tag{2}$$

I shall argue below that when $\alpha_0 > \alpha_{0c}$ unitarity again requires

$$\sigma_T(s) \underset{s\to\infty}{\to} 0 \tag{3}$$

which has the very important consequence that the *critical Pomeron is the only known theoretically consistent description* of *rising total cross-sections* (in the absence of massless vector particles). If this is the case then rising cross-sections are a very special phenomenon and their experimental observation is clearly telling us something. The question is therefore what property of the underlying strong interaction theory places the Pomeron at the critical point?

The super-critical RFT phase can be distinguished from the sub-critical phase by the following properties. The theory has a perturbation expansion in which

a) there is a bare Pomeron pole with intercept $\tilde{\Delta}_0$ $(= 1 - \tilde{\alpha}_0) > 0$ (this is of course, no different from the sub-critical phase).

b) There are singular (as functions of momentum transfer) Pomeron interactions proportional to $\tilde{\Delta}_0$. The singularities include those that would be produced if the Pomeron coupled through a fixed-pole associated with a two-vector state (and also through singularities associated with many-particle states whose mass goes to zero with $\tilde{\Delta}_0$).

c) There is a degenerate set of odd signature Reggeons producing vector particles with their mass and pair-wise coupling to the Pomeron also proportional to $\tilde{\Delta}_0$.

Since Reggeizing vector particles are directly associated with spontaneously broken gauge theories[8-11] we are immediately tempted to identify the RFT super-critical phase with the high-energy behaviour of such a theory. Clearly the above structure contains many (and perhaps all) of the essential features that we would expect to see if this identification can be made with the Pomeron generated as a bound state of two Reggeized vector particles.

The critical limit appears as a limit in which the vector particles ("gluons") *simultaneously* become *massless* and *decouple* from the Pomeron. Equating the decoupling with confinement we are led to the conjecture that the *critical Pomeron* and (hence) *rising total cross-sections are a direct consequence* of the *simultaneous occurrence* of *local gauge invariance and the confinement of gluons*.

This conjecture can be given further support by starting directly from the high-energy behaviour of a spontaneously broken theory[9,10,12,13]. A preliminary study indicates (although we will not elaborate this point here) that the RFT derived from such a theory[14] has a structure which coincides with that of our super-critical theory if the large transverse momentum of the Reggeized gluons is cut-off and the small momentum behaviour of the gluon potential generating the Pomeron is also cut off. If these cut-off's can be removed after the massless limit is taken, due to the combination of asymptotic freedom and confinement, then it seems that our conjecture will be verified.

Before discussing these exciting possibilities further I must unfortunately point out that at present my view of the RFT phase transition is far from being universally accepted by all workers in the field. The alternative point of view[15-17] that

$$\sigma_T \underset{s\to\infty}{\sim} [\ln s]^2 \qquad\qquad \alpha_0 > \alpha_{0c} \tag{4}$$

implies that the experimental observation of rising cross-sections is of little fundamental significance. However, I believe that the behaviour (4) is not consistent with unitarity and moreover, is not a solution of RFT in the neighbourhood of the strong coupling critical point. I will therefore try to explain my results on the RFT phase transition and why I think my point of view is singled out as correct by unitarity before enlarging further on the above conjecture.

It will be helpful to develop a general understanding of the problem by considering first a simple world where there is no impact parameter dimensions ($D = 0$) and all amplitudes depend on rapidity only.

$[\mathrm{RFT}]_0$

In this case the bare Pomeron states are the harmonic oscillator Fock space

$$| \rangle = \sum b_n \frac{(a^+)^n}{\sqrt{n!}} |0\rangle \quad , \quad [a, a^+] = 1 \quad , \quad a|0\rangle = 0 \tag{5}$$

We consider first the theory with just a triple Pomeron coupling (r_0). The RFT perturbation expansion of the one Pomeron Green's function is formally obtained from

$$\langle 0 | a\, e^{-Hy} a^+ | 0 \rangle \qquad y = \text{rapidity} \tag{6}$$

with

$$H(a^+, a) = \Delta_0 a^+ a + i r_0 a^+ (a^+ + a) a \qquad \Delta_0 = 1 - \alpha_0 \tag{7}$$

Now unfortunately H is a somewhat unusual Hamiltonian. Firstly

$$H \neq H^+ \Rightarrow H \text{ is } \underline{\text{non-hermitian}} \tag{8}$$

and even worse

$$H H^+ \neq H^+ H \Rightarrow H \text{ is } \underline{\text{non-normal}} \tag{9}$$

Consequently H cannot be diagonalized and it is not clear *a priori* that e^{-Hy} can be given any rigorous definition. In fact we shall see that the fact that H is *not* a nice hermitian operator defining to everyone's satisfaction an obvious space of eigenstates on which e^{-Hy} should be defined, is the source of all the arguments and confusion surrounding this problem.

I shall define e^{-Hy} directly in the Fock space by the following (non-rigorous) path-integral formalism[18]. A path-integral formalism is particularly suitable (and probabaly even essential) for discussing the strong-coupling phase-transition when $D \neq 0$ because of the applicability of the Wilson renormalization group transformation. We write

$$\sum_n |n\rangle\langle n| = \sum_n (a^+)^n \frac{|0\rangle\langle 0|\, a^n}{n!} \tag{10}$$

$$= \int \frac{dz\,dz^*}{2\pi i}\, e^{-|z|^2} |z\rangle\langle z| \,, \qquad |z\rangle = e^{za^+}|0\rangle \tag{11}$$

and *define*

$$\langle \bar{z}|\, e^{-Hy} |z\rangle = \lim_{N\to\infty} \langle \bar{z}| \prod^{N} e^{-\frac{Hy}{N}} |z\rangle \tag{12}$$

$$\langle \bar{z}|\, e^{-\frac{Hy}{N}} |z\rangle \simeq e^{\bar{z}z}\left[1 - \tfrac{y}{N} H(\bar{z},z)\right] \tag{13}$$

$$= e^{\bar{z}z - y/N\, H(\bar{z},z)} \left(1 + O\left(\tfrac{1}{N}\right)\right) \tag{14}$$

which gives

$$U_N(\bar{z},z_0,y) \simeq \int \prod_{k=1}^{N-1} \frac{dz_k dz_k^*}{2\pi i} \exp\left[\bar{z}z_{N-1} - z_{N-1}^* z_{N-1} + z_{N-1}^* z_{N-2} \cdots \right.$$
$$\left. + \cdots - z_1^* z_1 + z_1^* z - \varepsilon\left[H(\bar{z},z_{N-1}) + \cdots + H(z_1^*,z)\right]\right] \tag{15}$$

where $y = N\varepsilon$. We can then write formally

$$U(\bar{z},z,y) = \lim_{N\to\infty} U_N(\bar{z},z,y)$$
$$= \int d\bar{\psi}\, d\psi \exp\left[\tfrac{1}{2}(\bar{z}\,\psi(y) + z\,\psi(0)) - \int_0^Y dy \left(\tfrac{1}{2}\bar{\psi}\overleftrightarrow{\frac{\partial}{\partial y}}\psi \right.\right.$$
$$\left.\left. + \Delta_0\bar{\psi}\psi + i r_0 \bar{\psi}(\bar{\psi}+\psi)\psi\right)\right] \tag{16}$$

