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FORTRAN PROGRAMS FOR FAST MOMENTUM ANALYSIS IN THE CERN n-p CHARGE EXCHANGE EXPERIMENT

Yu. Galaktionov

ABSTRACT

The set of FORTRAN programs for fast momentum analysis of high momentum protons in reaction:

$$n + p \rightarrow p + n$$

is described; the method used is that proposed by Lechanoine et al.[1]. The accuracy of the calculation makes it possible to have a momentum resolution of ~ 0.1%. The analysis requires about 15 msec of CDC 6600 time per event.

1. GENERAL DESCRIPTION

The procedure set out below is, in fact, a FORTRAN representation of the method described by Lechanoine, Martin and Wind[1].

The programs have been written for a particular experiment, namely the CERN-Karlsruhe n-p charge exchange experiment, where one needed to have a momentum resolution that was good enough for 19-24 GeV protons; the field of the analysing magnet not being sufficiently homogeneous for using a straight forward trajectory calculation for each event, this would have required an unacceptable amount of computer time.

The computation time can be decreased if one first expresses in some way the momentum as a function of the measured track parameter (for example, the particle direction before the magnetic field and deflection angle), and then uses this expression when analysing the experimental data.

The particle trajectory is defined by a set of five numbers; four of them give the particle direction before it enters the magnetic field, and the fifth number is the deflection angle, i.e. the unknown momentum is a function of five variables:

$$p = p(x_i) \qquad i = 1, \ldots, 5 \, . \tag{1}$$

The procedure is such that at first the values of the function (1) are evaluated for different sets of arguments x_i, and than the function is expressed as a series of Chebyshev polynomials, so one gets:

$$\varphi = \sum_{i,k,\ell,m,n} C_{ik\ell mn} T_i(x_1) T_k(x_2) T_\ell(x_3) T_m(x_4) T_n(x_5) \, , \tag{2}$$

where i, k, ℓ, m, n run from 1 up to I, K, L, M, N, respectively. In order to have $p(x_i)$ expressed in terms of Chebyshev polynomials, the coefficients $C_{ik\ell mn}$ should be calculated. The number of these coefficients is:

$$\mathcal{M} = I \times K \times L \times M \times N \, ,$$

and I, K, L, M, N must be chosen in accordance with the required accuracy in momentum determination. These coefficients are calculated by the CHEKO program. The relationship (2) will be used for the data analysis.

When producing the coefficients, CHEKO first calculates the values of the function:

$$\Theta = \Theta(x_i, p) \qquad i = 1, \ldots, 4 , \qquad (3)$$

and then the p-values are found by inverse interpolation.

If the arguments of formula (3) are given, the calculation of Θ means simply the numerical integration of the equations of the particle movement in a given magnetic field (i.e. trajectory calculation). This is done by the TRACK subroutine. Trajectory calculation proceeds in steps along the beam direction, while the number of steps is given by the required accuracy in ϑ value.

For trajectory calculation, one needs to know the three components of the magnetic field at an arbitrary point of the space inside the magnet. These values of the field are given by the GENF subroutine, which uses the Fourier representation of the magnetic field taking into account Maxwell's equations.

The Fourier coefficients of the field have been computed beforehand by using the experimentally measured values of the main field component in two planes that are chosen in such a way as to be parallel to the pole surfaces. The analysis of the magnetic measurements, as well as the calculation of Fourier coefficients, is done by the ANFIELD program.

Thus the set of programs for the momentum analysis consists of the following main parts:

Routine name	Function
ANFIELD	reads the data of the measurements of the magnetic field from the tape; makes a check for wrong points; does Fourier analysis, calculating Fourier coefficients for GENF.
GENF	generates three field components B_X, B_Y, B_Z at the given point X, Y, Z.
TRACK	performs the numerical integration of the equations of motion, calling GENF when it needs the magnetic field value to be calculated.

(Cont.)

Routine name	Function
CHEKO	calculates Chebyshev coefficients using TRACK to make trajectories.
PROTRUN	carries out the momentum analysis of the events taken in the experiment, using the particle directions before and after the magnetic field region (8 numbers) as well as the SUMTF subroutine to sum up Chebyshev series.
SUMTF	makes a sum of Chebyshev series using coefficients from CHEKO.

We shall now describe some parts of these programs in more detail, with some results concerning the accuracy of the analysis.

2. THE TREATMENT OF THE MEASUREMENTS OF THE MAGNETIC FIELD AND FOURIER ANALYSIS OF THE MAGNETIC FIELD (PROGRAM ANFIELD)

The data of the measurements of the magnetic field are expressed in the same coordinate system as the one for the observations as indicated in Fig. 1. We have two identical beam transport magnets and their poles are shown in Fig. 1.

