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MODEL OF SUPERGRAVITY WITH MINIMAL ELECTROMAGNETIC INTERACTION

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Introduction.

The essential progress has been recently achieved in the construction of various models of supergravity [1-7] . The model of Refs. [2,4] proved to be renormalizable on the mass-shell in the one-loop and probably two-loop approximations [8] , while matter in models of Ref. [5] remains non-renormalizable [9] (on the mass-shell) similarly to the situation in the standard gravity. The latter is connected with the fact that in mentioned models matter is introduces in global supermultiplets without an extension of a local supergroup. Therefore it is desirable to construct models which would include matter by means of the extension of the local supergroup in the framework of a unique local multiplet. The present paper presents a step in this direction. Here we develop the model of supergravity, unifying the gravitational field (e_a^ν), electromagnetic field (A_ν) and the charged spin 3/2 field (ψ_ν , $\bar\psi_\nu$). The spin 3/2 field is complex, and corresponding supertransformations are described by complex spinor parameters. The introduction of the minimal interaction of the spin 3/2 field with the electromagnetic field in the framework of a local supergroup leads in particular to the appearance of the cosmological term and the massive term of the spin 3/2 field. *

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* In ref. [6] an analogous phenomenon takes place for the case of the Majorano fields with spins 3/2 and 1/2 and axial vectorial field.

The model of Ref.[7] is the particular case of our model when the minimal interaction is absent ($e=0$). We present below our method of obtaining the action and the form of supertransformations without the usage of the method of Ref.[7] which came to our knowledge after the main stages of the present work were completed.

In Sec.I the general scheme of obtaining the action and the supergroup and their explicit form are considered. In Sec. II the algebra of supertransformations is considered, which was obtained in Sec.I accounting the terms vanishing by virtue of equations of motion. In Appendix the method used in the body of the paper to verify the vanishing of the arbitrary structure of the higher order in ψ_ν is described and a much easier way to prove the invariance of the action under supertransformations is demonstrated.

I.

Action and Supertransformations.

Our goal is to construct a theory of supergravity including the complex spin 3/2 field and invariant under supertransformations containing complex spinor parameters. Such a theory can exist only if some additional boson fields are present (besides the gravitational one). We assumed that the electromagnetic field must be this additional boson field, because complex fields describe the charged objects. We have choosen the action (1.1) and supertransformations (1.2) as initial ones, these being the covariant generalizations (including the minimal electromagnetic interaction) of the action and the supergroup in the case of free fields. Further we succeeded in the modification of the action and supertransformations by terms containing no derivatives and linear in $F_{\nu\mu}$ in such a way as to achieve the invariance of the action under the supergroup in the lowest order in ψ_ν. Finally, adding the higher-order terms in ψ_ν we achieved the complete invariance of the action.

Thus the initial action and supertransformations are of the form [*]:

$$L^0 = -\frac{g^{1/2}}{4} F_{\mu\nu} F^{\mu\nu} - \frac{g^{1/2}}{4\varkappa^2} R(e) + \frac{i}{2} \epsilon^{\lambda\rho\mu\nu} \bar{\psi}_\lambda \gamma_5 \gamma_\mu D_\nu \psi_\rho \tag{1.1}$$

$$\delta^0 \psi_\nu = \varkappa^{-1} D_\nu \epsilon \tag{1.2}$$

where

$$\begin{aligned} D_\nu &= \partial_\nu + \tfrac{1}{2} \omega_{\nu,\alpha\beta} \sigma^{\alpha\beta} + ieA_\nu ; \\ F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu ; \\ \omega_{\nu,\alpha\beta} &= \tfrac{1}{2} \left[e^\mu_\alpha (\partial_\nu \beta_{\mu\beta} - \partial_\mu \beta_{\nu\beta}) - \partial_\delta (e^\rho_\alpha) e^\delta_\beta g_{\rho\nu} - (\alpha \leftrightarrow \beta) \right]; \\ \beta^\alpha_\mu &\equiv (e^\mu_\alpha)^{-1} ; \quad g^{\mu\nu} = e^\mu_\alpha e^{\nu\alpha} . \end{aligned} \tag{1.3}$$

The supertransformation for $\bar{\psi}_\nu$ follows from relations of the type (1.2) by the Diracts conjugation and will not be

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[*] Our metric $g^{diag}_{\nu\mu} = (+---)$; $\gamma_5 = \gamma_0\gamma_1\gamma_2\gamma_3$; $\{\gamma_\nu, \gamma_\mu\} = g_{\mu\nu}$; $\epsilon^{0123} = 1$; $\sigma^{\nu\mu} = \frac{1}{4}[\gamma^\nu, \gamma^\mu]_-$; $g = -\det|g_{\nu\mu}|$. We also use the designation: $\epsilon^{\lambda\nu\mu\rho\delta} \equiv g^{-1/2} \epsilon^{\nu\mu\rho\delta}$. $\epsilon^{\nu\mu\rho\delta}$ is the covariant quantity in contrast to $\epsilon^{\nu\mu\rho\delta}$.

further specially mentioned.

Performing the variation of (1.1) with the aid of (1.2) we obtain that in the lowest order in ψ_ν :

$$\delta^0 L^0 = -\frac{g^{1/2}}{4}[2 F_{\mu\rho} F_\nu{}^\rho - \frac{1}{2} g_{\mu\nu} F_{\delta\rho} F^{\delta\rho}]\delta g^{\mu\nu} + \delta[-\frac{g^{1/2} R(e)}{4\mathscr{X}^2}] + g^{1/2}(D_\mu F^{\mu\nu})\delta A_\nu + \frac{i}{4}\epsilon^{\lambda\rho\mu\nu}\bar{\psi}_\lambda \gamma_5 \gamma_\mu [D_\nu, D_\rho]\epsilon \tag{1.4}$$

The dependence on $\bar{\epsilon}$ is omitted in (1.4). Since complex ϵ are considered, ϵ and $\bar{\epsilon}$ are independent quantities, and variations of L with respect to ϵ and $\bar{\epsilon}$ may be considered independently. For the complete proof of the invariance of L it is sufficient to show that $\delta L = 0$ at $\bar{\epsilon} = 0, \epsilon \neq 0$ (formally), and use the hermiticity of L. Similarly to the case of Majorana spinors [2,4] , $\delta[-\frac{g^{1/2}}{4\mathscr{X}^2} R(e)]$ is compensated by the gravitational part of the commutator of covariant derivatives in the expression:

$$\frac{i}{4}\epsilon^{\lambda\rho\mu\nu}\bar{\psi}_\lambda \gamma_5 \gamma_\mu [D_\nu, D_\rho]_- \epsilon$$