This formalism has the difficulty that exp $- \varepsilon[H(z_i^*,z_{i-1})]$ is not in general bounded in (15) so that *a priori* even U_N may not exist. (This problem is not simply due to the non-hermiticity of H). Of course (15) can always be evaluated using (13) rather than (14). The real problem is to prove the existence of the limit $N \to \infty$ in a form which relates directly to the form (16) on which we make our manipulations. Probably the most direct way to do this would be to introduce a cut-off $|z_i| < N/y = \varepsilon$, $\forall_i$, although this has not been pursued in detail. If the integral is dominated by the region $z_i \sim z_{i+1}$ in the limit $N \to \infty$, as is expected (and is necessary if the Gaussian integration derivation[18] of the RFT perturbation expansion from (16) is to be justified) then the Cardy-Sugar contour deformation[19] can be used to ensure that the convergence of (16) due to the r_0 term holds for all values of Δ_0. This is what we shall assume from now on.

The difficulties with (16) that we shall worry about will be of a different nature. We wish to discuss the possibility of making changes of variables, that are analogous to vacuum shifts, in (16). However, since the Pomeron fields $\bar{\psi}$ and ψ are, respectively, pure creation and destruction operators we cannot use a formalism analogous to that used in relativistic field theory in which a change of vacuum is associated with the simultaneous creation of particles and anti-particles. There are no "anti-Pomerons" in RFT. We have Pomeron creation and destruction with the Pomeron number not conserved but there is no vacuum production or absorption. This situation is unique to RFT and will require a unique formalism.

First we note that we can rewrite U in the form

$$U(\bar{g},g,y_2-y_1) = \langle 0| e^{-H(Y_2-y_2)} e^{i\bar{g}a} e^{-H(y_2-y_1)} e^{iga^+} e^{-H(y_1-Y_1)} |0\rangle \tag{17}$$

and that provided U > 0 when $y \to \infty$ we can generalize this to

$$U(\bar{g},g,y_2-y_1) = \lim_{\substack{Y_2\to+\infty \\ Y_1\to-\infty}} \langle \bar{z}| e^{-H(Y_2-y_2)} e^{i\bar{g}a} e^{-H(y_2-y_1)} e^{iga^+} e^{-H(y_1-Y_1)} |z\rangle \tag{18}$$

which formally can be written as the path integral

$$\int d\bar{\psi}d\psi \exp\left[-\int_{-\infty}^{\infty}dy\, \bar{\psi}\overleftrightarrow{\frac{\partial}{\partial y}}\psi + \Delta_0\bar{\psi}\psi + ir_0\bar{\psi}(\bar{\psi}+\psi)\psi + ig\bar{\psi}\,\delta(y-y_1) + i\bar{g}\psi\,\delta(y-y_2)\right] \tag{19}$$

defined on the whole rapidity axis. The problem now is to interpret the freedom to vary z and $\bar{z}$ in (18) in terms of the freedom to change the boundary conditions and hence to make field shifts in (19). Note that in defining U by (18) we have implied that ψ is diagonalized at $y = Y_1$ $[\psi(Y_1) = z]$ and $\bar{\psi}$ is diagonalized at $y = Y_2$ $[\psi(Y_2) = \bar{z}]$. Therefore we can expect a shift in ψ to be well defined in the neighbourhood of $y = Y_1$ but not near $y = Y_2$ and vice versa for $\bar{\psi}$. That is a shift $(\delta\psi,\delta\bar{\psi})$ will be well defined in the limit (18) if

$$\delta\psi(y) = 0, \quad y = +\infty \quad , \quad \delta\bar{\psi}(y) = 0, \quad y = -\infty \tag{20}$$

An alternative way to describe this point is to note that if the operators $\bar{\psi}$ and ψ are to retain their creation, destruction property after the shift then $\bar{\psi}$ should be shifted by a field containing positive energies only and vice versa for ψ. That is we write

$$\delta\bar{\psi} = \frac{1}{2\pi i}\int_{-i\infty}^{+i\infty} dE\, e^{-Ey}\,\bar{f}(E) \quad ; \quad \delta\psi = \frac{1}{2\pi i}\int_{-i\infty}^{+i\infty} dE\, e^{-Ey} f(E) \tag{21}$$

or

$$\delta\bar{\psi} = \frac{1}{\pi} \int_0^{\infty} dE\, e^{-Ey} \text{disc}\, \bar{f}(E),\ y \to \infty\ ;\ \delta\psi = \frac{-1}{\pi} \int_0^{-\infty} dE\, e^{-Ey} \text{disc}\, f(E),\ y \to -\infty \tag{22}$$

where $\bar{f}$ has no singularities in the right half-plane and f has no singularities in the left half-plane.

We shall consider the perturbation expansion in the E-plane only and shall write all the RFT graphs as integrals over the real E-axis by closing contours using Cauchy's theorem. In this case the form (22) can be used for $\delta\bar{\psi}$ and $\delta\psi$ and translation invariance in rapidity can be restored by writing

$$\delta\bar{\psi} \to \delta\bar{\psi}(y-\bar{y}_0)\ ,\quad \delta\psi \to \delta\psi(y-y_0) \tag{23}$$

and averaging over pure imaginary y_0 and $\bar{y}_0$. [In fact the most elegant way to set up this formalism is to write the RFT path-integral in E-space from the start, with the integration over functions of the form (22). This is also necessary to discuss properly the Wilson renormalization group transformation which we shall come to shortly. However, we shall not discuss this here but in a forthcoming publication.] We write therefore

$$U(\bar{g}, g, y_2 - y_1) = \int d\bar{\psi}\, d\psi\, \exp\left[-\int_{-\infty}^{\infty} dy\, \mathcal{L}_{\bar{g}g}(\bar{\psi}, \psi)\right] \tag{24}$$

$$= \frac{\int_{-i\infty}^{+i\infty} dy_0\, d\bar{y}_0 \int d\bar{\psi}\, d\psi\, \exp\left[-\int_{-\infty}^{\infty} dy\, \mathcal{L}_{\bar{g}g}(\bar{\psi}+\delta\bar{\psi}, \psi+\delta\psi)\right]}{\int_{-i\infty}^{i\infty} dy_0\, d\bar{y}_0 \int d\bar{\psi}\, d\psi\, \exp\left[-\int_{-\infty}^{\infty} dy\, \mathcal{L}(\bar{\psi}+\delta\bar{\psi}, \psi+\delta\psi)\right]} \tag{25}$$

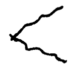
where $\mathcal{L}_{\bar{g}g}$ is defined by comparing (19) with (24).