ANFIELD reads the tape where the data of the magnetic field measurements are written. The measurements have been done for two planes that were parallel to the pole surfaces (Z_1 = -4.5 cm, Z_2 = +4.5 cm) *). In each plane, 216 × 41 values of the B_Z-component of the field in the 2 × 2 cm grid were measured. ANFIELD transforms the field to kilogauss, and by making a smooth curve through the neighbouring points it checks whether they are self-consistent or not. The deviation is considered as being too big if it is more than the prescribed limit set for the program.

*) In fact the plane Z = 0 has also been measured, but these data were not used by ANFIELD.

Along the X-axis the program takes 155 points (out of 216), which correspond to the field profile shown in Fig. 2.

In Y-direction, the measured field does not reach zero values at the ends of the interval, and ANFIELD makes parabolic extrapolation to zero so that the full number of points in Y-direction becomes equal to 51.

The units in both X and Y direction are changed in such a way that the field appears to be given at the intervals $0 \leq X \leq \pi$, $0 \leq Y \leq \pi$, while at the ends of the the intervals B_Z values are zero or practically zero. This provides fast convergence of the Fourier series[2]. ANFIELD then calculates Fourier coefficients, i.e. the program approximates the measured field values by the function:

$$B_Z(X,Y) = \sum_{i=1}^{i=2} \sum_{j=2}^{j=2} A_{ij} \cos(i-1)x \cos(j-1)y$$

$$+ \sum_{i=1}^{i=50} \sum_{j=1}^{j=30} B_{ij} \sin ix \sin jy$$

$$+ \sum_{i=1}^{i=50} \sum_{j=1}^{j=2} C_{ij} \sin ix \cos(j-1)y$$

$$+ \sum_{i=1}^{i=2} \sum_{j=1}^{j=30} D_{ij} \cos(i-1)x \sin jy \,. \qquad (4)$$

The coefficients A, B, C, D are found for each of two planes by making a χ^2 fit of function (4) to the data. The values of A and C turned out to be zero; so in the field generation program GENF, only B and C values are used, i.e. 1500 numbers B_{ij} and 60 numbers C_{ij} for each of the two planes.

3. MAGNETIC FIELD GENERATOR (GENF)

The program uses Fourier coefficients as well as Maxwell equations to calculate B_x, B_y, B_z components of the magnetic field at the point X, Y, Z. The coordinate system is the same as in the ANFIELD program, and $0 \leq X,Y \leq \pi$.

For calculation, the program uses the following formulas:

$$B_Z(X, Y, Z) = \sum_{i=1}^{i=50} \sum_{j=1}^{j=30} \sin ix \sin jy \left[(A'_{ij} + A''_{ij}) \frac{\cosh(\sqrt{i^2 + j^2}\, Z)}{2 \cosh(\sqrt{i^2 + j^2}\, Z_1)} \right.$$

$$\left. + (A'_{ij} - A''_{ij}) \frac{\sinh(\sqrt{i^2 + j^2}\, Z)}{\sinh(\sqrt{i^2 + j^2}\, Z_1)} \right]$$

$$+ \sum_{i=1}^{i=2} \sum_{j=1}^{j=30} \cos(i-1)x \sin jy \left[(D'_{ij} + D''_{ij}) \frac{\cosh[\sqrt{(i-1)^2 + j^2}\, Z]}{2 \cosh[\sqrt{(i-1)^2 + j^2}\, Z_1]} \right.$$

$$\left. + (D'_{ij} - D''_{ij}) \frac{\sinh[\sqrt{(i-1)^2 + j^2}\, Z]}{2 \sinh[\sqrt{(i-1)^2 + j^2}\, Z_1]} \right] ;$$

$$B_X(X, Y, Z) = \frac{1}{S_1} \left(\sum_{i=1}^{i=50} \sum_{j=1}^{j=30} \frac{i}{\sqrt{i^2 + j^2}} \cos ix \sin jy \left[(A'_{ij} + A''_{ij}) \frac{\sinh(\sqrt{i^2 + j^2}\, Z)}{2 \cosh(\sqrt{i^2 + j^2}\, Z_1)} \right. \right.$$

$$\left. + (A'_{ij} - A''_{ij}) \frac{\cosh(\sqrt{i^2 + j^2}\, Z)}{2 \sinh(\sqrt{i^2 + j^2}\, Z_1)} \right]$$

$$\left. + \{\text{similar terms with } D' \text{ and } D'' \text{ instead of } A' \text{ and } A''\} \right) ;$$

$$B_Y(X, Y, Z) = \frac{1}{S_2} \left(\sum_{i=1}^{i=50} \sum_{j=1}^{j=30} \frac{j}{\sqrt{i^2 + j^2}} \sin ix \cos jy \left[(A'_{ij} + A''_{ij}) \frac{\sinh \sqrt{i^2 + j^2}\, Z}{2 \cosh(\sqrt{i^2 + j^2}\, Z_1)} \right. \right.$$

$$\left. + (A'_{ij} - A''_{ij}) \frac{\cosh(\sqrt{i^2 + j^2}\, Z)}{2 \sinh(\sqrt{i^2 + j^2}\, Z_1)} \right]$$

$$\left. + \{\text{similar terms with } D' \text{ and } D'' \text{ instead of } A' \text{ and } A''\} \right) ;$$

where A', D', A'', D'' are the Fourier coefficients for the two planes $Z = Z_1$, $Z = Z_2$; $S_1 = 154/\pi$, $S_2 = 50/\pi$ are scaling factors.

