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ISR PERFORMANCE REPORT

Transverse intra-beam scattering

1. Conclusion

The beam blow-up due to transverse intra-beam scattering is much slower than the observed growth of the effective beam height at high currents (e.g. Run 300"P). It is compatible with observations at intermediate currents (e.g. Run 300'P).

2. Formulae

The rms betatron amplitudes of a coasting beam grow by intra-beam scattering. (C. Pellegrini; LNF-68/1.) The rate of change of the rms vertical betatron angle is

$$\dot{\Theta}^2 = \frac{\pi^{\frac{1}{2}}\, N\, r_o^{\,2}\, c}{V \langle x'^2 \rangle^{\frac{1}{2}} \gamma^5 \beta^3}\; F(\lambda_V)$$

Here, r_o, c, β and γ have their normal meanings. V is the betatron oscillation volume

$$V = 8\pi^2\, R\, \langle x^2 \rangle^{\frac{1}{2}}\, \langle z^2 \rangle^{\frac{1}{2}}$$

$\langle x^2 \rangle^{\frac{1}{2}}$ and $\langle z^2 \rangle^{\frac{1}{2}}$ are the rms betatron oscillation amplitudes, $\langle x' \rangle^{\frac{1}{2}}$ and $\langle z'^2 \rangle^{\frac{1}{2}}$ are the rms betatron angles, they are related by

$$\langle z'^2 \rangle^{\frac{1}{2}} = \frac{Q_V}{R} \langle z^2 \rangle^{\frac{1}{2}}$$

$$\langle x'^2 \rangle^{\frac{1}{2}} = \frac{Q_h}{R} \langle x^2 \rangle^{\frac{1}{2}}$$

λ_V is a cut-off parameter

$$\lambda_V = \left(\frac{2\bar{\lambda}_c}{\beta\gamma \quad x'^{2\ \frac{1}{2}}}\right)^2 \left(\frac{N}{V}\right)^{2/3}$$

and $\lambda\!\!\!^{-}_c = 2.103 \times 10^{-16}$ m is the proton Compton wavelength. The function $F(\lambda_v)$ is shown in Fig. 1.

N is the number of particles which are stacked within a momentum bite occupying the same radial width as $2\langle x^2\rangle^{\frac{1}{2}}$. This is a somewhat arbitrary choice due to the absence of a theory for a flat beam with many parallel equilibrium orbits.

3. Numerical example

Take the parameters of a shaved high current stack

$h_{eff} = 4$ mm (corresponds to an emittance of $0.37\pi\mu$radm)

$\langle x^2\rangle^{\frac{1}{2}} = 3$ mm (corresponds to an emittance of $2\pi\mu$radm)

$\gamma = 24$

10 A in 50 average radial mm's (hence $4\cdot 10^{12}$ protons/mm)

We calculate:

$$\langle z^2\rangle^{\frac{1}{2}} = 1.26 \text{ mm}$$

$$\langle x'^2\rangle^{\frac{1}{2}} = 0.175 \times 10^{-3}$$

$$N = 2.4 \times 10^{13}$$

$$V = 0.0447 \text{ m}^3$$

$$\lambda_v = 6.6 \times 10^{-17} \text{ m}$$

$$F(\lambda_v) = 50$$

$$\dot{\theta}^2 = 2.4 \times 10^{-14} \text{ s}^{-1}$$

To convert this into the growth rate of the effective height we multiply by the square of the vertical β-function at the crossing, $\beta_v = 13.8$ m, and by a factor of 4π, the ratio between h^2_{eff} and $\langle z^2\rangle$:

$$\dot{h}^2_{eff} = 5.7 \times 10^{-5} \text{ mm}^2 \text{ s}^{-1}$$

This means that the effective height grows from 4 mm to 4.25 mm within 10 hours. The initial doubling time $\tau_{\frac{1}{2}}$ for h_{eff} is

$$\tau_{\frac{1}{2}} = 2h_{eff}^2 \,/\, \dot{h}_{eff}^2 = 156 \text{ hours.}$$

4. Discussion

The value of $\dot{h}_{eff}^2$ calculated above is not high enough to explain the observed growth of the effective height. However, it is only about an order of magnitude below such values. So the question arises whether we can collect this factor by small changes in our data. If we reduce the emittances by factors a and b, we gain a factor ab which is not much. We may also change the way of calculating N, which is not well established, and gain a corresponding factor. This seems to be a more promising way of attack.

It would also be interesting to compare the diffusion rate due to intra-beam scattering with that for Coulomb scattering on the rest gas.

