

Study of oblate nuclear shapes and shape coexistence in neutron-deficient rare earth isotopes

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Abstract

We propose to investigate nuclear shapes and shape coexistence in neutron-deficient rare earth nuclei below the $N=82$ shell closure at the ISOLDE facility by employing Coulomb excitation of Nd, Sm, Gd, and Dy beams from the REX accelerator and the Miniball experiment. Nuclear shapes are expected to change rapidly in this region of the nuclear chart. The measurement of electric quadrupole moments of excited states and the transition rates between them serves as a stringent test of theoretical models and effective nucleon-nucleon interactions.

Introduction

The shape is one of the most fundamental properties of an atomic nucleus, along with its mass and radius. It is governed by the interplay of macroscopic, liquid-drop like properties of the nuclear matter and microscopic shell effects. In a nucleus with partially filled shells the valence nucleons tend to polarize the core towards a deformed mass distribution. The deformation can be described by a multipole expansion, with the quadrupole deformation being the most important deviation from spherical shape. Such quadrupole shapes can either have axial symmetry, in which case one distinguishes elongated (prolate) and flattened (oblate) shapes, or the deformation can be without axial symmetry resulting in different elongations along the three axes of the system, referred to as triaxial shape. In some areas of the nuclear chart the shape is very sensitive to structural effects and can change from one nucleus to its neighbor. In addition to shape changes with proton or neutron number, the shape can also change with excitation energy or angular momentum within the same nucleus. Such changes are caused by a rearrangement of the orbital configuration of the nucleons or by the dynamic response of the nuclear system to rotation. In some cases configurations corresponding to different shapes coexist at similar energies. The wave functions of such states can then mix according to the laws of quantum mechanics. The experimental observables most closely related to the nuclear shape are quadrupole moments of excited states and electromagnetic transition rates between them. The experimental measurement of these observables represents a stringent test for theoretical models, in particular in the case of shape coexistence and shape mixing.



To first order, describing the nuclear potential as a harmonic oscillator, the binding energy is independent of the sign of the deformation, and prolate and oblate shapes should be equally probable. In light nuclei prolate and oblate shapes occur indeed more or less equally. For heavier nuclei ($N, Z > 50$), where the shell structure changes from a harmonic oscillator type to a Mayer-Jensen type with intruder orbitals, a strong dominance of prolate shapes is observed, which has been related to the strength of the spin-orbit interaction relative to the radial term in the nuclear interaction [1]. Oblate shapes are then only expected when a major shell is almost filled due to the strong shape-driving effect of holes in the $\Omega=1/2$ orbitals [2]. This effect is seen for example in HFB calculations, which predict a dominance of oblate ground-state shapes just below the $N=82$, $N=126$, and $Z=82$ shell closures (see Fig. 1).