(see (1.4)) also in the case of complex ψ_ν and ϵ if one puts:

$$\delta e_\alpha^\nu = \frac{\mathscr{X}}{2i}[(\bar{\psi}_\alpha \gamma^\nu \epsilon) - (\bar{\epsilon}\gamma^\nu \psi_\alpha)] \tag{1.5}$$

Thus with the aid of (1.5) one finds from (1.4):

$$\delta^0 L^0 = -\frac{g^{1/2}}{4}\delta g^{\mu\nu}[2 F_{\mu\rho} F_\nu{}^\rho - \frac{1}{2} g_{\mu\nu} F_{\rho\delta} F^{\rho\delta}] + g^{1/2} D_\mu (F^{\mu\nu})\delta A_\nu - \frac{e}{2}\bar{\psi}_\nu \gamma_5 \gamma_\mu \tilde{F}^{\nu\mu}\epsilon \tag{1.6}$$

Here $\tilde{F}^{\nu\mu}$ is the dual pseudotensor:

$$\tilde{F}^{\nu\mu} = \frac{1}{2}\epsilon^{\tau\nu\mu\rho\delta} F_{\rho\delta} \tag{1.7}$$

and $D_\mu(F^{\mu\nu})$ is the standard gravitational covariant derivative of a tensor. In order to compensate in (1.6) for the terms, quadratic in $F_{\mu\nu}$, it is necessary to add terms linear in $F_{\mu\nu}$ to the supertransformations (1.2) and the action (1.1). Also one must put:

$$\delta A_\nu = \frac{i\eta}{2}\left[(\bar{\psi}_\nu \epsilon) - (\bar{\epsilon}\psi_\nu)\right] \tag{1.8}$$

In order to compensate in (1.6) for the term, proportional to $e\,F^{\lambda\rho}$, it is necessary to add terms independent of $F_{\mu\nu}$ and containing no derivatives to the action (1.1) and supertransformations (1.2), and also to add the cosmological term to the action.

Omitting the details of calculations we present the final expressions for the action and the supergroup in the lowest order in ψ_ν:

$$L^1 = -\frac{g^{1/2}}{4} F_{\mu\nu}F^{\mu\nu} - \frac{g^{1/2}}{4\varkappa^2} R(e) + \frac{i}{2}\epsilon^{\lambda\rho\mu\nu}\bar{\psi}_\lambda \gamma_5 \gamma_\mu D_\nu \psi_\rho$$
$$+ g^{1/2}\left\{2m\bar{\psi}_\nu \sigma^{\nu\mu}\psi_\mu + \alpha + \frac{\varkappa}{2i}\left[F^{\mu\nu}(\bar{\psi}_\mu\psi_\nu) - \tilde{F}^{\mu\nu}(\bar{\psi}_\mu\gamma_5\psi_\nu)\right]\right\} \tag{1.9a}$$

$$\delta^1\psi_\nu = \varkappa^{-1}\left[D_\nu\epsilon - im\gamma_\nu\epsilon\right] - \frac{\varkappa}{2}\left(F_{\nu\mu} + \gamma_5\tilde{F}_{\nu\mu}\right)\gamma^\mu\epsilon \tag{1.9b}$$

Parameters m and α are not arbitrary but connected with the electric charge e and the gravitational constant $\varkappa$ in the following way:

$$m = \frac{\eta e}{2\varkappa}; \quad \alpha = \frac{3e^2}{2\varkappa^4}; \quad \eta = \pm 1 \tag{1.10}$$

The presence of the factor η in Eqs. (18)-(1.10) reflects the independence of the theory of the sign of the charge e.

The invariance of the action (1.9a) under the transformations (1.5) (1.8) (1.9a) with the accuracy up to terms of the third order in ψ_ν and $\bar{\psi}_\nu$ may be verified by the direct computation using the identity: $D_\nu \tilde{F}^{\nu\mu} = 0$.

The next step is the determination of terms proportional to higher powers of ψ_ν and $\bar{\psi}_\nu$, which must be added to the action (1.9a) and supertransformations (1.9b) in order to achieve the complete invariance of the action under the supergroup. Our strategy is first to reveal the complete supertransformations $\delta\psi_\nu$ and then to obtain the additional terms in the action by the knowledge of $\delta\psi_\nu$.

For the determination of $\delta\psi_\nu$ we have considered the algebra of supetransformations.

In the lowest order in ψ_ν one easily finds from (1.5), (1.8), (1.9b):

$$[\delta_2,\delta_1]_- A_\nu = D_\nu \varphi + (D_\nu \xi^\mu) A_\mu + \xi^\mu D_\mu A_\nu \qquad (1.11)$$

$$[\delta_2,\delta_1]_- \beta_\nu^a = D_\nu \xi^a + \omega'^a{}_b \beta_\nu^b \equiv$$

$$\equiv \partial_\nu(\xi^\mu)\beta_\mu^a + \xi^\mu \partial_\mu(\beta_\nu^a) + (\xi^\mu \omega_\mu{}^a{}_l + \omega'^a{}_l)\beta_\nu^l \qquad (1.12)$$

where

$$\varphi = -\xi^\mu A_\mu + \varphi' \; ; \quad \omega^{ab} = \xi^\mu \omega_\mu{}^{ab} + \omega'^{ab} \qquad (1.13a)$$

$$\xi_\mu = \frac{i}{2}\left[(\bar{\epsilon}_2\gamma_\mu\epsilon_1) - (\bar{\epsilon}_1\gamma_\mu\epsilon_2)\right]; \quad \varphi' = \frac{i\eta}{2\varkappa}\left[(\bar{\epsilon}_2\epsilon_1) - (\bar{\epsilon}_1\epsilon_2)\right];$$

$$\omega'^{ab} = \left[2m(\bar{\epsilon}_2\sigma^{ab}\epsilon_1) + \frac{i\eta}{2}\left[F^{ab}(\bar{\epsilon}_2\epsilon_1) - \tilde{F}^{ab}(\bar{\epsilon}_2\gamma_5\epsilon_1)\right]\right] - [1\leftrightarrow 2] \qquad (1.13)$$

It follows from (1.11)-(1.13) that in this approximation the commutator of two supertransformations reduces to the general coordinate transformation with parameters ξ^μ, internal rotations with parameters ω^{ab} and gauge transformations with the parameter φ.