5. Relation between $\langle z^2 \rangle$ and h_{eff}^2

The growth rates for h_{eff}^2 depend critically on the factor between $\langle z^2 \rangle$ and h_{eff}^2 which was taken to be 4π. So here comes the derivation. The effective height is given by (S. van der Meer, ISR-PO/68-31):

$$h_{eff} = \frac{\int \rho_1 (z)\, dz \int \rho_2 (z)\, dz}{\int \rho_1 (z)\, \rho_2 (z)\, dz}$$

where ρ_1 and ρ_2 are the densities in the two beams. Assuming Gaussian distributions

$$\rho_1 (z) = \rho_2 (z) = \frac{1}{\langle z^2 \rangle^{\frac{1}{2}} \sqrt{2\pi}} \exp\left(- \frac{z^2}{2\langle z^2 \rangle}\right)$$

we get

$$h_{eff} = 2\sqrt{\pi}\, \langle z^2 \rangle$$

as claimed above.

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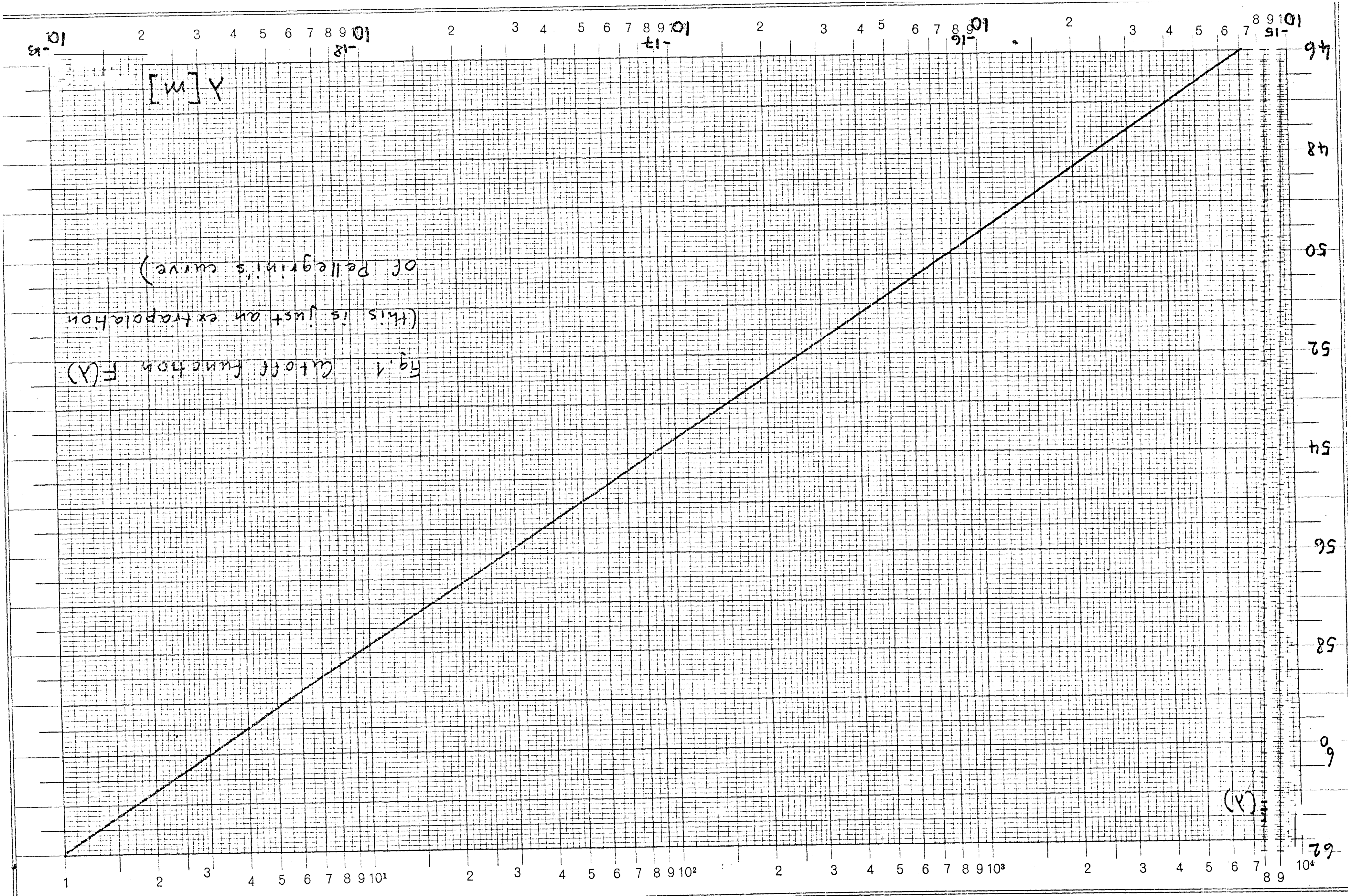

Fig. 1 Cutoff function F(λ)
(this is just an extrapolation
of Pellegrini's curve)
λ [m]
10^{-19}
10^{-18}
10^{-17}
10^{-16}
10^{-15}
46
48
50
52
54
56
58
60
62
(λ)