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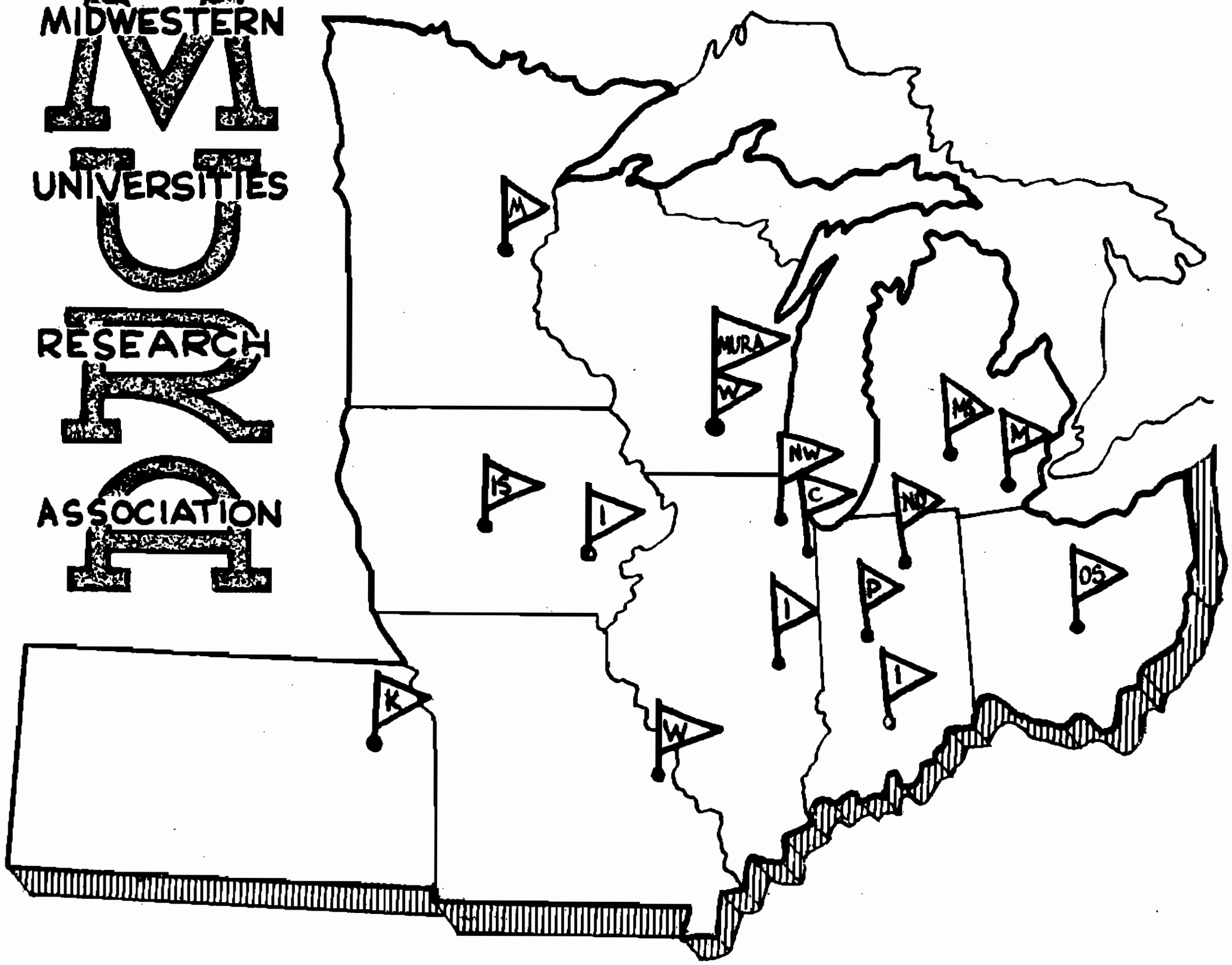

ON THE DESIGN OF SPIRAL SECTOR ACCELERATORS

REPORT **NUMBER** 336

ON THE DESIGN OF SPIRAL SECTOR ACCELERATORS

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ABSTRACT: The MURA IBM 704 computer was used to study the field variation in the median plane for Spiral Sector FFAG accelerators[1] with various parameters. The results of this study are summarized in the graphs of this report. It is hoped that by the use of these graphs one can readily arrive at approximate design parameters.

* Supported by Contract AEC # AT (11-1) - 384

** Student on leave from The Ohio State University, Columbus, Ohio

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Set A. f vs α For fixed β

Set B. β vs α For fixed f

Set C. $\frac{H_{max}}{\langle H \rangle}$ vs α For fixed β

Set D. f vs α For fixed (R , N , tune and gap)

Set E. $(f\beta)$ vs f For fixed α

Part I INTRODUCTION

The purpose of this report is to study some of the design problems of a spiral sector FFAG accelerator[1]. It is desired for larger nonlinear stability limits for betatron oscillations to have a large flutter[2] (f) , and for engineering reasons it is desired to have a large gap[3] (g) . Since for all other parameters remaining constant an increase in the gap causes a decrease in the flutter, one must have some method of finding the best compromise between the gap and the flutter.

The main problem treated in this report is that of finding a relationship between the flutter (f), the air /iron ratio (α), and the gap/sector ratio (β) in i , j , coordinates. The i and j coordinates are defined as:

$$i = a_m \xi \qquad \text{and} \qquad j = a_m \eta \qquad \text{where } \xi = -\frac{N\theta}{2\pi}$$

$$\eta = \frac{\sqrt{\frac{1}{w^2} + N^2}}{2\pi} y \qquad (4)$$

and a_m is the number of mesh points per sector. The ij coordinates are in the ξ, η plane which is a plane perpendicular to the spiral ridges.. The reason for introducing the ξ, η plane is that the problem can most nearly be approximated as a two dimensional problem in the ξ , η plane. In fact once the transformation to the ξ , η plane has been made, the dependence of f on κ , $\frac{1}{w}$ and N is very small. In other words $f \doteq f(\alpha, \beta)$ where $\beta = \beta(\frac{1}{w}, N)$ and $\alpha = \alpha(N)$. For example if the ξ , η plane is at a radius R the azimuthal displacement S will correspond to a displacement in the i direction equal to $a_m \frac{SN}{2\pi R}$, and a displacement z in the vetical direction will correspond to a displacement in the j direction equal to $a_m \frac{z\sqrt{\frac{1}{w^2} + N^2}}{2\pi R}$.

Although actually f is dependent upon κ , $\frac{1}{\omega}$ and N in a more complicated fashion then just the dependence introduced by the transformation, one must have an approximate value of f in order to find a value for $\frac{1}{\omega}$. Since f is only slightly dependent on κ , $\frac{1}{\omega}$, and N after the transformation has been made the work in this report assumes that $f = f(\alpha, \beta)$ where dependence of f on κ , $\frac{1}{\omega}$, and N is introduced only by $\beta = \beta(\frac{1}{\omega}, N)$ and $\alpha = \alpha(N)$. If a more accurate value of f is desired then one may obtain it from the MURA IBM 704 digital computer program Forocyl[5].