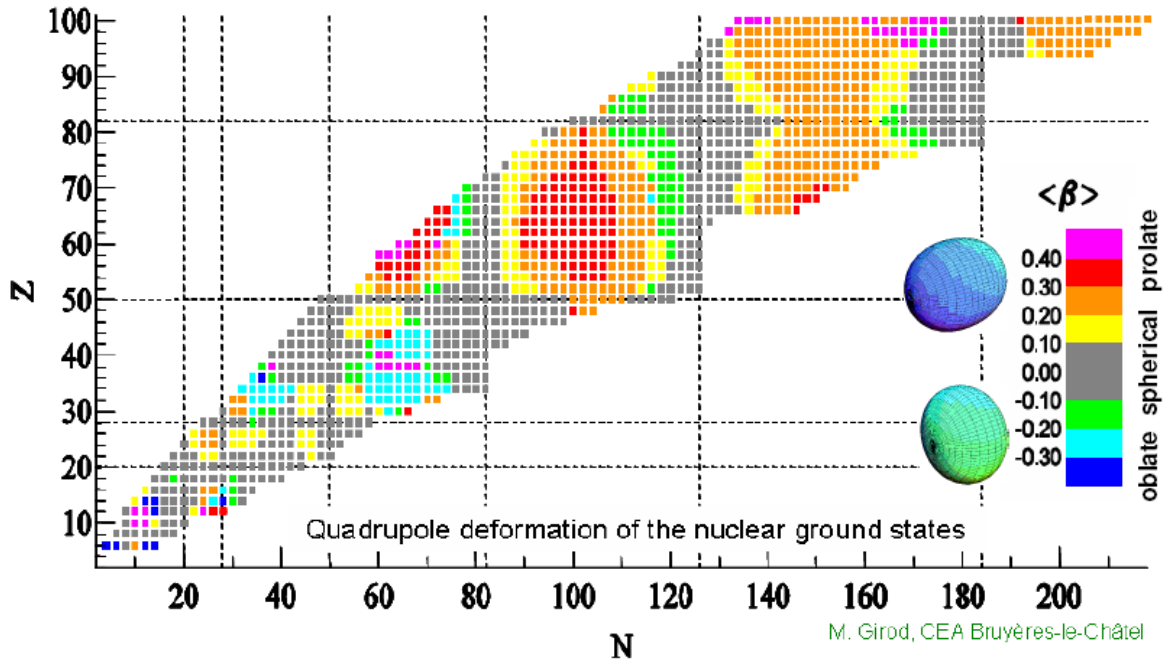


Fig.1. Nuclear chart showing the ground-state shapes predicted by a Hartree-Fock-Bogolyubov (HFB) calculation with the Gogny D1S effective interaction. The classical shell closures, or magic numbers, are marked by dotted lines. Large prolate deformations ($\beta > 0$) are found above the $Z=50$ and below the $N=82$ shell closures, with a small area of oblate shapes ($\beta < 0$) for $Z \geq 62$ and $N \approx 78$.

Apart from the fundamental question where in the nuclear chart oblate shapes can be found, nuclei with oblate ground-state shapes are also the best candidates to study the phenomenon of oblate-prolate shape coexistence, since prolate shapes become rapidly favored with angular momentum due to their higher moment of inertia, so that oblate shapes can only compete at low angular momentum. The aim of this proposal is to study nuclear shapes in the region of rare earth elements with $N \approx 78$ and $Z \geq 60$, for which oblate shapes are predicted near the ground state. Because the calculation of nuclear shapes and related observables is very sensitive to structural effects in this region of rapid change, the neutron-deficient rare earth nuclei are an ideal testing ground for theoretical models and the effective nucleon-nucleon interactions that they employ.

Present experimental knowledge

The neutron-deficient rare earth nuclei are accessible via heavy-ion induced fusion-evaporation reactions. Some of them, e.g. ^{142}Gd [3,4], have been studied in great detail up to high spins, where, among other phenomena, superdeformation [5] and magnetic rotational

(“shears”) bands [6] have been observed. However, even for nuclides which have been studied in detail at high spin, experimental studies of the shapes near the ground state are lacking completely. The experimental level schemes of the N=78 isotones from ^{138}Nd to ^{144}Dy are shown in Fig. 2. Yrast cascades are known from fusion-evaporation experiments; non-yrast states have been observed in β -decay studies except for ^{144}Dy . No B(E2) values between low-lying states are known for any of the nuclei in the region except for the B(E2; $2_1 \rightarrow 0_1$) value in $^{138}\text{Sm}_{76}$ [7].

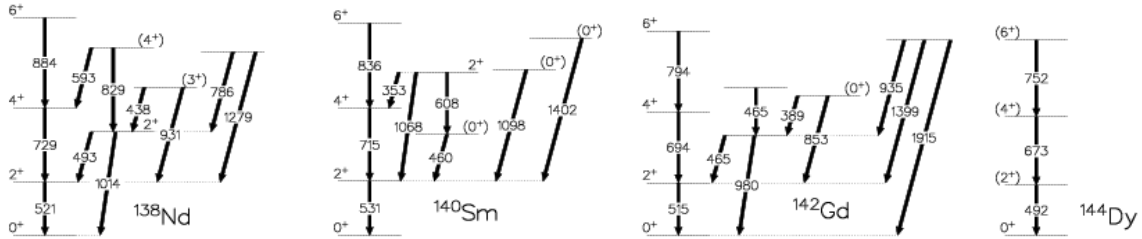


Fig.2. Experimental level schemes for the N=78 isotones ^{138}Nd [8], ^{140}Sm [9], ^{142}Gd [10], ^{144}Dy [11].

