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Estimates of CSR Instability Thresholds for Various Storage Rings

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Summary

We review the key predictions and conditions by several authors for the onset of longitudinal instabilities due to coherent synchrotron radiation (CSR), and evaluate them numerically for various storage rings, namely the KEKB High Energy Ring (HER) & Low Energy Ring (LER), SuperKEKB HER & LER, old and new designs of the SuperKEKB Damping Ring (DR), SuperB HER & LER, CLIC DR (2009 and 2010 design parameters), SLC DR, and ATF DR. We show that the theoretical uncertainty in the instability onset is at least at the level of 20-30% in bunch intensity. More importantly, we present some doubts about the general applicability for many of these storage rings of some commonly used formulae. To cast further light on these questions, an experiment at lower beam energy on the ATF Damping Ring is proposed.

1. Review of theoretical predictions and application to various storage rings

In 2002 a simple analytical theory for the onset of CSR instability was developed by Stupakov and Heifets [1]. Table 1 compares the parameters and criteria from this theory for the Super KEKB e+ DR (latest parameters [2]), the SuperKEKB HER/LER, and the ATF DR.

The "Stupakov-Heifets parameter" Λ is defined as [1]

$$\Lambda = \frac{N_b r_e \rho \sqrt{2\pi}}{C|\eta|\sigma_z \gamma \sigma_\delta^2}$$

Due to shielding by the conducting vacuum beam pipe, a necessary condition for instability is [1]

$$\frac{\rho}{b} \le \Lambda.$$

In Table 1, this condition is fulfilled only for the SuperKEKB LER, but not for SuperKEKB HER, SuperKEKB DR and ATF DR. Therefore the instability occurrence is not probable for the latter three storage rings. It is least probable for the SuperKEKB damping ring (with latest parameters).

Another theoretical cutoff is given by the bunch length: The microbunching CSR instability can develop only if [1]

$$\sigma_z \geq \frac{\rho}{2\Lambda^{3/2}}.$$

We note that this criterion is well fulfilled for all storage rings of Table 1. The condition is closest to the limit for the SuperKEKB damping ring. We note that Λ, on the right of the above equation, also contains the bunch length.

In addition, the theory of [1] can be applied only if the horizontal beam size is sufficiently small, namely if

$$\Lambda << \left(\frac{\rho^2}{\sigma_x \beta_x}\right)^{2/3}$$

Table 1

Beam and CSR-instability related parameters for four storage rings.

	SuperKEKB LER	SuperKEKB HER	SuperKEKB e+ DR	ATF DR
beam energy [GeV]	4	7	1.1	1.28
slip factor η	0.000274	0.000188	0.017	0.0019
rms momentum spread $\sigma_{\delta,\mathrm{rms}}$ [%]	0.08	0.065	0.055	0.06
bunch population [10^9]	90	65	37.5	10
circumference C [m]	3016	3016	135	138.6
bending radius ρ [m]	73.3	104.5	2.65	5.73
vert. beam pipe radius b [cm]	4.7	2.5	1.6	1.2?
Stupakov-Heifets parameter Λ	**1864**	**2905**	**67**	**339**
ρ/b	**1560**	**4180**	**166**	**478**
σ_z [cm]	**0.6**	**0.5**	**0.7**	**0.5**
$\rho/(2\Lambda^{3/2})$ [cm]	**0.05**	**0.03**	**0.24**	**0.05**
$N_b r_0/((2\pi)^{1/2}\sigma_z\sigma_\delta\,\gamma)$	0.0027	0.0016	0.0051	0.0015
β_x at bend [m]	10?	10?	1.5	3?
ε_x [nm]	3.2	5.0	2100→ 41	~1.5
σ_x at bend [μm]	179	224	248	67
$\rho^{4/3}/(\sigma_x\beta_x)^{2/3}$	20800	28800	710	2990
τ_x [ms]	37?	56?	11	17.2
C3	0.0128	0.0003	0.0033	0.0002
Q_s	-0.025	-0.025	-0.015	-0.0045
$N_{b,thr}$	$1.0\text{x}10^{11}$	$1.9\text{x}10^{11}$	$4.5\text{x}10^{10}$	$1.15\text{x}10^{10}$
$N_b/N_{b,thr}$	**0.89**	**0.35**	**0.83**	**0.86**
$\Lambda b/\rho$	**1.19**	**0.69**	**0.40**	**0.71**

