EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

ON MUON CAPTURE

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In this lecture I will briefly speak about some experimental preparations that are being made in order to improve our knowledge of the capture process

$$\mu^- + p \rightarrow N + \nu \qquad (1)$$

with the muon at rest.

At present the experimental situation is almost the same as it was in 1969; many of the things that I am going to say now, are in a series of lectures that I gave in 1969 at the SIN summer school in Leysin[1].

Let me summarize this situation.

The different experimental results obtained for process (1) are presented in Table 1 (taken from Ref. 2).

Before giving the conclusions drawn from the values of Table 1, it should be remembered that, so far, process (1) has been observed either with the muon bound to a proton in an atomic system μp (stopping negative muon in pure hydrogen gas), or with the muon bound in a molecular ion $p\mu p$*) (stopping negative muons in a pure liquid-hydrogen target). There are two possible independent values for the rate of process (1); the triplet capture rate Λ_t and the singlet capture rate Λ_s obtained, respectively, when the total spin of the μp initial system in process (1) is 1 (triplet) or 0 (singlet). In Table 1 Λ_{om} is the capture rate measured with the muon bound in an ortho-state molecule $(p\mu p)_o$; such a rate in terms of Λ_s and Λ_t is given by

$$\Lambda_{om} = \gamma_o\left[\frac{3}{4}\Lambda_s + \frac{1}{4}\Lambda_t\right], \qquad (2)$$

where γ_o is a parameter that is now well known[1]. The results for Λ_{om} are obtained by stopping negative muons in a pure liquid-hydrogen target and observing the escaping neutron [see formula (1)] only after a fraction of a microsecond: the capture rate Λ_{bc} is obtained by using a bubble chamber technique, and in principle it differs from Λ_{om} because one has also to take into account the fact that, before the muon is bound into a $p\mu p$ molecule, it spends some time in a μp atom (about 300 nsec). In interpreting or analysing the data of Table 1, the following assumptions are generally made:

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*) The $p\mu p$ molecule has only two orbital bound states: a para ground-state with rotational angular momentum $L = 0$, and an ortho state with $L = 1$; the energy difference between the two states is 148 eV.

i) the negative muons stopped in a hydrogen-gas target (say at 8 atm) will very quickly form the stable μp system (a few tens of nanoseconds) in the singlet state;

ii) the negative muon stopped in a (pure) liquid-hydrogen target will, after a certain time, form a $p\mu p$ molecule *only* in the ortho state $(p\mu p)_o$;

iii) the lifetime for the ortho $\to$ para transition of the $p\mu p$ molecule τ_{op} is much longer than the muon lifetime (2.2 μsec).

Assumptions (ii) and (iii) are rather crucial since the capture rate Λ_{pm} from a molecule in the para state $(p\mu p)_p$ is given by

$$\Lambda_{pm} = \gamma_p\left[\frac{3}{4}\Lambda_t + \frac{1}{4}\Lambda_s\right], \tag{3}$$

which can be quite different from Eq. (2).

Assumption (i) has been checked experimentally; we will come back to this question later.

Assumption (ii), according to the experts, is a quite safe assumption; however, it has not been experimentally verified.

Assumption (iii) has been partially checked experimentally; indeed, from the experimental results, we know that $\tau_{op} > 20$ μsec. Simple calculations also set a similar limit.

Finally, it should be noted that so far all measurements have been done by detecting the escaping neutron in process (1); therefore all experimental results rely on the possibility of being able to predict correctly the efficiency of the neutron-detecting device. With all these assumptions, it is now possible to condense the experimental results of Table 1, as shown in Fig. 1, where α is the fraction of the triplet state contained in the initial μp system for each experiment.

To continue, let us accept, as is currently done, that the interaction Hamiltonian H describing process (1) is a pure (V, A) type, and that second-class currents are absent; in this case, to write H we still need at least four parameters: g_V^μ, g_A^μ, g_P^μ, g_M^μ, called respectively the polar vector, the axial vector, the induced pseudoscalar, and the weak magnetic coupling constants.