The level scheme of ^{138}Nd is typical for a γ -soft nucleus: The energies of the yrast cascade suggest a transitional character (far from a rigid well-deformed rotor), and the regular cascade built on the 2_2^+ state can be interpreted as a γ -vibrational band. However, conclusions about the sign of the deformation, i.e. prolate or oblate character, cannot be drawn. In ^{140}Sm the state at 990 keV excitation energy has been tentatively assigned as an excited 0^+ state [12]. If this assignment is correct, the very low-lying 0^+ state could be interpreted as a sign of shape coexistence. The structure of the non-yrast states in ^{142}Gd is unclear, and experimental information on excited states in ^{144}Dy is sparse. From the known excited states in the neighboring N=76 isotones it is obvious that the collectivity increases very rapidly away from the N=82 shell closure, but again, no conclusions can be drawn about the sign of the deformation.

The possibility to accelerate beams of rare-earth isotopes at the REX-ISOLDE facility for Coulomb excitation experiments represents a unique opportunity to obtain not only B(E2) values, but also quadrupole moments including their sign. Coulomb excitation of ISOL beams at low energy is the only experimental technique which can provide electric quadrupole moments of short-lived excited states in unstable nuclei. Such data provides much-needed benchmarks for state-of-the-art nuclear structure theory.

Theoretical predictions

The HFB calculations with Gogny D1S interaction shown in Fig.1 predict strongly deformed prolate shapes in the deformed region above $Z=50$ and below $N=82$, except for a small region of oblate shapes for the most proton-rich N=78 and N=76 isotones. Relativistic mean-field (RMF) calculations with the NL-SH parameterization of the RMF Lagrangian find a similar shape transition from prolate for $N>82$ to spherical at $N=82$ to oblate at $N=78$ and back to prolate for $N<78$ [13]. This shape transition is illustrated in Fig.3, which is taken from Ref.[13] and which also shows deformation parameters based on the empiric mass formulae FRDM and ETF-SI.

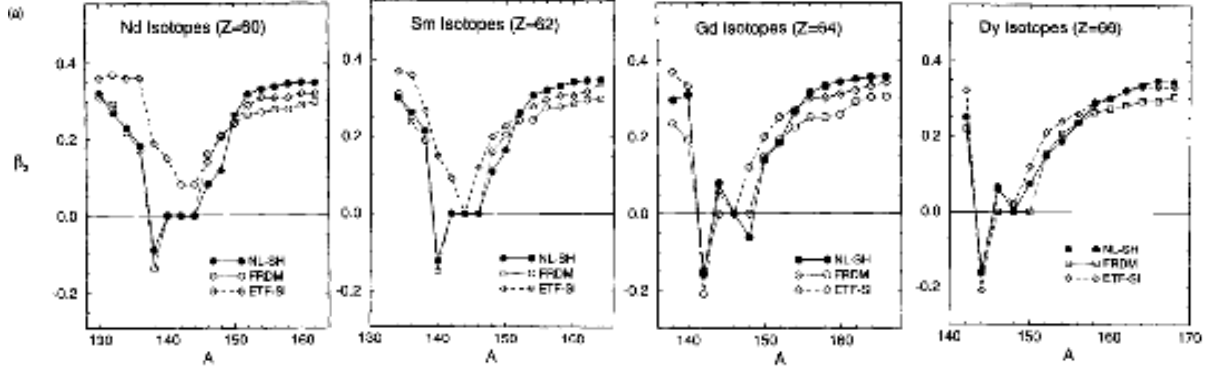


Fig. 3. Quadrupole deformation β_2 from RMF calculations using the NL-SH effective interaction [13].

The rapid shape transition predicted by the different mean-field calculations suggests that shape coexistence may be found in the transitional region. To account for configuration mixing, correlations beyond the mean field have to be considered in the calculations. Measuring spectroscopic properties of shape coexisting states represents a particularly stringent test of such configuration-mixing calculations. Significant progress has been achieved in recent years in the description of collective nuclear excitations in models which include such correlations. Configuration mixing can be described by introducing fluctuations in the collective degrees of freedom with the so-called Generator Coordinate Method (GCM). The GCM formalism is relatively straight-forward if only axial shapes are considered. For the description of non-axial deformations, however, also rotations about the three Euler angles have to be taken into account. Full GCM calculations in five dimensions have only very recently been attempted for the light system ^{24}Mg [14]. The task can be facilitated by assuming Gaussian overlap between the HFB basis states. The so-called Gaussian overlap approximation (GOA) makes configuration mixing calculations feasible also in heavier systems [15]. We have performed HFB-based configuration mixing calculations using the Gogny D1S interaction and the GCM-GOA approach comprising axial and non-axial quadrupole deformations to investigate the sensitivity of the model to the shape effects described above.

