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EVOLUTIONARY ALGORITHMS IN THE DESIGN OF CRAB CAVITIES

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Abstract

The design of RF cavities is a multivariate multi-objective problem. Manual optimisation is poorly suited to this class of investigation, and the use of numerical methods results in a non-differentiable problem. Thus the only reliable optimisation algorithms employ heuristic methods. Using an evolutionary algorithm guided by Pareto ranking methods, a crab cavity design can be optimised for transverse voltage (V_T) while maintaining acceptable surface fields and the correct operating frequency.

Evolutionary algorithms are an example of a parallel meta-heuristic search technique inspired by natural evolution. They allow complex, epistatic (non-linear) and multimodal (multiple optima and/or sub-optima) optimization problems to be efficiently explored. Using the concept of domination the solutions can be ordered into Pareto fronts. The first of which contains a set of cavity designs for which no one objective (e.g. the transverse voltage) can be improved without decrementing other objectives.

EVOLUTIONARY ALGORITHMS (EA)

Evolutionary algorithms are an example of a parallel meta-heuristic search technique inspired by natural evolution. Unlike many optimisation methods, EAs work from a population of solutions. Recombination operators are applied to share information, and mutation operators to explore new regions of the search space. This allows for complex, epistatic (non-linear) and multimodal (multiple optima and/or sub-optima) optimization problems to be efficiently explored. [1]

Real-coded EAs (RCEA) are used in a wide range of scientific applications and their characteristics are well understood. They are well suited to performing optimizations in problems with a high dimensionality. RCEAs are distinct from genetic algorithms in that they manipulate the parameters numerically rather than encoding them as binary information. [1]

The problem is represented using the abstraction of individuals in populations. Each individual has a decision vector (the inputs to the simulation or calculation) and a solution vector (the outputs) and so maps the independent variables onto the dependant. It is the manipulation of these vectors which the EA performs.

A basic optimisation algorithm of this kind uses 4 distinct blocks (see fig 1). An initialiser which produces the initial population of guess decision vectors. The calculation block which evaluates the decision vector to find the solution vector. The selection block which determines the best individuals (using a property known as fitness) to take part in the final block, recombination. This stage combines the decision vectors of the best of the population to create an individual which is expected to be "fitter". The final three stages are then repeated until a stop condition is met.

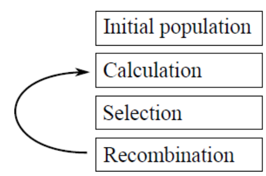

Figure 1: Basic evolutionary Algorithm Structure.

If a single objective is to be used then the fitness is defined to be this objective. However if there are multiple objectives this definition is more complex.

Multiple Objective EAs (MOEAs)

In any multi-objective simulation there exists a set of solutions which are demonstrably superior to all other solutions. Such a set is known as a Pareto optimal set, critically no member of this set can be said to be superior to any other. [3]

Classical methods, such as objective weighting, often converged to a single value of the Pareto optimal set. Modern Multiple Objective EAs (MOEAs) allow the full front to be found and are often based on the concept of domination (see fig 2):

$$\boldsymbol{a} \preccurlyeq \boldsymbol{b}$$

where $\preccurlyeq$ denotes all components are greater than or equal to and **a** and **b** are solution vectors.

These vectors can be arranged into Pareto fronts (solution vectors where no one characteristic can be further optimised without detrimental effect on the other characteristics) the first of which is the Pareto optimal set and contains no dominated individuals.

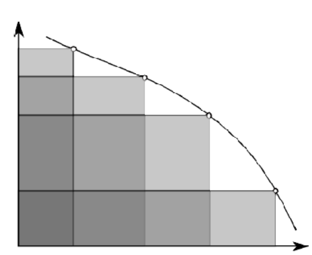

Figure 2: The effect of $\boldsymbol{a} \preccurlyeq \boldsymbol{b}$ on the space behind a series of points. The darker colours represent a more highly dominated area.

SPEA2 [2] (Strength Pareto EA 2) and NSGA-II [3] (Non-dominated Sorting Genetic Algorithm*) use the concept of domination to simultaneously converge on a set of Pareto-optimal† solutions. A single solution can then be selected from the set by an informed design engineer. The two methods show differing convergence characteristics depending on the type of problem. In some complex problems, it is possible for the algorithm to converge to a Pareto-optimum set which is not truly optimal.