Thus $\delta\psi_\nu$ (1.9b) must be supplemented by such terms of higher orders in ψ_ν, $\bar{\psi}_\nu$ as to obtain:

$$[\delta_2,\delta_1]_-\psi_\nu = D_\nu(\xi^\mu\psi_\mu) + \xi^\mu(D_\mu\psi_\nu) + \frac{1}{2}\omega'^{ab}\sigma_{ab}\psi_\nu - ie\varphi'\psi_\nu +$$
$$+\varkappa^{-1}\left[D_\nu(\epsilon_{1,2}) - im\gamma_\nu\epsilon_{1,2}\right] - \frac{\eta}{2}(F_{\nu\mu} + \gamma_5\tilde{F}_{\nu\mu})\gamma^\mu\epsilon_{1,2} + S_\nu + \qquad (1.14)$$

+ terms of higher order in ψ_ν, $\bar{\psi}_\nu$.

Here S_ν is the notation for terms proportional to the equations of motion for ψ_ν and $\bar{\psi}_\nu$. The presence of such terms was indicated in Refs. [2,3], they will be discussed in more detail below. The explicit form of S_ν and $\epsilon_{1,2}$ is to be found. We note that φ' and ω'ab, rather than φ and ω_{ab}, enter explicitely (1.14), because the term $\xi^\mu\omega_{\mu,ab}\sigma^{ab}$ from ω_{ab} and the term $-\xi^\mu A_\mu$ from φ provide for the needed correction of the covariant derivative in the term $\xi^\mu D_\mu\psi_\nu$.

Requiring that $\delta\psi_\nu$ should not contain derivatives of ψ_ν in higher orders, what is necessary for the locality of the action, one finds after cumbersome calculations:

$$\epsilon_{1,2} = -\varkappa\,\xi^\mu\psi_\mu = -\frac{i\varkappa}{2}\left[(\bar{\epsilon}_2\gamma^\mu\epsilon_1) - (\bar{\epsilon}_1\gamma^\mu\epsilon_2)\right] \qquad (1.15)$$

and $\delta\psi_\nu = \delta^1\psi_\nu + \delta^2\psi_\nu$ where

$$\delta^2\psi_\nu = \frac{i\varkappa}{4}\{[2\phi_{\mu\nu,\eta} + \phi_{\mu\eta,\nu}]\delta^{\eta\mu}\epsilon + 2(\overline{\psi}_\rho\psi_\rho)\delta^{\rho\eta}\gamma_\nu\epsilon\} \tag{1.16}$$

and $\delta^1\psi_\nu$ is contained in (9.6). In Eq. (1.16) the designation is used:

$$\phi_{\mu\nu,\eta} = \frac{1}{2}[(\overline{\psi}_\mu\gamma_\eta\psi_\nu) - (\overline{\psi}_\nu\gamma_\eta\psi_\mu)] \tag{1.17a}$$

The following designations will be of use below:

$$\phi_{\mu\nu} = \frac{1}{2}[(\overline{\psi}_\mu\psi_\nu) - (\overline{\psi}_\nu\psi_\mu)]; \quad \phi^5_{\mu\nu} = \frac{1}{2}[(\overline{\psi}_\mu\gamma_5\psi_\nu) - (\overline{\psi}_\nu\gamma_5\psi_\mu)] \tag{1.17b}$$

It can be shown, that besides (1.16) no other additions of higher orders in ψ_ν can arise in $\delta\psi_\nu$. On the basis of the complete knowledge of $\delta\psi_\nu$, we may consider δL^1 in the third order in ψ_ν, $\overline{\psi}_\nu$. It turns out that terms with derivatives of ψ_ν, $\overline{\psi}_\nu$ are absent in δL^1 and terms with derivatives of ϵ take the form:

$$-\frac{\delta^R L_4}{\delta\psi_\nu}\varkappa^{-1}(D_\nu\epsilon)$$

where L_4 is explicitely given by Eq. (1.18a), (see below).

Thus, introducing $L = L^1 + L_4$ and verifying directly (see Appendix) the vanishing of all other structures of the third and fifth order in ψ_ν without derivatives, we finally find that the action:

$$L = L_{3/2} + L_4 - \frac{g^{1/2}}{4}F_{\mu\nu}F^{\mu\nu} - \frac{g^{1/2}}{4\varkappa^2}[R(\Gamma) - 24m^2] \tag{1.18}$$

where

$$L_{3/2} = \frac{i}{2}\epsilon^{\lambda\rho\mu\nu}\overline{\psi}_\lambda\gamma_5\gamma_\mu D_\nu\psi_\rho + 2mg^{1/2}\overline{\psi}_\nu\delta^{\nu\mu}\psi_\mu +$$
$$+ \frac{\varkappa}{2i}g^{1/2}[F^{\mu\nu}\phi_{\mu\nu} - \widetilde{F}^{\mu\nu}\phi^5_{\mu\nu}]$$

$$L_4 = -\frac{\varkappa^2 q m}{16}\left\{2\, \varphi_{\lambda\rho;s}\, \phi^{\lambda\rho;s} + \phi_{\lambda\rho;s}\, \phi^{\lambda\rho;s} - 4\, \varphi_{\lambda\rho;}{}^{\rho}\, \phi^{\lambda\mu}{}_{;\mu} - \right.$$

$$\left. - 4\, \phi_{\nu\mu}\, \phi^{\nu\mu} + 2\, \epsilon^{\tau\lambda\rho\mu\nu}\, \varphi_{\nu\rho}\, \phi^{5}_{\lambda\mu}\right\} \qquad (1.18a)$$

$$m = \frac{qe}{2\varkappa}; \quad \alpha = \frac{3e^2}{2\varkappa^4}; \quad \eta^2 = 1 \qquad (1.10)$$

is invariant under supertransformations of the following form:

$$\delta\psi_\nu = \varkappa^{-1}\left[D_\nu\epsilon - i m \gamma_\nu \epsilon\right] - \frac{\eta}{2}\left(F_{\nu\mu} + \gamma_5 \widetilde{F}_{\nu\mu}\right)\gamma^\mu \epsilon +$$

$$+ \frac{i\varkappa}{4}\left\{\left[2\, \phi_{\mu\nu;\eta} + \phi_{\mu\eta;\nu}\right] 8^{\eta\mu}\epsilon + 2\, \phi_{\rho\delta}\, 8^{\rho\delta}\gamma_\nu \epsilon\right\} \qquad (1.19)$$

$$\delta \beta_\nu^a = \frac{i\varkappa}{2}\left[(\bar\psi_\nu \gamma^a \epsilon) - (\bar\epsilon \gamma^a \psi_\nu)\right] \qquad (1.5)$$

$$\delta A_\nu = \frac{i\eta}{2}\left[(\bar\psi_\nu \epsilon) - (\bar\epsilon \psi_\nu)\right] \qquad (1.8)$$