This condition is always fulfilled, as Table 1 demonstrates. Another condition must also hold [1]:

$$\frac{N_b r_e}{\sqrt{2\pi}\gamma\sigma_z\sigma_\delta} << 1.$$

This ensures that the effect of velocity spread remains negligible. The last condition above is also always fulfilled for all cases examined.

Yet another condition for the validity of the underlying model is

$$C3 \equiv \left(\frac{N_b r_e}{\sqrt{2\pi}\gamma\sigma_z}\right)^5 \left(\frac{2a}{\eta\rho}\right)^3 \frac{1}{\sigma_\delta^8}\frac{1}{\pi^{2/3}} \geq 1,$$

which refers to the interaction with a continuous mode spectrum, rather than with a single mode, and to the "instability of higher-order modes where the shielding effect of the walls can be neglected" [3]. Actually this condition is ***not at all fulfilled for any of the cases considered in Table 1***! It is not evident that or if Reference [3] provides an alternative expression for the instability threshold for the case that the above condition is not fulfilled (it is possibly contained in Eq. (20) of [3], and based on Landau damping), but [3] does present a condition for when, in the single-mode case, no threshold is expected and the beam should be unstable (see later).

Recently, at IPAC'10, an alternative prediction of the CSR instability threshold has been obtained by numerical solution of both the Vlasov-Fokker-Planck equation of the linearized Vlasov equation [4], assuming the shielding from two parallel plates, which reads:

$$N_{b,thr} \approx \left(\frac{2\pi Q_s \gamma\sigma_{\delta 0}}{r_e}\right)\left(\frac{\sigma_z^{4/3}}{\rho^{1/3}}\right)\left(0.5 + 0.12\frac{\sigma_{z0}\rho^{1/2}}{b^{3/2}}\right).$$

The second term in the last round brackets represents the shielding effect. The intensity threshold from this formula can be compared with the design bunch intensity. Interestingly, this formula contains the synchrotron tune, which seems plausible, while all the others above do not. Nevertheless, the predictions are often similar, but not always. For example, in Table 2, which compares the results for SuperKEKB LER & HER with those of SuperB LER & HER and two versions of the CLIC DR [5], for the SuperB LER the parameter ρ/b is smaller than Λ, which should be safe according to [1], and, yet, the above equation, from [4], predicts instability. The 2010 version of the CLIC DR appears more stable than the previous one.

We can conclude that both the shielding by the beam pipe and the finite bunch length will prevent any CSR microbunching instability in the KEKB Damping Ring (see the last two rows in Table 1). The instability is also unlikely to appear in the ATF for present operating conditions.

At the threshold the inequality (26) of Reference [3], which we have rewritten as "C3≥1", is not fulfilled for any of the example storage rings considered so far. This could mean that only a single isolated mode should drive the CSR instability in all these cases, and arguably that neither the formalism of [1] nor the one of [4] is applicable. More specifically, we can ask: (1) Is the treatment of [4] still applicable for cases where C3≥1 (Eq. (26) of [3]) is not fulfilled? (2) Or must the codes of [4] not be used? Or, (3), should the paper [3] not be interpreted in this sense?

Table 2

Beam and CSR-instability related parameters for five storage rings.

