

## “Strongly” Interacting Mirror Dark Matter

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### Abstract

Mirror baryons, being invisible in terms of ordinary photons, could constitute a viable dark matter candidate. If mirror baryons constitute at least a reasonable fraction of dark matter, the mirror world is dominated by helium. It is pretty possible that ordinary and mirror particles have common forces mediated by the gauge boson of some additional symmetry group. The immediate implication of such kind of interaction would be the mixing of neutral ordinary bosons and their counterparts. Exploiting the one boson exchange concept of strong interactions we calculate a possible event rate, which could be induced by mirror helium populating the halo through the meson mirror- meson oscillations exchange.

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# 1 Introduction

The old hypothesis [1] that there may exist a mirror world, a hidden parallel sector of particles and interactions which is the exact duplicate of our observable world, has attracted a significant interest over the past years in view of interesting implications for the particle physics and cosmology (see e.g. [2, 3] for reviews). Such a theory is based on the product  $G \times G'$  of two identical gauge factors with identical particle contents, where ordinary (O) particles belonging to  $G$  are singlets of  $G'$ , and mirror (M) particles belonging to  $G'$  are singlets of  $G$ . Mirror parity under the proper interchange of  $G \leftrightarrow G'$  and the respective matter fields [4] renders the Lagrangians of two sectors identical. The two worlds can be viewed as parallel branes in a higher dimensional space, with O-particles localized on one brane and the M-particles on another brane, while gravity propagates in the bulk. Such a setup can be realized in the string theory context.

Besides gravity, two sectors could communicate by other means. In particular, any neutral O-particle, elementary or composite, could have a mixing with its M-twin. E.g. kinetic mixing of ordinary and mirror photons [5], mass mixing between ordinary and mirror neutrinos [6] and neutrons [7, 8], ordinary mesons with neutral mesons [?], etc. Such mixings may be induced by effective interactions between O- and M-particles mediated by messengers between two sectors, e.g. gauge singlets or some extra gauge bosons interacting with both sectors [9].

If the mirror sector exists, then the Universe should contain along with the ordinary photons, electrons, nucleons etc., also their mirror partners with exactly the same masses and exactly the same microphysics. However, two sectors must have different cosmological evolutions: in particular, they never had to be in equilibrium with each other. In fact, the BBN constraints require that M-sector must have smaller temperature than O-sector,  $T' < T$ . In this way, the contribution of mirror degrees of freedom to the Hubble expansion rate, equivalent to an effective number of extra neutrinos  $\Delta N_\nu = 6.14 \cdot x^4$ , where  $x = T'/T$ , can be rendered small enough. E.g. the bound  $\Delta N_\nu < 0.4$  implies  $x < 0.5$ , and for  $x = 0.3$  we have  $\Delta N_\nu \simeq 0.05$ . This can be achieved by demanding that [10]:

(A) at the end of inflation the O- and M-sectors are (re)heated in a non-symmetric way,  $T_R > T'_R$ , which can naturally occur in the context of certain inflationary models;

(B) after (re)heating, at  $T < T_R$ , the possible particle processes between O- and M-sectors are too slow to establish equilibrium between them, so that both systems evolve adiabatically and the temperature asymmetry  $T'/T$  remains nearly constant in all subsequent epochs until the present days. Therefore, although the two sectors have the same microphysics, the cosmology of the early mirror world can be very different from the standard one as far as the crucial epochs like baryogenesis, nucleosynthesis, etc. are concerned. Any of these epochs is related to an instant when the rate of the relevant particle process  $\Gamma(T)$ , which is generically a function of the temperature, becomes equal to the Hubble expansion rate  $H(T)$ . Obviously, in the M-sector these events take place earlier than in the O-sector, and as a rule, the relevant processes in the former freeze out at larger

temperatures than in the latter.

Mirror baryons, being invisible in terms of the ordinary photons, could constitute a viable dark matter candidate [10] and this possibility could shed a new light to the baryon and dark matter coincidence problem. First, the M-baryons have the same mass as the ordinary ones,  $m'_B = m_B$ . And second, the unified mechanism can be envisaged which generates the comparable baryon asymmetries in both O- and M-sectors, via  $B-L$  violating scattering processes that transform the ordinary particles into the mirror ones. It is natural that these processes violate also CP due to complex coupling constants. And finally, their departure from equilibrium is already implied by the above condition (B). Therefore, all three Sakharov's conditions for baryogenesis can be naturally satisfied. In addition, the mirror baryon density can be generated somewhat bigger than that of ordinary baryons,  $n'_B \geq n_B$ , since the mirror sector is cooler than the ordinary one and hence the out-of-equilibrium conditions should be better fulfilled there.

An intriguing possibility is related to the fact that neutral ordinary particles, elementary or composite, can have a mixing with its mirror counterparts. In particular, photon could have kinetic mixing with mirror photon, neutrons could mix with mirror neutrons, ordinary mesons with neutral mesons, etc. Such mixing can be induced by an effective interaction between O- and M-fields mediated by some heavy gauge singlet particles, or heavy bosons interacting with both sectors.