The accuracy in field values depends on the number of Fourier coefficients taken into account. An example of GENF output is shown in Fig. 3, where the curve represents the result of a calculation of the B_Z-component in the mid-plane of the magnet where the field has also been measured but these measurements were not used when calculating the Fourier coefficients. The points in Fig. 3 represent the measured field values. The difference between calculated and measured field values is about 10^{-3} for B_Z and of the order of several per cent for B_X and B_Y. If one does not need this accuracy one can decrease the number of Fourier coefficients by setting on upper limit for the value of a coefficient to be taken into account. The dependence of the computing time on this upper limit C is shown in Fig. 4.

4. THE TRAJECTORY CALCULATION (TRACK)

TRACK uses a coordinate system as shown in Fig. 5. When TRACK needs a field value to be calculated it calls subroutine MAGF which performs coordinate transformation and reflects the Z-axis if necessary to get a field value in the second magnet. Trajectory calculation is done in steps along the Z-axis, and the main problem here is how to choose these steps in order on the one hand to have good enough accuracy in deflection angle ϑ and on the other hand to spend as little computer time as possible. These steps (the grid) in Z-direction have been chosen in the following way. First by using a very fine grid (2 cm) several trajectories which came through the most inhomogeneous field were calculated. The Θ values thus obtained were used as reference points when ajusting the grid to have the accuracy in ϑ of $\sim 0.25\%$. The results for the different grids are given in the table. The value of the upper limit C for GENF (see Section 2) has been chosen at the same time. In the table are given results for two types of trajectories: those coming through the most inhomogeneous field (peripheral) and those coming through central parts of the field (central).

Table

Number of steps in grid	Deflection angle (mrad)		
	Peripheral trajectory		Central trajectory
	I	II	
150*)	89.43	88.47	81.92
17	89.35	88.36	81.60
19	89.20	88.31	81.80
14	89.55		
10	90.45		

*) That gives exact values of Θ.

With the grid of 19 steps one reaches an accuracy of ϑ of $\sim 0.25\%$ and $\sim 0.13\%$ for the peripheral and central trajectories correspondingly.

5. THE CALCULATION OF COEFFICIENTS IN CHEBYSHEV EXPANSION (CHEKO)

The particle momentum is given as a function of five variables x_i $(i = 1, \ldots, 5)$, where $x_1, \ldots, x_4$ are the x, y coordinates of the particle trajectory in two planes before the magnet, namely $Z_1 = -750$ cm, $Z_2 = -330$ cm (see Fig. 5), and x_5 is the deflection angle. CHEKO calculates three-dimensional*) Chebyshev expansion coefficients $C_{ik\ell}$ of the function:

$$p(x_1, x_2, x_3) = \sum_{i=1}^{i=N_1} \sum_{j=1}^{j=N_2} \sum_{k=1}^{k=N_3} C_{ik\ell} T_i(x_1) T_j(x_2) T_k(x_3) \qquad (5)$$

where x_1 and x_2 are the x-coordinates of the track in the planes Z_1 and Z_2, $x_3 = \mathrm{tg}\,\Theta$, and $T_i(x) = \cos\,(i \arccos x)$ are Chebyshev polynomials.

*) In FORTRAN one has variables with a maximum of three subscripts.

When calculating $C_{ik\ell}$ the variables are normalized in the interval $[-1, 1]$ *), i.e. if $x_{io} - \Delta x_i \leq x_i \leq x_{io} + \Delta x_i$, then:

$$-1 \leq \frac{x_i - x_{io}}{\Delta x_i} \leq +1 \; .$$

For each variable x_i one takes N_i points according to Chebyshev distribution:

$$x_{ki} = \cos \frac{2k - 1}{2N_i \pi}$$

(in our case $N_1 = N_2 = N_3 = 5$), and for all the combinations x_1, x_2, x_3 one performs the trajectory calculation. When calculating a trajectory the momentum p is used as x_3 i.e. one calculates the values of the function:

$$\theta = \theta(x_1, x_2, x_3) \; ,$$

then by making the inverse interpolation the corresponding values of p are found:

$$p = p(x_1, x_2, x_3) \qquad \text{where} \qquad x_3 = \text{tg}\, \vartheta \; .$$

CHEKO calculates $5 \times 5 \times 5 = 125$ values of p and using them it finds $C_{ik\ell}$.