Part II THEORY

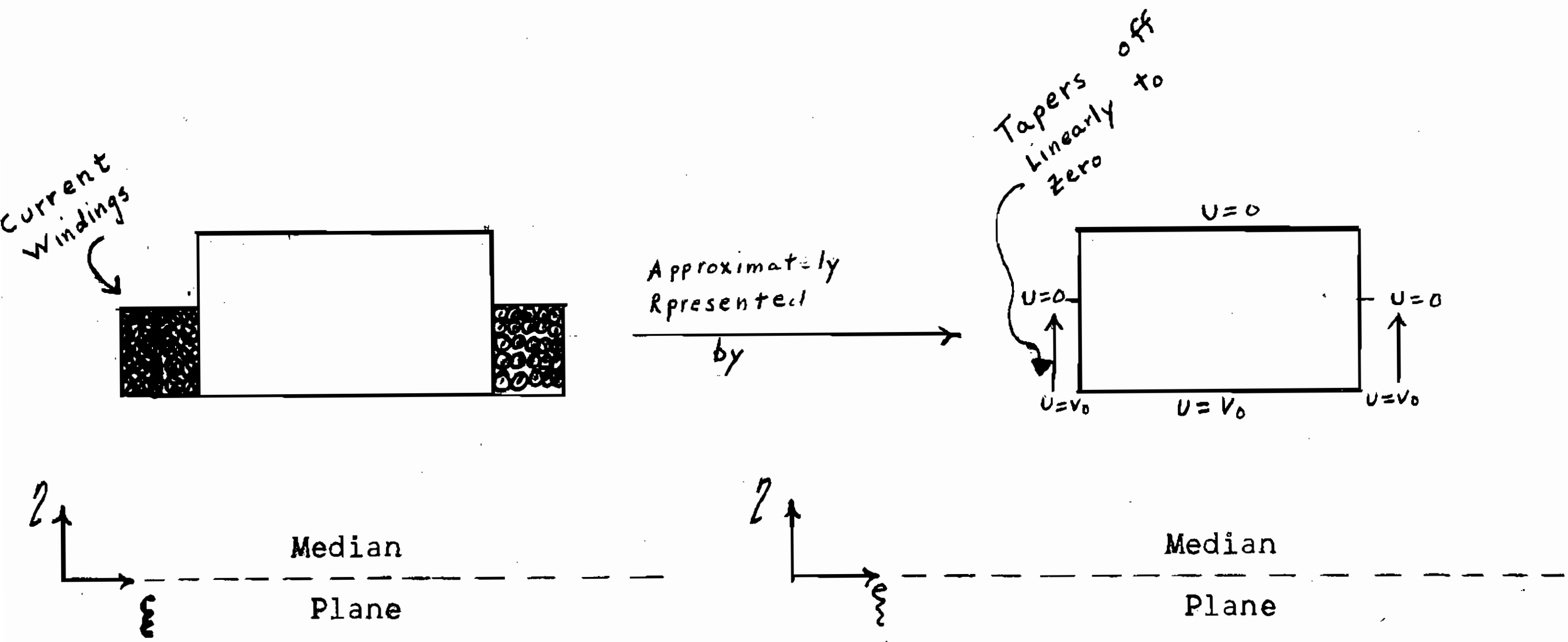
A. Graphs

The first set of graphs (# A)[6] were obtained (as explained in detail below), by using the program Forocyl. They show the relationship between the flutter (f), and the air/iron ratio (α), for various values of the gap/sector ratio (β) in ξ-η coordinates. From set # A the graphs of set # B were obtained; these graphs show the relationship between α and β for different values of f. The third set (# C) are graphs of $\frac{H_{max}}{\langle H \rangle}$ vs α, for various values of β, and also was obtained by using the Forocyl program.

The other graphs were obtained from the first two sets by methods which are explained in Parts II- C and II- D. Graph set # D shows the relationship between the flutter and the air/iron ratio for several spiral sector machines with specific radii, tune[7], number of sectors, and gap. Set # E contains graphs of $f\beta$ vs f for several values of α. Set # E was obtained directly from set # A and the significance of # E is explained in Part II- C.

B. Type of Magnetic Pole

The type of magnetic pole which was used for obtaining the graphs in this report is shown below. The bottom surface of the pole was assummed to be at a constant potential V_0 while the potential on the sides of the poles was assummed to taper off linearly to zero at a point half way up the pole. This type of pole is an approximation of a pole with current windings half way up the side. The thickness of the pole was assummed to be approximately six tenths of the gap distance.



C. Design Study

In order to study more fully the design problems of a spiral sector FFAG accelerator one must select values for some of the parameters, and then study the relationship between the other parameters. For example if one desires to study the flutter and gap relationship then one should pick values for the tune and number of sectors for the machine (8).

When one has picked these values, the values for $\frac{f}{\omega}$ and κ can be roughly determined from the necktie diagram (9). For small values of N, the necktie diagram is not very accurate in predicting tune and when one has finally arrived at a desirable design he should find the actual values of σ_x and σ_y which correspond to the values of $\frac{f}{\omega}$ and k used. This is done by using the digital computer program Forfix Point (10) which in turn uses the fields printed out by the program Forocyl.

After one has obtained the values for $\frac{f}{\omega}$ and k a value for the inside radius R_i should be picked (it is usually desirable for the radius to be as small as possible). Using the value of $\frac{f}{\omega}$ and the value of the gap one can calculate a numerical value of β (11) for various values of f. Thus one can obtain the value of β associated with particular values of flutter, gap, radius, tune, and number of sectors. From graph set # B one can determine the value of α which corresponds to these values of f and β.

It is now possible to plot a graph of flutter vs α for a fixed tune, number of sectors radius, and gap. One can then change any or all of these four parameters and obtain a set of graphs of f vs α. This has been done for several sets of values for these parameters and is called Set # D in this report.

After one has selected the parameters which are most desirable from Set # D, the outer radius (R_o) can be calculated by the formula:

$$R_o = R_i \left(\frac{P_o}{P_i}\right)^{\frac{1}{(k+1)}}$$

where P is the momentum of the particle and the subscripts i and o stand for inner and outer respectively. The average magnetic field ($\langle H \rangle$) can be obtained by the formula[12]: $\langle H \rangle = \frac{100}{3} \frac{[T(T+2E_r)]^{1/2}}{R}$

where T is the kinetic and E_r the rest mass energy in Mev the radius R is in meters, and the magnetic field is in gauss.

Since both a large flutter and a large gap are desired one can define the point where the value of $\frac{f}{\omega}$ times the gap is maximum as the optimum point. It becomes desirable, as explained below, to have graphs of ($f\beta$) vs f for various values of α . This was done for several values of α (Set # E) and the graphs were obtained directly from graph Set # A. For a particular value of the gap $= g$ the value of ℓ for the gap is equal to β (ie: $\beta = \frac{\sqrt{\frac{1}{\omega^2} + N^2}}{2\pi} g/R$). If $N \ll \frac{1}{\omega}$ then $\beta \approx \left(\frac{1}{2\pi R}\right)\frac{g}{\omega}$ and $f\beta \doteq \left(\frac{1}{2\pi R}\right)\left(\frac{f}{\omega}\right)(g)$. Thus $f\beta$ is proportional to $\left(\frac{f}{\omega}\right)(g)$. Thus for a fixed air/iron ratio one can find the value of f which corresponds to the maximum value of $\left(\frac{f}{\omega}\right)(g)$ from graph Set # E.

D. Examples

1. The first example used is one of a small machine into which electrons are injected at an energy of 100 Kev and accelerated to an energy of 50 Mev. The values of the various parameters are listed below.

$\sigma_x = .766\pi$ $\quad \sigma_y = .1\pi$ $\quad E_i = .1$ Mev $\quad E_o = 50$ Mev

$N = 8$ $\quad R_i = 1.75$ m $\quad$ gap = 10 cm

gap

From the necktie and Tables (13) $\longrightarrow$

$\frac{f}{\omega} = \underline{28}$ $\quad k = \underline{9.4}$

$\left(\frac{P_o}{P_i}\right) = 150 \longrightarrow R_o = 1.75\,(150)^{(1/10.4)} = \underline{2.83 \text{ meters}}$

radical aperature $= R_o - R_i = 1.08$

$\langle H \rangle$ at $R_i = \frac{100}{3} \frac{[.1(.1+1.02)]^{1/2}}{1.75} = 6.37$ gauss

$\langle H \rangle$ at $R_o = \frac{100}{3} \frac{[50(50+1.02)]^{1/2}}{2.83} = 596$ gauss

For $f = 1 \longrightarrow \frac{1}{\omega} = 28$

$$\eta = \frac{\sqrt{\frac{1}{\omega^2} + N^2}}{2\pi}\, y = 4.63\, y$$

value of 10 cm gap in y coordinate $= \frac{10 \text{ cm}}{175 \text{ cm}} = .0572$

η for 10 cm gap $= .265 = \beta =$ gap/sector ratio

From graph of $f = 1$ (# B-5) the value of α corresponding to $\beta = .265$ is $\alpha = 2.09$