## 2 BBN in mirror sector and mirror dark matter content

It can also be shown that the BBN epoch in the mirror world proceeds differently from the ordinary one, and it predicts different abundancies of primordial elements [10]. It is well known that primordial abundancies of the light elements depend on the baryon to photon density ratio  $\eta = n_B/n_\gamma$ , and the observational data well agree with the WMAP result  $\eta \simeq 6 \times 10^{-10}$ . As far as  $T' \ll T$ , the universe expansion rate at the ordinary BBN epoch ( $T \sim 1$  MeV) is determined by the O-matter density itself, and thus for the ordinary observer it would be very difficult to detect the contribution of M-sector: the latter is equivalent to  $\Delta N_\nu \approx 6.14x^4$  and hence it is negligible for  $x \ll 1$ . As for nucleosynthesis epoch in M-sector, the contribution of O-world instead is dramatic: it is equivalent to  $\Delta N'_\nu \approx 6.14x^{-4} \gg 1$ . Therefore, mirror observer which measures the abundancies of mirror light elements should immediately observe discrepancy between the universe expansion rate and the M-matter density at his BBN epoch ( $T' \sim 1$  MeV) as far as the former is determined by O-matter density which is invisible for the mirror observer. The result for mirror  ${}^4\text{He}$  also depends on the mirror baryon to photon density ratio  $\eta' = n'_B/n'_\gamma$ . Recalling that  $\eta' = (\beta/x^3)\eta$ , we see that  $\eta' \gg \eta$  unless  $\beta = n'_B/n_B \ll x^3$ . However, if  $\beta > 1$ , we expect that mirror helium mass fraction  $Y'_4$  would be considerably larger than the observable  $Y_4 \simeq 0.24$ . Namely, direct calculations show that for  $x$  varying

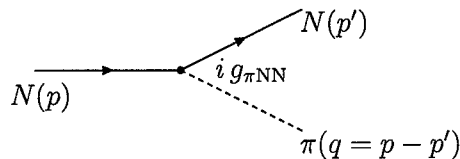
from 0.6 to 0.1,  $Y'_4$  would varie in the range  $Y'_4 = 0.5 - 0.8$ . Therefore, if M-baryons constitute dark matter or at least its reasonable fraction, the M-world is dominantly helium world while the heavier elements can also present with significant abundances.

The ‘helium’ nature of the mirror universe should have a strong impact on the processes of the star formation and evolution in the mirror sector [?].

### 3 Direct detection of "strongly" interacting mirror nuclei

#### 3.1 Meson - mirror meson occillating cross section

In 1935 Yukawa proposed [13] that the force which acts between two nucleons may be produced by exchange of mesons. This idea extended naturally the quantum electrodynamics, which describes the force acting between charged particles in terms of the exchange of photons. A diagrammatic representation of the the vertex of meson exchange is shown in figure bellow



and can be associated with the following effective Lagrangian

$$\Delta\mathcal{L} = i g_{\pi NN} \pi^\alpha \bar{N} \gamma_5 \sigma_\alpha N. \quad (1)$$

Here  $g_{\pi NN} = 13.5$  is the strong coupling constant at zero momentum transferred,  $N$  is the SU(2) doublet isospin  $(p, n)$ ,  $\pi^\alpha$  ( $\alpha = 3$  so that  $\pi^3 = \pi^0$ ) is the meson field. This implies that one meson is transferred between two interacting nucleons. The intermediate state in the process of nuclon- nuclon interaction contains an extra meson as opposed to the initial and final state. This situation is of course permitted in quantum mechanics because of the uncertainty prinsiple. However, this state can only exist for a finit time interval compatible with the energy uncertainty regulated by the pion mass.

If we assume a mixing between the ordinary pions  $\pi$  and their mirror counterparts  $\pi'$ , one can count on a possible impact on results of direct detect experiments if the mirror dark matter is the infered non-baryonic dark matted in the Universe. Indeed, let us consider the possible pion - mirror pion mixing due to a small mass term  $\mu^2 (\bar{\pi}\pi' + \bar{\pi}'\pi)$  where  $\mu^2 = 2 m_\pi \delta m$ . In low-energy QCD the pion is created by the axial current  $J_\mu^5 = \bar{q} \gamma_\mu \gamma^5 (\sigma_3/2) q$ , where  $q = (u, d)$ . We can parametrize this matrix element between the vacuum state  $|0\rangle$  and the pion state  $|\pi(p)\rangle$ , by writing

$$\langle \pi(p) | J_\mu^5(x) | 0 \rangle = i f_\pi p_\mu e^{i p_\mu x^\mu} \quad (2)$$

where  $f_\pi \approx 93$  MeV is the pion decay constant, which can be determined from the rate of  $\pi^\pm$  decay through the weak interaction [15]. Therefore for  $\pi - \pi'$  mixing we have the following low-energy vertex

