FUTURE DIRECTIONS FOR ISOCHRONOUS CYCLOTRONS

J. Reginald Richardson
TRIUMF and UCLA*

05 DEC. 1978

Abstract

There are many directions which isochronous cyclotrons will take in the future. Some of the directions are very certain such as the use of cyclotrons as booster and/or initial accelerators for heavy ions and their commercial/medical use as neutron sources and sources of radioisotopes. On the other hand, there are several other exciting possibilities such as energy-multiplier rings for existing facilities, including the possibility of multi-ring cyclotrons of the superconducting type for use as kaon factories. Also one of the existing intermediate-energy cyclotrons could be used as an injector for a fast-cycling synchroton in a kaon factory. Some of these possibilities are described.

Introduction

The canonical crystal ball would be extremely useful in assembling this paper. In its absence I must try to classify the future directions for isochronous cyclotrons into three categories. They are:

1. Almost certain. Here I refer to cyclotrons (both normal and superconducting) for the acceleration of heavy ions up to 20 MeV/nucleon. They may be injectors for other cyclotrons or other accelerators, or they may be the second or final accelerator in a series of two or three. The commercial applications of isochronous cyclotrons also belong in this category, either for the production of radioisotopes or for neutron therapy, or both.

2. Probable. Relativistic heavy ions. These are cyclotrons for the acceleration of very heavy ions to energies greater than 20 MeV/nucleon.

3. Very desirable but no concrete proposals yet. This category includes the use of intermediate-energy isochronous cyclotrons as injectors for larger superconducting cyclotrons or other accelerators which in turn will act as kaon factories. Also included here are possible ring cyclotrons to increase the energy of existing facilities and new ideas for isochronous cyclotrons.

Heavy Ion Cyclotrons

Most of the cyclotrons which have done research with heavy ions were originally designed primarily for the acceleration of light ions. Outstanding exceptions to this statement are the dual cyclotrons at JINR, Dubna and the ALICE combination at Orsay. Now it appears that the open radial sector ring-type cyclotron pioneered by Indiana is particularly suitable for the acceleration of heavy ions because of the ease of injection and extraction of the ions. For the acceleration of heavy ions there is a tremendous advantage in using two or more stages of acceleration because of the desirability of changing the charge state of the ions in order to maximize the final energy and intensity. An important justification for the cyclotrons of this type which have been built recently or are under construction or design lies in their heavy ion capability.

Nevertheless, it appears that the wave of the future in heavy ion cyclotrons is embodied in the superconducting versions. There are presently two cyclotrons of this type under construction—one at Chalk River,[1] the other at Michigan State[2]—and there is one under intensive design and model testing at Milan and a second at Michigan State. At Chalk River the cyclotron will act as a post-accelerator for a tandem while at Michigan State the cyclotron will act as injector for a second superconducting cyclotron. There are interesting differences in the two designs as shown in Table I, and later designers will follow closely the results achieved at the two laboratories.

In these superconducting cyclotrons only the main magnet coils are cooled to cryogenic temperatures. The remainder of the cyclotron is at normal temperature. Access to the median plane is quite restricted compared to ordinary cyclotrons because of the high flux density and the need for a circumferential yoke. Figures 1 and 2 show the design and magnet of the Chalk River project while Figs. 3 and 4 refer to the Michigan State facility. Figure 5 illustrates the design of the K = 800 cyclotron (also superconducting) which will act as the final stage in the Michigan State coupled cyclotron project. In the Michigan State plans the K = 500 cyclotron will first be operated alone with an internal ion

Table I. Comparison of the two superconducting heavy ion cyclotrons presently under construction.

	Chalk River	Michigan State
K in $E/A = K(Q/A)^2$	520	500
Maximum radius (mm)	650	667
Injection radius (mm)	160	internal ion source
Hill field (T)	5	5
Number of sectors	4	3
Variation in radial profile of magnetic field	two pairs of main coils + 104 trim rods	two pairs of main coils + trim coils
Main coils (MA turns)	6	6
Number of dees	4	3
Radio-frequency range (MHz)	31-62	9-27 or 30-65 stand alone
Peak voltage dee to ground	100 kV	100 kV
Acceleration mode	0 or π with h = 2, 4 or 6	0 with h = 2, 3 & 4 & 3 ϕ
Extraction	1 electrostatic deflector + magnetic channel	3 electrostatic deflectors

*TRIUMF, Vancouver, B.C., Canada V6T 1W5

source, producing ions with energy from 1 MeV/nucleon for uranium to 50 MeV/nucleon for light ions. Later, when the K = 800 cyclotron has been finished, the output will rise to 15 MeV/nucleon for uranium and 200 MeV/nucleon for light ions.