We claim now that the shift formalism implied by (22), (23) and (25) is what we require. It enables us to make rapidity-dependent shifts independently in $\bar{\psi}$ and ψ which preserve the creation and destruction operator character of these operators. This formalism replaces the constant field shift formalism used in a relativistic (or any Lagrangian) formalism to discuss a change of vacuum.

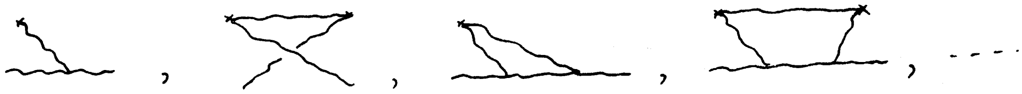
It can be checked that (24) and (25) are indeed equal in perturbation theory by considering a simple shift of the form

$$\delta\psi(y) = z\,\Theta(-y)\ ,\qquad \delta\bar{\psi} = \bar{z}\,\Theta(y) \tag{26}$$

This shift introduces new vertices (with obvious y-dependence) into the usual RFT graphs [note that the averaging over y_0 and $\bar{y}_0$ is well-defined in the Fourier transform of (26)]. The vertices are

 $\sim \Delta_0 z + i r_0 \frac{z^2}{2}$,  $\sim i r_0 z$,  $\sim \Delta_0 \bar{z} + i r_0 \frac{\bar{z}^2}{2}$, etc. (27)

which generate new Feynman diagrams for the Pomeron propagator, for example

 (28)

only some of which are non-zero after the y_0 and $\bar{y}_0$ integrations. However, the non-zero diagrams cancel among themselves order by order in $\bar{z}$ and z and so the original perturbation expansion is recovered.

Having defined the freedom of "choice of the vacuum" in (19) we can now discuss the addition of an external source term to H (analogous to the addition of a linear source term to $\lambda\phi^4$ -- a device well-known to be useful for the study of the phase-transition in this theory). Again the creation and destruction operator character of $\bar{\psi}$ and ψ is preserved only if we add a source of the form (22). For simplicity we choose the form (26) and consider

$$U(\bar{y}, y, y_2 - y_1, s) = \int d\bar{\varphi}\, d\varphi \, \exp\Big[-\int_{-\infty}^{\infty} dy \, \mathcal{L}_{\bar{y}y}(\bar{\varphi}, \varphi) + is\,\theta(-y)\,\bar{\varphi} + is\,\theta(y)\,\varphi\Big] \tag{29}$$

We then make a shift of the form (26) to cancel the source term in (29), that is we choose

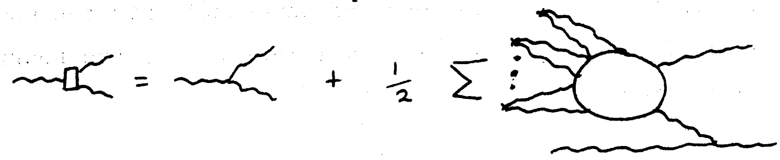
$$\bar{z} = z = -\frac{\Delta_0 + \sqrt{\Delta_0^2 + 2 r_0 s}}{2 i r_0} \tag{30}$$

The integration over y_0 and $\bar{y}_0$ eliminates those diagrams which are sensitive to the lack of translational invariance of the source. The remaining diagrams can be summarized by the following new RFT rules:

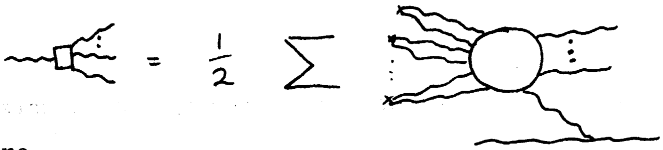
i) there is a shifted bare propagator

 $= \left[E - \sqrt{\Delta_0^2 + 2 r_0 s}\right]^{-1}$ (31)

ii) There are new Pomeron production vertices



(32)



(33)

where

 $= \dfrac{-\Delta_0 + \sqrt{\Delta_0^2 + 2r_0 s}}{2} \times \delta(E)$ (34)

and the circles are the usual RFT Green's functions (with just a triple Pomeron interaction and the new bare propagator) evaluated in old-fashioned perturbation theory with the time-ordering implied by the diagrams.

We now observe that this new perturbation expansion is well defined for all real values of Δ_0, including $\Delta_0 \to -\infty$. Therefore, we have a definition of $[\mathrm{RFT}]_0$ for all values of Δ_0 by taking the limit $s \to 0$ at fixed Δ_0. However, it is clear from (31) and (34) that this definition of $[\mathrm{RFT}]_0$ is *continuous* but *non-analytic* at $\Delta_0 = 0$. Only for $s \neq 0$, is this form of the theory the analytic continuation in Δ_0.

For $s \neq 0$, the singularity structure in the Δ_0-plane is as shown in Fig. 1. Our definition of $[\mathrm{RFT}]_0$ is to analytically continue between the two complex branch points at $\Delta_0 = \pm i\sqrt{2r_0 s}$. To obtain the $s = 0$ analytic continuation as a limit in which $s \to 0$ we should analytically continue around the two branch points in the same direction so that their phases add rather than subtract, as shown in Fig. 2. We shall compare these two analytic continuations again after discussing an alternative formulation of $[\mathrm{RFT}]_0$ due to Ciafaloni, Le Bellac and Rossi (CLR)[20].

The CLR formalism

Those authors first make a similarity transformation and define

$$\tilde{H} = F^{-1} H F \qquad F = (iz)^{\frac{1}{4}} \exp\left[-\tfrac{1}{4}\left(z + \tfrac{\Delta_0}{i r_0}\right)^2\right] \tag{35}$$

writing

$$\tilde{H}\left(z, \tfrac{d}{dz}\right) = F^{-1}(z)\, H\left(z, \tfrac{d}{dz}\right) F(z) \tag{36}$$

$\tilde{H}$ defines a hermitian operator when restricted to the half-axis $z = -iq$, $q > 0$. CLR then define

$$e^{-Hy} = F e^{-\tilde{H}y} F^{-1} \tag{37}$$

$$= F \sum_n |\phi_n\rangle\langle\phi_n| e^{-E_n y} F^{-1} \tag{38}$$

where ϕ_n and E_n are eigenfunctions and eigenvalues of $\tilde{H}$ in the space of L^2 functions on the half-axis. The $\{F\phi_n\}$ are clearly eigenfunctions of H. The general asymptotic behaviour for an eigenfunction $\psi(z)$ of H is either

$$\psi(z) \underset{z\to\infty}{\sim} \frac{1}{z} \exp\left[-\frac{z^2}{2} - \frac{\Delta_0}{ir_0} z\right] \tag{39}$$

or

$$\psi(z) \underset{z\to\infty}{\sim} \text{constant} \tag{40}$$

it is straightforward to show that normalizability of ψ in the Fock space requires that the behaviour (39) be absent along the direction

$$\Delta_0 \, \mathrm{Im}\, z < 0 \tag{41}$$

Therefore normalizability in the Fock space is equivalent to normalizability in the L^2 space for $\Delta_0 > 0$. (In fact Ciafaloni has recently shown[21)] that our path-integral formulation and the definition (38) of e^{-Hy} coincide for $\Delta_0 > 0$.) CLR also adopted (38) as a definition of e^{-Hy} for $\Delta_0 < 0$, since they were able to prove that this definition *is* the analytic continuation in Δ_0. From our previous discussion it is clear that for $\Delta_0 < 0$ therefore, the CLR formalism is equivalent to the alternative analytic continuation (when $s \neq 0$) that we discussed and so does not coincide with our formulation of $[\mathrm{RFT}]_0$.