	SuperKEKB LER	SuperKEKB HER	SuperB LER	SuperB HER	CLIC DR	
					2009	2010
beam energy [GeV]	4	7	4.18	6.7	2.86	
slip factor η	0.000274	0.000188	0.00042	0.00040	6.5x10^{-5}	8x10^{-5}
rms momentum spread $\sigma_{\delta,rms}$ [%]	0.08	0.065	0.066	0.062	0.11	0.13
bunch population [10^9]	90	65	57.4	57.4	4.1	
circumference C [m]	3016	3016	1258	1258	493	421
bending radius ρ [m]	73.3	104.5	29.3 (13.75)	80.5 (165)	6.9	6.84
vert. beam pipe radius b [cm]	4.7	2.5	2.0	2.5	0.9	1.0
Stupakov-Heifets parameter Λ	**1864**	**2905**	**1254**	**2557**	**915**	**387**
ρ/b	**1560**	**4180**	**1465**	**3220**	**767**	**684**
σ_z [cm]	**0.6**	**0.5**	**0.5**	**0.5**	**0.1**	**0.16**
$\rho/(2\,\Lambda^{3/2})$ [cm]	**0.05**	**0.03**	**0.03**	**0.03**	**0.01**	**0.04**
$N_b r_0/((2\pi)^{1/2}\sigma_z\sigma_\delta\,\gamma)$	0.0027	0.0016	0.0024	0.0016	0.0007	0.0004
β_x at bend [m]	10?	10?	6?	2?	0.2	
ε_x [nm]	3.2	5.0	2.41	2.0	0.09	
σ_x at bend [μm]	179	224	120	63	4	
$\rho^{4/3}/(\sigma_x\beta_x)^{2/3}$	20800	28800	11230	137900	146600	144900
τ_x [ms]	37?	56?	44	29	1.62	1.88
C3	0.0128	0.0003	0.0042	0.0001	0.0052	0.0001
Q_s	-0.025	-0.025	-0.01	-0.01	-0.009	-0.0076
$N_{b,thr}$	1.0x10^{11}	1.9x10^{11}	5.5x10^{10}	6.7x10^{10}	5.7x10^{9}	1.2x10^{10}
$N_b/N_{b,thr}$	**0.89**	**0.35**	**1.04**	**0.85**	**0.72**	**0.33**
$\Lambda b/\rho$	**1.19**	**0.69**	**0.86**	**0.79**	**1.19**	**0.57**

2. CSR instability in the ATF Damping Ring – a possible experiment

Could one make the CSR instability appear in the ATF? The threshold strongly depends on the momentum spread through the parameter Λ. The relative momentum spread decreases in proportion to the beam energy. So does the bunch length if the RF voltage is varied in proportion to the beam energy. The geometric transverse emittance scales with the second power of energy, and the damping time with the inverse third power. Table 3 shows an example evaluation of CSR effects for the ATF damping ring at 1.00 GeV instead of 1.28 GeV. At this energy Λ clearly exceeds r/b so that instability is expected, provided the assumed horizontal emittance and longitudinal parameters could be preserved (or achieved) for 10^{10}

particles per bunch. Probably such experiment at the ATF Damping Ring would require operating on the coupling resonance to suppress the effect of intrabeam scattering, which might otherwise hide increases in energy spread and bunch length due to CSR.

The predictions for the onset of instability (in terms of ATF beam energy) are slightly different depending on which criterion we use. This is illustrated in Figure 1. In any case, lowering the beam energy of the ATF to 1.1 or 1.0 GeV should lead to CSR instability at a bunch population of 10^{10}, according to [1] and [4]. Interestingly, the ATF Damping Ring initially operated at 0.96 GeV beam energy, in 1997 [6].

Table 3

Beam and CSR-instability related parameters for the ATF DR at two different energies.