$$\begin{array}{ccc} \pi(q) & & \pi'(q) \\ \text{-----} & \times & \text{-----} \\ & \mu_\pi^2 & \end{array}$$

$$\mu^2 = \frac{1}{\mathcal{M}^2} \langle \pi | J^{\mu 5} | 0 \rangle \langle 0 | J'_\mu{}^5 | \pi' \rangle = \frac{f_\pi f'_\pi m_\pi m'_\pi}{\mathcal{M}^2}, \quad (3)$$

where  $\mathcal{M}$  is the common (mirror-ordinary world) gauge boson mass. Considering a symmetric mirror sector we expect that the decay constants and the pion masses are degenerate in both sectors and thus we obtain the following estimation for the mass mixing term between  $\pi$  and  $\pi'$

$$\delta m \simeq \left( \frac{10 \text{ TeV}}{\mathcal{M}} \right)^2 5.6 \times 10^{-3} \text{ eV}. \quad (4)$$

Once the  $\pi - \pi'$  oscillation phenomena is quantified, one can consider the process of one  $\pi - \pi'$  exchange between an ordinary  $N$  and mirror  $N'$  nuclon. The effective lagrangian can be parametrized as follows

$$\mathcal{L}_{\text{eff}}^\pi = g_{\pi NN}^2 \bar{N} \gamma_5 \sigma_3 N \frac{\mu_\pi^2}{m_\pi^4} \bar{N}' \gamma_5 \sigma_3 N' = G_\pi \bar{N} \gamma_5 \sigma_3 N \bar{N}' \gamma_5 \sigma_3 N'. \quad (5)$$

Here, in analogy with the Fermi's theory of weak interactions we define

$$G_\pi = \left( \frac{2 g_{\pi NN}^2}{m_\pi^3} \right) \delta m_\pi \simeq 1.5 \times 10^{-5} \left( \frac{\delta m_\pi}{10^{-10} \text{ GeV}} \right) \text{ GeV}^{-2} \quad (6)$$

If  $\delta m \sim 10^{-10}$  GeV we expect that the interaction between ordinary and mirror matter is weak ( $G_\pi \simeq G_F$ ).

Let us now consider a halo of mirror nuclei,  $A'$  with atomic number  $Z'$  scattered on an ordinary nucleus,  $A$  with atomic number  $Z$  of a direct detect experiment

$$N'(A, Z)' + N(A, Z)_{\text{at rest}} \longrightarrow N'(A, Z)' + N(A, Z)_{\text{recoil}}. \quad (7)$$

One should take into account the fact that the  $A'$ 's move in the halo with velocities determined by their velocity distribution function  $f(v)$ , and that the differential cross section of (7) depends on  $f(v)$  through an elastic nuclear form factor  $F(q)$  [17]:

$$d\sigma = \frac{1}{v^2} \left( \frac{\sigma_0}{4m_r^2} \right) F^2(q) dq^2. \quad (8)$$

Here  $\sigma_0$  is the total cross section ignoring the form factor suppression

$$m_r = \frac{m_A m_{A'}}{m_A + m_{A'}} \quad (9)$$

is the reduced mass and  $q$  the transferred 3-momentum

$$q = \sqrt{2m_A Q} \quad (10)$$

as a function of the energy deposited in the detector  $Q$ . By means of classical mechanics, the transferred momentum can be expressed as

$$q = 2 \left[ m_A \left( \frac{m_{A'} v}{m_{A'} + m_A} \right) \right] \sin \left( \frac{\theta_{\text{CM}}}{2} \right) = 2m_r v \sqrt{\frac{1 - \cos \theta_{\text{CM}}}{2}}, \quad (11)$$

where  $\theta_{\text{CM}}$  is the scattering angle in the center-of-momentum frame. Since

$$1 \leq 1 - \cos \theta_{\text{CM}} \leq 2 \quad (12)$$

for a given deposited energy  $Q$ , we obtain

$$Q = \frac{2m_r^2 v_{\text{min}}^2}{m_A}. \quad (13)$$

Therefore the minimal incoming velocity of incident A's that can deposit the energy  $Q$  in the detector can be expressed as

$$v_{\text{min}}(Q) = \alpha \sqrt{Q}, \quad (14)$$

where

$$\alpha = \sqrt{\frac{m_A}{2m_r}}. \quad (15)$$

Let us consider a scalar interaction mediated by oscillation of pion mirror pion. The cross section from the Lagrangian (5) now can be written

$$\frac{d\sigma^\pi(v, Q)}{dq^2} = \frac{1}{\pi v^2} |\mathcal{M}|_\pi^2 F^2(q) \quad (16)$$

where

$$|\mathcal{M}|_\pi^2 = |\langle A(p') A'(k') | \mathcal{L}_{\text{eff}} | A(p) A'(k) \rangle|^2. \quad (17)$$

A commonly used approximation for the nuclear spin independent form factor [14], inspired by the Woods-Saxon nuclear density profile,

$$F_{\text{WS}}^2(q) = \left[ \frac{3j_1(qR_1)}{qR_1} \right]^2 e^{-(qs)^2}. \quad (18)$$

Here  $j_1(x)$  is a spherical Bessel function, and

$$R_1 = \sqrt{R_A^2 - 5s^2} \quad (19)$$

with

$$R_A \simeq 1.2 A^{1/3} \text{ fm}, \quad s \simeq 1 \text{ fm}, \quad (20)$$

where  $A$  is the atomic mass number of the target nucleus.