The expressions (1.18) (1.19) (1.9a) may be simplified by the introduction of the covariant derivative with the torsion [4]. Let us introduce the following designations:

$$\hat{\omega}_{\nu a b} = \omega_{\nu, a b} + \frac{i\varkappa^2}{2}\left[\varphi_{\nu a;b} - \phi_{\nu b;a} - \phi_{ab;\nu}\right] \qquad (1.20)$$

$$\hat{F}_{\nu\mu} = F_{\nu\mu} + i\,\eta\varkappa\, \phi_{\nu\mu} \qquad (1.21)$$

It is also convenient to introduce the supercovariant derivative [*]:

$$\varkappa^{-1}\hat{D}_\nu\epsilon = \varkappa^{-1}[\tilde{D}_\nu\epsilon - im\gamma_\nu\epsilon] - \frac{f}{2}(\hat{F}_{\nu\mu} + \gamma_5\hat{\tilde{F}}_{\nu\mu})\gamma^\mu\epsilon \tag{1.22}$$

where $\tilde{D}_\nu$ is the gravitational covariant derivative with the torsion:

$$\tilde{D}_\nu\epsilon = \partial_\nu\epsilon + \frac{1}{2}\hat{\omega}_{\nu,\alpha\beta}\sigma^{\alpha\beta}\epsilon + ieA_\nu\epsilon$$

Eq. (1.19) may be now rewritten as:

$$\delta\psi_\nu = \varkappa^{-1}\hat{D}_\nu\epsilon \tag{2.23}$$

The expressions (1.18) and (1.9a) also simplify in regard to (1.22). The action may be verified to take the form:

$$L = -\frac{g^{1/2}}{4}F_{\mu\nu}F^{\mu\nu} - \frac{g^{1/2}}{4\varkappa^2}[R(e^\mu_\alpha, \hat{\omega}_{\nu,\alpha\beta}) - 24m^2] +$$
$$+ \frac{i}{4}\epsilon^{\lambda\rho\mu\nu}[\bar{\psi}_\lambda\gamma_5\gamma_\mu\hat{D}_\nu\psi_\rho - \bar{\psi}_\lambda\hat{\bar{D}}_\nu\gamma_5\gamma_\mu\psi_\rho] -$$
$$- \frac{1}{4}\varkappa^2 g^{1/2}[\Phi_{\nu\mu}\Phi^{\nu\mu} - \frac{1}{2}\epsilon^{\tau\lambda\rho\mu\nu}(\bar{\psi}_\nu\psi_\rho)(\bar{\psi}_\lambda\gamma_5\psi_\rho)] \tag{1.24}$$

II

THE ALGEBRA OF SUPERTRANSFORMATIONS.

In Ref. [3] the algebra of supertransformations has been considered for the supergravity theory [2,4] including the Majorana spin 3/2 field and the gravitational field. We shall consider the generalization of this algebra to the case considered in the present paper.

The relations (1.11)-(1.15) describe the algebra of supertransformations in the lowest order in ψ_ν. Using the exact form of the group (1.19) (1.5) (1.8) and designations (1.20), (1.21), we find that in the agreement with general requirements:

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[*] The analogous expression for the particular case $e = 0$ (and consequently $m = 0$) is contained also in [7].

$$[\delta_2,\delta_1] A_\nu = D_\nu\varphi + D_\nu(\xi^\mu) A_\mu + \xi^\mu D_\mu A_\nu + \frac{i\varkappa}{2}[(\bar{\psi}_\nu \epsilon_{12}) - (\bar{\epsilon}_{12}\psi_\nu)] \tag{2.1}$$

$$[\delta_2,\delta_1]_- \beta^\alpha_\nu = \tilde{D}_\nu(\xi^\alpha) + \hat{\omega}'^\alpha{}_\nu \tag{2.2}$$

Here $\tilde{D}_\nu$ is the covariant derivative with the torsion:

$$\tilde{D}_\nu \xi^\alpha = \partial_\nu \xi^\alpha + \hat{\omega}_{\nu,}{}^{\alpha\beta}\xi_\beta \tag{2.3}$$

and $\hat{\omega}'_{\alpha\beta}$ coincides with $\omega_{\alpha\beta}$ in (1.13) after the replacement of $F_{\nu\mu}$ by $\hat{F}_{\nu\mu}$.

One obtains from (2.2):

$$[\delta_2,\delta_1]_- \beta^\alpha_\nu = \delta^g_{12}\beta^\alpha_\nu + \hat{\omega}^\alpha{}_\nu + \frac{i\varkappa}{2}[(\bar{\psi}_\nu)^\alpha\epsilon_{12}) - (\bar{\epsilon}_{12})^\alpha\psi_\nu)] \tag{2.4}$$

where $\delta^g_{12}\beta^\alpha_\nu$ is the general coordinate transformation β^α_ν with parameters ξ^μ, and $\hat{\omega}_{\alpha\beta}$ coincides with $\omega_{\alpha\beta}$ in (1.13) if one substitutes $\hat{\omega}_{\nu,}{}^{\alpha\beta}$ instead of $\omega_{\nu,}{}^{\alpha\beta}$ and $\hat{F}_{\nu\mu}$ instead of $F_{\nu\mu}$, i.e.:

$$\hat{\omega}^{\alpha\beta} = \xi^\mu \hat{\omega}_{\mu,}{}^{\alpha\beta} + \Big\{[2m(\bar{\epsilon}_2\sigma^{\alpha\beta}\epsilon_1) + \\ + \frac{i g\varkappa}{2}[\hat{F}^{\alpha\beta}(\bar{\epsilon}_2\epsilon_1) - \tilde{\hat{F}}^{\alpha\beta}(\bar{\epsilon}_2\gamma_5\epsilon_1)]] - (1\leftrightarrow 2)\Big\} \tag{2.5}$$

In an analogous manner one finds after cumbersome calculations:

$$[\delta_2,\delta_1]_- \psi_\nu = \delta^g_{12}\psi_\nu + \frac{1}{2}\hat{\omega}^{\alpha\beta}\sigma_{\alpha\beta}\psi_\nu - ie\varphi\psi_\nu + \varkappa^{-1}\hat{D}_\nu\epsilon_{12} + S_\nu \tag{2.6}$$

where $\hat{D}_\nu$ is the supercovariant derivative (1.22) and S_ν denotes the mentioned-above terms, proportional to the equations of motion for ψ_ν and $\bar{\psi}_\nu$ [2,3], (the explicit form of S_ν is given by Eq. (2.15)). Eq. (2.6) may be also represented as:

$$[\delta_2,\delta_1]_- \psi_\nu = D_\nu(\xi^\mu)\psi_\mu + \xi^\mu(\tilde{D}_\mu\psi_\nu) + \frac{1}{2}\hat{\omega}'^{\alpha\beta}\sigma_{\alpha\beta}\psi_\nu \ldots \\ - ie\varphi'\psi_\nu + \varkappa^{-1}\hat{D}_\nu\epsilon_{12} + S_\nu$$

where

$$D_\nu(\xi^\mu) = \partial_\nu(\xi^\mu) - \Gamma^\mu_{\nu\rho}\xi^\rho \tag{2.8}$$

$$\tilde{D}_\nu \psi_\mu = \partial_\nu \psi_\mu + \Gamma^\rho_{\nu\mu}\psi_\rho + \frac{1}{2}\hat{\omega}_{\nu,ab}\,\delta^{ab}\psi_\mu + ieA_\nu\psi_\mu \tag{2.9}$$

The Christoffel symbols in (2.8) and (2.9) are symmetric (no torsion).

