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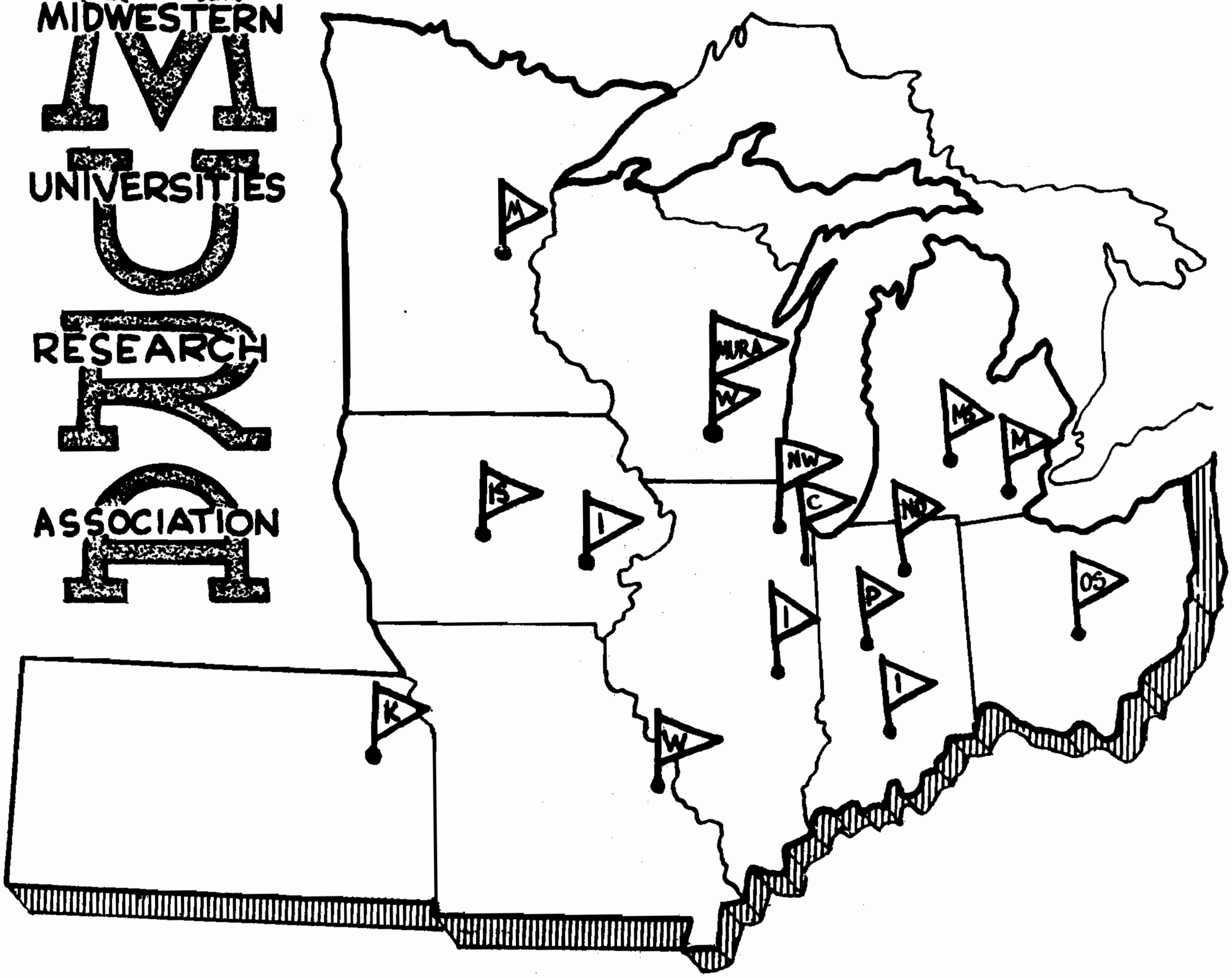