By using values of $f = .7, .8, .9, 1,$ and 1.1 one can by the same procedure obtain the data in the table below and also the curve for graph # D- 1.

f	.7	.8	.9	1	1.1
β	.372	.326	.292	.265	.243
α	1.69	1.83	1.96	2.09	2.24

2. The second example used is one of a large machine into which protons are injected at an energy of 5 Mev and accelerated to an energy of 15 Bev. The values of the various parameters are listed below:

$\sigma_x = .57\pi$ $\sigma_y = .255\pi$ $E_i = 5$ Mev $E_o = 15{,}000$ Mev

$N = 38$ $R_i = 71.5m$ gap $= 20$ cm

From the necktie $\rightarrow$ $k = 92.3$ $\frac{f}{\omega} = 409$

$$\left(\frac{P_o}{P_i}\right) = 167 \longrightarrow R_o = 71.5\,(167)^{\frac{1}{92.3}} = 75.6m$$

Radical Aperature $= R_o - R_i = 4.0$ meters

$$\langle H\rangle \text{ at } R_i = \frac{100}{3}\,\frac{[5(5+1876)]^{1/2}}{71.5} = 45.3 \text{ gauss}$$

$$\langle H\rangle \text{ at } R_o = \frac{100}{3}\,\frac{[15000\,(15000+1876)]^{1/2}}{75.5} = 7{,}020 \text{ gauss}$$

For $f = 1 \longrightarrow \frac{1}{\omega} = 409$

$$\zeta = \frac{\sqrt{\frac{1}{\omega^2} + N^2}}{2\pi}\, y = 65.3\, y$$

value of 20 cm gap in y coordinate $= \frac{20 \text{ cm}}{7{,}150} = .00028$

ζ for 20 cm gap $= .178 = \beta =$ gap/sector ratio

From graph of $f = 1$ (# B- 5) the value of α corresponding to $\beta = .178$ is $\alpha = 1.29$

By using values of $f = .8, .9, 1, 1.1$ and 1.2 one can by the procedure obtain the data in the table below and also the curve for graph # D- 6.

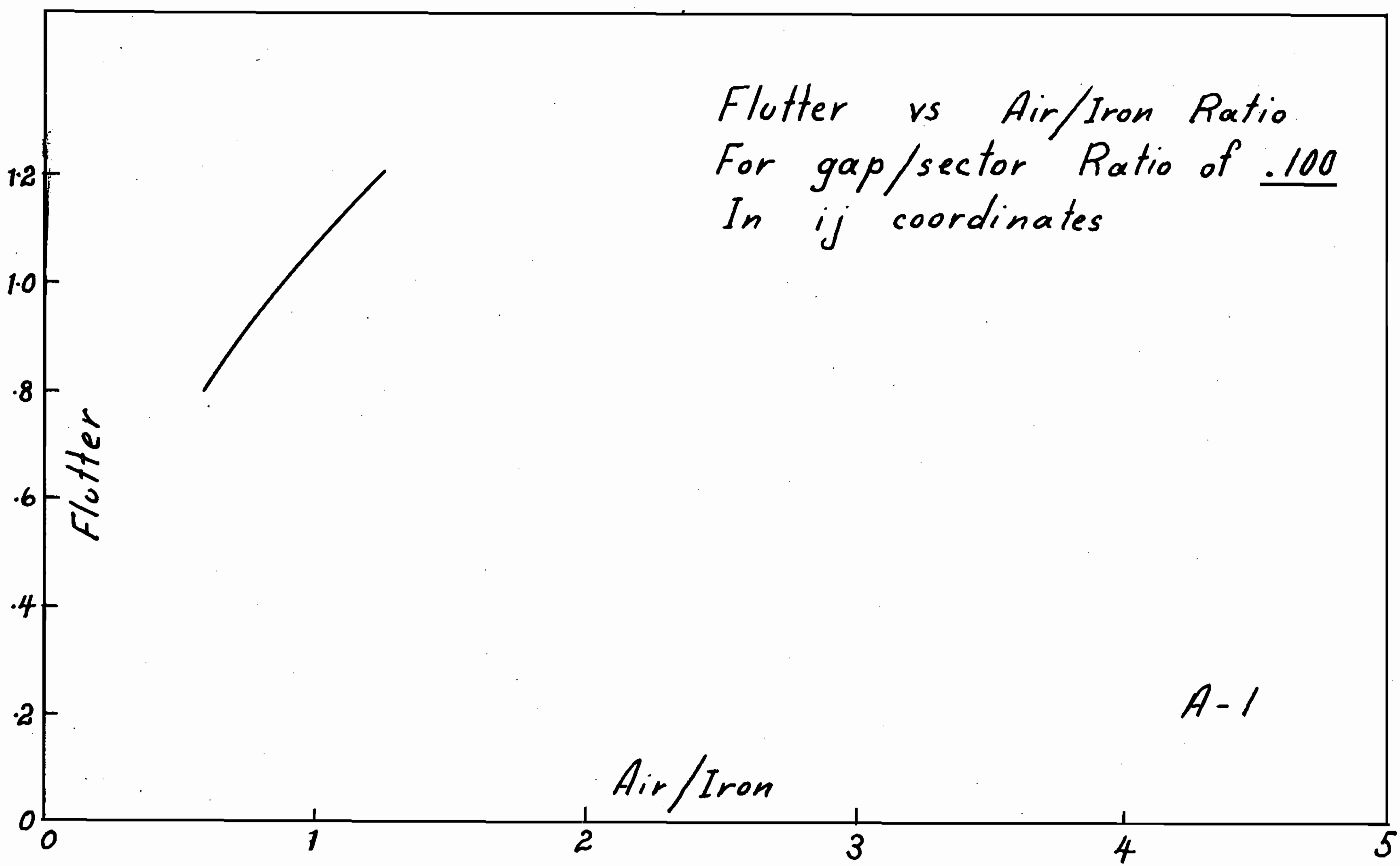
f	.8	.9	1.0	1.1	1.2
β	.228	.203	.178	.166	.155
α	1.09	1.17	1.29	1.48	1.69