ATF damping ring	nominal	lower energy
beam energy [GeV]	**1.28**	**1.00**
slip factor η	0.0019	
rms momentum spread $\sigma_{\delta,\mathrm{rms}}$ [%]	0.06	0.047
bunch population [10^9]	10	10
circumference C [m]	138.6	
bending radius ρ [m]	5.73	
vert. beam pipe radius b [cm]	1.2?	
Stupakov-Heifets parameter Λ	**339**	**906**
ρ/b	**478**	
σ_z [cm]	**0.5**	**0.39**
$\rho/(2\,\Lambda^{3/2})$ [cm]	**0.05**	**0.011**
$N_b r_0/((2\pi)^{1/2}\sigma_z\sigma_\delta\,\gamma)$	0.0015	0.0031
β_x at bend [m]	3?	
ε_x [nm]	~1.5	0.9
σ_x at bend [μm]	67	52
$\rho^{4/3}/(\sigma_x\beta_x)^{2/3}$	2990	3540
τ_x [ms]	17.2	36.1
C3	0.0017	0.0141
Q_s	-0.0045	
$N_{b,thr}$	$1.15\mathrm{x}10^{10}$	$4.3\mathrm{x}10^{9}$
$N_b/N_{b,thr}$	0.86	**2.32**
$\Lambda b/\rho$	0.71	**1.90**

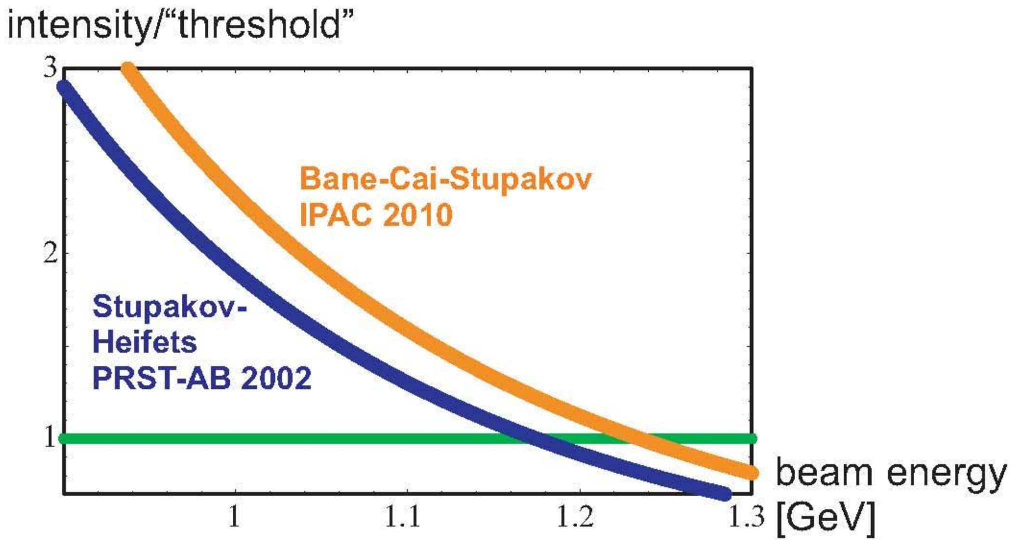

Figure 1 Ratio of bunch intensity over threshold for CSR instability according to two different theoretical formulations as a function of beam energy in the ATF Damping Ring. A nominal bunch intensity of 10^{10} electrons is assumed.

3. CSR stability for SuperKEKB Damping Ring designs

The parameters of the latest version of the SuperKEKB DR can be compared with those of an earlier version [7,8]. This is shown in Table 4. For the earlier version of the SuperKEKB DR all indicators and theories suggest that the CSR instability should occur. This is consistent with the finding of Ref. [7]. Remarkably this is one of the rare cases where the inequality (26) of [3], $C3 \geq 1$, is actually fulfilled. Therefore, we can be certain that the formalism of [1] and the result of [4] are valid for the old SuperKEKB DR and that this ring would indeed have been CSR-unstable, had it been constructed.

Table 4

Beam and CSR-instability related parameters for the present and an old version of the SuperKEKB damping ring.