The calculations reproduce the experimentally known excitation energies well. As an example, the calculated level scheme for ^{144}Dy is shown in Fig.4. The calculated level schemes of all $N=78$ isotones are similar and resemble the experimental one of ^{138}Nd . Our calculations confirm their γ -soft character. The lowest-lying states can be grouped into three bands (only states with positive parity are calculated): the ground-state band, a γ -vibrational band, and a band based on an excited 0^+ state with larger deformation of opposite sign compared to the ground-state band. The collectivity is increasing slowly with proton number Z and rapidly with decreasing neutron number N . The most interesting result concerns the sign of the quadrupole moments: Along the chain of $N=78$ isotones the sign of the quadrupole moments changes from $Z=62$ to $Z=64$. The signs are consistent with prolate ground-state and γ bands in ^{138}Nd and ^{140}Sm , and oblate ground-state and γ bands in ^{142}Gd and ^{144}Dy . While γ vibrations are among the most commonly encountered excitation modes in deformed nuclei, there exists to our knowledge no direct evidence for γ -vibrational bands built on oblate shapes. Results of our calculations are presented in Table 1. Note that the 2_2^+ states are found with predominant $K=2$ character, resulting in opposite sign of $Q_s(2_2^+)$ compared to $Q_s(2_1^+)$, even though the intrinsic shape is the same. The change of signs from ^{140}Sm to ^{142}Gd , on the other hand, reflects a change of the intrinsic shape. The last column of Table 1 shows the spectroscopic quadrupole moments extracted from the theoretical $B(E2)$ values using the relation between $B(E2)$ and Q_s of the rotational model. As can be seen, the quadrupole

moments are significantly smaller than the rotational values, in particular for ^{140}Sm and ^{142}Gd , for which the mixing of prolate and oblate configurations is expected to be strongest.

One would expect to observe low-lying 0^+ states, such as the one tentatively assigned in ^{140}Sm [12], in the nuclei near the predicted shape transition. The calculations find the lowest 0^+ states at excitation energies of about 2 MeV. However, there are examples also in other mass regions where the GCM-GOA calculations have overestimated the excitation energies of 0^+ states. To investigate the character of the 990 keV state in ^{140}Sm and to search for low-lying 0^+ states in neighboring nuclei is therefore of importance. If 0^+ states indeed exist at such low excitation energies, Coulomb excitation experiments will be able to populate and identify them.

	$B(E2; 2_1^+ \rightarrow 0_1^+)$ [$e^2\text{fm}^4$]	$B(E2; 4_1^+ \rightarrow 2_1^+)$ [$e^2\text{fm}^4$]	$Q_s(2_1^+)$ [efm^2]	$Q_s(2_2^+)$ [efm^2]	$Q_s(4_1^+)$ [efm^2]	$Q_{\text{RM}}(2_1^+)$ [efm^2]
^{138}Nd	1736	2853	-30	+31	-31	± 84
^{140}Sm	2055	3344	-12	+12	-15	± 92
^{142}Gd	2376	3847	14	-17	8	± 99
^{144}Dy	2743	4476	37	-44	29	± 106

Table 1. $B(E2)$ values and spectroscopic quadrupole moments from the configuration-mixing calculations with Gogny D1S interaction. The last column shows the spectroscopic quadrupole moment of the 2_1^+ states calculated from the $B(E2)$ values using the rotational model.

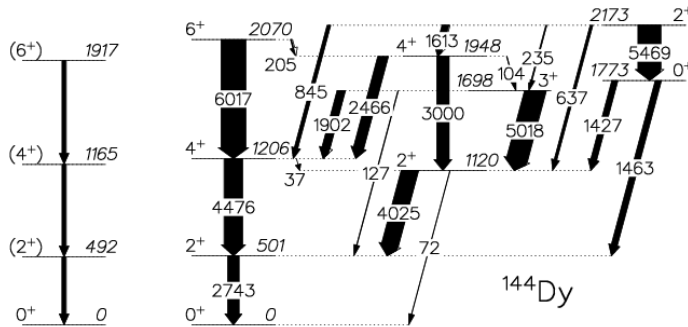


Fig.4. Experimental and calculated level scheme of ^{144}Dy . The labels of the transitions give the $B(E2)$ values in $e^2\text{fm}^4$, and the width of the arrows is proportional to these values. The calculated quadrupole moments suggest oblate character for the ground-state and γ -vibrational bands and a well-deformed prolate rotational band built on the 0_2^+ state. No $B(E2)$ values are known experimentally.