For analytical evaluations one shall use the simpler exponential form factor introduced in [18] and [19]:

$$F_{\text{ex}}^2(Q) = e^{-Q/Q_0}, \quad (21)$$

where  $Q_0$  is the nuclear coherence energy,

$$Q_0 = \frac{1.5}{m_N R_0^2} \quad (22)$$

and,

$$R_0 = \left[ 0.3 + 0.91 \left( \frac{m_N}{\text{GeV}} \right)^{1/3} \right] \text{ fm} \quad (23)$$

is the radius of the nucleus.

The amplitude (17), in the non relativistic limit ( $E \ll m_{A'}$  or  $q \simeq 0$ ), can be expressed via minimal projections of isospin of interacting  $A$  and  $A'$  nuclei as follows

$$|\mathcal{M}|_\pi^2 \simeq \frac{[G_\pi(A - 2Z)(A' - 2Z')]^2}{2}. \quad (24)$$

Therefore the total cross section ignoring the formfactor suppression can be obtained by integration

$$\sigma_0^\pi = \int_0^{q_{\text{max}}^2} \frac{d\sigma^\pi(v, E_R)}{dq^2} dq^2 = 4 \frac{[G_\pi(A - 2Z)(A - 2Z)]^2 m_r^2}{\pi} \quad (25)$$

where  $q_{\text{max}}^2 = 4m_r v^2$  is the maximal transferred momentum. Notice that only odd isospin O or M-nuclei yield  $d\sigma/dq^2 \neq 0$ . Therefore, if only  ${}^4\text{He}$  populate the dark matter content of the halo (realistic case for symmetric mirror world  $m_A = m_{A'}$ ), no scattering mediated by oscillating mirror pion to ordinary pion exchange on a target nuclei of a detector is possible. However, we could consider strong interaction mediated by other neutral oscillating mesons, like the  $\eta$  meson. If we consider this particle as a isospin singlet, the effective Lagrangian that describes this interaction is

$$\mathcal{L}_{\text{eff}}^\eta = g_{\eta NN}^2 \bar{N} \gamma_5 \mathcal{I} N \frac{\mu_\eta^2}{m_\eta^4} \bar{N}' \gamma_5 \mathcal{I} N' \quad (26)$$

with  $g_{\eta NN} \simeq g_{\pi NN}$  and thus the matrix element in nonrelativistic limit becomes

$$|\mathcal{M}|_\eta^2 \simeq [G_\eta AA']^2 / 2. \quad (27)$$

This implies that the total  $\eta$  meson exchange cross section without form factor suppression can be expressed as

$$\sigma_0^\eta = \int_0^{q_{\max}^2} \frac{d\sigma^\eta(v, E_R)}{dq^2} dq^2 = 4 \frac{[G_\pi AA']^2 m_\pi^2}{\pi} \quad (28)$$

Notice that for any mirror nuclear the one  $\eta$  meson exchange cross section is not vanished. However, due to the larger mass of the  $\eta$  meson ( $m_\eta \simeq 550$  MeV) compared to the mass of  $\pi$  ( $m_\pi \simeq 135$  MeV), this cross section is suppressed as follows

$$G_\eta = \left( \frac{2 g_{\pi NN}^2}{m_\eta^3} \right) \delta m_\eta \simeq 2.2 \times 10^{-7} \left( \frac{\delta m_\eta}{10^{-10} \text{ GeV}} \right) \text{ GeV}^{-2}. \quad (29)$$

If we consider  $\delta m_\pi \sim \delta m_\eta$ , we find that  $G_\eta \sim 10^{-2} G_\pi$  and thus in this approximation the ratio between the total cross section is

$$\frac{\sigma_0^\eta}{\sigma_0^\pi} \sim 10^{-4} \left| \frac{AA'}{(A-2Z)(A-2Z)'} \right|^2 \quad (30)$$

For instance, if only protons compose the M-matter (realistic case for asymmetric mirror world  $m_N \simeq 5 m_N$ ) this ratio yields  $10^{-4} A^2 / (A-2Z)^2$ .