	SuperKEKB e+ DR	SuperKEKB e+ DR OLD DESIGN
beam energy [GeV]	1.1	1.0
slip factor η	0.017	0.00343
rms momentum spread $\sigma_{\delta,\mathrm{rms}}$ [%]	0.055	0.054
bunch population [10^9]	37.5	37.5
circumference C [m]	135	135.5
bending radius ρ [m]	2.65	2.2
vert. beam pipe radius b [cm]	1.6	1.4?
Stupakov-Heifets parameter Λ	**67**	**430**
ρ/b	**166**	**129**
σ_z [cm]	**0.7**	**0.51**
$\rho/(2\,\Lambda^{3/2})$ [cm]	**0.24**	**0.012**
$N_b r_0/((2\pi)^{1/2}\sigma_z\sigma_\delta\,\gamma)$	0.0051	0.078
β_x at bend [m]	1.5	1.5?
ε_x [nm]	2100→ 41	2100→ 41
σ_x at bend [μm]	248	248?
$\rho^{4/3}/(\sigma_x\beta_x)^{2/3}$	710	553
τ_x [ms]	11	11
C3	0.0033	7.68??
Q_s	-0.015	-0.00788
$N_{b,thr}$	**$4.5\mathrm{x}10^{10}$**	**$1.1\mathrm{x}10^{10}$**
$N_b/N_{b,thr}$	0.83	**3.27**
$\Lambda b/\rho$	0.40	**3.32**

4. Instability threshold in the single-mode regime

For all other cases, according to [3] in the single-mode driven case Landau damping can be neglected and there is always an instability if (rewriting the condition "$|\mu|>\eta\omega_0\delta_0$" below Eq. (23) in Ref. [4])

$$C3b \equiv \frac{\left(\dfrac{r_e N_b 2a}{\gamma\sqrt{a\rho\sigma_z}}\right)^{1/3}}{\left(\eta\pi\sqrt{\dfrac{\rho}{a}}\sigma_\delta\right)} >> 1.$$

This condition appears to be easily fulfilled for all storage rings considered here, in particular for the SuperKEKB and SuperB HER & LER rings, for both versions of the CLIC DR and for the old and new versions of the SuperKEKB Damping Ring (in case of the old SuperKEKB DR design, however, this single-mode theory does not apply, since its parameters lie in the mode-overlap regime), which would imply that the HER and LER rings for SuperKEB and SuperB, the two CLIC DR examples, and the new SuperKEKB damping ring are CSR unstable, despite opposite predictions from [1] and [4]. The condition "C3b>>1" is also fulfilled for the ATF Damping Ring at either 1.28 or 1.0 GeV, as is illustrated in Table 5.

Table 5

Condition "C3b" evaluated for various storage rings. If C3b <1 the beam could be stable in the single-mode model, which is not the case for any of the rings considered.

	SuperKEKB LER	SuperKEKB HER	SuperKEKB DR	ATF	ATF at 1 GeV	SuperB LER	SuperB HER	CLIC DR	
								2009	2010
C3b	**239**	**175**	**27**	**75**	**113**	**177**	**100**	**3851**	**540**

5. Benchmarking against other existing storage rings: SLC DR, KEKB HER and LER

Table 6 presents an assessment for yet another three rings, namely the SLC Damping Ring and the present KEKB LER and HER. For the SLC Damping Ring the Stupakov-Heifets parameter Λ is smaller than ρ/b. The threshold according to the 2010 Bane-Cai-Stupakov paper is two times above the design bunch intensity. Stability is expected from these two criteria. However, according to the 2003 paper [3] the SLC DR should have operated in the single-mode instability regime (C3<1), and since C3b>1 the single mode instability would not have been Landau damped. Indeed the SLC rings have been plagued longitudinal "microwave" instabilities [9], to which CSR might have contributed. The present KEKB LER could be unstable according to the 2002 Heifets-Stupakov paper [1]. It also represents the only other case (together with the old SuperKEKB DR design) where C3>1 and therefore the formalism of the 2002 paper should indeed apply. There has been evidence for CSR adding to the longitudinal impedance in this ring [10].