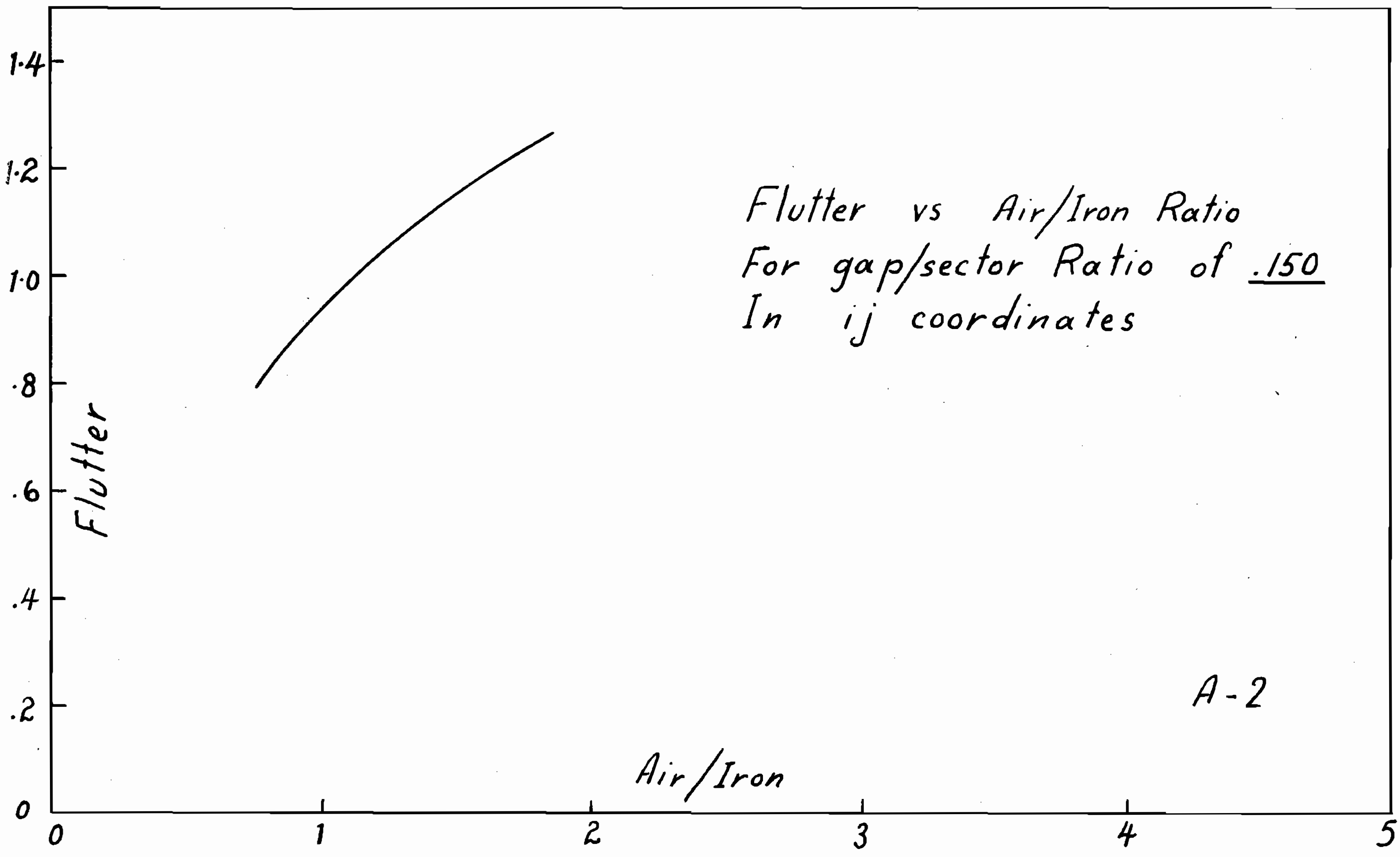
Part III REFERENCES AND NOTES

1. Symon, Kerst, Jones, Laslett and Terwilliger
   Phys. Rev. 103, p. 1837- 1859, (1956)
2. Flutter $= \sqrt{2\left[\frac{\langle B^2\rangle}{\langle B\rangle^2} - 1\right]}$ where B is the magnetic flux density.
3. The gap is the vertical distance between the magnetic poles.
4. y is a dimensionaless variable equal to the vertical coordinate divided by the radius, ie: $y = \frac{z}{R}$
5. J. N. Snyder, MURA-221
6. Many of these graphs had already been obtained by T. Ohkawa and the author has merely supplemented them.
7. The tune of the accelerator refers to the values of σ_x and σ_y. The σ_y used by the author is identical to the σ_z used in other reports, and σ_x is identical to σ_r.
8. The tune and number of sectors determine the values of $\frac{f}{\omega}$ and k. Since the value of the gap in ξ, ζ coordinates is dependent on the value of $\frac{1}{\omega}$ which in turn depends on the value of the flutter, one can see how the value of the gap/sector ratio in ξ, ζ coordinates is dependent on the flutter for a fixed tune and number of sectors.
9. L. J. Laslett MURA- 75 p. 35
10. J. N. Snyder MURA- 231
11. Since the value of the gap in the ξ coordinate is equal to a_m times the value of ζ for the gap, and a_m is equal to the sector length in the ξ coordinate; it then follows that the gap/sector ratio (β) must be equal to the value of ζ for the gap.
12. M. S. Livingston, High Energy Accelerators
    Interscience Tracts on Physics and Astronomy 2 (1954) p. 20
13. G. Belford, L. J. Laslett, J. N. Snyder MURA Table 6