Thus the commutator of two supertransformations leads to internal rotations with parameters $\hat{\omega}_{ab}$ (2.5), general coordinate transformations with parameters ξ^μ (1.13), gauge transformations with the parameter φ (1.13) and supertransformations with the parameter $\epsilon_{1,2}$ (1.15). The new fenomenon in comparance to the algebra of Ref.[3] (besides the appearance of gauge transformations) is the appearance of terms proportional to m and $\hat{F}_{\mu\nu}$ in the parameters $\hat{\omega}_{ab}$ of internal rotations.

Let us consider now the terms S_ν in (2.6) in more detail. Since the action (1.18) is invariant under supertransformations (1.19), (1.6) , (1.8) and also under internal rotations, general coordinate transformations and gauge transformations, it follows from (2.,1), (2.4), (2.6), that the action (1.18) must be invariant under the transformations of the form:

$$\delta\psi_\nu = S_\nu\,; \quad \delta\bar{\psi}_\nu = \bar{S}_\nu\,. \tag{2.10}$$

This is indeed so, although the transformations (2.10) are not connected with any symmetry. An arbitrary action is invariant under transformations of the type (2.10). Let us explain this fact. Let there be an arbitrary action, depending on a set of fields $\varphi_\alpha(x)$. Let us define the transformation:

$$\delta \varphi_\alpha = \int dx' \left[A_{\alpha\beta}(x,x') \varphi_\beta(x') \right] \tag{2.11}$$

where $A_{\alpha\beta}(x,x')$ is the infinitesimal matrix generally depending on φ_α and satisfying the condition

$$A_{\alpha\beta}(x,x') = -\eta A_{\beta\alpha}(x',x) \tag{2.12}$$

where $\eta = -1$ for fermion α and β, and $\eta = 1$ for other α and β. The quantity $\varphi_\beta(x)$ in (2.11) is defined as:

$$\varphi_\beta(x) \equiv \frac{\delta^L S}{\delta \varphi_\beta(x)} \tag{2.13}$$

It may be easily verfiied, thatthe action S is invariant under the transformation (2.11):

$$\delta S = \int A_{\alpha\beta}(x,x') \frac{\delta^L S}{\delta \varphi_\beta(x')} \frac{\delta^L S}{\delta \varphi_\alpha(x)} dx\, dx' \equiv 0 \tag{2.14}$$

The transformation (2.10) in our theory is just of the form (2.11), the Grassman fields ψ_v, $\overline{\psi}_v$ playing the role of φ_α. The corresponding matrix $A_{\alpha\beta}(x,x')$ is of the form:

$$A_{\alpha\beta}(x,x') = \hat{A}_{\alpha\beta}(x)\, \delta(x-x')$$

and

$$\hat{A}_{\alpha\beta}(x) = \hat{A}_{\beta\alpha}(x)$$

The explicit form of S_v is:

$$S_\nu = \frac{g^{m}}{2}\Big\{-(\bar{\epsilon}_2\gamma_5\epsilon_1)\gamma_5\gamma_\mu\gamma_\nu\varphi^\mu + \frac{1}{2}(\bar{\epsilon}_2\gamma_5\gamma_\mu\varphi^\mu)\gamma_5\gamma_\nu\epsilon_1 - -(\bar{\epsilon}_2\gamma_5\gamma_\nu\varphi^\mu)\gamma_5\gamma_\mu\epsilon_1 + (\bar{\epsilon}_2\gamma_5\varphi^\mu)\gamma_5\partial_{\nu\mu}\epsilon_1 + \tag{2.15}$$

$$+(\bar{\epsilon}_2\gamma_5\partial_{\nu\mu}\varphi^\mu)\gamma_5\epsilon_1 - \frac{1}{2}(\bar{\epsilon}_2\gamma_5\varphi_\nu)\gamma_5\epsilon_1 + +(\bar{\varphi}^s\partial_{\mu\nu}\gamma_5\gamma_5\epsilon_2)\gamma_5\gamma^\mu\epsilon_1 + +(\bar{\varphi}^\mu\gamma_5\epsilon_2)\gamma_5\partial_{\nu\mu}\epsilon_1 + \frac{1}{2}(\bar{\varphi}^\mu\gamma_\nu\gamma_\mu\epsilon_2)\epsilon_1\Big\} - -\{1\leftrightarrow 2\}$$

where

$$\varphi^\mu = \frac{\delta^L L}{\delta \bar{\psi}_\mu}\ ;\ \bar{\varphi}^\mu = \frac{\delta^R L}{\delta \psi_\mu}\ . \tag{2.16}$$

Performing the variation (2.10) of the action we obtain:

$$\delta^{s_\nu} L = \bar{\varphi}^\mu S_\mu + \bar{S}_\mu \varphi^\mu \tag{2.17}$$

Using (2.15) we verify that:

$$\delta^{s_\nu} L = 0 \tag{2.18}$$

Eq. (2.18) is the relation of the type of (2.14).

The properties of the transformations (2.11) (we shall call such a transformations "non-gauge" ones) will be considered in more detail in an other article. Here we formulate (without proving) the statement is connected with the group structure of this transformations.

If there exists any infinitesimal transformation

$$\delta\,\varphi_\alpha(x) = f_\alpha(\varphi, x) \tag{2.19}$$

under which the action S is invariant, then, the commutator of the transformation (2.19) with an arbitrary "non-gauge" transformation (correspondings to the actions) will again give some "non-gauge" transformation (2.11). Hence it immediately follows that supertransformations of Refs. [2-4] and the present paper form a group together with "non-gauge" transformations. This statement is true for any other gauge theory too. However the gauge theories, considered earlier, usually formed a group by themselves and therefore constituted the subgroup of a group including gauge and "non-gauge" transformations, but supertransformations do not form a subroup by themselves. When the equations of motion are satisfied, the "non-gauge" transformations become trivial and the supertransformations form a group.

CONCLUSION.