Proposed experimental program

We propose to exploit the unique capability of the ISOLDE facility to produce re-accelerated beams of neutron-deficient rare earth elements for Coulomb excitation experiments to study the predicted shape transition and shape coexistence in this region of the nuclear chart. The required isotopes can be produced by proton-induced fragmentation using for example Ta targets. Yield measurements are available for some isotopes and show that they can be produced with sufficient intensity, e.g. for ^{140}Sm ($2 \cdot 10^8$ ions/ μC), ^{138}Sm ($5 \cdot 10^6$ ions/ μC), or ^{142}Gd ($1.2 \cdot 10^6$ ions/ μC) [16]. The production of sufficiently pure rare earth beams with the ISOL method is very challenging due to their similar chemical properties, but can be achieved by resonant laser ionization. The development of rare earth beams at ISOLDE is on-going and first tests have given promising results. Laser ionization schemes have been developed at ISOLDE for Nd and Dy [17]. Ionization schemes exist also for Sm and Gd, but have not yet been tested at ISOLDE. Previous experiments, e.g. with Hg beams (IS452), have shown that a sufficient charge breeding efficiency of the EBIS and transmission of the REX accelerator can be achieved for such heavy beams, resulting in overall transmission efficiencies of 1% or more.

The beams will be accelerated to the maximum attainable energy of 2.95 MeV/u and Coulomb excited on a heavy target such as ^{206}Pb , ^{208}Pb , or an even-even Pt isotope. The final choice of the target material depends on the purity of the beam that is achieved. If beam contaminants cause too many γ -ray transitions in the spectrum which may partly overlap with transitions following target excitation, it will be better to use ^{208}Pb and normalize the Coulomb excitation yields to the Rutherford cross section. Otherwise it is advantageous to observe the Coulomb excitation of the target nucleus for normalization purposes. The scattered projectiles and recoiling target nuclei will be detected in a highly segmented double-sided silicon detector placed about 25 mm downstream from the target and covering scattering angles between $\sim 15^\circ$ and $\sim 50^\circ$ degrees in the laboratory system. The kinematics of the reaction allows distinguishing projectile and target nuclei in the detector. The MINIBALL array of segmented germanium detectors will be used to detect γ rays following the Coulomb excitation.

Due to the high Z of both projectile and target nuclei the Coulomb excitation cross sections are large even at the relatively low beam energy. Using the experimentally known excitation energies of the states and the $B(E2)$ values and quadrupole moments from our GCM-GOA calculations, we estimate the Coulomb excitation cross sections for the 2_1^+ states to be 4.9 b (^{138}Nd), 5.2 b (^{140}Sm), 6.0 b (^{142}Gd), and 7.4 b (^{144}Dy) for a ^{208}Pb target and the angular range covered by the silicon detector. The cross sections for the 4_1^+ states are estimated to increase from 140 mb in ^{138}Nd to 240 mb in ^{144}Dy . The cross sections for the 2_2^+ states depend strongly on their structure, but can be expected to be of similar magnitude as the ones for the 4_1^+ states. The decreasing production yields with increasing Z will hence be partly compensated by the higher cross sections. Due to the higher collectivity, the cross sections will be even higher for the $N=76$ isotones, rendering measurements possible. For example, we estimate the cross section to populate the 2_1^+ states in ^{138}Sm to be 10.8 b. With such large cross sections it will be possible to exploit the angular dependence of the cross sections in a differential measurement, where the so-called reorientation effect gives access to the diagonal matrix elements and hence to the spectroscopic quadrupole moments. Our previous experiments with radioactive Kr beams at GANIL [18], for which the cross sections were similar, have shown that such measurements are feasible with beam intensities as low as 10^4 /s.

The goal of the proposed experiments is to measure the quadrupole moments at least of the 2_1^+ states and the transitional matrix elements, i.e. the $B(E2)$ values, between the 2_1^+ , 2_2^+ , 4_1^+ states. Furthermore it will be possible to identify low-lying 0_2^+ states, such as the one proposed in ^{140}Sm . The sensitivity of the measurement to the reorientation effect and hence to the quadrupole moment of the 2_1^+ state is illustrated in Fig.5 for the example of ^{144}Dy .