### 3.2 Rate estimations

In general, the differential scattering event rate (per unit detector mass) should be written as [17]

$$\begin{aligned} dR &= \frac{\rho_0}{m_{A'} m_A} \int v f_1(v) d\sigma dv \\ &= \left( \frac{\rho_0 \sigma_0}{2m_{A'} m_\pi^2} \right) F^2(Q) \int \left[ \frac{f_1(v)}{v} \right] dv dQ, \end{aligned} \quad (31)$$

where  $f_1(v) = 4\pi v^2 f(v)$  is the one-dimensional velocity distribution function of WIMPs impinging on the detector,  $v$  is the absolute value of the WIMP velocity in the Earth rest frame, and we have to integrate over all possible incoming velocities. Therefore the differential event rate for elastic  $A'$ - $A$  nucleus scattering can be rewritten as

$$\frac{dR}{dQ} = \mathcal{K} F^2(Q) \int_{v_{\min}}^{\infty} \left[ \frac{f_1(v)}{v} \right] dv, \quad (32)$$

where the constant coefficient  $\mathcal{K}$  is defined as

$$\mathcal{K} \equiv \frac{\rho_0 \sigma_0}{2m_{A'} m_\pi^2}. \quad (33)$$



Here we assume that the detector essentially only consists of nuclei of a single isotope. If the detector contains several different nuclei (e.g., NaI as in the DAMA detector [16] see the next section), the right-hand side of Eq.(32) has to be replaced by a sum of terms, each term describing the contribution of one isotope.

Finally, the total event rate per unit time per unit mass of detector material can be expressed as

$$R = \int_{Q_{\text{thre}}}^{\infty} \left( \frac{dR}{dQ} \right) dQ, \quad (34)$$

where  $Q_{\text{thre}}$  is the threshold energy of the detector.

For the simplest halo model, the canonical isothermal spherical halo, with the assumption that the WIMPs trapped in the galactic field have attained thermal equilibrium with a Maxwellian velocity distribution, the velocity distribution function is given by [17]

$$f_{\text{Gau}}(v) = \left( \frac{1}{\pi^{3/2} v_0^3} \right) e^{-v^2/v_0^2}, \quad (35)$$

where  $v_0$  is the orbital velocity of the Sun in the Galactic frame:

$$v_0 \simeq 220 \text{ km/s}, \quad (36)$$

which characterizes the velocity of all virialized objects in the Solar vicinity. Then the normalized one-dimensional velocity distribution function has been obtained as [17]

$$f_{1\text{Gau}}(v) = \frac{4}{\sqrt{\pi}} \left( \frac{v^2}{v_0^3} \right) e^{-v^2/v_0^2}. \quad (37)$$

When we take into account the orbital motion of the Solar system around the Galaxy, as well as that of the Earth around the Sun, this velocity distribution function should be modified to [17]

$$f_{1,\text{sh}}(v, v_e) = \frac{1}{\sqrt{\pi}} \left( \frac{v}{v_e v_0} \right) \left[ e^{-(v-v_e)^2/v_0^2} - e^{-(v+v_e)^2/v_0^2} \right] \quad (38)$$

with

$$v_e(t) = v_0 \left[ 1.05 + 0.07 \cos \left( \frac{2\pi(t - t_p)}{1 \text{ yr}} \right) \right], \quad (39)$$

where  $t_p \simeq$  June 2nd is the date on which the velocity of the Earth relative to the WIMP halo is maximal [19].

Substituting the shifted Maxwellian velocity distribution function in Eq.(38) into Eq.(32), the theoretically expected scattering spectrum can be obtained as

$$\left( \frac{dR}{dQ} \right)_{\text{sh}} = \mathcal{K} \left( \frac{1}{2v_e} \right) F^2(Q) \left[ \text{erf} \left( \frac{\alpha\sqrt{Q}+v_e}{v_0} \right) - \text{erf} \left( \frac{\alpha\sqrt{Q}-v_e}{v_0} \right) \right]. \quad (40)$$

Here  $\text{erf}(x)$  is the error function, defined as

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt.$$

For the case with the exponential form factor  $F_{\text{ex}}^2(Q)$  given in Eq.(21),

$$R_{\text{sh,ex}}(Q_{\text{thre}}) = \frac{\rho_0 \sigma_0}{m_\chi m_N} \left( \frac{v_0^2}{2v_e} \right) \left( \frac{\beta^2}{1 - \beta^2} \right) \times \left\{ e^{-(1-\beta^2)\alpha^2 Q_{\text{thre}}/v_0^2 \beta^2} [\text{erf}(S_+) - \text{erf}(S_-)] - \beta e^{-(1-\beta^2)v_e^2/v_0^2} [\text{erf}(T_+) - \text{erf}(T_-)] \right\}, \quad (41)$$

where

$$\beta \equiv \left( 1 + \frac{v_0^2}{\alpha^2 Q_0} \right)^{-1/2} = \left( 1 + 1.622 \times 10^{-5} \frac{A'^2 A^2}{(A' + A)^2 R_0^2 [\text{fm}]^2} \right)^{-1/2}, \quad (42)$$

$$S_{\pm} \equiv \frac{\alpha \sqrt{Q_{\text{thre}} \pm v_e}}{v_0}, \quad (43)$$

and

$$T_{\pm} \equiv \frac{\alpha \sqrt{Q_{\text{thre}} \pm \beta^2 v_e}}{v_0 \beta}. \quad (44)$$