We emphasize at this point that while the CLR formulation gives a smooth continuation in Δ_0 which can be made completely rigorous, this is done at a cost. The cost is the loss of a Fock space formulation of the Hamiltonian for large y. The states appearing in the spectral representation (38) are *non-normalizable* in the bare Pomeron Fock space for $\Delta_0 < 0$. Our formalism is based on the requirement that in the E-plane (which describes the large y limit) we can describe the Hamiltonian in terms of bare Fock space creation and destruction operators. Clearly the new Pomeron vertices appearing in (32) and (33) imply that we can formulate a new RFT Hamiltonian for $\Delta_0 < 0$ which involves an infinite number of Fock-space interactions all of which are expressed as power series in (r_0/Δ_0). This formalism enables us to prove the Reggeon unitarity of our theory for $[\mathrm{RFT}]_0$ in the supercritical phase as we shall discuss shortly. Since the CLR formalism is based on the L^2 space and not the Fock space for $\Delta_0 < 0$, all contact with bare Pomeron states, i.e. Regge pole states, and hence Reggeon unitarity, is lost when the impact parameter dimensions are added.

Finally we note the spectrum of singularities in the E-plane of the two definitions shown in Fig. 3. The striking points of these spectra are clearly the discontinuity at $\Delta_0 = 0$ (which in a sense exists even in the CLR formalism because of the normalizability problem) and the degeneracy ("tunnelling effect") in the CLR approach which appears as $\Delta_0 \to -\infty$. The tunnelling effect has also been found[22,23] in other previous formulations of $[\mathrm{RFT}]_0$ and is the basis of all arguments that (4) holds in $[\mathrm{RFT}]_2$. It is argued that the phase-transition in $[\mathrm{RFT}]_0$ is effectively at $\Delta_0 = -\infty$ and therefore can be studied for $D \neq 0$ by keeping the vacuum and the nearly degenerate state from the D = 0 theory and building a spin-model on an impact parameter lattice[15,16]. However, we shall argue that the analogue of the *strong coupling* phase-transition for $D \neq 0$ occurs in the D = 0 theory not at $\Delta_0 = -\infty$ but at $\Delta_0 = 0$. This brings us to ...

The phase-transition in $[\mathrm{RFT}]_D$

We consider now the most general problem in which there are D impact parameter dimensions (in the physical world D = 2) and all possible Pomeron interactions. That is we consider

$$\mathcal{U}_D(g_1, g_2, y_1 - y_2, \underline{x}_1 - \underline{x}_2) = \int d\bar{\psi}\, d\psi \, \exp - \int_{-\infty}^{\infty} dy \int_{|x|>\Lambda} d^D\underline{x} \; \mathcal{L}^D_{g_1 g_2}(\bar{\psi}, \psi) \tag{42}$$

where

$$\begin{aligned} \mathcal{L}^D_{g_1 g_2} = {} & \tfrac{1}{2} \bar{\psi} \overleftrightarrow{\frac{\partial}{\partial y}} \psi + \alpha_0 \nabla\bar{\psi} . \nabla\psi + \Delta_0 \bar{\psi}\psi + i r_0 \bar{\psi}(\bar{\psi}+\psi)\psi + \lambda_4^1 \bar{\psi}^2 \psi^2 \\ & + \lambda_4^2 \bar{\psi}(\bar{\psi}^2 + \psi^2)\psi + \lambda_5^1 (\bar{\psi}^2 \psi^3 + \bar{\psi}^3 \psi^2) + \cdots \\ & + i g_1 \delta(y - y_1)\, \delta^D(\underline{x} - \underline{x}_1) + i g_2 \delta(y - y_2)\, \delta^D(\underline{x} - \underline{x}_2) \end{aligned} \tag{43}$$

Eq. (42) can be regarded as the most general solution of the Reggeon unitarity equations for the Pomeron. To begin a discussion of the phase-transition we should define a Wilson renormalization group transformation on (42) which enables us to eliminate the "irrelevant" variables in (43). The usual procedure would be to emphasize the very large impact parameter by writing

$$\psi(\underline{x}) = \psi_1(\underline{x}) + \psi_2(\underline{x}) \quad , \quad \begin{array}{ll} \psi_1 = 0 & \Lambda < |\underline{x}| < \Lambda' \\ \psi_2 = 0 & \Lambda' < |\underline{x}| \end{array} \tag{44}$$

and similarly for $\bar{\psi}$ and then integrating over ψ_2 in (42) leading to new parameters Δ_0', r_0', $\lambda_n^{j\prime}$, etc. Dimensional analysis alone shows that in general in the limit $\Lambda' \to \infty$

$$\Delta_0' \sim \Delta_0 \, , \quad r_0' \sim \left(\frac{\Lambda}{\Lambda'}\right)^{\frac{D}{2}} r_0 \, , \quad \lambda_n^{j} \sim \left(\frac{\Lambda}{\Lambda'}\right)^{(n-2)\frac{D}{2}} , \quad \alpha_0' \sim \left(\frac{\Lambda}{\Lambda'}\right)^{2} \alpha_0 , \; \cdots \tag{45}$$

where n is the total number of $\bar{\psi}$ or ψ fields which λ_n^j couples.

There are two related problems here. Firstly all couplings, except Δ_0 are driven to zero in the limit $\lambda' \to \infty$ by this transformation. To see at least the triple Pomeron interaction enhanced as we would like we must also interprete out the low-rapidity fields in (42). This leads to

$$\Delta_0' \sim \left(\frac{\Lambda'}{\Lambda}\right) \Delta_0 \, , \quad r_0' \sim \left(\frac{\Lambda}{\Lambda'}\right)^{\frac{D}{2}-2} r_0 \, , \quad \lambda_n^{j} \sim \left(\frac{\Lambda}{\Lambda'}\right)^{(n-2)\frac{D}{2}-2} , \quad \alpha_0' \sim \left(\frac{\Lambda}{\Lambda'}\right)^{0} , \; \cdots \tag{46}$$

For $D > 2$ taking $\lambda' \sim \infty$, will now remove all but the triple Pomeron coupling. Also there will be a phase-transition at any special set of initial values of the parameters for which the scaling of Δ_0 is avoided and replaced by $\Delta_0' = 0$. This is the usual argument for considering only the triple Pomeron coupling near the phase-transition. Note now that *it does not work for $D = 0$, all couplings have the same scaling behaviour* in this case.