After having said this, let me now give you one way of proceeding further with the data presented in Table 1.

a) Let us assume first for $(g_M/g_V)^\mu$ the value suggested from the CVC hypothesis. In this case it can be shown that, writing

$$r = \frac{g_A^\mu}{g_V^\mu} ; \qquad f = \frac{g_P^\mu}{g_A^\mu} , \tag{4}$$

the data of Fig. 1 necessarily imply, *irrespective* of g_V^μ and for *any* choice of the parameters r and f, that the (V, A) interaction *is* of the type (V − βA) with β *positive* and obeying the limitation $0 < \beta < 2$. It is important to realize that this result has been obtained using only data from hydrogen, and is therefore without complications arising from nuclear physics.

b) Then let us assume that from the experimentally found branching ratio BR, with

$$BR = \frac{\pi \to e\nu}{\pi \to \mu\nu} , \tag{5}$$

we can deduce the equality between the absolute values

$$\left| g_A^e \right| = \left| g_A^\mu \right| . \tag{6}$$

Then it is possible to analyse the data of Fig. 1 in terms of only the two ratios: g_A^μ/g_V^μ, g_P^μ/g_V^μ; if we do this we find that the data of Table 1 define the dashed region of Fig. 2 which, to my mind, synthesizes with a maximum of generality the consequences of the data of Table 1.

c) To continue, let us assume the equality

$$g_A^\mu/g_V^\mu = g_A^e/g_V^e = -1.23 \pm 0.01 , \tag{7}$$

the numerical value being taken from the results of *the neutron-beta decay*.

In this case, from the data of Fig. 1 we get

$$g_P^\mu/g_V^\mu = 12.2 \pm 2.0 . \tag{8}$$

This value is in good agreement with that obtained by calculations assuming the OPED or the PCAC scheme.

From the general analysis presented, it appears that the data show that Λ_s is quite smaller than Λ_t; in particular, assuming (7), we get (see Table 1):

$$F = \frac{\Lambda_t}{\Lambda_s} \simeq 2.3\% \ . \qquad (9)$$

Figure 2 also shows how important it is to be able to measure in process (1) the two independent quantities Λ_s, Λ_t; it would thus be possible to obtain simultaneously information on g_A^μ/g_V^μ and g_P^μ/g_V^μ without making use of quantities inferred from beta-decay processes.

At this point we are in a position to discuss some experiments that I know of that are being planned or that are now going on, in different laboratories, in order to improve our knowledge of process (1). The planned measurements can be divided into the following categories according to their aim:

i) Experiments planned in order to check some of the assumptions usually made for interpreting data similar to those of Table 1.

ii) Experiments planned in order to obtain a more precise value of Λ_s (directly or through Λ_{om}), improving both the statistical and the systematic errors.

iii) Experiments planned in order to arrive, in the near future, at a direct measurement of the capture rate Λ_t.

Let us first consider the Saclay-CERN Collaboration experiment (Duclos et al.) done at the Saclay electron linear accelerator (600 MeV). The aim of this group is to measure the rate Λ_{om} (and deduce Λ_s), stopping negative muons in a pure liquid-hydrogen target (protonium). The novelty here is to measure Λ_{om} by comparing, at a level of a few 10^{-5}, the μ^- disappearance rate λ_- with the μ^+ disappearance rate λ_+ detecting the muon-decaying electron; apart from small corrections due to the fact that the negative muon is in a bound state, assuming that the TPC theorem is valid, the difference $D = \lambda_- - \lambda_+$ gives directly Λ_{om}. Obviously the difficulty here is that D/λ_+ is of the order of 10^{-3}.

The electron linear accelerator, which can supply muon bunches 2-3 μsec long, is particularly suitable for this type of measurement since the electrons can be detected from the muon decay, after the muon bunch. The liquid-hydrogen target set-up is similar to the one used in the first measurement of Λ_{om}: the aim is to obtain Λ_{om} with a precision 2-3 times better than the one now available. Moreover, since with this method the capture rate is measured without the detection of the escaping neutron, in process (1), this measurement offers a check on some types of systematic errors that are always present in the case when the neutron is detected.

The second aim of this experiment is to check the assumption that τ_{op} is much larger than the muon lifetime. This is done by detecting the neutrons emitted in process (1) (the absolute efficiency for neutron detection is not needed here) and measuring their time distribution. In fact it is possible to show that if the pμp ortho molecule decays into a para pμp molecule at a rate $1/\tau_{op}$, the yield $Y_n(t)$ of neutrons emitted because of the capture process (1) is no longer an exponential but is given by

$$Y_n(t) \propto \exp(-\lambda_0 t) \left[(\Lambda_{om} - \Lambda_{pm}) \exp(-t/\tau_{op}) + \Lambda_{pm} \right] , \qquad (10)$$

where $\lambda_0 = 4.5 \times 10^5 \text{ sec}^{-1}$ in the decay rate of the free muon. With this measurement it is hoped to prove that $\tau_{op} > 70$ μsec.