Considering, for example, the light- $A'$  case and using  $F^2(Q) \approx 1$ , the total event rate for the shifted Maxwellian distribution in Eq.(38) can be found as

$$R_{\text{sh}}(Q_{\text{thre}}) = \frac{\rho_0 \sigma_0}{m_{A'} m_A} \left( \frac{v_0^2}{2v_e} \right) \left\{ \left( \frac{1}{2} - S_+ S_- \right) [\text{erf}(S_+) - \text{erf}(S_-)] + \frac{1}{\sqrt{\pi}} (S_+ e^{-S_+^2} - S_- e^{-S_-^2}) \right\}. \quad (45)$$

For the case of  $Q_{\text{thre}} = 0$ , Eq.(45) can be reduced directly to

$$R_{\text{sh}}(Q_{\text{thre}} = 0) = \frac{\rho_0 \sigma_0}{m_{A'} m_A} \left[ \left( \frac{v_0^2}{2v_e} + v_e \right) \text{erf} \left( \frac{v_e}{v_0} \right) + \left( \frac{v_0}{\sqrt{\pi}} \right) e^{-v_e^2/v_0^2} \right], \quad (46)$$

while in the limit of  $Q_{\text{thre}} = 0$ , Eq.(41) takes the form

$$R_{\text{sh,ex}}(Q_{\text{thre}} = 0) = \frac{\rho_0 \sigma_0}{m_\chi m_N} \left( \frac{v_0^2}{v_e} \right) \left( \frac{\beta^2}{1 - \beta^2} \right) \left[ \text{erf} \left( \frac{v_e}{v_0} \right) - \beta e^{-(1-\beta^2)v_e^2/v_0^2} \text{erf} \left( \frac{\beta v_e}{v_0} \right) \right]. \quad (47)$$

Now, once known the differential particle density

$$dn = \frac{n_\xi}{k} f(v, v_E) d^3v \quad (48)$$

where  $n_\xi = \xi \rho_0 / m_{N'}$  is the mean Dark Matter particle number density,  $\xi \leq 1$  is the fractional amount of local WIMP density,  $v_E$  is the Earth (target) velocity respect to the Dark matter distribution and  $k$  is a normalization constant such that

$$\int_0^{v_{\text{esc}}} dn = \frac{\xi \rho_0}{m_{N'}} = n_\xi, \quad (49)$$

one can write the energy distribution of the recoil rate ( $R$ ) in the form

$$\frac{dR^i(v_E, v_{\text{esc}})}{dE_R} = N_T \int_{v_{\text{min}}(E_R)}^{v_{\text{esc}}} \frac{d\sigma^i(v, E_R)}{dE_R} v dn \quad (50)$$

where  $i = (\pi, \eta)$ ,  $N_T = N_0/A$  is the number of O-target nuclei,

$$v_{\text{min}}(E_R) = \left( \frac{m_N E_R}{2 m_{N'}^2} \right)^{\frac{1}{2}} = \left( \frac{E_R}{E_0 r} \right)^{\frac{1}{2}} v_0; \quad E_0 = \frac{1}{2} m_{N'} v_0^2 \quad (51)$$

is the minimal WIMP velocity providing  $E_R$  recoil energy,  $v_0 \sim 10^{-3}$  is the most probability one and  $v_{\text{esc}}$  is the local Galactic escape velocity. From equations (??)-(??)

$$\frac{d\sigma^i(v, E_R)}{dE_R} = \frac{\sigma_0^i m_N}{2 m_{N'}^2} \frac{1}{v^2} F_{\text{SI}}^2(E_R) \quad (52)$$

and thus the energy distribution of the recoil rate becomes

$$\begin{aligned} \frac{dR^i(v_E, v_{\text{esc}})}{dE_R} &= \left( \frac{2 N_0}{\pi^{\frac{1}{2}} A} n_\xi \sigma_0^i v_0 \right) \left( \frac{1}{\frac{1}{2} m_{N'} v_0^2 r} \right) \frac{1}{2\pi v_0^2} \frac{k_0}{k} I(E_R) F_{\text{SI}}^2(E_R) \\ &= \frac{R_0^i}{E_0 r} \frac{1}{2\pi v_0^2} \frac{k_0}{k} I(E_R) F_{\text{SI}}^2(E_R) \end{aligned} \quad (53)$$

with  $k_0 = (\pi v_0^2)^{\frac{3}{2}}$  normalization constant for maxewellian velocity distribution at  $v_{\text{esc}} = \infty$  and

$$I(E_R) = \int_{v_{\text{min}}(E_R)}^{v_{\text{esc}}} \frac{f(v, v_E)}{v} d^3v. \quad (54)$$