Table 6

Beam and CSR-instability related parameters for the SLC DR and for the present KEKB LER and HER.

	SLC DR	KEKB LER	KEKB HER
beam energy [GeV]	1.19	3.5	8.0
slip factor η	0.0147	0.00033	0.00034
rms momentum spread $\sigma_{\delta,rms}$ [%]	0.09	0.073	0.067
bunch population [10^9]	40	65	47
circumference C [m]	35.28	3016	3016
bending radius ρ [m]	2.04	15.5	104
vert. beam pipe radius b [cm]	1	4.7	2.5
Stupakov-Heifets parameter Λ	**90**	**389**	**952**
ρ/b	**204**	**330**	**4160**
σ_z [cm]	**0.65**	**0.5**	**0.5**
$\rho/(2\Lambda^{3/2})$ [cm]	**0.12**	**0.10**	**0.18**
$N_b r_0/((2\pi)^{1/2}\sigma_z\sigma_\delta\,\gamma)$	0.0033	0.0039	0.001
β_x at bend [m]	1.5	15	15
ε_x [nm]	15	18	24
σ_x at bend [μm]	150	520	600
$\rho^{4/3}/(\sigma_x\beta_x)^{2/3}$	699	983	11300
τ_x [ms]	3.7	90	45
C3	0.0001	**1.5**	4x10^{-6}
Q_s	-0.012	-0.025	-0.021
$N_{b,thr}$	**8.7x10^{10}**	**7.0x10^{10}**	**1.8x10^{11}**
$N_b/N_{b,thr}$	**0.46**	**0.92**	**0.26**
$\Lambda b/\rho$	**0.44**	**1.18**	**0.23**
C3b	**17**	**609**	**81**

6. Conclusions

Following their 2003 paper [3], the 2002 formulae from Stupakov & Heifets [1] might not be applicable in most of the cases considered, the old SuperKEKB DR design and the present KEKB LER being the sole exceptions where the standard theory based on continuous modes would apply.

It is unclear if, but unlikely that, the IPAC10 result from Bane, Cai and Stupakov [4] is applicable to those machines for which the condition C3≥1 of [3] is not fulfilled. However, if we apply their results regardless, the SuperKEKB LER, HER and DR are predicted to be stable with respect to CSR, while the SuperB LER might be marginally unstable (but only 4% above threshold).

For most machines the 2003 condition C3≥1 [3] is not fulfilled, so that they should operate in the single-mode CSR regime. In all these cases (the SuperKEKB and SuperB HER and LER, both CLIC DR designs, the ATF, the KEKB HER, and the new design of the SuperKEKB DR) the beam is predicted to be unstable (C3b>>1) without threshold and with negligible Landau damping according to [3]. As stated before, for two other machines the 2003 condition is fulfilled so that the standard theories and results of [1] and [4] should apply, namely for the present KEKB LER and for the old design of the SuperKEKB DR. The latter is predicted to be clearly unstable according to all theories applied. The present KEKB LER could be unstable from the 2002 formulae [1], but it should be 8% below the CSR instability threshold from the result of [4]. In any case the KEKB LER has been operating close to the CSR stability limit.

We can safely conclude that the old SuperKEB DR design has clearly been unstable, and that the ATF DR should be unstable for beam energies below 1 GeV. For all other accelerators the various formulae and theories lead to conflicting predictions. Trusting [1] and [4], we would expect that the new SuperKEKB DR design, the CLIC DR, the SLC DR and the ATF at 1.28 GeV are stable with regard to CSR.

According to all theories and papers examined here, the CSR instability could be studied in the ATF Damping Ring by lowering the ATF ring energy from 1.28 GeV to 1.0 GeV or below.

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