Although this new type of cyclotron—the superconducting heavy ion cyclotron—is probably not as convenient or as easy to commission as the Indiana type of open sector, they are much less expensive in capital and in power, and so it is likely that most heavy ion cyclotrons that are built in the future will be of the superconducting type.

Commercial Cyclotrons

In this category we include two general types of situations:

1. The cyclotron is completely produced by a commercial company and is used by another company (or the same company) for the production of radioisotopes or other services for sale—see Table II.

2. The cyclotron is completely produced by a commercial company and is sold to another institution, which may use it for research—usually in biomedical applications—see Table III.

In type 1 situations[3] the private organizations which own their own cyclotrons produce and market such long-lived nuclides as ^{67}Ga, ^{81}Rb, ^{111}In, ^{123}I and ^{201}Tl. The shorter-lived positron emitters ^{11}C, ^{13}N, ^{15}O and ^{18}F are usually produced by research institutions and used on site in type 2 situations. These isotopes are particularly interesting because they are positron emitters—suitable for use with several recently developed annihilation coincidence tomographs and because three of them have stable isotopes which form the basic constituents of organic compounds.

A number of radiation therapy trials are under way using neutrons produced by cyclotrons. Perhaps the most extensive program is that conducted by the U.K. Medical Research Council at Hammersmith.[4] This program has been under way since 1966 and has produced clinically interesting results, using the $^9Be(d,n)$ neutrons. It has been shown recently, however, that with proper filtering of the low-energy neutrons, the $^9Be(p,n)$ reaction produces a more penetrating neutron beam than the $^9Be(d,n)$ reaction for the same momentum of bombarding particle.[5] This means that simple, compact cyclotrons can probably be effective, at least for exploratory trials in neutron therapy.

There seems little reason to doubt that the number of cyclotrons produced commercially both for situations of type 1 and type 2 will continue to grow in the years ahead.

Kaon Factories

Isochronous cyclotrons may be used as injectors for post-accelerators which accelerate protons up to energies suitable for kaon production (5-10 GeV). Since presently operating slow synchrotrons accelerate currents of a few tenths of a microampere, it should be possible to increase the production of low- and medium-energy kaons by at least two orders of magnitude. The post-accelerators themselves could be superconducting isochronous cyclotrons or fast-cycling synchrotrons.

Since I am most familiar with TRIUMF as a possible injector for a kaon factory, I will use that facility as an example to consider. A paper[6] has been presented

Table II. Commercial cyclotron installations.

Location	Cyclotron Manufacturer	Installation
Philips-Duphar Ltd. Petten, Holland	Philips	1964
Radiochemical Centre Amersham, U.K.	Philips	1966
New England Nuclear Corp. Boston, MA, U.S.A.	Cyclotron Corp. (CS-22)	1970
Medi-Physics Inc. California, U.S.A.	Cyclotron Corp. (CS-22)	1971
Medi-Physics Inc. California, U.S.A.	Cyclotron Corp. (CS-22)	1972
New England Nuclear Corp. Boston, MA, U.S.A.	Cyclotron Corp. (CS-30)	1977
New England Nuclear Corp. Boston, MA, U.S.A.	Cyclotron Corp. (CS-30)	1978
Nihon Medi-Physics Osaka, Japan	Cyclotron Corp. (CS-30)	1974

Table III. Cyclotrons at medical research institutions.