In order to take into account the dependence of the momentum p on y_1 and y_2 coordinates (see Fig. 5), one chooses N_4 and N_5 points correspondingly in these two dimensions (in our case $N_4 = 3$, $N_5 = 5$) and for each combination of y_1, y_2 the calculation of $C_{ik\ell}$ is repeated. Thus CHEKO produces $3 \times 5 = 15$ sets of coefficients $C_{ik\ell}$ i.e. the full number of Chebyshev coefficients is 1875. In the y_1 and y_2 directions the points are chosen to be equally spaced; therefore when making momentum analysis one needs to do linear interpolation between 2×2 values of p obtained by the formula (5).

6. THE ACCURACY OF THE METHOD

Fifty tracks of the same momentum (19 GeV/c) were randomly taken to check the accuracy of the method. The result is shown in Fig. 6. The

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*) That means that one has to know beforehand both the lower and upper limits for each variable.

difference between the calculated value of p and 19 GeV/c is less than 0.1% for 75% of the events.

If one takes the number of C_{ikl} to be somewhat less, for example $4 \times 4 \times 4$ for $C_{ik\ell}$, the resolution goes down to $\sim$ 1%.

7. THE COMPUTING TIME

To run the ANFIELD program one needs ~ 100 sec of CDC 6400; GENF takes ~ 150 msec of 6600 to produce one field point; TRACK needs ~ 20 sec of 6400 per trajectory. CHEKO calculates in one run 625 coefficients and it takes ~ 5000 sec of 6600. That means to produce the whole amount of coefficients one needs about 4.5 hours of 6600 time. After the coefficients have been computed it takes 16 msec of 6600 time to have one event momentum analysed.

ACKNOWLEDGEMENTS

I would like to thank Dr. H. Wind for some FORTRAN routines which have been written by him, and especially for his very useful and informative contribution to our discussions. I am also grateful to Prof. R. Hartung for the improved version of the TRACK subroutine.

* * *

REFERENCES

1) C. Lechanoine, M. Martin and H. Wind, Nucl.Instr.Meth. 69, 122 (1969).

2) C. Lancros, Applied analysis (Prentice hall, Inc., N.Y. 1956).

Figure captions

Fig. 1 : Coordinate system of the magnetic measurements.

Fig. 2 : The field profile taken by ANFIELD (see text).

Fig. 3 : Comparison of the calculated and measured magnetic field (B_Z-component).

Fig. 4 : The dependence of computing time used by field generator (GENF) on the upper limit C of Fourier coefficients.

Fig. 5 : Coordinate system used by TRACK subroutine, also shown two planes Z_1 and Z_2 where input coordinates for trajectory calculation are defined.

Fig. 6 : Calculated momentum distribution for 50 randomly taken tracks of 19 GeV/c.