The second problem with this formalism which now arises is that it is difficult to be sure that the Lagrangian obtained after the renormalization group transformation has the same form as the initial Lagrangian (43) when the rapidity is involved (there may be $\bar{\psi}^2$, ψ^2 terms for example). This problem can be solved by first Fourier transforming the whole formalism to $(E,\underline{k})$ space and replacing (44) by a separation of small and large $(E,\underline{k})$ fields. In order to obtain (46) it is essential to integrate out both large $\underline{k}$ and large E fields. To do this in a well-defined way, which preserves the general form of the Lagrangian, we need the formulation of the path-integral in terms of fields of the form (22) referred to earlier. For our present purposes we shall simply assume that the $(E,\underline{k})$ renormalization group transformation can be defined.

For $D \neq 0$ then, we can study the phase-transition by studying the triple Pomeron theory with the α_0' term in (43) added to (7) and with $\Delta_0 \sim 0$. We now note that the formalism (21)-(34) can be applied immediately to the theory with $D \neq 0$, with the momentum transfer dependence of the theory playing a secondary role. If a source s is added to the theory, the phase-transition disappears for real Δ_0. The singularity structure of the theory is again that shown in Fig. 1. The phase-transition is in effect now at the branch-points $\Delta_0 = \pm i\sqrt{2r_0 s}$ and the addition of the impact parameter dimension to the theory simply means that these singularities are more complicated than in the $D = 0$ theory because there is scaling behaviour. If we define the theory by analytic continuation in Δ_0 with $s \neq 0$, as we did for $[RFT]_0$, taking $s = 0$ afterwards, *then* we have the following properties of the theory:

i) The theory is given by the perturbation expansion (31)-(33) with momentum transfer dependence added to all propagators, for $\Delta_0 \to -\infty$ the Pomeron intercept $\sim |\Delta_0|$.

ii) The theory satisfies Reggeon unitarity, since this property is satisfied perturbatively -- *note* that this property holds for $s \neq 0$ also. The analytic continuation *preserves* unitarity. This is the main justification for the continuation procedure.

iii) The theory satisfies the same scaling behaviour when $\Delta_0 \to 0_-$, as when $\Delta_0 \to 0_+$. This follows from the renormalization group analysis of Abarbanel, Bronzan, Schwimmer and Sugar[24] (ABSS).

It should be clear that we are simply carrying through the initial programme of ABSS to study the phase-transition in close analogy with the usual analysis of the phase-transition in $\lambda\phi^4$, this is by the addition of an external source ("external magnetic field" in the language of the Ising model) to the theory. We have shown that this programme *can* be carried through with consistent (and strong!) results *provided* that the process of adding the source is properly defined.

This then is our view of the phase-transition. We shall discuss the new phase further shortly. Let us discuss now how the alternative view (4) is arrived at. The main point is that the phase-transition is seen as occurring at $\Delta_0 = -\infty$ when $D = 0$. The near degeneracy in the CLR version of the $D = 0$ theory becomes a degeneracy when the impact parameter dimension is added[15,16]. The behaviour (4) is associated with an "expanding disc" in impact parameter space inside of which there is a transition from the old vacuum to the new degenerate state. The new phase is therefore pictured as involving two degenerate but communicating vacua. We have already remarked that this analysis loses all contact with Reggeon unitarity. Essentially this happens once the CLR continuation through $\Delta_0 = 0$ is made in $D = 0$. For us $\Delta_0 = 0$ is much closer to the phase-transition point when $D = 0$ and there is only a triple Pomeron coupling. This becomes clearer if we discuss the addition of higher couplings to the $D = 0$ theory.

First we recall that from (46) there is no justification for ignoring higher couplings when D = 0. But can there be a phase-transition in one dimension of rapidity? The answer is yes. Because the theory is non-relativistic there are no infra-red problems preventing the definition of a "massless" theory as in a relativistic theory such as $\lambda\phi^4$. Further, if we compare the sum-rules[25] giving the bare intercept or mass of the strong-coupling critical theory we note a significant difference for RFT and $\lambda\phi^4$

RFT

$$\delta\Delta_0 = -\left(\frac{r_0}{(\alpha_0')^{\frac{D}{4}}}\right)^{\frac{4}{4-D}} \int_0^\infty \frac{dx}{x^2}\left[Z^{-1}(x) - 1\right] \quad , \quad Z(x) \underset{x\to\infty}{\sim} x^{-\eta} \tag{47}$$

$\lambda\phi^4$

$$\delta m_0^2 = (-g_0)^{\frac{2}{4-D}} \int_0^\infty \frac{dx}{x^2}\left[Z^{-1}(x) - 1\right] \quad , \quad Z(x) \sim x^{-\eta/2} \tag{48}$$

where z(x) is a renormalization constant in both cases and x is a dimensionless renormalization point with $x \sim \infty$ being the "infra-red" or low-momentum region. The point we wish to make is that in $\lambda\phi^4$ the critical exponent η is *positive* while in RFT it is *negative*. Therefore as $D \to 1$, η increases in $\lambda\phi^4$ and the phase-transition disappears ($\delta m_0^2 \to \infty$) due to an *infra-red divergence* of (48). In (47) there can be no infra-red divergence with $\eta < 0$ and so the strong-coupling critical theory which is insensitive to the ultra-violet behaviour of the theory should have a smooth continuation to D = 0, we simply have to include higher-order couplings because of (46).

Suppose we add a four-Pomeron coupling of the form

$$\delta H_4 = 2\lambda_4^1 \bar{\psi}^2\psi^2 + \lambda_4^2 \bar{\psi}\left(\bar{\psi}^2 + \psi^2\right)\psi \tag{49}$$

to (7). If we carry through the analog of the manipulations (29)-(34) and set s = 0 we obtain the new bare intercept (for $\Delta_0 < 0$)

$$\tilde{\Delta}_0 = \frac{(-ir_0 + X)X}{2\lambda_4^2} \quad , \quad X = \left(-r_0^2 - 4\lambda_4^2\Delta_0\right)^{\frac{1}{2}} \tag{50}$$

a two-Pomeron source term [analogous to (34)]

$$\tilde{s} = \tilde{\Delta}_0 + 2\left(\lambda_4^1 - \lambda_4^2\right)\left[\frac{-ir_0 + X}{2\lambda_4^2}\right]^2 \tag{51}$$

and a new triple Pomeron coupling

$$\tilde{r}_0 = \left(\frac{\lambda_4^2 - \lambda_4^1}{\lambda_4^2} \right) i r_0 + \lambda_4^1 x \tag{52}$$

We now observe that

$$\frac{\tilde{s}}{\tilde{\Delta}_0} \underset{\Delta_0 \to -\infty}{\sim} \frac{\lambda_4^1}{\lambda_4^2} \tag{53}$$

and consequently if $\lambda_4^1 > \lambda_4^2$ is is meaningless to treat $\tilde{s}$ perturbatively with respect to $\tilde{\Delta}_0$ in this limit. We conclude that our formalism does not apply unless $\lambda_4^2 \sim \lambda_4^1$, when, not surprisingly, the Fock space normalizability problem again occurs when $\Delta_0 = 0$.