Location	Cyclotron Manufacturer	Installation
Hammersmith Hospital London, U.K.	Medical Research Council	1955
Mallinckrodt Institute of Radiology St. Louis, MO, U.S.A.	Allis Chalmers	1965
Massachusetts General Hospital Boston, MA, U.S.A.	Allis Chalmers	1967
Sloan-Kettering Institute for Cancer Research New York, NY, U.S.A.	Cyclotron Corp. (CS-15)	1967
Argonne Cancer Research Hospital Chicago, IL, U.S.A.	Cyclotron Corp. (CS-15)	1969
Service Hospitalier Frederic Joliot Orsay, France	Thompson-CSF (Compact)	1971
University of California Los Angeles, CA, U.S.A.	Cyclotron Corp. (CS-22)	1971
Inst. fur Nuklearmedizin Heidelberg, Germany	A.E.G. (Compact)	1972
Mount Sinai Hospital Miami, FL, U.S.A.	Cyclotron Corp. (CS-22)	1972
Inst. de Engenharia Nuclear Rio de Janeiro, Brazil	Cyclotron Corp. (CV-28)	1973
Inst. of Medical Sciences University of Tokyo Tokyo, Japan	Cyclotron Corp. (CS-30)	1973
Inst. of Radiological Sciences Chiba, Japan	Thompson-CSD (70)	1974
University of Liege Liege, Belgium	Thompson-CSF	1974
Klinikum Essen, Germany	Cyclotron Corp. (CV-28)	1975
Medizinische Hochschule Germany	Scanditronix (MC-35)	1975
Kernforschungsanlage Julich, Germany	Cyclotron Corp. (CV-28)	1975
Western General Hospital Edinburgh, Scotland	Cyclotron Corp. (CS-30)	1975
Mallincrodt Inst. of Radiology St. Louis, MO, U.S.A.	Cyclotron Corp. (CS-15)	1978

NOTE: I am indebted to George Hendry for the information in Tables II and III.

at this conference on the possible design of a superconducting cyclotron at 8 GeV. Using an isochronous cyclotron as an "after-burner" has the advantage that its time structure can be matched quite closely to the time structure of TRIUMF. On the other hand, the primary difficulty in the use of a fast-cycling synchrotron as an after-burner lies in the matching of its varying time structure to the continuous beam of TRIUMF.

The heart of the problem consists of the extraction of the beam from TRIUMF and its injection into the synchrotron. The two fundamental limitations on beam intensity are:

1. space charge
2. admittance of the synchrotron vs. the emittance of TRIUMF (phase space ratio)

The space charge limit is usually most serious at injection into a synchrotron and is given approximately by the expression[7]

$$(\nu^2 - \nu_0^2)_{incoherent} = -\frac{2}{\pi} \frac{N \, r_p \, R}{B \, \beta^2 \gamma^3 \, b(a+b)} \; .$$

Here ν denotes either the radial or axial tune with space charge and ν_0 is the same tune without space charge. In order to avoid integral or half-integral resonances in ν_r, ν_z or $\nu_r + \nu_z$ it is necessary that $|\nu - \nu_0| < 1/2$. B is the branching factor, $2\pi R$ is the circumference of the orbit, $r_p = 1.54 \times 10^{-16}$ cm, a and b are the semi-axes of the beam cross section, and N is the number of protons in the synchrotron pulse. This relation leads to an estimate of the space charge limit

$$N \lessgtr \frac{\pi}{2} \frac{\nu B \, \beta^2 \gamma^3 \, b(a+b)}{r_p R} \; .$$

For an example synchrotron $R = 50$ m, $b = 30$ mm, $a = 50$ mm, $B = 1/3$ and $\nu = 10$, one finds $N \lessgtr 3 \times 10^{13}$ p/pulse. Scaling up from the performance of the Brookhaven AGS according to the higher injection energy from TRIUMF, one arrives at the limit $N = 4 \times 10^{13}$ p/pulse.

H⁻ Injection

Since TRIUMF accelerates H⁻ ions to 180-525 MeV it is natural to consider the possibility of extracting the H⁻ ions (as negative ions) from TRIUMF and injecting them into the synchrotron by stripping to H⁺ ions near the equilibrium orbit of the synchrotron. Such a system is under construction at the Rutherford Laboratory for the strong neutron source (SNS) and has been proposed at LAMPF as the WNR storage ring.

Injection by stripping avoids most problems with the ratio of the phase space in the two accelerators, leaving space charge as the most important limit on the intensity. The ratio of the lengths of the equilibrium orbits in the two machines is approximately 7. Assuming that 100 μA of H⁻ ions can be extracted from TRIUMF, using the $\nu_r = 3/2$ resonance (as is done at SIN), one has approximately 2.6×10^7 ions per cyclotron pulse and 3.8×10^5 pulses spread over 16 ms to give 10^{13} protons per synchrotron cycle. With a synchrotron repetition rate of 20 Hz we obtain 2×10^{14} protons s⁻¹. The synchrotron magnets would have a dc bias with flat-topping—the 20 Hz repetition rate will allow the application of third harmonic at 60 Hz.