$R_0^i$  is conventionally expressed in units  $\text{kg}^{-1} \text{d}^{-1}$  or tru (total rate unit). Normalized to  $\rho_0 = 0.3 \text{ GeV c}^{-2} \text{cm}^{-3}$  and  $v_0 = 230 \text{ km s}^{-1}$ ,  $R_0^i$  becomes

$$R_0^i = \frac{405 \xi}{A m_{N'}} \left( \frac{\sigma_0^i}{1 \text{ pb}} \right) \left( \frac{\rho_0}{0.3 \text{ GeV c}^{-2} \text{cm}^{-3}} \right) \left( \frac{v_0}{230 \text{ km s}^{-1}} \right) \text{ tru} \quad (55)$$

Now we can calculate the unmodified nuclear recoil spectrum ( $v_E = 0$  and  $v_{\text{esc}} = \infty$ ) considering a maxwellian velocity distribution

$$f(v, v_E) = \exp \left[ -\frac{(v + v_E)^2}{v_0^2} \right]. \quad (56)$$

It is straightforward to verify that  $I(E_R) = 2\pi v_0^2 \exp \left( -\frac{v_{\text{min}}^2}{v_0^2} \right)$  and  $k = k_0$ , so

$$\frac{dR^i(0, \infty)}{dE_R} = \frac{R_0^i}{E_0 r} \exp \left( -\frac{E_R}{E_0 r} \right) F_{\text{SI}}^2(E_R). \quad (57)$$

In the realistic model  $v_E \simeq 244 + 15 \cos(2\pi y)$  km s<sup>-1</sup> ( $y$  is the elapsed time from March 2nd in years) and, as  $v_{\text{esc}} = 600$  km s<sup>-1</sup> is much greater than  $v_0$  ( $\exp[-v_{\text{esc}}^2/v_0^2] \simeq 0$ ), one can consider in a well approximation  $v_{\text{esc}} = \infty$ . For practical use the behavior of equation (53) is well fitted by

$$\frac{dR^i(v_E, \infty)}{dE_R} = c_1 \frac{R_0^i}{E_0 r} \exp \left( -c_2 \frac{E_R}{E_0 r} \right) F_{\text{SI}}^2(E_R). \quad (58)$$

where  $c_1, c_2$  are fitting constant of order unity ( $c_1 = 0.751, c_2 = 0.561$ ).

## 4 DAMA normalization

Symmetric Mirror	I - <sup>4</sup> He'	Na - <sup>4</sup> He'
$\sigma_0^\pi$ (pb)	0	0
$\sigma_0^\eta$ (pb)	$2 \times 10^5 \left( \frac{\delta m_\pi}{10^{-10} \text{ GeV}} \right)^2$	$3 \times 10^3 \left( \frac{\delta m_\pi}{10^{-10} \text{ GeV}} \right)^2$
$E_R$ (keV)	$1.5 \times 10^{-1}$	$6.5 \times 10^{-1}$
$S^\pi$ (cpd/kg/keV)	0	0
$S^\eta$ (cpd/kg/keV)	$25 \left( \frac{\delta m_\eta}{10^{-10} \text{ GeV}} \right)^2 \xi$	$811 \left( \frac{\delta m_\eta}{10^{-10} \text{ GeV}} \right)^2 \xi$

Table (1): Theoretical residual in the energy window 2-6 keV expected for scattering between ordinary particle and symmetric mirror one ( $m_N = m_N$ ). We have also reported the total cross section and the recoil energy.

Asymmetric Mirror	I - p'	Na - p'
$\sigma_0^\pi$ (pb)	$8 \times 10^7 \left( \frac{\delta m_\pi}{10^{-10} \text{ GeV}} \right)^2$	$9 \times 10^4 \left( \frac{\delta m_\pi}{10^{-10} \text{ GeV}} \right)^2$
$\sigma_0^\eta$ (pb)	$6 \times 10^5 \left( \frac{\delta m_\pi}{10^{-10} \text{ GeV}} \right)^2$	$10^4 \left( \frac{\delta m_\pi}{10^{-10} \text{ GeV}} \right)^2$
$E_R$ (keV)	$2.3 \times 10^{-1}$	$9.4 \times 10^{-1}$
$S^\pi$ (cpd/kg/keV)	$10^5 \left( \frac{\delta m_\pi}{10^{-10} \text{ GeV}} \right)^2 \xi$	$3 \times 10^4 \left( \frac{\delta m_\pi}{10^{-10} \text{ GeV}} \right)^2 \xi$
$S^\eta$ (cpd/kg/keV)	$10^3 \left( \frac{\delta m_\eta}{10^{-10} \text{ GeV}} \right)^2 \xi$	$3 \times 10^3 \left( \frac{\delta m_\eta}{10^{-10} \text{ GeV}} \right)^2 \xi$

Table (2): Theoretical residual in the energy window 2-6 keV expected for scattering between ordinary particle and asymmetric mirror one ( $m_{N'} \simeq 5 m_N$ ). We have also reported the total cross section and the recoil energy.