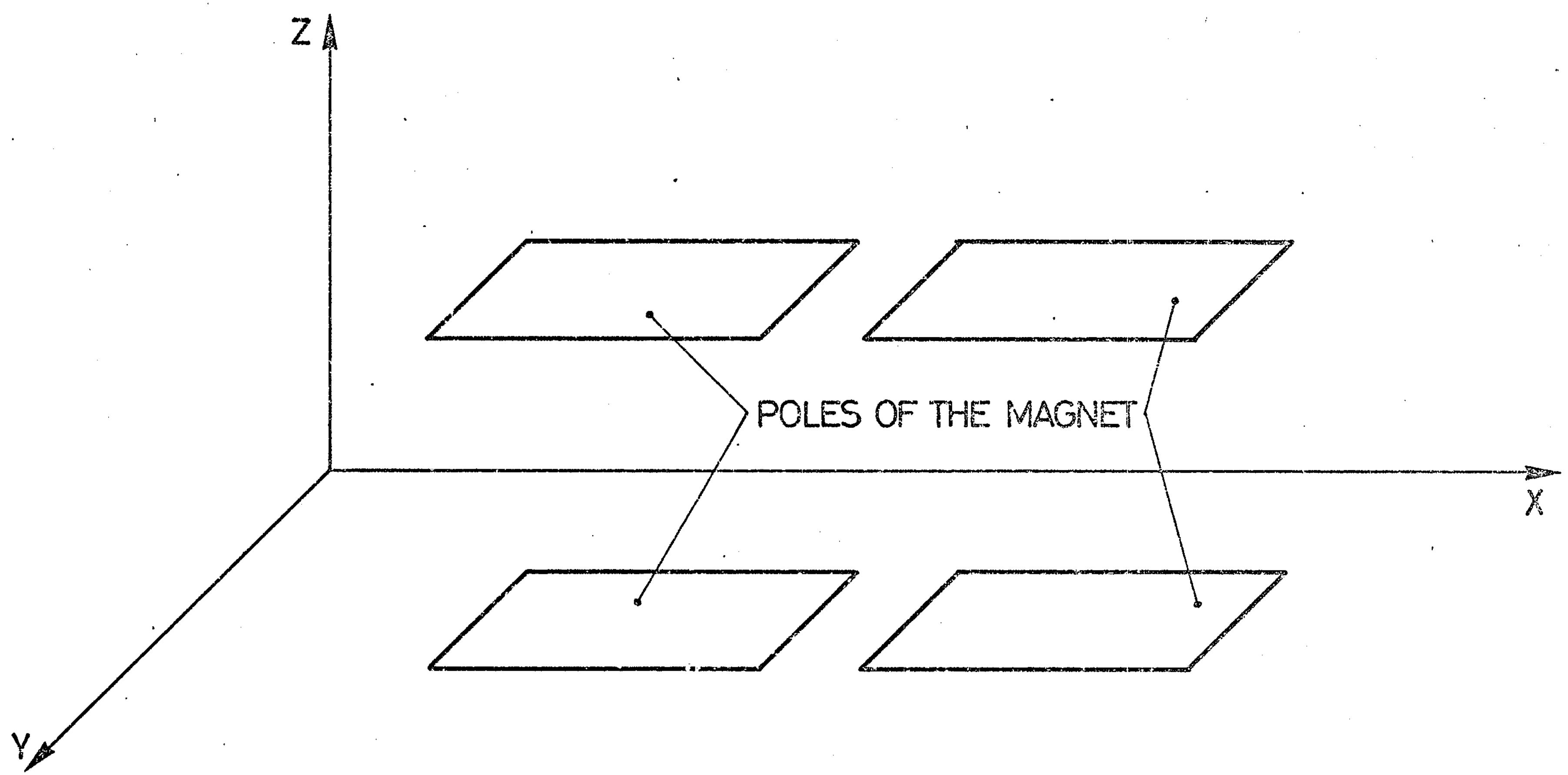

FIG.1

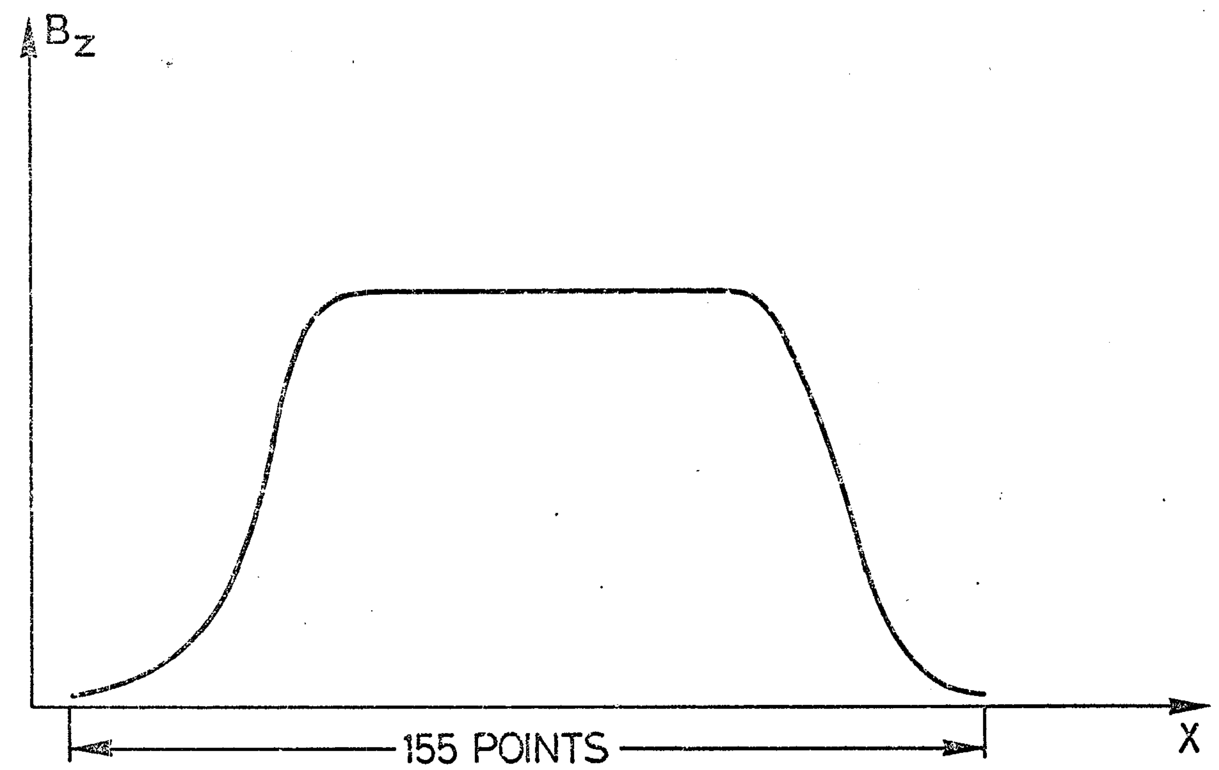