The use of an EA can return surprising results, such as the Tech-X simulation of a Periodic Band Gap cavity. [4]

CRAB CAVITIES

In particle accelerators a non zero crossing angle can be used to increase luminosity and avoid parasitic collisions. To achieve the increase in luminosity before collision the bunches must be rotated so they collide head on. This is achieved using the dipole mode of a cavity. The phase of this mode is such that the magnetic field in the centre of the cavity deflects the leading edge of the bunch in the opposite direction to the trailing edge, while leaving the centre unperturbed, and thus the bunch is rotated. [6] The electric field in the cavity iris also contributes to this deflecting effect (see fig 3).

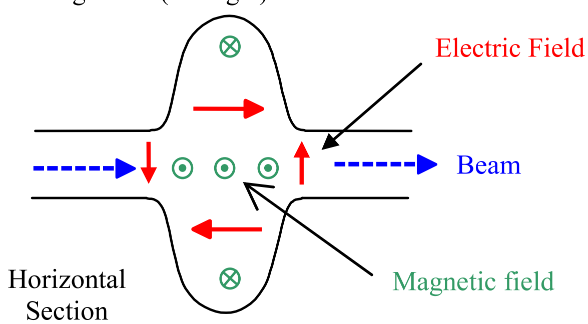

Figure 3: The fields used in a crab cavity to deflect the leading and trailing edges of the bunch.

There are multiple figures of merit which can be defined for such a cavity based on the magnitude of the deflecting "kick" and the peak fields. These figures of merit will make up the solution vector.

Transverse voltage

The deflecting "kick" delivered by a crab cavity, although it is delivered through a combination of the electric and magnetic fields, is defined using the transverse voltage (V_T). This can be found from a longitudinal voltage using Panofski-Wenzel theorem [5]:

$$V_T = \frac{V_z(d)c}{\omega d}$$

where $V_z(d)$ is the voltage in the direction of propagation at position d, c is the speed of light and ω is the angular frequency of the mode.

V_T is of course strongly related to the stored energy. As in accelerating cavities the R/Q is a suitable figure of merit. The transverse R/Q is defined by using the standard accelerator definition for longitudinal R/Q applied to V_T: [6]

$$R/Q_T = \frac{{V_T}^2}{\omega U}$$

where U is the stored energy.

Peak fields

The peak magnetic fields (H_{pk}) are constrained if a superconducting crabbing cavity is to be used. If the peak fields increase beyond a certain limit then the cavity will quench. In order to normalise for the accelerating field the ratio of peak field to accelerating voltage is used: [6]

$$\frac{H_{pk}}{V_T}$$

A high peak electric field (E_{pk}) increases the risk of field emission and also must be minimised and kept below a defined limit: [6]

$$\frac{E_{pk}}{V_T}$$

Frequency Targeting

The frequency of the cavity must meet the specification. Using a Pareto based MOEA allows the algorithm to tune the cavity while simultaneously optimising the other parameters. Without intervention it will however produce the optimum H_{pk}/V_T and R/Q_T for a range of frequencies.

To strongly target a suitable frequency, Pareto Search Pressure focusing can be used (see fig 4). [7] In NSGA-II the fitness of the individuals is defined by the "Pareto front" which contains them. [3] This fitness can be manipulated by demoting the individuals by one front (and so in fitness) if they do not meet certain criteria. In this case the individuals can be demoted if they are not at, or within a tolerance of, the target frequency. This approach effectively allows off target cavities with low E_{pk}/V_T, H_{pk}/V_T and high R/Q_T to be tuned to the correct frequency.

* Although NSGA uses the term "Genetic Algorithm", It is in reality a fitness assignment system that can be, and is, applied to other forms of EA.

† A Pareto-optima is a solution to a multi-objective problem, where no one objective can be improved without detrimental effects on other solutions. A Pareto-optimal set are a set of such solutions, no one of which can be considered "better" than any other.