Finally, we calculate the theoretical amplitude for a particular detector. This amplitude is given by the following relation

$$S_j^i = \frac{1}{\Delta E_j} \int_{E_{\min}}^{E_{\max}} \frac{dR^i(v_E, \infty)}{dE_R} dE_R \quad (59)$$

where  $E_{\min}$  and  $E_{\max}$  are respectively the low and high window threshold and  $\Delta E_j = E_{\max} - E_{\min}$  is the  $j$ -esime energy bin. In the tables above we show the residual (59) gives rise to the scattering between M-particle and O-NaI (DAMA detector) in the energy window 2-6 keV for symmetric ( $A' = 4$ ,  $Z' = 2$ ) and asymmetric mirror world ( $A' = Z' = 1$ ). The DAMA results, give an evidence for the presence of an annual modulation of the measured rate of the single-hit events in the lowest energy region. In fact, fitting the experimental points with modulated cosine-like function  $A \cos \omega(t - t_0)$ , a modulation amplitude equal to  $(0.0200 \pm 0.0032)$  cpd/kg/keV,  $t = (140 \pm 22)$  days and  $T = \frac{2\pi}{\omega} = (1.00 \pm 0.01)$  year are obtained. The period and phase agree with those expected in case of an effect induced by Dark Matter particles in the galactic halo ( $T = 1$  year and  $t_0$  roughly at  $\simeq 152.5^{th}$  day of the year). If really mirror matter is a possible Dark Matter candidate we should have  $\delta m_i \sim 10^{-12} \div 10^{-13}$  GeV for symmetric M-world or  $\delta m_i \sim 10^{-13} \div 10^{-14}$  GeV for asymmetric one. This estimation is in agreement with the upper limit of  $\pi - \pi'$  mass difference estimation ( $\delta m_\pi \lesssim 1.6 \times 10^{-12}$  GeV).

## References

- [1] For the very first papers discussing mirror parity and mirror particles, see T. D. Lee and C. N. Yang, *Phys. Rev.* **104**, 254 (1956); I. Yu. Kobzarev, L. B. Okun and I. Ya. Pomeranchuk, *Sov. J. Nucl. Phys.* **3**, 837 (1966); For a hystorical overview, see L.B. Okun, hep-ph/0606202
- [2] Z. Berezhiani, "Through the Looking-Glass: Alice's Adventures in Mirror World," hep-ph/0508233; Z. Berezhiani, *Int. J. Mod. Phys. A* **19**, 3775 (2004) [hep-ph/0312335]
- [3] R. Foot, *Int. J. Mod. Phys. A* **19**, 3807 (2004) [astro-ph/0309330]
- [4] R. Foot, H. Lew and R. R. Volkas, *Phys. Lett. B* **272**, 67 (1991)
- [5] B. Holdom, *Phys. Lett. B* **66**, 196 (1986);  
S. L. Glashow, *Phys. Lett. B* **167**, 35 (1986);  
E. D. Carlson and S. L. Glashow, *Phys. Lett. B* **193**, 168 (1987)

- [6] R. Foot, H. Lew and R.R. Volkas, *Mod. Phys. Lett. A* **7**, 2567 (1992);  
E.K. Akhmedov, Z.G. Berezhiani and G. Senjanović, *Phys. Rev. Lett.* **69**, 3013 (1992);  
R. Foot and R.R. Volkas, *Phys. Rev. D* **52**, 6595 (1995);  
Z.G. Berezhiani and R.N. Mohapatra, *Phys. Rev. D* **52**, 6607 (1995);  
Z. Silagadze, *Phys. At. Nucl.* **60**, 272 (1997)
- [7] Z. Berezhiani and L. Bento, *Phys. Rev. Lett.* **96**, 081801 (2006) [hep-ph/0507031];  
rays,” *Phys. Lett. B* **635**, 253 (2006) [hep-ph/0602227];  
R.N. Mohapatra, S. Nasri and S. Nussinov, *Phys. Lett. B* **627**, 124 (2005) [hep-ph/0508109]
- [8] Z. Berezhiani, arXiv:0804.2088 [hep-ph].
- [9] Z. Berezhiani, *Phys. Lett. B* **417**, 287 (1998)
- [10] Z. Berezhiani, D. Comelli and F. Villante, *Phys. Lett. B* **503**, 362 (2001)
- [11] M. S. Atiya et al., *Phys.Rev.Lett.*66:2189-2192,1991.
- [12] H.W. Atherton et al., *Phys.Lett.*B158:81-84,1985.
- [13] H. Yukawa, *Proc. Phys. Math. Soc. Jap.* **17** (1935) 48.
- [14] J. Engel, *Phys. Lett. B* **264**, 114 (1991).
- [15] Review of Particle Physics, *J. Phys. G: Nucl. Part. Phys.* **33**, 2 (2006).
- [16] See <http://people.roma2.infn.it/~dama/web/home.html>.
- [17] G. Jungman, M. Kamionkowski, and K. Griest, *Phys. Rep.* **267**, 195 (1996).
- [18] S. P. Ahlen et al., *Phys. Lett. B* **195**, 603 (1987).
- [19] K. Freese, J. Frieman, and A. Gould, *Phys. Rev. D* **37**, 3388 (1988).