ADIABATIC DAMPING OF LARGE PHASE OSCILLATIONS

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ADIABATIC DAMPING OF LARGE PHASE OSCILLATIONS

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The phase equation $\frac{d}{dt}\left(\frac{E_s\dot{\varphi}}{-K}\right) + \frac{h\omega_0^2 eV}{2\pi}(\sin\varphi - \sin\varphi_0) = 0$
can be derived from the Hamiltonian $H = \frac{1}{2}\left(\frac{-K}{E_s}\right)p^2 + \frac{h\omega_0^2 eV}{2\pi}[u(\varphi_0) - u(\varphi)] = W$
where $p = \frac{E_s\dot{\varphi}}{-K}$; $K = \alpha - \left(\frac{Mc^2}{E_s}\right)^2$; $u(\varphi) = \cos\varphi + \varphi\sin\varphi_0$

Stable phase oscillations occur about the stable phase φ_0
if the maximum phase $\varphi_m < \pi - \varphi_0$

The action of a stable oscillation is $J = \oint p\,d\varphi = \sqrt{\frac{h\omega_0^2 eV}{2\pi}\frac{E_s}{-K}\cos\varphi_0}\;F(\varphi_m,\varphi_0)$

where $F(\varphi_m,\varphi_0) = \oint\sqrt{\frac{u(\varphi)-u(\varphi_m)}{\cos\varphi_0}}\,d\varphi$

The phase oscillation angular frequency $\Omega = \frac{2\pi W}{J} = \omega_0\sqrt{\frac{h eV\cos\varphi_0(-K)\pi}{E_s}}\;\frac{G}{F}$

The maximum relative energy spread $\frac{\Delta E}{E_s} = \beta_s\sqrt{\frac{eV\cos\varphi_0}{-K\pi h E_s}}\cdot\sqrt{G}$

The " " momentum spread $\frac{\Delta P}{P_s} = \frac{1}{\beta_s^2}\;\frac{\Delta E}{E_s}$

The " " radial " $\frac{\Delta r}{r_s} = \frac{\alpha}{\beta_s^2}\;\frac{\Delta E}{E_s}$

The functions F, $\frac{G}{F}$, $\sqrt{G}$ are tabulated below, for $\varphi_0 = 30^\circ$

J is invariant under adiabatic change of its parameters. Thus given certain conditions, say $\varphi_m + E_s$ at injection, we can use the table to compute the change in φ_m and hence $\frac{\Delta E}{E_s}$ as E_s rises.

As soon as $\varphi_m - \varphi_0$ has damped down to less than 30° the formulae in Courant and Snyder's paper are valid.

NOTE. $\frac{\Delta E}{E_s}$ gives the maximum tolerable spread in injection energy in terms of E_{inj}, $\frac{eV}{h}$, φ_0, φ_m for sudden application of the r.f. It is a more stringent requirement in most cases than that requiring the particles to stay within the doughnut. Symbols have the same meaning as in Courant's and Snyder's paper.

$\varphi_0 = 30^\circ$

$\varphi_m - \varphi_0$	$\varphi_m - \varphi_0$	$F(\varphi_m, \varphi_0)$	$G(\varphi_m, \varphi)$	$\sqrt{G(\varphi_m, \varphi_0}$	$\frac{G}{F}$
0°	0 rad	.0000	.0000	.0000	.225
10°	.1745	.065	.0148	.1217	.225
20°	.3490	.251	.0561	.2369	.224
30°	.5235	.539	.1203	.3468	.224
40°	.6980	.909	.2020	.4494	.224
45°		1.090	.2477	.4977	.223
50°	.8725	1.339	.2956	.5437	.221
60°	1.047	1.807	.3954	.6288	.219
75°	1.309	2.532	.5431	.7370	.214
90°	1.571	3.258	.6704	.8188	.206
105°	1.833	3.793	.7584	.8609	.200
120°	2.094	4.049	.7908	.8893	.176