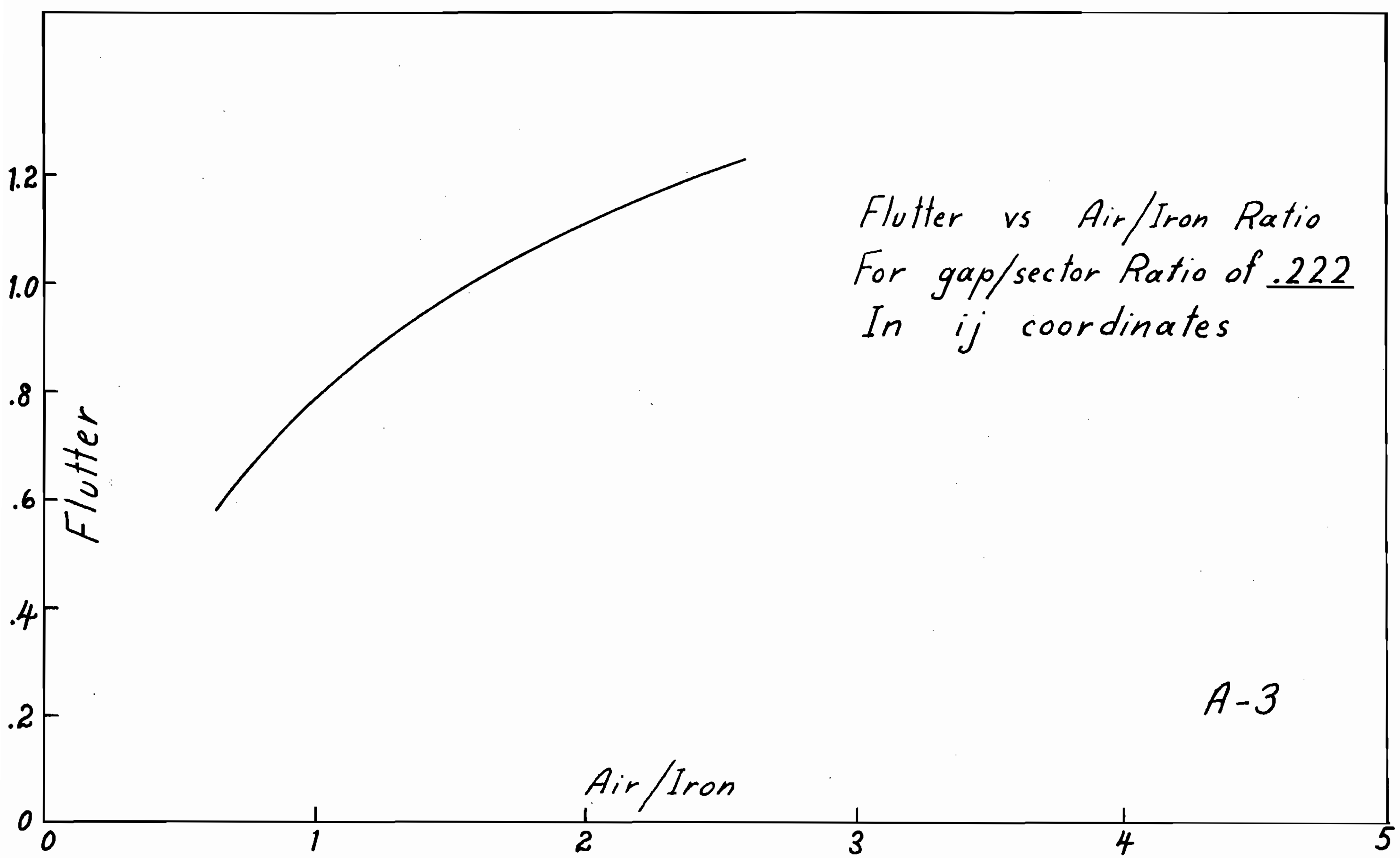
Part IV ACKNOWLEDGEMENTS

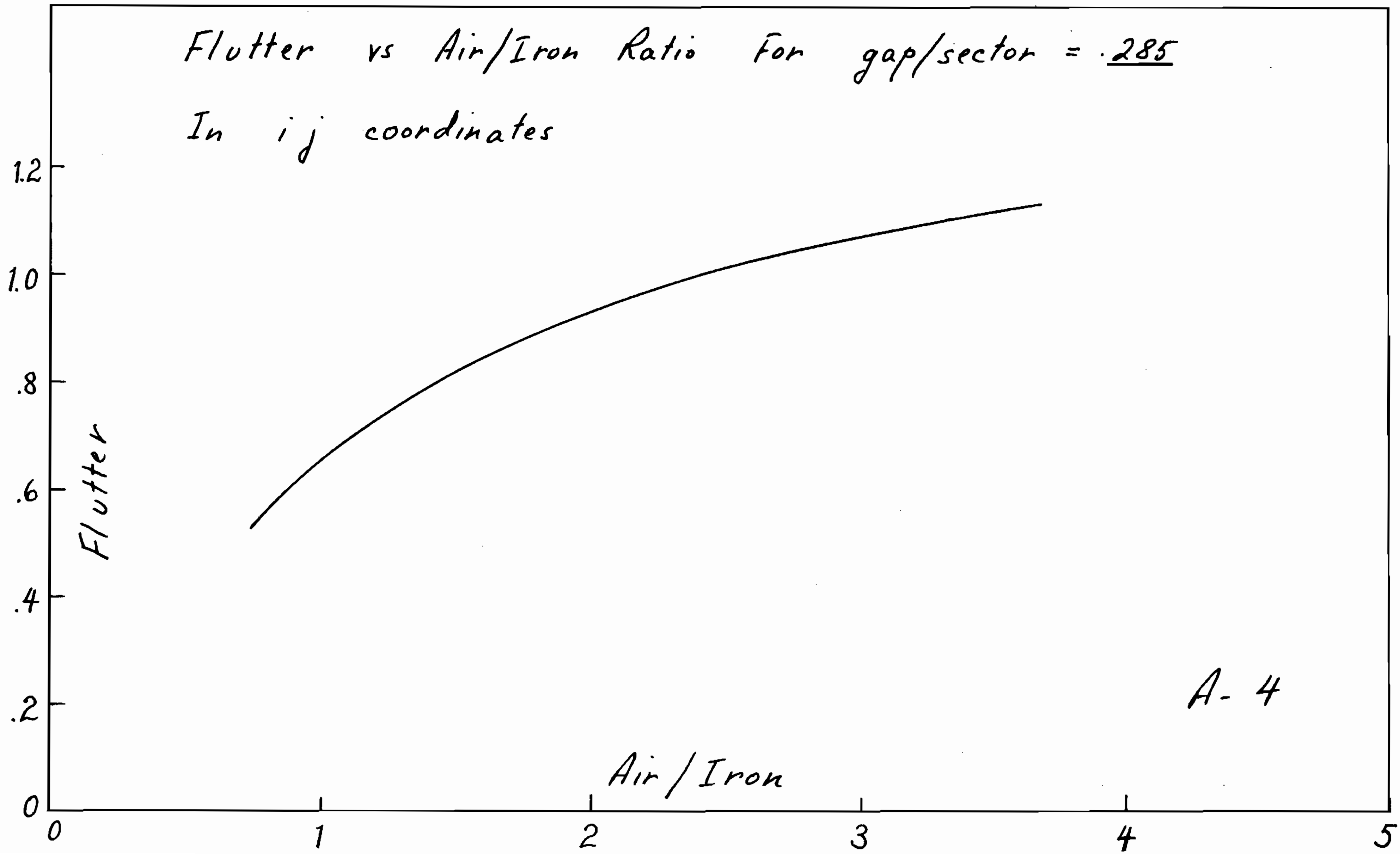
The author would like to thank most sincerely Dr. A. M. Sessler for his encouragement and many constructive suggestions during the progress of this work. He would also like to express his most sincere gratitude to T. Ohkawa who not only had many constructive suggestions but also supplied many of the graphs. It is also a pleasure to thank J. Anderson for his work in connection with the computer.

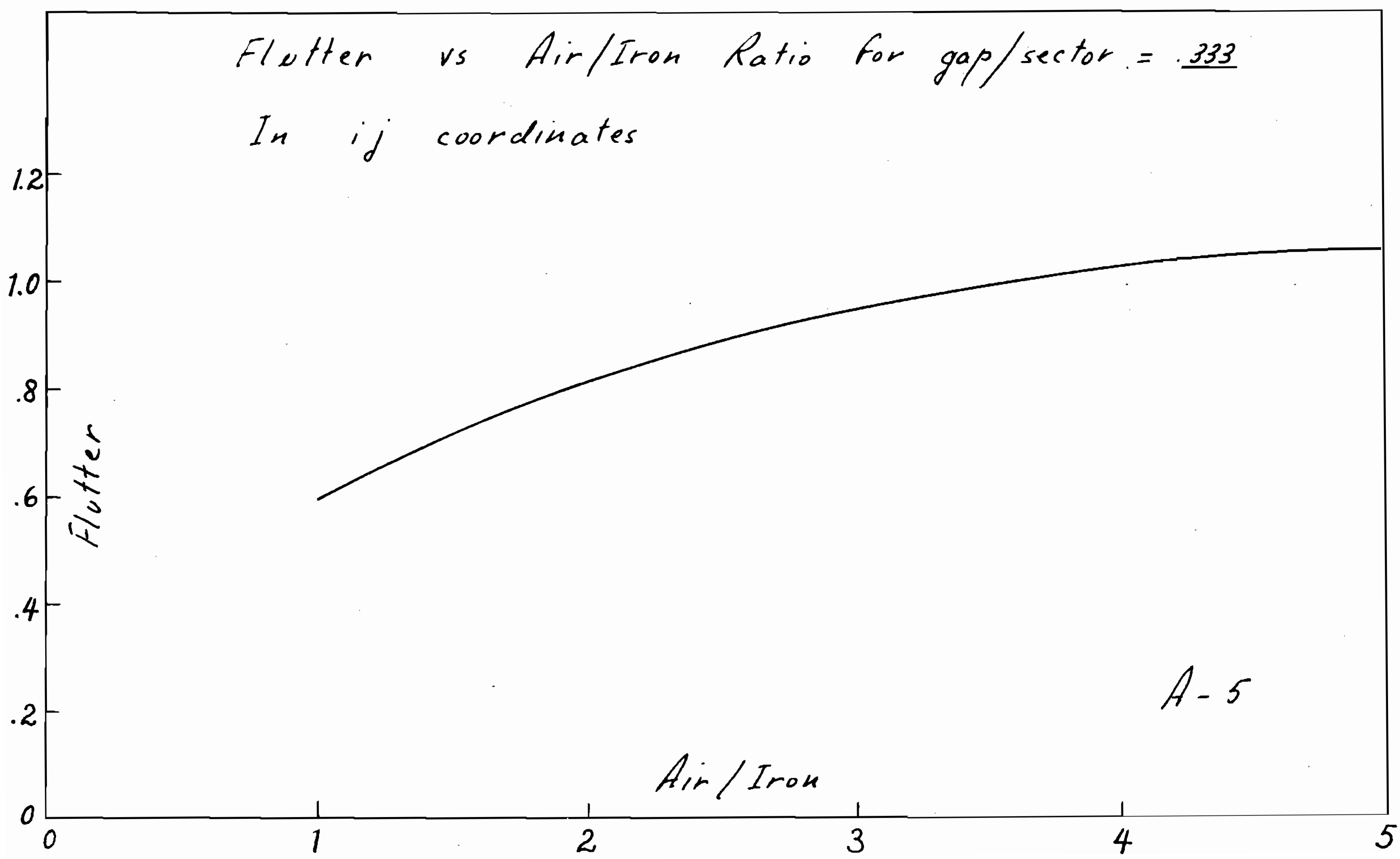

Flutter vs Air/Iron Ratio
For gap/sector Ratio of .100
In ij coordinates
Flutter
Air/Iron
0
·2
·4
·6
·8
1·0
1·2
0
1
2
3
4
5
A-1