There are a number of ways in which the phase space of the synchrotron can be filled by stripping. One possibility is mechanically moving the stripper across the synchrotron aperture—together with vertical and horizontal steering of the H⁻ beam. Another possibility is to use fast bumper magnets as will be used at the Rutherford Laboratory. There are approximately

10,000 synchrotron turns in the 16 ms injection time. With 2 mm wide stripper and a horizontal aperture of 50 mm, this means that the particles will traverse the stripping foil some 400 times. This should not cause any difficulties in multiple scattering, although the stripper lifetime might be a problem. The momentum compaction would be approximately $\Delta p/p = 2 \times 10^{-3}/$mm.

Ideally, the synchrotron RF should be at the same frequency as the cyclotron (23 MHz) increasing by about 30% to the time of beam extraction. Although difficult, this would result in almost 100% capture into the synchrotron and is worth considerable effort to achieve it. For a final energy of 8 GeV, the energy gain should be 1 MeV per synchrotron turn. This could be provided by 12 RF stations.

The general design of the synchrotron might well be of the "missing magnet" separated function type, but further details would be premature at this time.

H⁺ Injection

If extraction of the beam from TRIUMF is obtained by stripping, Liouville's theorem cannot be circumvented, and we must concern ourselves with the acceptance of the synchrotron compared to the emittance of TRIUMF. The acceptance of the synchrotron is given for the radial direction by

$$A_R = \frac{\pi \, a^2 \nu_r}{FR}$$

and similarly for the y direction; with $F \simeq 2$, $A_R \simeq 200$ mm-mrad for the example, while $A_y \simeq 72$ mm-mrad. In the beam direction the phase space ratio is simply

$$\frac{R_{synchrotron}}{R_{cyclotron}} = 7$$

unless additional cyclotron pulses are inserted into the synchrotron. Taking the emittance of the TRIUMF cyclotron as 2π mm-mrad, the total phase space ratio is 25,000. This ratio would allow the insertion of $25 \times 10^3 \times 5.2 \times 10^8 = 1.3 \times 10^{13}$ p/pulse into the synchrotron, and with a pulse repetition rate of 20 Hz the acceleration of some 2.6×10^{14} p s⁻¹ is possible. The number 5.2×10^8 corresponds to the number of particles in one cyclotron turn at 400 μA. This current appears to be tolerable at 450 MeV.

Instead of injecting each cyclotron pulse or turn in succession into the synchrotron, it is desirable from the point of view of capture efficiency and the RF system to collect or stack about 100 turns in the outer edge of the cyclotron and then inject them all at once into the synchrotron. A scheme which looks feasible makes use of the decelerated beam previously predicted[8] and observed at TRIUMF.[9] By tailoring the magnetic field near $R = 7.5$ m (say) so that it drops from 0.2% to 0.6% below the isochronous field, it is possible to cause the H⁻ ions to fall out of phase with the RF and then start back toward the centre of the cyclotron. In fact, in a radial interval of 25 mm all the ions with an initial spread of 0.3 in $\sin \alpha$ will have about 50 turns of acceleration and 50 turns of deceleration (α is the phase with respect to the RF). In the 100 turns the total loss due to stripping (both electromagnetic and gas) is less than 2% at 450 MeV.

The importance of this stacking technique is that it makes possible the time matching of the two accelerators in such a way that total beam losses should not exceed 5% in extraction from the cyclotron and injection and capture into the synchrotron. The orbit period of the 50 m synchrotron is 1.5 μs, and so the 100 turn stack is pulsed axially by an electric field so that the ions intersect a stripper over a period of

2 rotations or 10 RF cycles (0.44 μs). In this way the protons are injected into the synchrotron, at the correct synchrotron phase for complete capture, with a repetition period of 21 μs (14 synchrotron turns). Injection appears to be well within the state of the art, using fast bumper magnets and RF acceleration to move the equilibrium orbit away from the injection system. The energy spread in the macropulse from the cyclotron would be 5 MeV, corresponding to a radial spread in the synchrotron of 5 mm. 200 macropulses per synchrotron cycle would be adequate to achieve an intensity of 2×10^{14} p s^{-1} with a frequency of 20 Hz. The total injection period would be 200 × 2.2 μs = 4.4 ms out of a total cycle time of 50 ms. Accommodating this long injection time will require some "flat-topping" of the magnet cycle—together with the usual dc bias. It should be noted that the stacking and stripping technique described here will improve the phase space ratio by a considerable factor over that calculated above. Some of the parameters of the injection-capture system are shown in Table IV.