Here we want to discuss the massive and cosmological terms present in the action. The massive term of (1.18), (1.9a):

$$2m\bar{\psi}_\nu \gamma^{\nu\mu}\psi_\mu \qquad \left(m = \frac{\gamma c}{2\alpha}\right)$$

exactly coincides with the massive term in the conventional Rarita-Schwinger equation. However the conclusion, that the theory describes the massive spin 3/2 particle, is wrong. The reason is that the presence of the supergroup leaves only four (complex) independent degrees of freedom of the field ψ_ν , this corresponding to the massless complex spin 3/2 field. We think that the explanation of the paradox is the following. In the quantum theory one must introduce the space of in-states, while the interaction adiabatically switches off at the inifinity. The interaction being switched off, we have electric charge $e = 0$. Hence it follows (see Eq. (1.10)), that in the in-space $m = 0$ and $\lambda = 0$, i.e. the massive and cosmological terms are absent. The absence of these terms in the in-limit is also necessary for the preservation of the supergroup in this limit (the completeness of the in-space). The usual way of constructing the in-space cannot be applied in the present case because of the presence of the cosmological term.

APPENDIX

The direct verification of the cancellation of various structures in the variation of the action in higher orders in ψ_v and $\bar{\psi}_v$ meets the difficulty that the notation of these structures is ambiguous (the Fierz identities). When performing the calculations we used the following method. Let there be an expression:

$$A = \sum_i (\bar{\psi} A_i \psi)(\bar{\psi} B_i \epsilon) \tag{A.I}$$

(For simplicity we confined ourselves in (A.1) to the third order in ψ_v, $\bar{\psi}_v$ and omitted the indices of ψ_v and $\bar{\psi}_v$). In order that:

$$A = 0 \tag{A.2}$$

it is necessary and sufficient that the relation

$$tr\left[\Gamma\left(\frac{\delta^R}{\delta\bar{\psi}}\left(\frac{\delta^L A}{\delta\epsilon}\right)\right)\right] = 0 \tag{A.3}$$

should take place, the matrix Γ being arbitrary. When verifying the relations of the type (A.3) the notation ambiguities do not arise. It is convenient to choose for calculations the following complete set of matrices: 1 , γ_5 , $\sigma_{\nu\mu}$, $\gamma_\mu, \gamma_5\gamma_\mu$.

The invariance of the action under supertransformations and the verification of the superalgebra can be proved much easier if one uses eqs. (1.20)-(1.22). Now we shall use this method to prove the invariance of the action under supertransformations in more detail.

We shall consider $\hat{\omega}_{\nu,\alpha\beta}$ (1.20) and $\hat{F}_{\nu\mu}$ (1.21) only as suitable designations, but not as independent fields. There exists a possibility to consider $\hat{\omega}_{\nu,\alpha\beta}$ as an independent field (in an analogy to the method of [4]) because $\hat{\omega}_{\nu,\alpha\beta}$ (1.20) is the solution of the corresponding equation of motion:

$$\frac{\delta L}{\delta \omega_{\nu,\alpha\beta}}\bigg|_{\omega_{\nu,\alpha\beta}=\hat{\omega}_{\nu,\alpha\beta}} = 0 \qquad \text{(A.4)}$$

Due to (A.4) we can omit the terms with $\delta\omega_\nu^{\alpha\beta}$ in δL , because they are identically equal to zero.

$$\delta_\omega L = \frac{\delta L}{\delta\hat{\omega}_{\nu,\alpha\beta}}\,\delta\hat{\omega}_{\nu,\alpha\beta} = 0 \qquad \text{(A.5)}$$

This observation simplifes all calculations.

We shall use the action (1.18) in a slightly different form:

$$L = -\frac{g^{1/2}}{4}\hat{F}_{\alpha\beta}\hat{F}^{\alpha\beta} - \frac{g^{1/2}}{4\mathscr{X}^2}\left[R(e,\hat{\omega}) - 24m^2\right] + \\ + \frac{i}{4}\epsilon^{\lambda\rho\mu\nu}\left[\bar{\Psi}_\lambda\gamma_5\gamma_\mu\tilde{D}_\nu\Psi_\rho - \bar{\Psi}_\lambda\overleftarrow{\tilde{D}}_\nu\gamma_5\gamma_\mu\Psi_\rho\right] + \\ + g^{1/2}\left[\frac{i\eta\mathscr{X}}{2}\,\Phi^5_{\nu\rho}\hat{F}^{\nu\rho} + \frac{\mathscr{X}^2}{8}\epsilon^{\tau\lambda\mu\nu\rho}(\bar{\Psi}_\nu\Psi_\rho)(\bar{\Psi}_\lambda\gamma_5\Psi_\mu)\right] \qquad \text{(A.6)}$$

where

$$\tilde{D}_\nu = \partial_\nu + \frac{1}{2}\hat{\omega}_{\nu,\alpha\beta}\,\sigma^{\alpha\beta} + ieA_\nu - im\gamma_\nu \qquad \text{(A.7)}$$

We have obviously:

$$\hat{D}_\nu = \tilde{D}_\nu + \frac{\eta\mathscr{X}}{2}\left(\hat{F}^{\alpha\beta}\sigma_{\alpha\beta}\right)\gamma_\nu \qquad \text{(A.8)}$$

It is easy to verify, that

$$\delta\hat{F}_{\alpha\beta} = \frac{i\eta}{2}\left[(\bar{\Psi}_\beta\hat{D}_\alpha\epsilon) - (\bar{\epsilon}\overleftarrow{\hat{D}}_\alpha\Psi_\beta)\right] - (\alpha \leftrightarrow \beta) \qquad \text{(A.9)}$$

In eq. (A.9) the part of the supercovariant derivative corresponding to the vectorial part of ψ_α, $\bar{\psi}_\alpha$ is the usual gravitational covariant derivative without torsion, acting on vectorial index. In other words:

$$\hat{D}_\nu \psi_\alpha = \left[\partial_\nu + \frac{1}{2}\hat{\omega}_{\nu,ab}\sigma^{ab} + ieA_\nu - im\gamma_\nu + \frac{i\varkappa}{2}(\hat{F}_{ab}\sigma^{ab}\gamma_5)\gamma_\nu\right]\psi_\alpha + \omega_{\nu,\alpha}{}^{b}\psi_b \qquad (A.10)$$

($\omega_{\nu,ab}$ is defined by Eq. (1.3)).

Now we consider the variation of L (A.6). We must use the definition (1.20) only after the calculation of δL is done, because we use eq. (A.5).