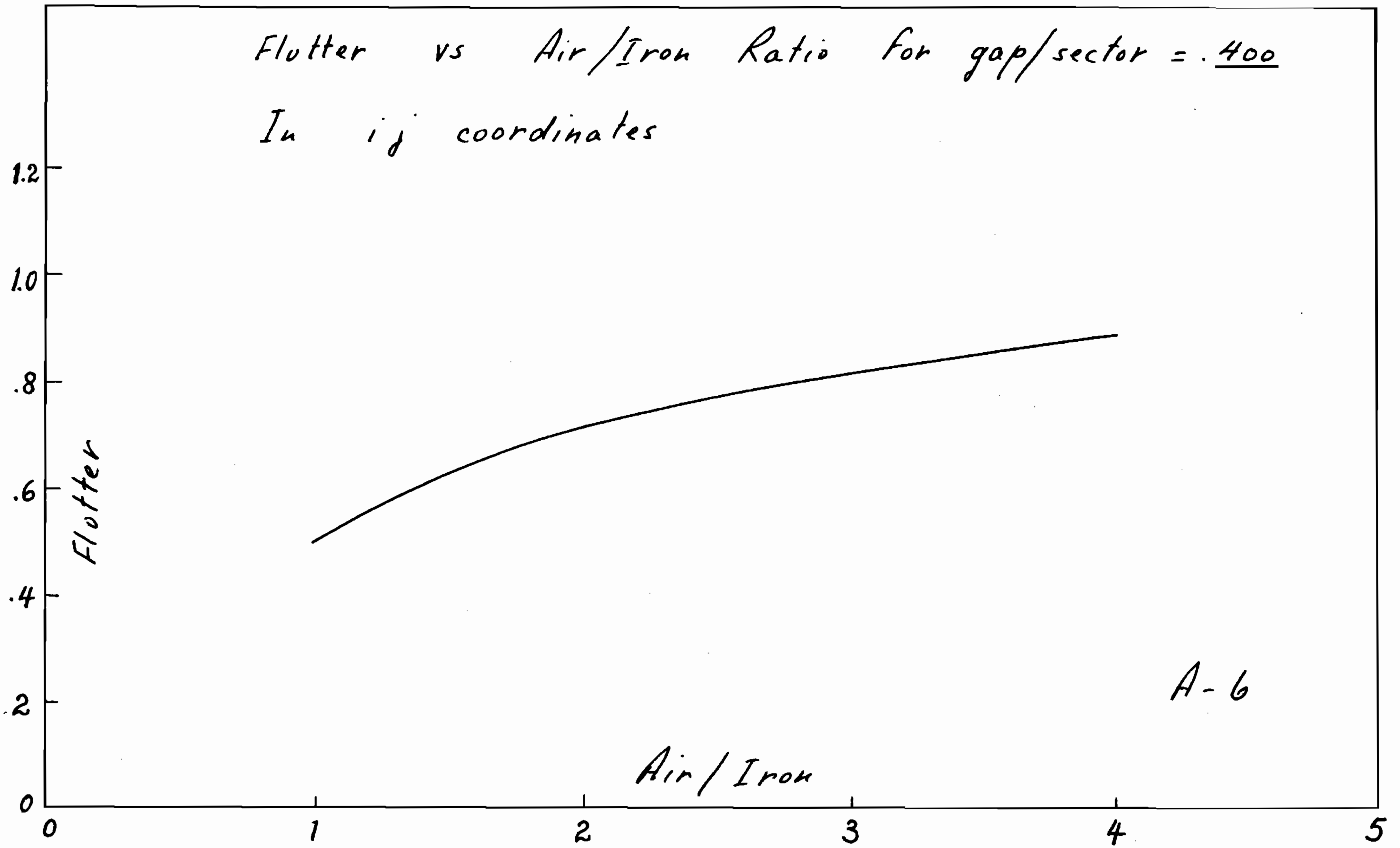
Flutter vs Air/Iron Ratio
For gap/sector Ratio of .150
In ij coordinates
Flutter
Air/Iron
1.4
1.2
1.0
.8
.6
.4
.2
0
0
1
2
3
4
5
A-2


Flutter vs Air/Iron Ratio
For gap/sector Ratio of .222
In ij coordinates
Flutter
Air/Iron
1.2
1.0
.8
.6
.4
.2
0
0
1
2
3
4
5
A-3

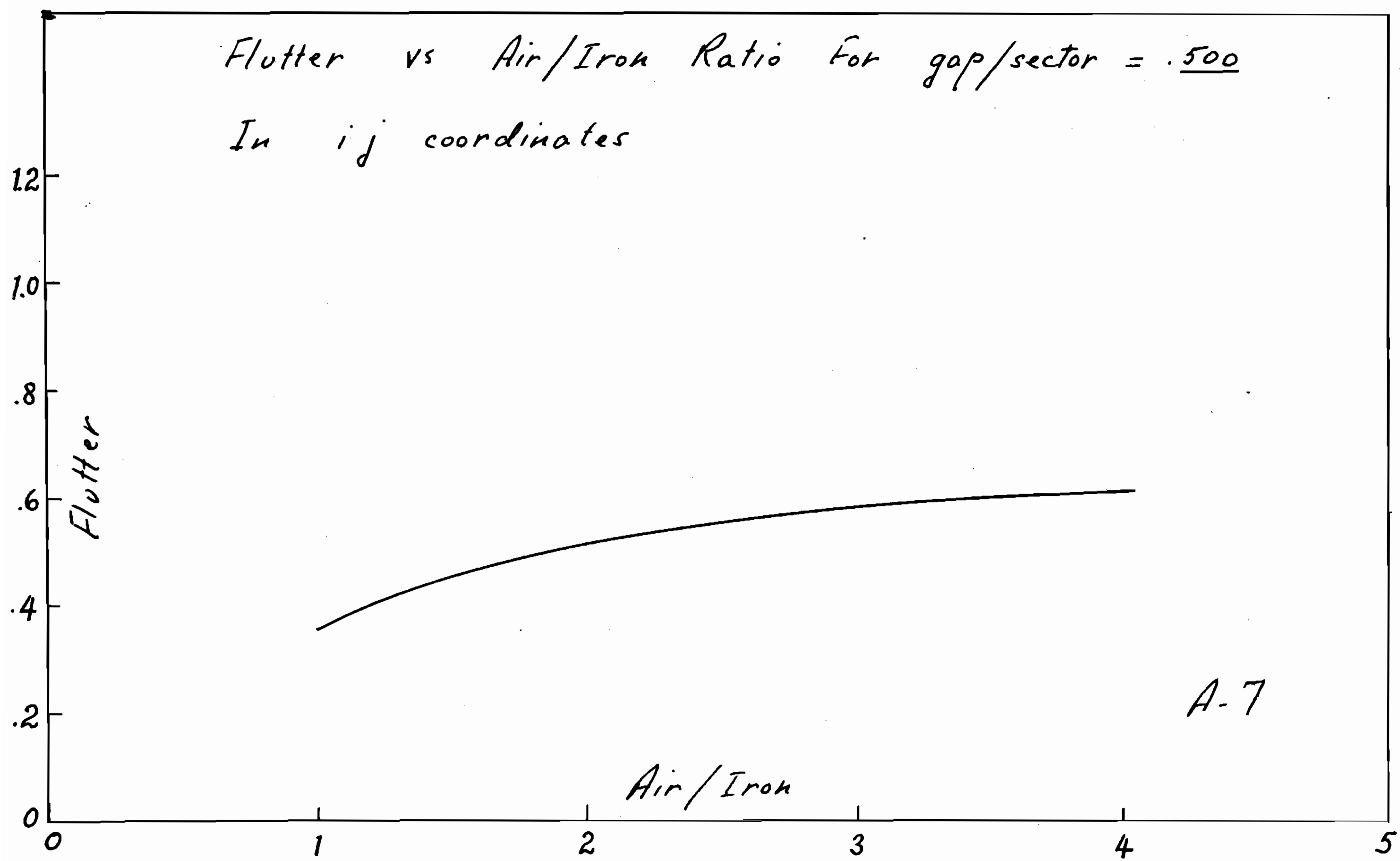

Flutter vs Air/Iron Ratio for gap/sector = .285
In ij coordinates
1.2
1.0
.8
.6
.4
.2
0
Flutter
0
1
2
3
4
5
Air/Iron
A- 4