FIG. 2

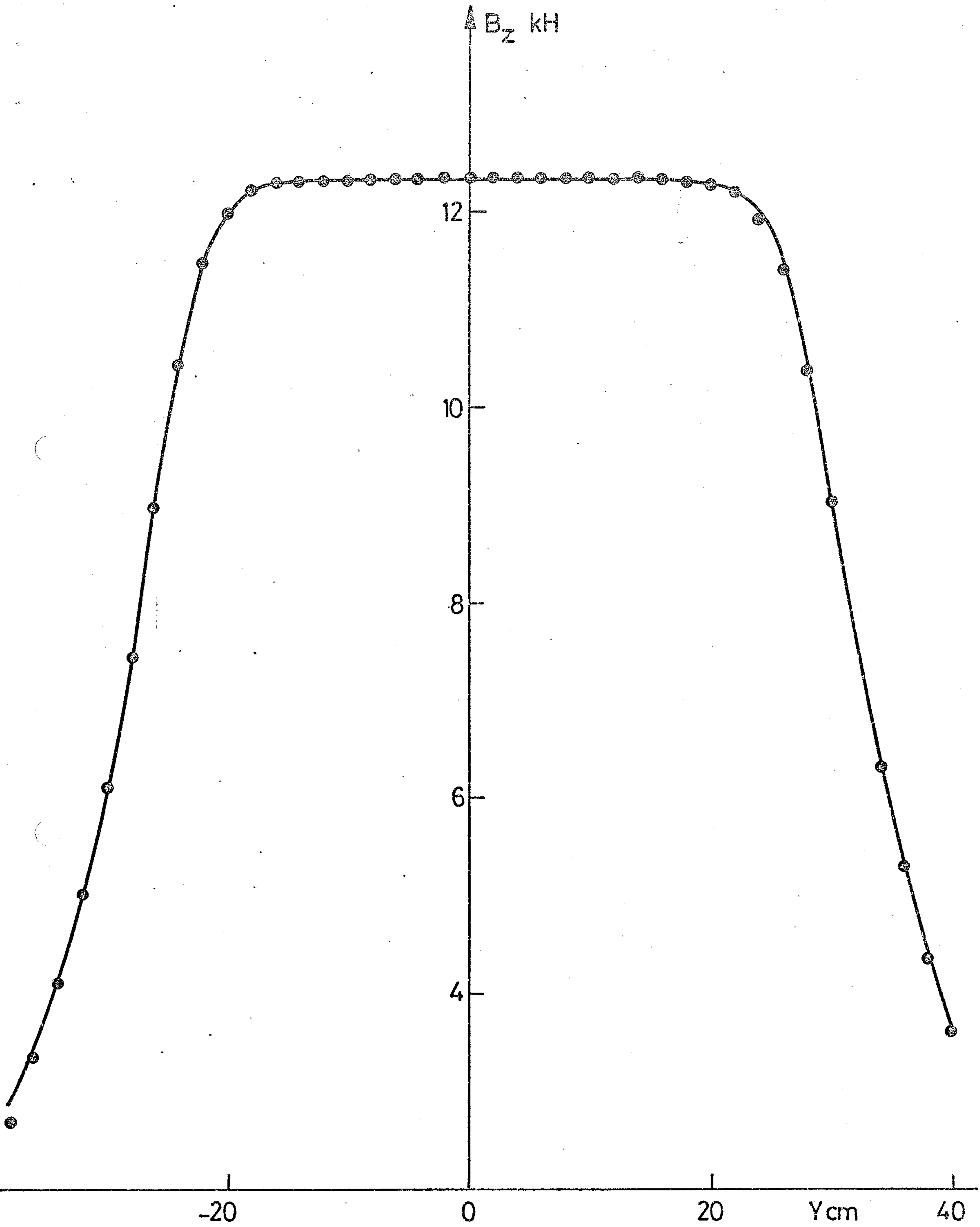

FIG.3