Table IV. TRIUMF → fast cycling synchrotron.

Number of stacked turns - TRIUMF	~100
Energy spread of extracted beam	450 ± 2.5 MeV
Injected beam microstructure	10 each pulses of 14 ns
macrostructure	0.4 μs every 21 μs
Beam loss - stacking in TRIUMF	<2%
Phase space ratio	~10^5
Synchrotron frequency	20 MHz
Space charge limit	8×10^{14} p s^{-1}
Phase space limit	10^{15} p s^{-1}
Synchrotron orbit time	1.5-1.14 μs
Number of macropulses per synchrotron cycle	200
Injection time	4.4 ms
Mean synchrotron radius	50 m
Average energy gain per turn	0.45 MeV
Radio frequency	0.67-0.93 MHz
Beam loss - extraction, injection and capture	<5%
Final energy	8-10 GeV

Superconducting Cyclotrons

Some conceptual design work has been done at TRIUMF on a high-energy cyclotron to use as a post-accelerator for a kaon factory. Since this work is very preliminary and is reported at this conference,[6] I will confine myself here to some general remarks on the subject. An important problem in trying to design a cyclotron for energies in the 5-10 GeV range is the rapidly increasing change in energy with radius expressed by

$$\frac{\Delta E}{\Delta R} = \frac{E_0}{R_c} \beta \gamma^3,$$

where E_0 is the rest energy, $R_c = c/\omega_I$ and ω_I is the resonant ion frequency. It can be seen that in going from 450 MeV to 8 GeV this parameter has gone up by a factor of about 300. Of course, the radius can be increased (decreasing the average magnetic field), but there are practical limitations to this move. On the other hand, the magnetic rigidity of the particles has increased by a factor of 9 and the hill field has increased by a factor of 8.7, so the net effect is not as serious as seems at first sight.

Another problem is the large spiral angle which will probably be necessary for the axial focusing at high energies. In this connection, however, there is an advantage arising from the use of superconducting separated magnets (essential to minimize capital investment and power usage). This advantage comes from making use of a fraction of the return flux along a "gully" between the hill and an adjacent valley. This

flux is led back to the median plane by part of the iron yoke, and it has the advantage of making the hill edge very steep and large. The flutter factor F^2 in the axial focusing of the particles can be increased by as much as 50% in this way. Still, if a maximum energy of 8 GeV is required, the tangent of the spiral angle ε will probably be tan ε ≃ 6 compared to tan ε ≃ 3 for TRIUMF. The important parameter here is d(tan ε)/dε = cos^{-2}ε which increases by a factor of 4. On the other hand, the angular deviation of the actual motion from the equilibrium orbit determines the significance of this increase. Consider the radial oscillations about the equilibrium orbit—the maximum angular deviation $(\mu_{max})^2 \sim \nu_r/\beta\gamma R$ so $(\mu_{max})_{8\ GeV} \simeq 0.6(\mu_{max})_{0.5\ GeV}$. Thus the sensitivity to alignment on the 8 GeV cyclotron would be some 2.5 times as great as on TRIUMF. This is not a serious limitation.

The advantage in going up in energy from 450 MeV to 8 GeV in two steps is rather clear-cut from the point of view of capital cost and power consumption (both magnet and RF). Ideally the first cyclotron would accelerate at the TRIUMF RF of 23 MHz and ω_I = 23/5 MHz from γ = 1.5 to γ = 4 with a radially increasing dee voltage such that the phase spread of the ions would decrease by a factor of 3. The second post-accelerator would be a cyclotron with ω_I = 23/10 MHz and an RF of 69 MHz, allowing the use of small SIN-type cavities. The advantages of this scheme would have to be weighed against the additional extraction and injection required over the use of a single large cyclotron at ω_I = 23/10 MHz.

The TRIUMF group is continuing work on these various alternative possibilities for a kaon factory.