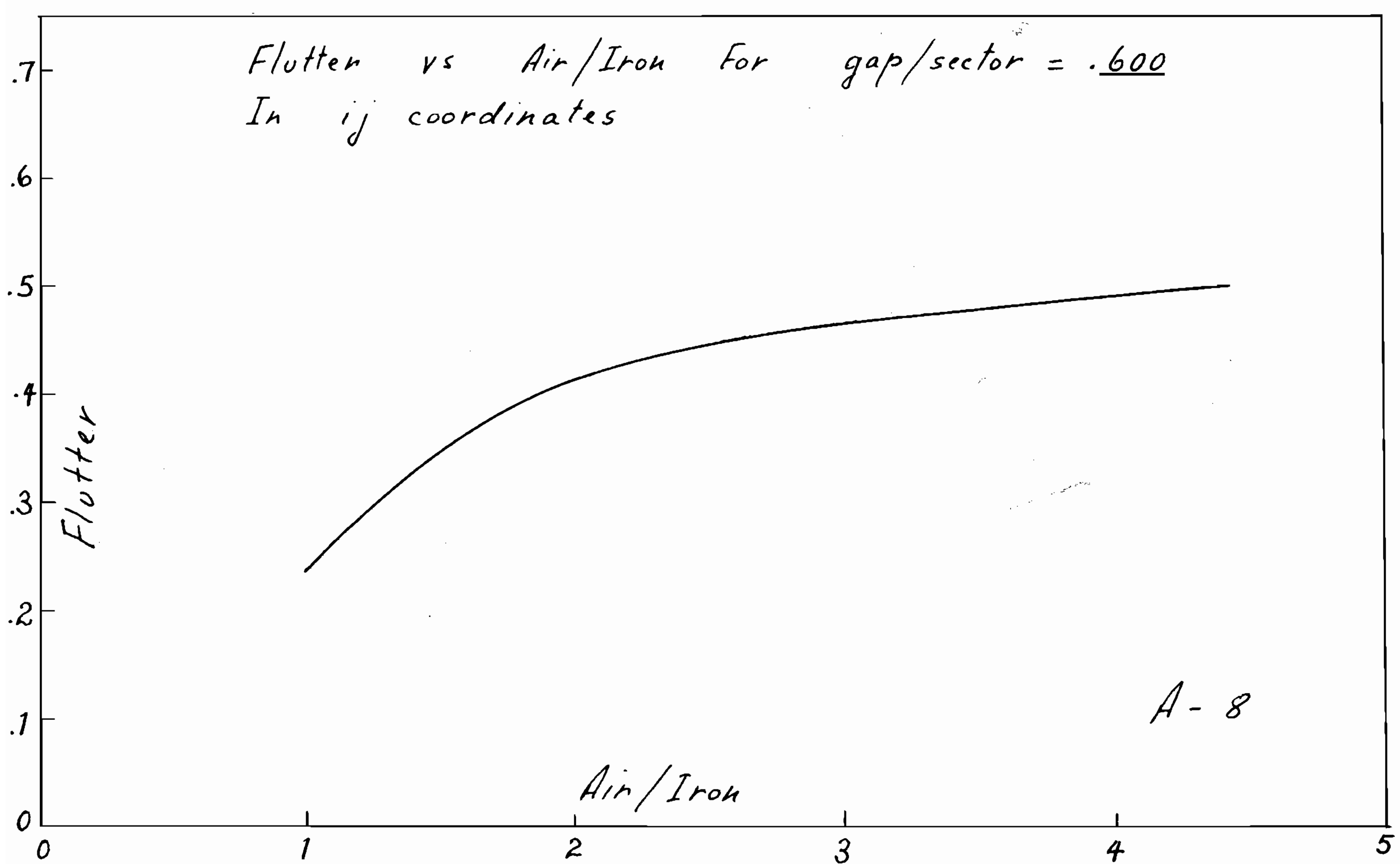
Flutter vs Air/Iron Ratio for gap/sector = .333
In ij coordinates
Flutter
1.2
1.0
.8
.6
.4
.2
0
0
1
2
3
4
5
Air/Iron
A-5

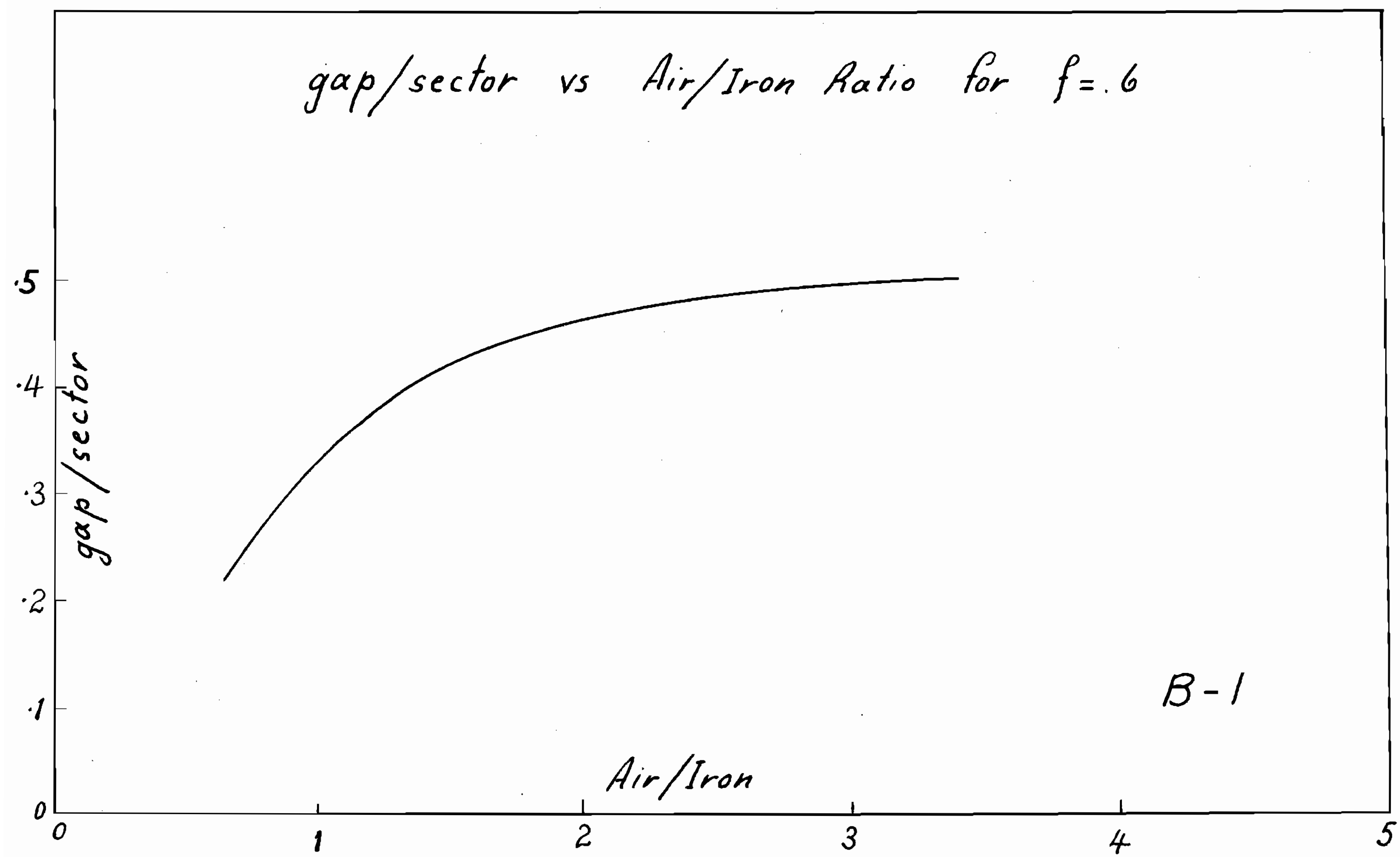


Flotter vs Air/Iron Ratio for gap/sector = .400
In ij coordinates
1.2
1.0
.8
.6
.4
.2
0
Flotter
Air/Iron
0
1
2
3
4
5
A-6