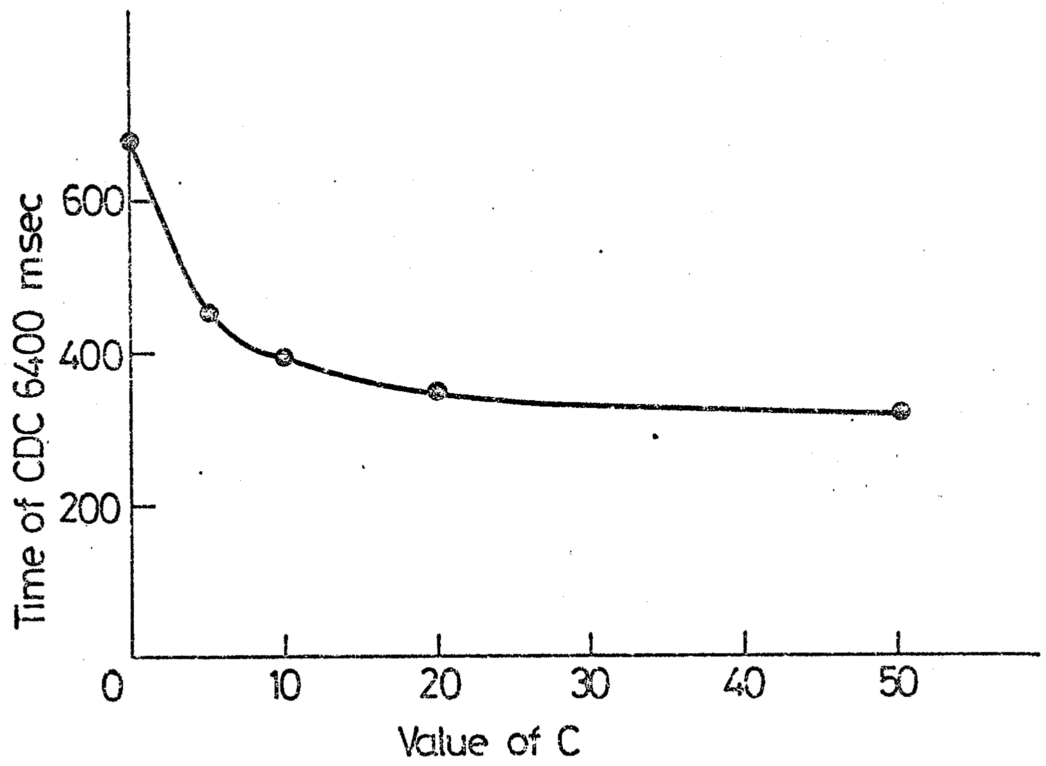

FIG.4

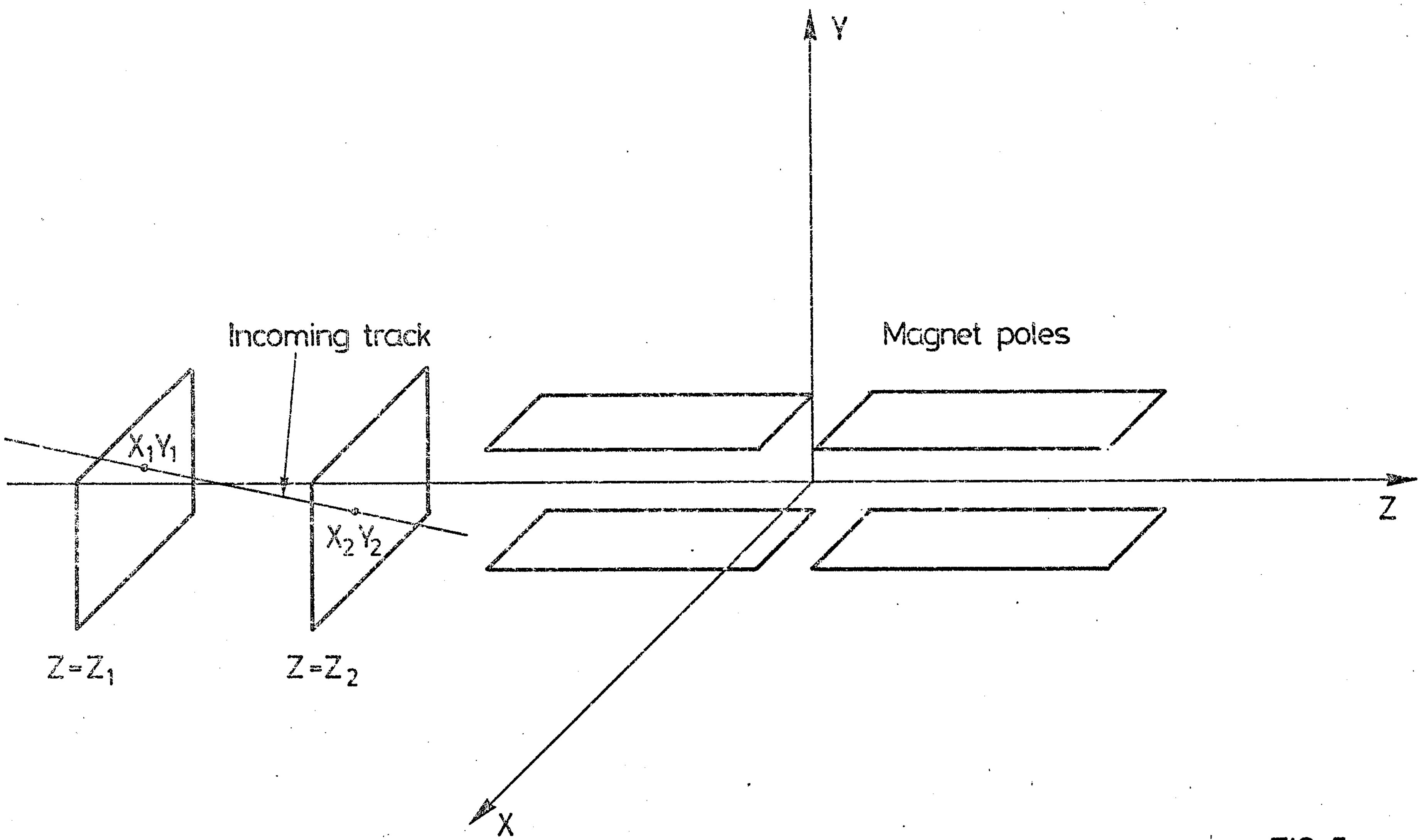