A second experiment to measure Λ_s (stopping negative muons in a pure hydrogen target) is planned at SIN by a Vienna-SIN Collaboration (Breunlich et al.): actually, one of the main aims of this collaboration is to measure the neutron-neutron S-wave scattering length $a_{n,n}$ by looking at the neutrons coming out when a negative muon is captured by a deuteron. Experimentally they proceed in a more "classical" way in the sense that the identification of the capture process (1) is made by detecting the escaping 5.2 MeV neutron.

It should be added that the group is extremely qualified in neutron detection techniques as they have worked with neutrons for a long time, and having confidence in their superior "expertise" with neutron detectors and the high muon fluxes available at SIN, they hope to improve the knowledge of Λ_s.

I now wish to speak about an interesting experiment being carried out at CERN by a Bologna University Group (Bertin et al.).

It has already been said that by stopping negative muons in hydrogen at low density (e.g. 8 atm, 300 K) one gets only muonic atoms μp (the molecular formation pμp being too slow). Now it happens that the μp's initially formed in a statistical mixture of triplet and singlet (3/4 triplet and 1/4 singlet) will, because of the collisions with the neighbouring hydrogen molecules, in a time $\bar{t}$ (hopefully short) become all singlet. The length of $\bar{t}$ will depend on the cross-section for the process

$$(\mu p)_{F_1} + H_2 \rightarrow (\mu p)_{F_2} + H_2 , \tag{11}$$

where F_1 and F_2 are the two possible spin states 1, 0 of the 1S state atom μp *); moreover, it will also depend on the hydrogen pressure and on the energy E_i with which the μp is initially formed.

In order to measure the cross-section for process (11), muons are stopped in a hydrogen-gas target in which many layers of thin aluminium foils are placed; the μp atom, diffusing throughout the hydrogen, may eventually reach the surface of one of the aluminium foils, where the following transfer process will then occur:

$$\mu^- p + Al \rightarrow (\mu Al)^* \rightarrow (\mu Al) + X\ (\simeq 300\ keV) . \tag{12}$$

The detection of the hard, typical X-ray emitted in process (12) will give the time of arrival of the muon at the aluminium surface. The distribution time of these X-rays will depend (apart from the muon lifetime) on σ_{F_1,F_2} and on E_i; therefore the experiment can give information about these quantities.

From the theory we know that $\sigma_{0,0}$ is much smaller than $\sigma_{1,0}$ (by about 60 times); therefore the distribution time of the X-rays from process (12) could tell us how long it takes, in the hydrogen-gas target, for the μp system to become a singlet state. In a previous experiment, a CERN-Bologna Collaboration found that the experimental cross-section was compatible with the theoretical value $\sigma_{0,0}$, showing that in a very short time (less than 100 nsec) the atomic system μp formed in a 20 atm hydrogen gas target was all transformed into a singlet system. However, in an earlier experiment a group from Dubna, using a diffusion chamber technique, found the cross-section to be compatible with the theoretical value of $\sigma_{1,0}$. This new measurement will clarify the situation on this point and, moreover, should be able to give better information regarding E_i. The measurements will be performed down to a very low value of the hydrogen pressure (it is hoped to reach 1 atm). It has to be remarked that a knowledge of these parameters ($\sigma_{0,0}$, $\sigma_{1,0}$ and E_i) is not only necessary in order to clarify the situation for Λ_s, but also to prepare the ground for the measurement of Λ_t in the near future.

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*) The difference between the hyperfine states is 0.18 eV, i.e. much bigger than the average thermal energy at room temperature.

And now we come to the last part of this lecture, which is to present to you some planned attempt to measure Λ_t. The method used so far is the one described in the lecture given at Leysin in 1969[1].