In contrast the CLR formalism is improved[21,22] if $\lambda_4^2 = 0$ and $\lambda_4^1 \neq 0$. In this case the analytic continuation in Δ_0 stays within the Fock space for $\Delta_0 < 0$ and the near degeneracy at $\Delta_0 = -\infty$ becomes an exact degeneracy at the "magic value"

$$\Delta_{0M} = - \frac{r_0^2}{2\lambda_4^1} \tag{54}$$

the spin model can be set up at this point for $D \neq 0$ with complete consistency. Further Bronzan and Sugar[17] have solved the $D = 1$ theory exactly at this point and showed that the expanding disc does indeed occur. Bronzan[26] has also shown that this magic-value persists when $\lambda_4^2 \neq 0$ although direct contact with the spin-model is lost as λ_4^2 increases. When $\lambda_4^2 = \lambda_4^1$ the magic value is

$$\Delta_{0M} = \frac{-r_0^2}{4\lambda_4^1} + O\left(\lambda_4^1\right) \tag{55}$$

which implies $x \sim 0$ in (50)-(52). Note that this implies that $\tilde{\Delta}_0 \sim 0$ *and* $\tilde{r}_0 \sim 0$. This shows that the magic value corresponds to a second critical point of the theory as Δ_0 is varied. The spectrum of our theory when $\lambda_4^1 = \lambda_4^2$ is shown in Fig. 4. Since $\tilde{r}_0 \sim 0$ we conclude that the magic value critical point is *essentially* the *weak coupling* critical point[1,2] when $D \neq 0$. Beyond the magic value it is impossible to define a real bare Pomeron propagator from our formalism. The effective potential defining $\tilde{\Delta}_0$, $\tilde{s}$, etc., has two complex stationary points. We conclude that we cannot find a Fock space formalism (leading to a Reggeon unitary solution when $D \neq 0$) beyond the magic value.

Our formalism also applies for any higher-order interactions of the form

$$\delta H = \bar{\psi} \left[: f(\bar{\psi} + \psi) : \right] \psi \tag{56}$$

where :: indicates normal ordering. In such a case there will always be a Fock space normalization problem (or equivalently a divergence problem in the path-integral) when $\Delta_0 = 0$ and our formalism will define a non-analytic solution at this point. That the renormalized Pomeron intercept should reach one at this point provides a single constraint on the infinity of couplings appearing in f. When this is satisfied we believe we have the true strong-coupling critical Pomeron for D = 0. That we should have to consider interactions of the form (54) when D = 0 is clear when we consider the behaviour of the higher-order couplings near the strong coupling critical point when $D \neq 0$. The relevant couplings are those defined *after* the renormalization group transformation. These are dominated by the triple Pomeron interaction at small (but not zero) E and $\underline{k}$ and it is not difficult to see that they will indeed have the form (56).

In general the "magic-value" critical point will be associated with the first stationary point of the classical Hamiltonian beyond $\Delta_0 = 0$. It will involve two zeros of the Hamiltonian meeting and becoming complex conjugate with the (shifted) triple Pomeron coupling (in our formalism) vanishing. In contrast the strong coupling critical point involves one zero crossing the fixed zeros at $\bar{\psi}$, $\psi = 0$ with no vanishing of the triple Pomeron coupling. These two distinct kinds of critical point should describe two *distinct* phase transitions with the behaviour at the critical points describing the old "strong" and "weak" coupling Pomerons[1,2], respectively.

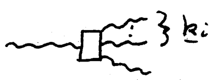
There are in fact many reasons to believe that the phase-transition associated with the expanding disc involves a weak-coupling Pomeron. (Note that there may still be scaling properties at the critical point since the triple Pomeron vertex will not be strictly analytic when $D \neq 0$ as envisaged in the original weak coupling solution). The tunnelling effect when D = 0 involves the isolation of the one Pomeron state indicating a suppression of the two and higher Pomeron states by suppression of the triple Pomeron vertex. The expanding disc is very similar to that obtained by simple eikonalization and in fact it seems likely (to me at least) that the limit as $r_0 \to 0$ of the complete expanding disc s-matrix is the eikonal. The eikonalization of the weak-coupling Pomeron is, of course, hardly surprising since this involves an essentially isolated pole passing through unit intercept.

We anticipate finally that Fig. 4 carries over to D = 2 as in Fig. 5, with the saturation of the Froissart bound (4) occurring beyond the second critical point. Our formalism defines a Reggeon unitary solution up to the second weak coupling point if we stay within the class of higher-order couplings given by (56). We believe that a solution satisfying t-channel unitarity cannot be found beyond this point. The weak-coupling Pomeron has, of course, run into many difficulties with s-channel unitarity which have still to be completely consistently overcome[2].

We argue therefore that the strong coupling critical Pomeron is the only known completely consistent theoretical description of rising total cross-sections.

The second phase and cut RFT

The striking property of the new phase, as defined by the perturbation expansion (31)-(33) with $\alpha_0' \underline{k}^2$ added to the bare propagator, is that the new Pomeron interaction vertices have a non-trivial momentum dependence. For example the exchanged propagator shown gives a pole in vertices of the form



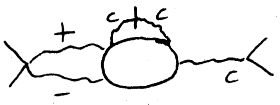
$$\sim \frac{1}{\alpha_0' \underline{k}_i^2 + |\Delta_0|} \tag{57}$$

where $\underline{k}_i$ is the transverse momentum carried by one or more produced Pomerons. This illustrates that the singularities occur for negative $\underline{k}_i^2$, or positive $t_i = -\underline{k}_i^2$ and so they do not occur in the high-energy scattering region but approach it as $|\Delta_0| \to 0$. In fact it is clear from the generation of those singularities by zero energy sources that it is as if there were negative energy states ("anti-Pomerons") in the theory which do not appear as intermediate states but appear only inside transition vertices. This situation is precisely what would occur if there were vector particles in the underlying theory with a mass of the order of magnitude of $(|\Delta_0|/\alpha_0')^{\frac{1}{2}}$. There will be many j-plane singularities in such a theory (associated with all possible combinations of Reggeons and many-particle phase-space singularities) which appear on the physical sheet of the j-plane for positive t and which for negative t are on the unphysical sheets of Reggeon cuts and lie in the right-half j-plane (that is have negative energy). Such singularities would be expected to produce singularities of the Pomeron interaction vertices of the same nature as those appearing in our super-critical RFT. The simplest example is the Gribov-Pomeranchuk fixed pole at j = 1. When coupling a single Pomeron this would produce just a singularity of the form (57). It seems pointless to try to exactly identify such effects without first studying in more detail the form of the RFT derived from the underlying vector theory. Also all particles that become massless as $|\Delta_0| \to 0$ (including Higgs scalars and fermions, for example) will give extra singular momentum dependence to Pomeron interaction vertices and also to the trajectory function. Thus further complicating the problem. However, since it can further be shown that the singularities of our vertex functions do not produce any physical sheet singularities, but only produce new singularities on the unphysical sheets of the multi-Pomeron cuts, it seems that they do have all of the right properties to be produced by vector (and other) particles becoming massless as $|\Delta_0| \to 0$. Further evidence that our super-critical RFT does indeed contain vector particles is obtained as follows.