It will be suitable to separate the action (A.6) in two parts: $L = L_0 + L_1$, and determine the variation of these parts independently. Here

$$L_0 = -\frac{g^{1/2}}{4\varkappa^2}\left[R(e,\hat{\omega}) - 24m^2\right] + \frac{i}{4}\epsilon^{\lambda\rho\mu\nu}\left[\bar{\psi}_\lambda\gamma_5\gamma_\mu\tilde{D}_\nu\psi_\rho - \bar{\psi}_\lambda\overleftarrow{\tilde{D}}_\nu\gamma_5\gamma_\mu\psi_\rho\right] \qquad (A.11)$$

$$L_1 = -\frac{g^{1/2}}{4}\hat{F}_{ab}\hat{F}^{ab} + g^{1/2}\left[\frac{i\varkappa}{2}\phi^5_{\nu\rho}\hat{F}^{\nu\rho} + \frac{\varkappa^2}{8}\epsilon^{\tau\lambda\mu\nu\rho}(\bar{\psi}_\nu\psi_\rho)(\bar{\psi}_\lambda\gamma_5\psi_\mu)\right] \qquad (A.12)$$

Using the fact that

$$[\tilde{D}_\nu, \tilde{D}_\mu] = \frac{1}{2}R_{\nu\mu,ab}\sigma^{ab} - 4m^2\sigma_{\nu\mu} - imC^{a}_{\nu\mu}\gamma_a + ieF_{\nu\mu} \qquad (A.13)$$

where $C^{\alpha}_{\nu\mu}$ is the torsion:

$$C^{\alpha}_{\nu\mu} = [\partial_\nu \beta^{\alpha}_{\mu} - \partial_\mu \beta^{\alpha}_{\nu} + \hat{\omega}_{\nu,}{}^{\alpha}{}_{\mu} - \hat{\omega}_{\mu,}{}^{\alpha}{}_{\nu}] \equiv -i\mathscr{L}^2 \Phi_{\nu\mu,}{}^{\alpha} , \tag{A.14}$$

using the method analogous to that of Ref.[4] we obtain with the aid of Ricci identities

$$\delta L_0 = \delta^0 L_0 + \delta^1 L_0 \tag{A.15}$$

where

$$\begin{aligned} \delta^0 L_0 = & -\frac{ig}{4} \epsilon^{\lambda\rho\mu\nu} \bar{\Psi}_\lambda \widetilde{D}_\nu \gamma_5 \gamma_\mu \hat{F}^{\alpha\beta} \sigma_{\alpha\beta} \gamma_\rho \epsilon - \\ & - \frac{e}{2\mathscr{L}} F^{\lambda\mu} (\bar{\Psi}_\lambda \gamma_5 \gamma_\mu \epsilon) \end{aligned} \tag{A.16}$$

$$\begin{aligned} \delta^1 L_0 = -\frac{\mathscr{L}}{4} \epsilon^{\lambda\rho\mu\nu} \{ & (\bar{\Psi}_\mu \gamma^\alpha \epsilon)(\bar{\Psi}_\lambda \gamma_5 \gamma_\alpha \widetilde{D}_\nu \Psi_\rho) - \\ & - \frac{i}{2\mathscr{L}} \delta C^{\alpha}_{\nu\mu} (\bar{\Psi}_\lambda \gamma_5 \gamma_\alpha \Psi_\rho) + \frac{i}{2\mathscr{L}^2} G^{\alpha}_{\nu\mu} (\bar{\Psi}_\lambda \widetilde{D}_\rho \gamma_5 \gamma_\alpha \epsilon) - \\ & - \frac{m}{\mathscr{L}^2} [C_{\nu\mu,\rho} (\bar{\Psi}_\lambda \gamma_5 \epsilon) + C^{\alpha}_{\nu\rho} (\bar{\Psi}_\lambda \gamma_5 \gamma_\mu \gamma_\alpha \epsilon)] + \\ & + \frac{ige}{\mathscr{L}} (\bar{\Psi}_\nu \epsilon)(\bar{\Psi}_\lambda \gamma_5 \gamma_\mu \Psi_\rho) - im (\bar{\Psi}_\nu \gamma^\alpha \epsilon)(\bar{\Psi}_\lambda \gamma_5 \gamma_\mu \gamma_\alpha \Psi_\rho) + \\ & + \frac{ig}{4\mathscr{L}} C^{c}_{\nu\mu} \bar{\Psi}_\lambda \gamma_5 \gamma_c \hat{F}^{\alpha\beta} \sigma_{\alpha\beta} \gamma_\rho \epsilon \} \end{aligned} \tag{A.17}$$

After simple straighteforward calculations we also obtain.

$$\delta L_1 = \delta^0 L_1 + \delta^1 L_1 \tag{A.18}$$

where

$$\delta^0 L_1 = \frac{i\eta g^{1/2}}{2}[(\bar{\psi}_\nu \gamma_5 \hat{D}_\rho \epsilon)\hat{\tilde{F}}^{\nu\rho} - (\bar{\psi}_\delta \hat{D}_\alpha \epsilon)\hat{F}^{\alpha\delta}] - \qquad (A.19)$$
$$- \frac{i\mathscr{L}}{8} g^{1/2}(\bar{\psi}_\nu \gamma^\nu \epsilon)\hat{F}_{\alpha\delta}\hat{F}^{\alpha\delta}$$

$$\delta^1 L_1 = -\frac{g^{1/2}\mathscr{L}}{2}\tilde{\phi}^{5\nu\rho}(\bar{\psi}_\rho \hat{\tilde{D}}_\nu \epsilon) - \frac{\mathscr{L}^2\eta}{4}\epsilon^{\nu\rho\Lambda\mu}\phi^5_{\nu\rho}(\bar{\psi}_\Lambda \gamma^\alpha \epsilon)\hat{F}_{\alpha\mu} + \qquad (A.20)$$
$$+ \frac{\mathscr{L}}{8}\epsilon^{\Lambda\mu\nu\rho}[(\bar{\psi}_\nu \hat{D}_\rho \epsilon)\phi^5_{\Lambda\mu} + \phi_{\nu\rho}(\bar{\psi}_\Lambda \gamma_5 \hat{D}_\mu \epsilon)]$$

Using the following simple identities

$$D_\nu \tilde{F}^{\nu\mu} = 0 \ ; \quad \hat{F}_{\nu\mu}\hat{\tilde{F}}^\mu{}_\rho = \hat{\tilde{F}}_{\nu\mu}\hat{F}^\mu{}_\rho \ ;$$
$$\hat{F}^{\alpha\nu}\hat{F}_{\alpha\mu} - \hat{\tilde{F}}^{\alpha\nu}\hat{\tilde{F}}_{\alpha\mu} - \frac{1}{2}g^\nu_\mu \hat{F}^{\alpha\delta}\hat{F}_{\alpha\delta} = 0 \ ;$$
$$\epsilon^{\Lambda\rho\mu\nu}\gamma_\mu(\hat{F}^{\alpha\delta}\sigma_{\alpha\delta})\gamma_\nu = 2(\hat{F}^{\Lambda\rho} + \gamma_5 \hat{\tilde{F}}^{\Lambda\rho}) \ ;$$

one may easily verify that

$$\delta^0 L_0 + \delta^0 L_1 = -\frac{\mathscr{L}}{4}\epsilon^{\Lambda\rho\mu\nu}[(\bar{\psi}_\Lambda \gamma_5 \tilde{D}_\rho \epsilon) + (\bar{\psi}_\Lambda \bar{\tilde{D}}_\rho \gamma_5 \epsilon)]\phi_{\mu\nu} \qquad (A.21)$$