It can safely be shown that at a sufficiently low pressure P_0, the μp atoms will be in a statistical mixture of their hyperfine states of singlet and triplet. In this condition the capture rate Λ_{st} will be

$$\Lambda_{st} = \frac{3}{4}\Lambda_t + \frac{1}{4}\Lambda_s , \tag{13}$$

whereas, as we have seen, at a few atm we measure essentially Λ_s. Therefore comparing the experimental capture rate obtained at different pressures (starting from P_0 up to a few atm), it is possible to infer the ratio (9) from the ratio

$$\frac{\Lambda_{st}}{\Lambda_s} = \frac{3}{4}\frac{\Lambda_t}{\Lambda_s} + \frac{1}{4} . \tag{14}$$

It is clear, however, that to guess the pressure P_0 correctly, we have to have some information on σ_{F_1,F_2} and E_i; which is why, as emphasized by Bertin et al., the results that will be obtained by the Bologna Group will be very important for the planning and interpretation of some of these experiments.

A Columbia University group at Nevis, led by Prof. S.C. Wu, is planning an experiment to measure the capture rate (1) stopping negative muons in a hydrogen-gas target in which the pressure can be varied over a very wide range. The capture process will be identified by the detection of the outgoing 5.2 MeV neutron: at present, I think, they are waiting for the cyclotron to deliver stronger muon beams.

Another group, at SIN (Hofer et al.), is seriously considering the measurement of the ratio (14): here also, the detection of the capture process will be done by looking at the outgoing neutron in process (1). What makes their plans so very hopeful, is that this group has succeeded in using a new technique which has enabled them to stop negative muons in a hydrogen-gas target at a pressure as low as a few mm of Hg, at a rate not much lower than what they have achieved with a hydrogen target at 1 atm. This is the famous magnetic muon bottle; I think that during this School we will hear directly from one of the authors. Certainly, at that small pressure, the μp systems formed must be in a statistical mixture of triplet and singlet.

These are the plans for the measurement of Λ_t that I know of. Before closing the lecture, however, I would like to speak again about the Saclay-CERN Collaboration experiment (Duclos et al.). We have seen that by looking at the neutron yield $Y_n(t)$ and at the electron yield $Y_e(t)$ (the muons being stopped in a pure liquid-hydrogen target), this group hopes to raise the known experimental limit on τ_{op} to a value much higher than 20 μsec.

What happens if an unexpected situation arises? For instance, if the value is found to be near 20 μsec? The following two things can be said:

i) The theoretical value for Λ_{om} would have to change; for instance, we would have to accept as the prediction for Λ_{om} the value put in parentheses in Table 1 for the case τ_{op} = 20 μsec.

ii) Since the ratio F is considered to be very much smaller than 1, then from Eqs. (2), (3), (9), and (10) we obtain for the neutron yield $Y_n(t)$:

$$Y_n(t) \propto \Lambda_{op}\left\{\exp(-\lambda_0 t) + 2(1 - 4F)\exp\left[-(1/\tau_{op} + \lambda_0)t\right]\right\} \qquad (15)$$

(all capture rates have been dropped from the exponentials in the first approximation).

So (if τ_{op} is sufficiently small), we can imagine fitting the experimental data to (15) in order to deduce the coefficient 2(1 - 4F): if we have sufficient statistics to deduce this coefficient to better than, say, 5%, this means that we can deduce from the data a value for the ratio F.

REFERENCES

1) E. Zavattini, Muon capture in hydrogen and deuterium, Proc. of the SIN Sommerschule, Leysin, 1969.

2) See, for example, E. Zavattini, Muon capture, *in* Muon physics (Academic Press Inc., NY, 1975), Vol. II, p. 219.

Table 1

Summary of experimental results on muon-capture rate measurements in hydrogen and their latest theoretical predictions [a)]

Authors	Experimental values (sec^{-1})			Theoretical values (sec^{-1})					
	Λ_s	Λ_{om}	Λ_{bc}	Λ_s	Λ_t	Λ_{om}	Λ_{bc}	Λ_{pm}	$\Lambda_{st} = \frac{3}{4}\Lambda_t + \frac{1}{4}\Lambda_s$
Chicago [b)]			428 ± 85			522 ± 10 (506)			
Columbia I [c)]		515 ± 85					493 ± 4 (461)		
CERN-Bologna [d)]			450 ± 50			522 ± 10 (506)			
Columbia II [e)]		464 ± 42					493 ± 4 (461)		
CERN-Bologna II [f)]	651 ± 57			650 ± 2	14.82 ± 0.05			198	174
Dubna [g)]	686 ± 88								

a) The theoretical values reported here are taken from Pascual (1969). The value assumed for the ratio (g_A/g_V) is the one obtained by Christensen et al. (1967). The values in parentheses are the expected rates assuming for τ_{op} the superior limit (see text).

b) J.H. Doede and R. Hildebrand, cited by Rubbia (1963); see Bertolini et al. (d).

c) Bleser et al., Phys. Rev. 132, 2679 (1962).

d) Bertolini et al. (1962), cited by Rubbia *in* Proc. Internat. Conf. on High-Energy Physics, CERN, 1962 (CERN, Geneva, 1962), p. 421.

e) Rothberg et al., Phys. Rev. 132, 2664 (1963).

f) Alberigi Quaranta et al., Phys. Rev. 177, 2118 (1969).

g) Bystristky et al., Dubna Preprint-DI-7300 (1973).