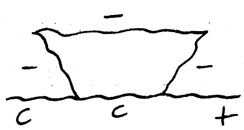
We have emphasized that our super-critical solution satisfies t-channel unitarity in the form of Reggeon unitarity but we have not discussed the s-channel unitarity of the theory. Clearly a Pomeron pole with intercept below one will not produce any blatant conflict with unitarity. But we can examine the detailed satisfaction of unitarity (at least the inclusive sum rules) by considering the *cut RFT*[27,28]. This is equivalent to the AGK cutting rules[29] which have a very general basis (probably they can be derived directly from analyticity and unitarity). In this theory we have three Reggeon fields ψ_+, ψ_-, ψ_c with the Lagrangian

$$\begin{aligned}\mathcal{L}_c = {} & \mathcal{L}_0(\bar\psi_+,\psi_+) + \mathcal{L}_0(\bar\psi_-,\psi_-) + \mathcal{L}_0(\bar\psi_c,\psi_c) + i r_0 \bar\psi_+(\bar\psi_+ + \psi_+)\psi_+ \\ & - i r_0 \bar\psi_-(\bar\psi_- + \psi_-)\psi_- + \sqrt{2}\, r_0 \bar\psi_c(\bar\psi_c + \psi_c)\psi_c + \frac{i}{\sqrt{2}} r_0 (\bar\psi_c\psi_+\psi_- + \bar\psi_+\bar\psi_-\psi_c) \\ & + i r_0(\bar\psi_c\psi_c\psi_+ + \psi_c\bar\psi_+\psi_c) - i r_0(\bar\psi_c\psi_c\psi_- + \bar\psi_c\bar\psi_-\psi_c)\end{aligned} \tag{58}$$

The Green's functions $\langle\bar\psi_c\psi_c\rangle$, $\langle\bar\psi_+,\psi_+\rangle$, $\langle\bar\psi_-,\psi_-\rangle$ give respectively the cut amplitude, the amplitude above the cut and the amplitude below the cut. Inclusive cross-sections are given by, for example, in the central region

 +  + . . .

The inclusive sum rules are satisfied by this formalism. The equality of $\langle\bar\psi_c,\psi_c\rangle$, $\langle\bar\psi_+\psi_+\rangle$, $\langle\bar\psi_-\psi_-\rangle$ can be traced back to a permutation symmetry of (58) which is evident after a field transformation given by Cardy and Suryani[27]. The above super-critical formalism can be applied directly to $\mathcal{L}_c$ and a solution involving a perturbation expansion analogous to (31)-(33) can be found. It is much more complicated since all three fields have to be shifted. The permutation symmetry is not broken but transitions such as $\langle\bar\psi_c\psi_+\rangle$ appear, given for example by graphs such as



and new amplitudes have to be defined by a diagonalization which does break the permutation symmetry. The new cut and physical amplitudes are defined uniquely by the requirement that they are equal for all Δ_0 and continuously equal to their subcritical equivalents at the critical point. Examining the Reggeon unitarity content of these amplitudes at the one loop level we find that *in addition* to the negative two-Pomeron cut contribution corresponding to the triple Pomeron vertex (32) there is *a positive two-Reggeon cut* term involving singular vertices corresponding to a degenerate set (at least two) of odd-signature Reggeons coupling to the Pomeron (the momentum dependence of the vertices essentially gives the correct signature factors).

The complete structure of the super-critical cut RFT has still to be determined but we can already draw the conclusion that requiring s-channel unitarity (in the form of the inclusive sum rules) in the super-critical phase requires odd-signature Reggeons to accompany the singular Pomeron vertices required by t-channel unitarity. We have thus arrived at the complete structure described in the introduction.

In conclusion it is clear that at the very least making direct contact between the high-energy behaviour of Yang-Mills theories (spontaneously broken) and super-critical RFT would be very helpful in understanding the complex structure of both formalisms. As we have said our preliminary study of the problem encourages us to believe very strongly that we really are studying the same phenomenon from different points of view. It is very important in this context that the high-energy behaviour of the Yang-Mills theories can be studied directly from multiparticle dispersion relations. The Reggeization of the vector mesons in these theories essentially means that all of the multiparticle angular momentum theory including Reggeon unitarity (and hence RFT) based on the multiparticle dispersion relations[4)] must be applicable in the neighbourhood of $J = 1$, $t = 0$ and so must be applicable to the Pomeron. Once super-critical RFT is seen to include odd-signature Reggeons and it is observed that spontaneously broken non-Abelian gauge theories are simply Reggeizing vector mesons satisfying perturbative unitarity the connection of the two formalisms seems inevitable.

Finally we make the obvious comment that the establishment of our conjecture would imply that rising total cross-sections can be added to Bjorken scaling as experimental evidence for an unbroken non-Abelian gauge theory of the strong interactions. [A connection between the two phenomena has been suggested previously by Gribov[30)], but on the basis of (apparently) very different arguments.] We also note that while it has been previously argued that a Pomeron constructed from confined gluons is a very good phenomenological candidate giving approximately constant total cross-sections[31,32)], we believe our conjecture gives for the first time a possible profound explanation of *why the Pomeron intercept is exactly one*.

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Figure captions

Fig. 1 : Singularity structure in the Δ_0 plane.

Fig. 2 : Alternative analytic continuation.

Fig. 3 : Spectra of $[\mathrm{RFT}]_0$ with just a triple Pomeron coupling.

Fig. 4 : Spectrum of our $[\mathrm{RFT}]_0$ when a four Pomeron coupling is present.

Fig. 5 : Spectrum of our $[\mathrm{RFT}]_D$ when a four Pomeron coupling is present.