After the reduction of the first term in (A.17) to the form: $(\bar{\psi}\tilde{D}\epsilon)(\bar{\psi}\psi) + (\bar{\psi}\hat{\tilde{D}}\epsilon)(\bar{\psi}\psi)$ (by means of the integration by parts and Eierz rearrangement) and the evaluation of δL (using (A.21), (A.17), (A.20)) one finds that

$$\delta L = \frac{\mathscr{L}}{8}\epsilon^{\Lambda\rho\mu\nu}\{(\bar{\psi}_\mu \gamma^\alpha \tilde{D}_\nu \epsilon)(\bar{\psi}_\Lambda \gamma_5 \gamma_\alpha \psi_\rho) +$$
$$+ (\bar{\psi}_\nu \tilde{D}_\rho \epsilon)(\bar{\psi}_\Lambda \gamma_5 \psi_\mu) - (\bar{\psi}_\Lambda \gamma_\mu \epsilon)(\bar{\psi}_\nu \psi_\rho) +$$
$$+ \frac{\eta\mathscr{L}}{2}[\phi_{\nu\rho}(\bar{\psi}_\Lambda \gamma_5 \hat{F}^{\alpha\delta}\sigma_{\alpha\delta}\gamma_\mu \epsilon) - (\bar{\psi}_\nu \hat{F}^{\alpha\delta}\sigma_{\alpha\delta}\gamma_\rho \epsilon)\phi^5_{\Lambda\mu} - \qquad (A.22)$$
$$- \phi_{\nu\mu,}{}^c(\bar{\psi}_\Lambda \gamma_5 \gamma_c \hat{F}^{\alpha\delta}\sigma_{\alpha\delta}\gamma_\rho \epsilon] +$$
$$+ 2im[(\bar{\psi}_\nu \gamma_\rho \psi_\mu)(\bar{\psi}_\Lambda \gamma_5 \epsilon) - (\bar{\psi}_\nu \epsilon)(\bar{\psi}_\Lambda \gamma_5 \gamma_\mu \psi_\rho) -$$
$$- (\bar{\psi}_\nu \gamma^\alpha \epsilon)(\bar{\psi}_\Lambda \gamma_5 \gamma_\alpha \gamma_\mu \psi_\rho) + (\bar{\psi}_\mu \gamma^\alpha \gamma_\nu \epsilon)(\bar{\psi}_\Lambda \gamma_5 \gamma_\alpha \psi_\rho) +$$
$$+ (\bar{\psi}_\nu \gamma^\alpha \psi_\rho)(\bar{\psi}_\Lambda \gamma_5 \gamma_\alpha \gamma_\mu \epsilon)]\}$$

Finally, by virtue of Eierz identities

$$\epsilon^{\lambda\rho\mu\nu}\left[(\bar{\psi}_\nu\gamma^\alpha\varphi_1)(\bar{\psi}_\lambda\gamma_5\gamma_\alpha\varphi_2)+(\bar{\psi}_\nu\varphi_1)(\bar{\psi}_\lambda\gamma_5\varphi_2)+(\bar{\psi}_\nu\gamma_5\varphi_1)(\bar{\psi}_\lambda\varphi_2)\right]=0$$

$$\epsilon^{\lambda\rho\mu\nu}\left[(\bar{\psi}_\mu\gamma^\alpha\varphi_1)(\bar{\psi}_\lambda\gamma_5\gamma_\alpha\varphi_2)-(\bar{\psi}_\mu\gamma^\alpha\varphi_2)(\bar{\psi}_\lambda\gamma_5\gamma_\alpha\varphi_1)\right]=0$$

δL obviously vanishes.

Thus we have shown that the action (A.6)(or (1.18)) is invariant under supertransformations (1.19), (1.5), (1.8). Using the designations (1.20) (1.21) we avoid dealing with terms of fifth order in ψ_ν in δL.

In an analogous manner it is possible to verify the structure of the superalgebra. For these calculations the following fact is of use:

$$\delta\hat{\omega}_{\nu,ab}\big|_{\bar{\epsilon}=0}=\frac{i\varkappa}{4}\left[(\bar{\psi}_a\hat{D}_\nu\gamma_b\epsilon)+(\bar{\psi}_\nu\hat{D}_b\gamma_a\epsilon)+(\bar{\psi}_a\hat{D}_b\gamma_\nu\epsilon)\right]-(a\leftrightarrow b)-2m\varkappa(\bar{\psi}_\nu\sigma_{ab}\epsilon)-\frac{i\varkappa}{2}\left[\hat{F}_{ab}(\bar{\psi}_\nu\epsilon)-\tilde{F}_{ab}(\bar{\psi}_\nu\gamma_5\epsilon)\right] \tag{A.23}$$

It is also remarkable that equations of motion for ψ_ν are of the following simple form:

$$\epsilon^{\lambda\rho\mu\nu}\gamma_5\gamma_\mu\hat{D}_\nu\psi_\rho=0 \tag{A.24}$$

REFERENCES

1. V.Akulov, D.Volkov, V.Soroka. Pisma JETP, 22, 396 (1975); D.Volkov, V.Soroka. Russian Journal Teor. Mat. Fiz., 20, 291, (1974).
2. D.Z.Freedman, P. van Nieuwenhuizen and S.Ferrara. Phys. Rev. D13, 3241, (1976).
3. D.Z.Freedman, P. van Nieuwenhuizen. Phys. Rev. D14, 921, (1971).
4. S.Deser, B.Zumino. Phys. Lett., B62, 335, (1976).
5. S.Ferrara, F.Gliozzi,J.Sherk and P.van Nieuwenhuizen. Ecole Normall Superieure preprint PTENS 76/19.
6. B. de Wit. Institute-Lorentz, University of Leiden. preprint (September, 1976).
7. S.Ferrara and P. van Nieuwenhjizen. Stony Brook preprint ITP-SB-76-48 (October 1976).
8. M.T.Grisaru, Brandeis preprint (1976).
9. P. van Nieuwenhuizen and J.A.M.Vermaseren. Stony Brook preprint ITP-SB-76-44 (September 1976).

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