Figure captions

Fig. 1 : Experimental results of Table 1 plotted as a function of the fraction α of the triplet state present in the initial μp system. The solid line is the expected value assuming equality (7), and for g_P^μ/g_V^μ the expected value from OPED.

Fig. 2 : Region for the pair g_A^μ/g_V^μ, g_P^μ/g_V^μ allowed by the observed value of the capture rate of Table 1. The arrow indicates values (7).

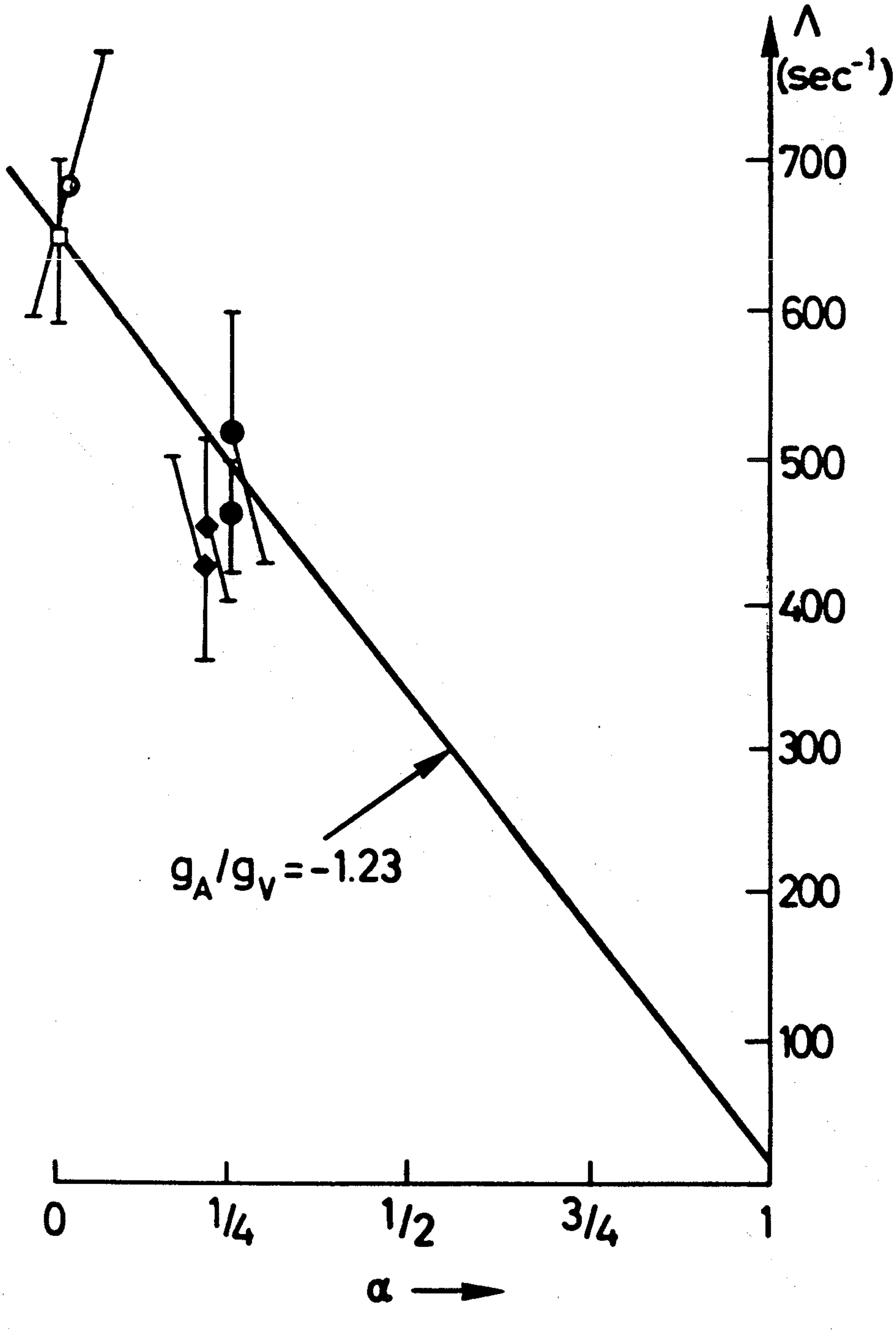

Fig. 1

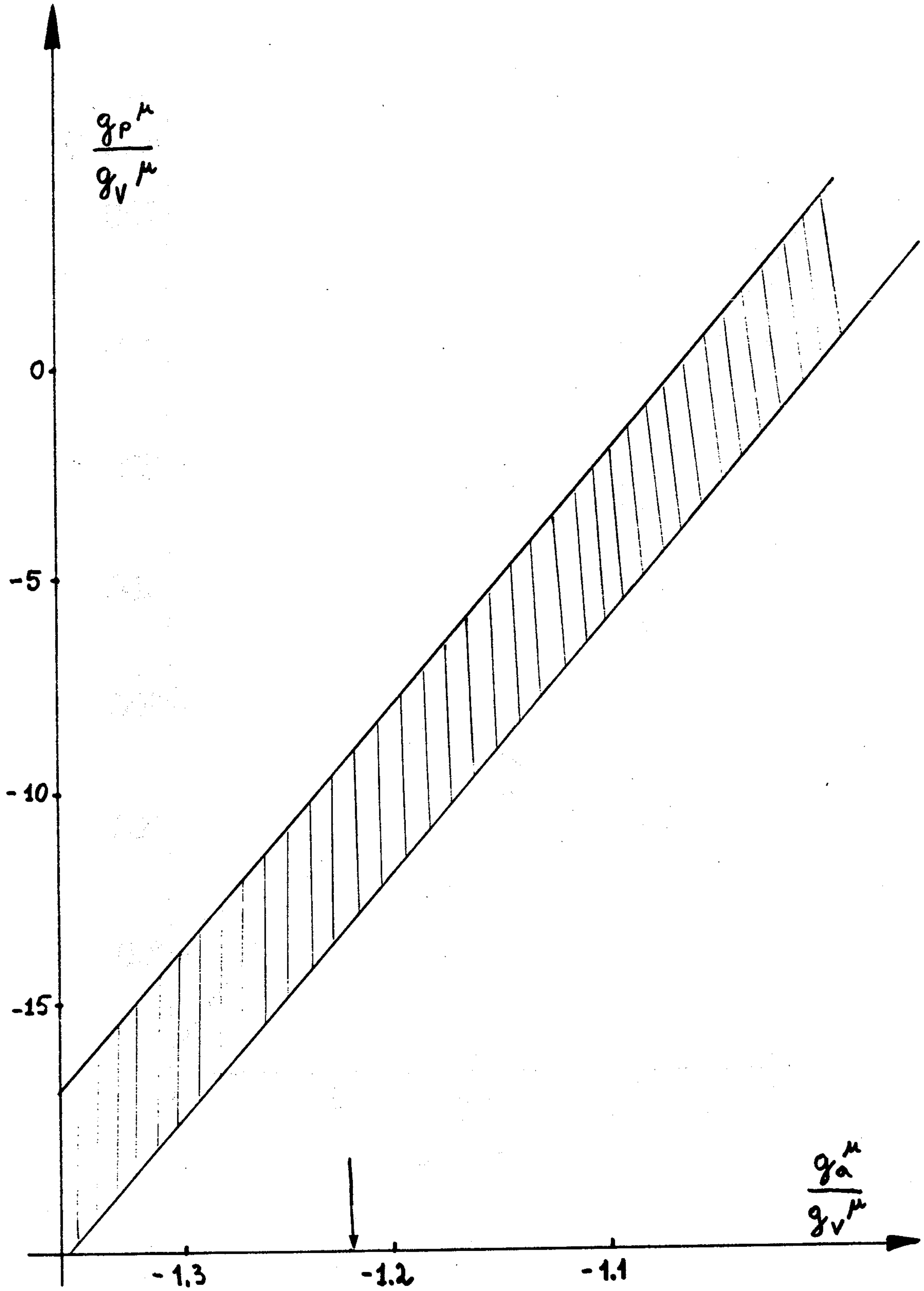

Fig. 2