Other Interesting Ideas

The Indiana group has been considering a possible ring cyclotron to increase the energy of their multi-particle facility by a factor of about three so that it is called an "energy tripler". The resulting energy would be 300 MeV for deuterons, 600 MeV for ^{4}He, 800 MeV ^{3}He, 1000 MeV ^{6}Li, etc. The upper proton limit is set by focusing restraints and varies from 380 MeV for a four-sector design to 500 MeV for a six-sector version. At the present time these concepts are all preliminary and in the form of simple sketches. All the magnet sectors have straight edges, but they do not necessarily intersect in the centre. One interesting version, not yet completely explored, makes use of six superconducting magnets with a total weight of 500 tons and a circumference only half that of the present Indiana cyclotron magnet. This is to be contrasted with the 4-sector normal temperature design where some 2000 tons of steel are required. Plots of ν_z vs. ν_r show that for protons in general $4/3 < \nu_r < 2$ and $0.1 < \nu_z < 1$. No decision has been made as to the future of this effort.

R.W. Müller of Darmstadt has proposed a "Racetrack Cyclotron"[10] which is designed to act as a post-accelerator for a linac. The main objective of this work was to produce an isochronous cyclotron design which could be efficiently coupled to the time structure of an injection linac. The result is shown in Fig. 6. The orbits are cotangential rather than concentric, and a high harmonic number h > 20 is used to obtain small velocity increments for the ions when h is changed. Müller has considered the orbital stability in both the axial and horizontal directions and has particularly investigated the crossing of the ν_z = 0.5 resonance under which condition an energy multiplication factor of 16 can be achieved. It should be noted that this design has a lower energy stability limit as well as an upper one, in particular stability limits for E/A given by 10 < E/A < 100 MeV/nucleon are

possible. As an example, a machine with K = 900 MeV would have a mass of 4000 tons vs. 8000 tons for a conventional design.

Conclusion

It is clear from this brief survey that isochronous cyclotrons have a future varying from the certain to the possible. The really fascinating experience will be to watch the possible converted to the certain.

It is a pleasure to acknowledge the receipt of valuable information from Henry Blosser, Mike Craddock, George Hendry, John Ormrod and the SNS group at the Rutherford Laboratory.

References

1. J.H. Ormrod et al., IEEE Trans. NS-24, 1093 (1977).
2. H.G. Blosser, Proc. Seventh International Conference on Cyclotrons and Their Applications (Birkhäuser, Basel, 1975), p.584.
3. M.A. Chaudri, ibid., p.447.
4. M. Catterall, Int. Journal of Radiation Oncology, Biology, Physics 3, 247 (1977).
5. W.M. Quam, S.W. Johnson, G.O. Hendry, J.L. Tom, P.H. Heintz, and R.B. Theus, Physics in Medicine and Biology 23, No. 1 (1978).
6. M.K. Craddock, C.J. Kost, and J.R. Richardson, Paper C-6, this conference.
7. E.D. Courant, Ann. Rev. Nucl. Sci. 18, 435 (1968).
8. J.R. Richardson, Nucl. Instr. & Meth. 24, 493 (1963).
9. M.K. Craddock, E.W. Blackmore, G. Dutto, C.J. Kost, G.H. Mackenzie, and P. Schmor, IEEE Trans. NS-24 1615 (1977).
10. R.W. Mueller, "The Racetrack Cyclotron", paper presented at the Berlin Cyclotron Meeting, April 5-7, 1978 (unpublished).
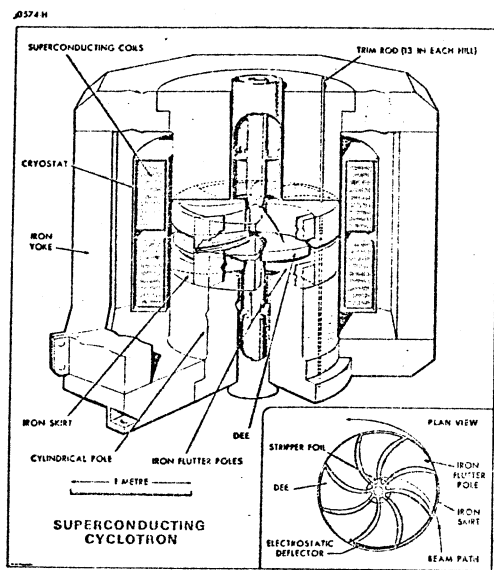
Fig. 1. The superconducting cyclotron under construction at Chalk River. Note the iron trim rods to be used in adjusting the radial profile of the magnetic field.
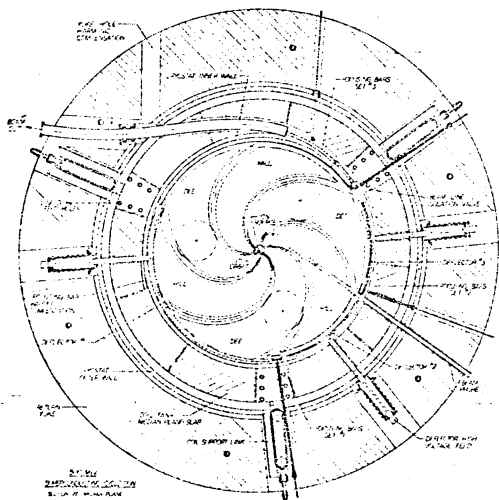