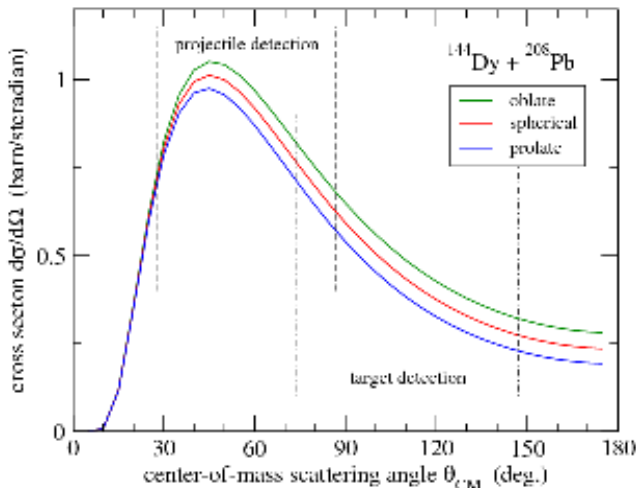


Fig. 5. Differential Coulomb excitation cross section for the first 2^+ state in ^{144}Dy populated on a ^{208}Pb target at 2.9 MeV/u, based on the quadrupole moments and $B(E2)$ values from the Gogny calculation (green curve). If the quadrupole moment is set to zero, the red curve is obtained, if the sign is inverted (to prolate shape), the blue curve is found. This illustrates the sensitivity of the measurement to the reorientation effect. The vertical lines show the angular range covered by the silicon detector.

Further testing is needed in order to establish reliable numbers for the beam intensities and purities. While the intensities can be estimated based on previous yield measurements, the beam purities are more difficult to predict. Improvement of the purity is expected to come from using a new GdB₆ cavity, which will be tested soon for Nd and which should suppress unwanted surface ionization of isobaric contaminants also for other rare earth elements. The order and schedule of the experimental program will depend on the outcome of these beam tests. We propose to investigate the predicted shape transition and shape coexistence in this region in a first experiment using two different beams, ideally ¹⁴⁰Sm and ¹⁴²Gd. A measurement with ¹⁴⁴Dy beam will be more challenging due to the lower expected production yield, but constitutes a main goal of the experimental program. We envisage performing a measurement on ¹⁴⁴Dy as the second step in our program. A measurement with less exotic ¹³⁸Nd would be more easily achievable and could be an alternative if problems are encountered with Sm or Gd during the beam tests.

The ISOLDE data base lists production yields of up to $2 \cdot 10^8$ ions/ μ C for ¹⁴⁰Sm and $1.2 \cdot 10^6$ ions/ μ C for ¹⁴²Gd. If we assume an average proton current of 2 μ A and an overall REX efficiency of 1%, we would expect the beam intensities at Miniball to be up to $4 \cdot 10^6$ /s and $2.4 \cdot 10^5$ /s for ¹⁴⁰Sm and ¹⁴²Gd, respectively. However, experience shows that the maximum yields listed in the ISOLDE data base are not always sustainable over extended periods of time. We judge intensities of $4 \cdot 10^5$ /s for ¹⁴⁰Sm and $2 \cdot 10^4$ for ¹⁴²Gd at Miniball to be more realistic. With these intensities and the cross sections estimated above, and assuming a Miniball efficiency of 8%, we expect to observe 14000 γ rays from the 2⁺ and 460 from the 4⁺ state per 8 hour shift in ¹⁴⁰Sm. For ¹⁴²Gd the respective numbers are 800 and 25 per shift, respectively. To perform a successful measurement we estimate to need 6 shifts with ¹⁴⁰Sm beam and 15 shifts with ¹⁴²Gd. In addition, 3 shifts are needed for each case for tuning and to account for losses due to experimental problems. To summarize, we ask for the the following beam time:

Beam	Min. intensity*	Target material	Ion source	Shifts
¹⁴⁰ Sm	$4 \cdot 10^5$	Ta	RILIS	6+3
¹⁴² Gd	$2 \cdot 10^4$	Ta	RILIS	15+3

* at Miniball

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