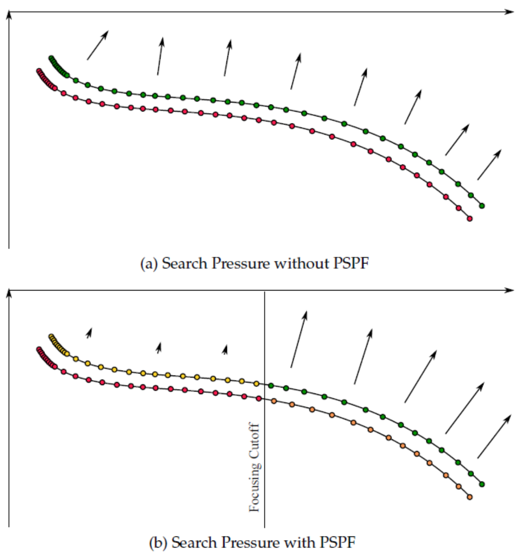

Figure 4: PSPF (Pareto Search Pressure Focusing) alters the search pressure (denoted by arrows), increasing it above the focusing cut off.

To calculate these figures of merit the simulation code must be capable of accurately calculating the resonant frequency of the dipole mode; E_z along a line; the energy stored in the cavity; and the peak magnetic and electric fields on the surface. From these the figures of merit can be calculated to create the solution vector:

$$Solution\ Vector = \left[-|F - F_T|, R/Q_T, -\frac{E_{pk}}{V_T}, -\frac{H_{pk}}{V_T}\right]$$

Where F is the dipole frequency, F_T is the target frequency and a maximisation algorithm is used.

The decision vector will be made up of the various dimensions which are variable in the chosen cavity geometry.

Optimisation

The advantage of analysis using the domination concept can be seen when E_{pk}/V_T and H_{pk}/V_T are plotted for a range of existing cavity designs and hand optimised examples (see fig 5). A clear front can be seen at the bottom left of the problem (as this is a minimisation optimisation).

Multi Objective Evolutionary Algorithms clearly have the potential to improve the design of crabbing cavities. Supplied with a suitable parameterised model they can supply a trade off curve between E_{pk}/V_T and H_{pk}/V_T for a given design. This would allow the comparison of the advantages of cavity features rather than particular examples.

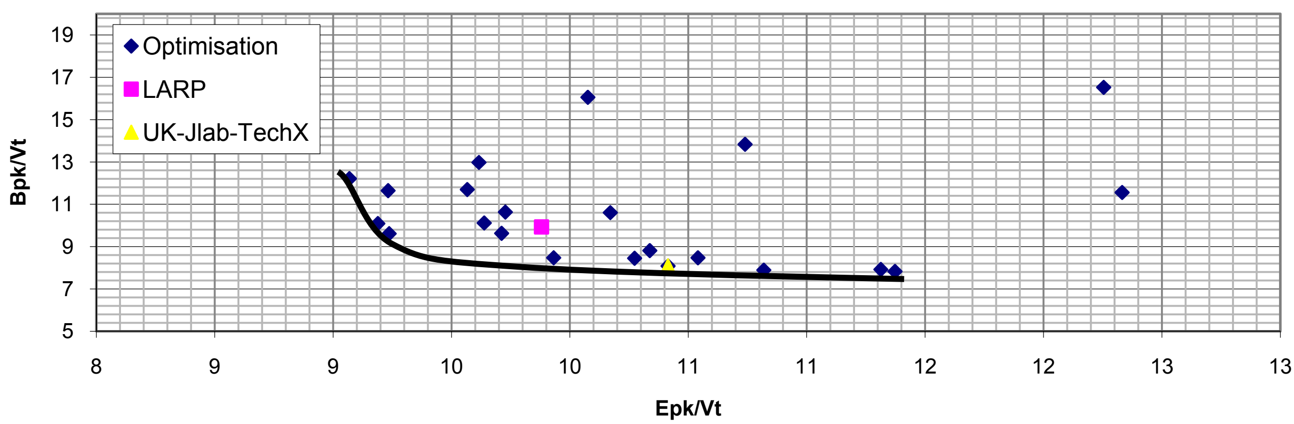

Figure 5: An example of how the concept of domination and Pareto fronts can be applied to crab cavity properties which are to be minimised. An approximate Pareto optimal set for this data (proposed crabbing cavities for LHC) is marked.

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