Fig. 2. Photograph of the completed magnet for the Chalk River superconducting cyclotron. A 13 MV tandem will be used as injector.

Fig. 3. Sectional view of the K = 500 cyclotron under construction at Michigan State. Note the three dees which will sometimes be used in three phase.
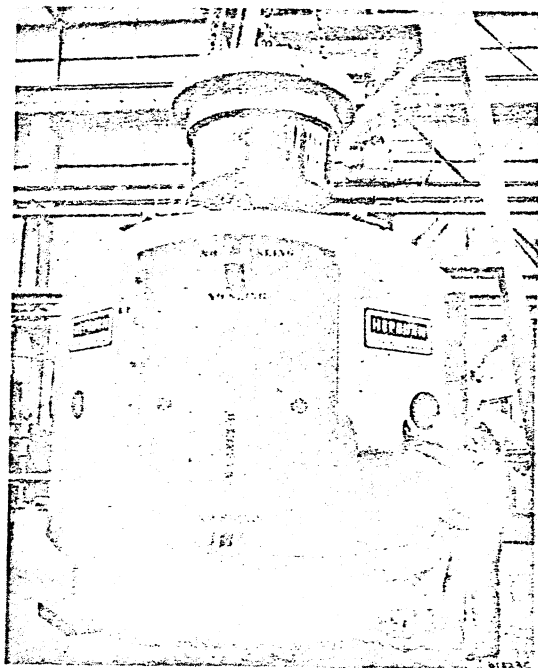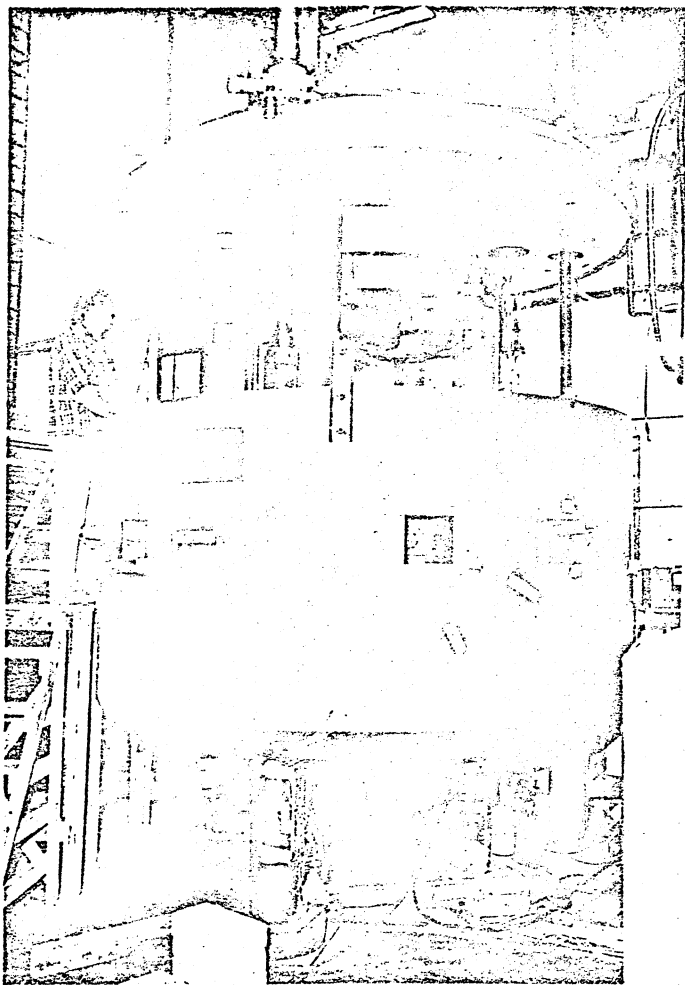
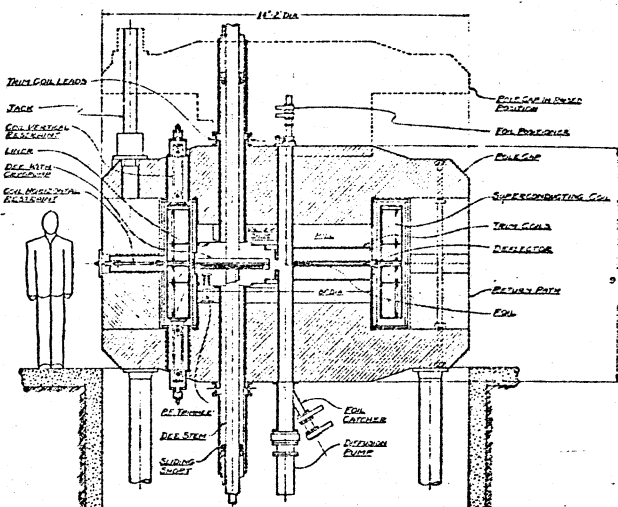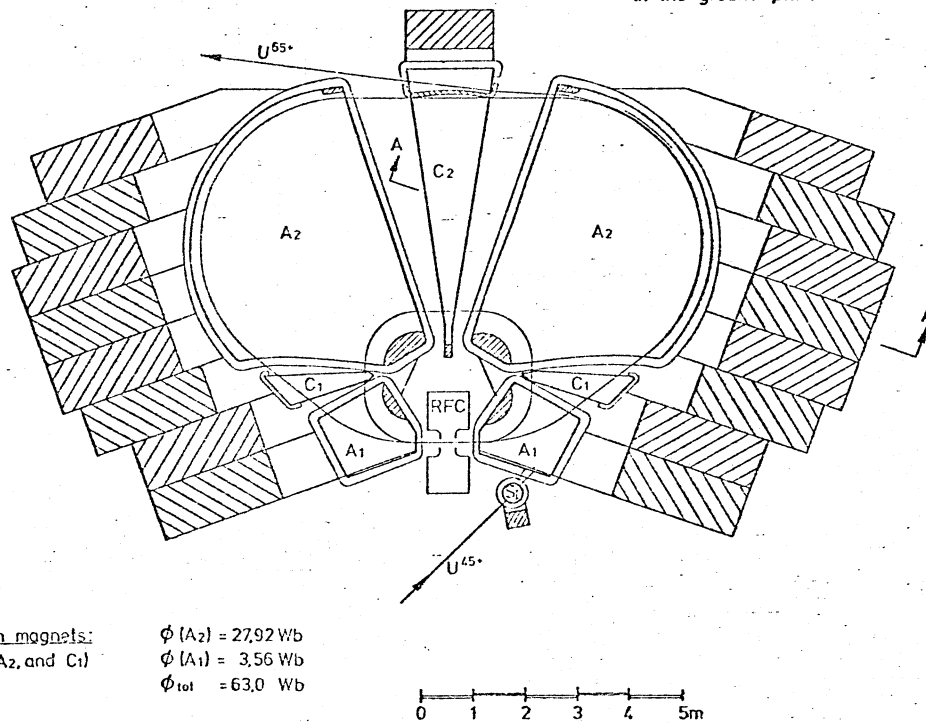Fig. 4. The K = 500 superconducting magnet at Michigan State with top raised.

Fig. 5. A vertical section showing the K = 800 superconducting cyclotron to be built as a post-accelerator for the K = 500 machine.

The vacuum chamber is not shown in the ground plan.

Main magnets:
(A₁, A₂, and C₁)

ϕ (A₂) = 27.92 Wb
ϕ (A₁) = 3.56 Wb
ϕ_{tot} = 63.0 Wb

0   1   2   3   4   5m

Fig. 6. The Racetrack Cyclotron of R.W. Müller. The special features of this design include a single gap re-entrant cavity and acceleration on a high harmonic number.