FIG. 5

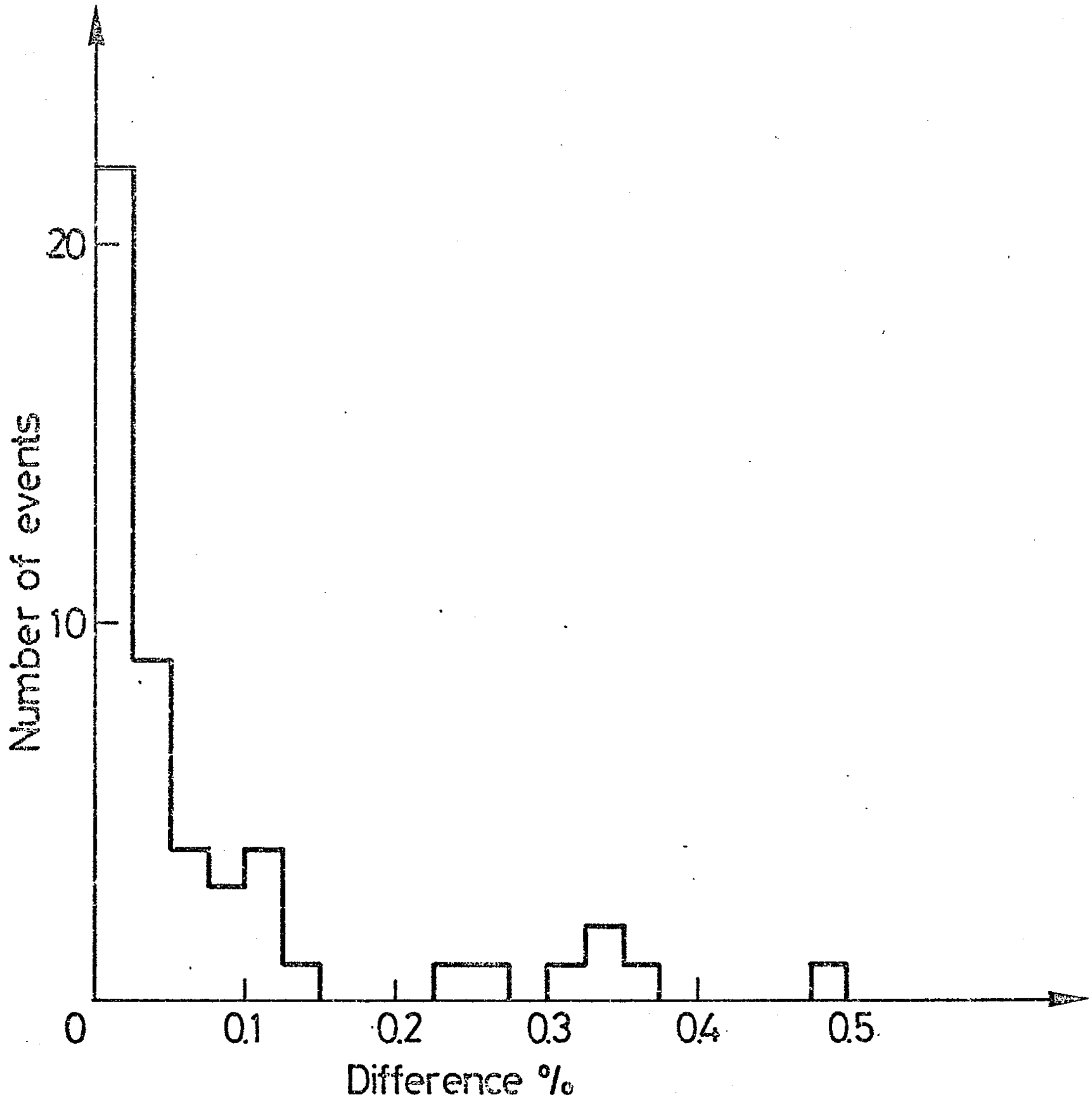

FIG. 6