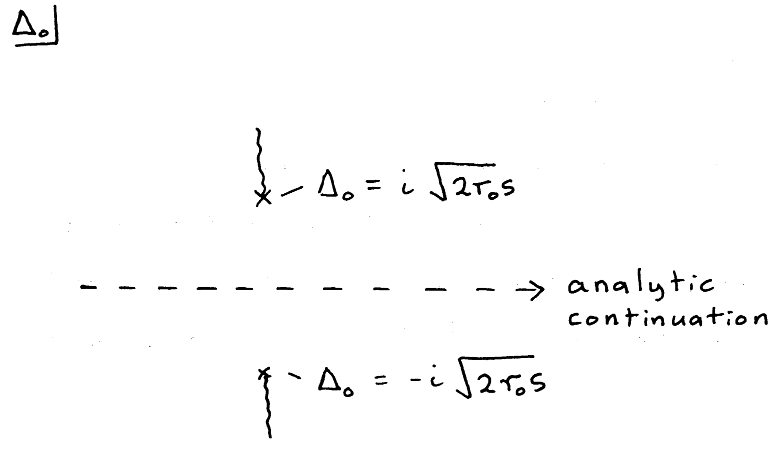

Fig. 1

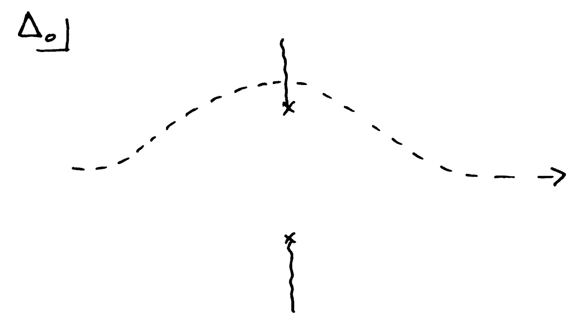

Fig. 2

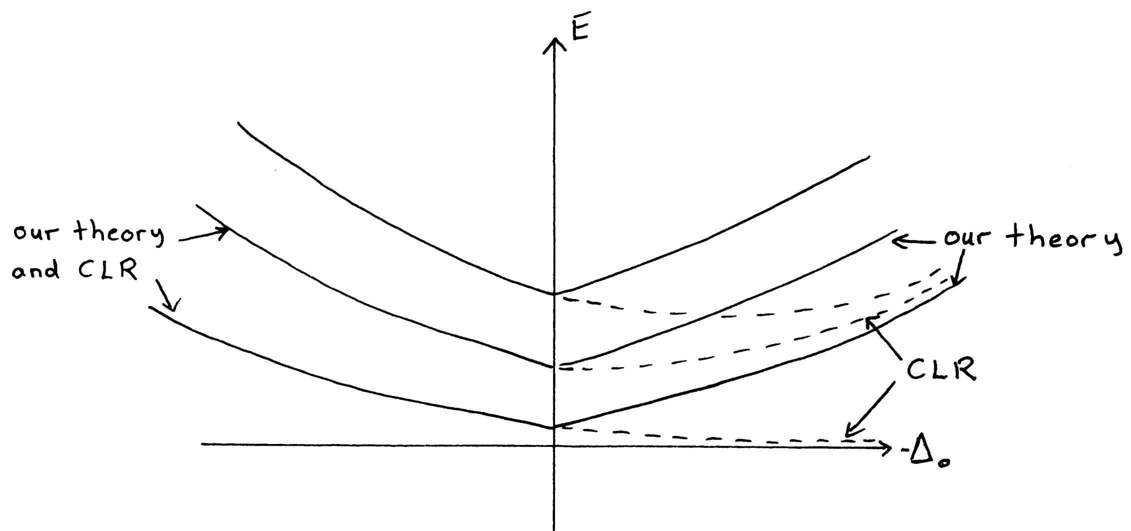

Fig. 3

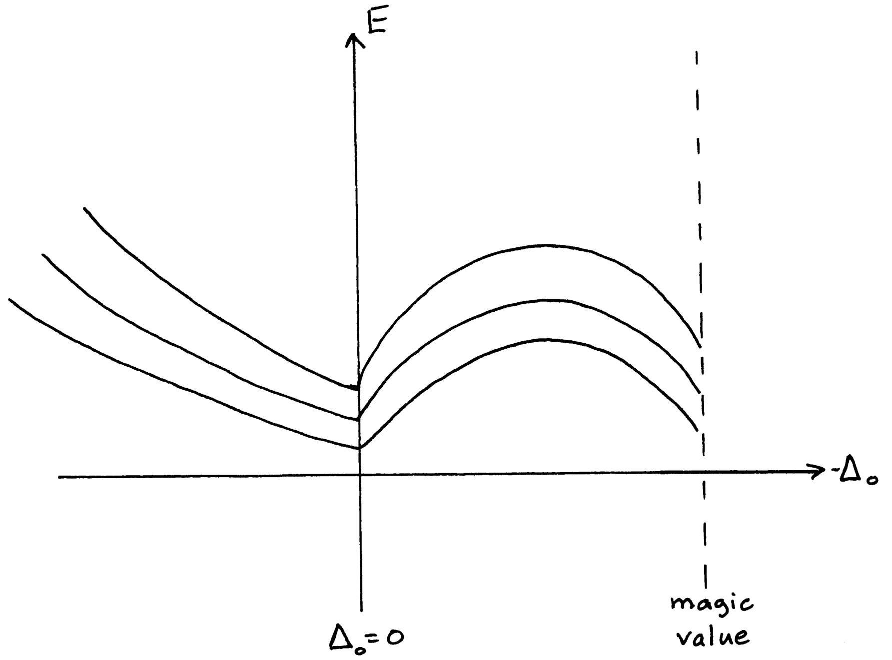
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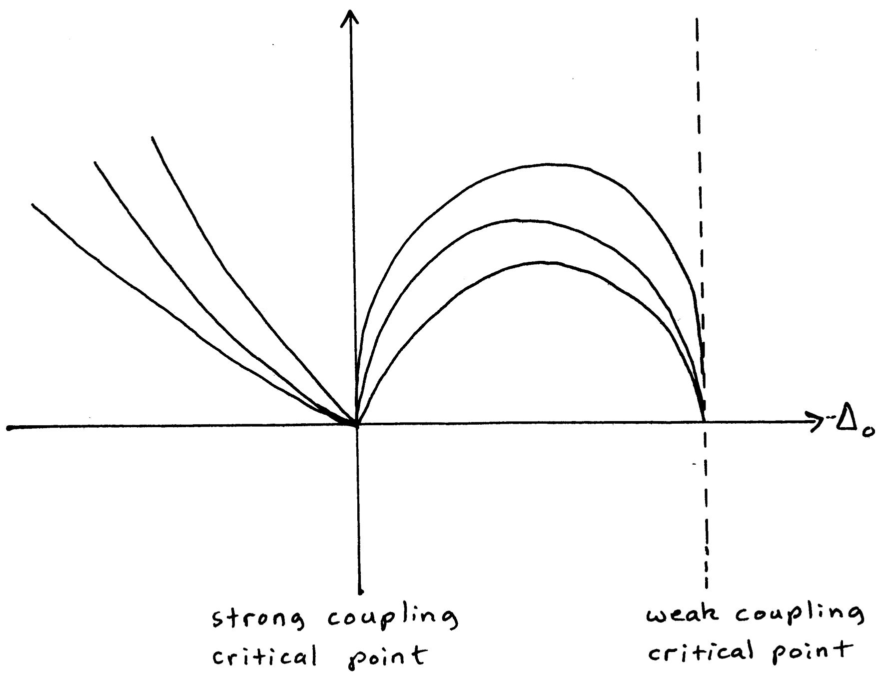
Fig. 4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strong coupling
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weak coupling
critical point

Fig. 5