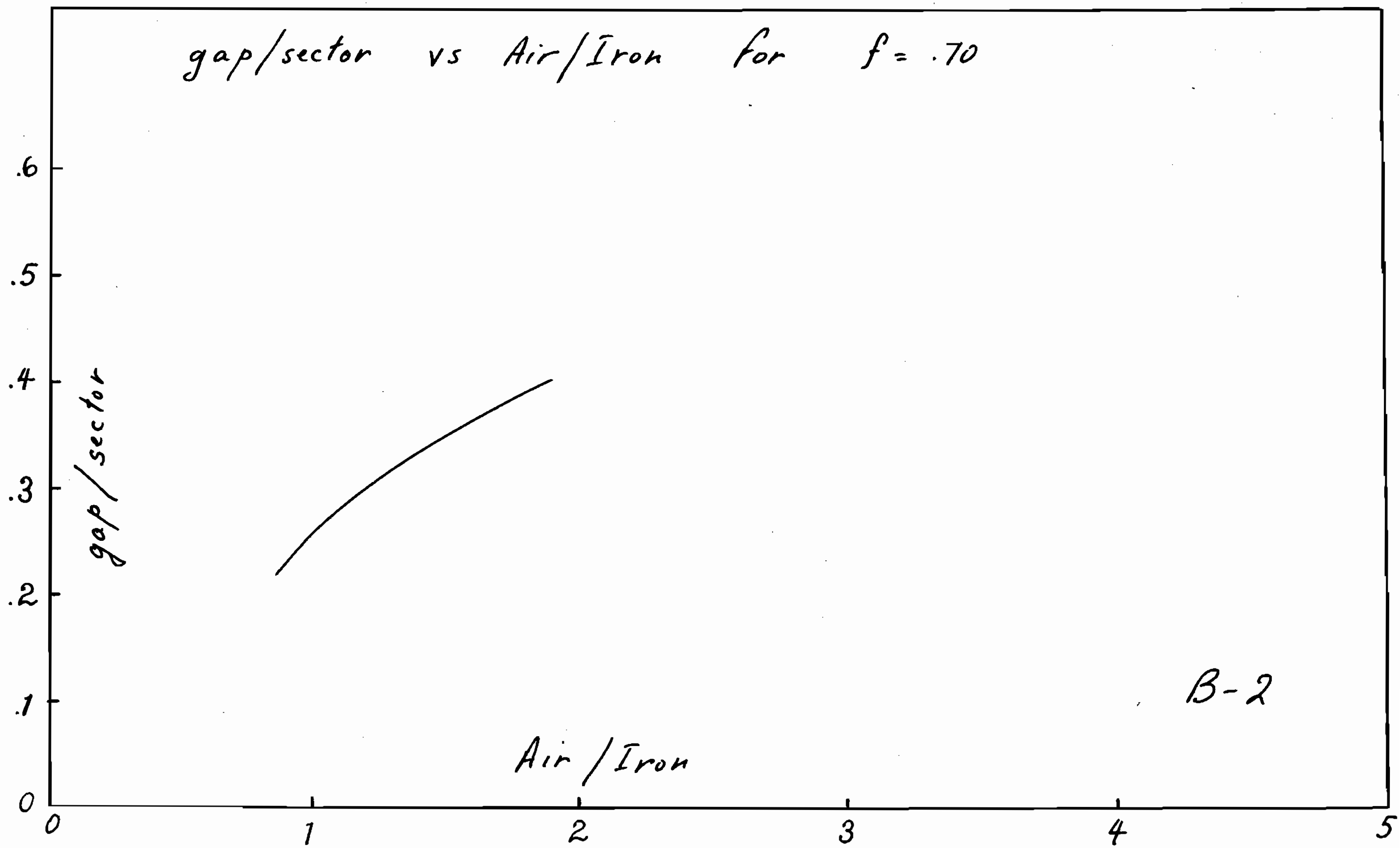
Flutter vs Air/Iron Ratio for gap/sector = .500
In ij coordinates
1.2
1.0
.8
.6
.4
.2
0
Flutter
Air/Iron
0
1
2
3
4
5
A-7

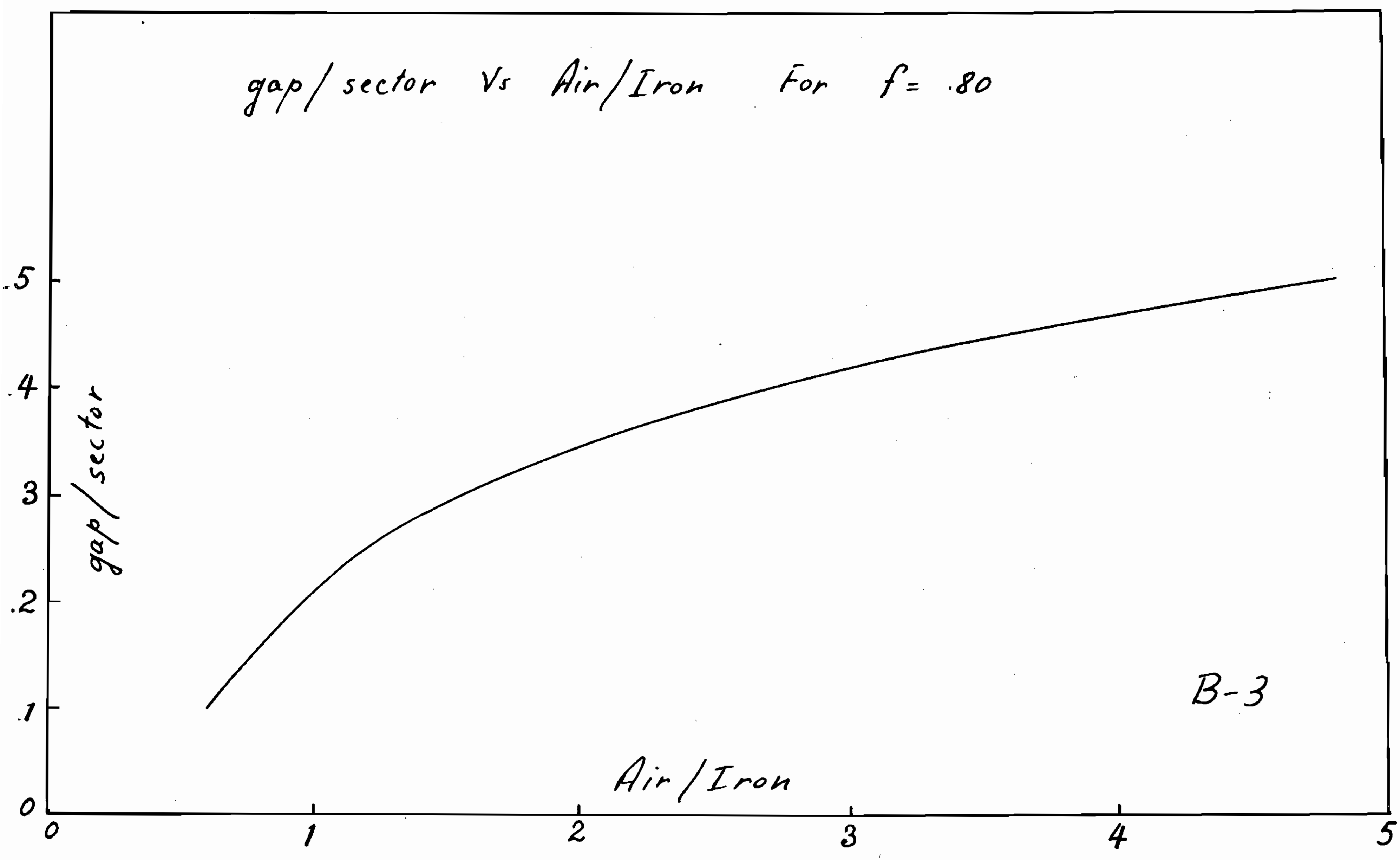

Flutter vs Air/Iron For gap/sector = .600
In ij coordinates
Flutter
Air/Iron
.7
.6
.5
.4
.3
.2
.1
0
0
1
2
3
4
5
A-8

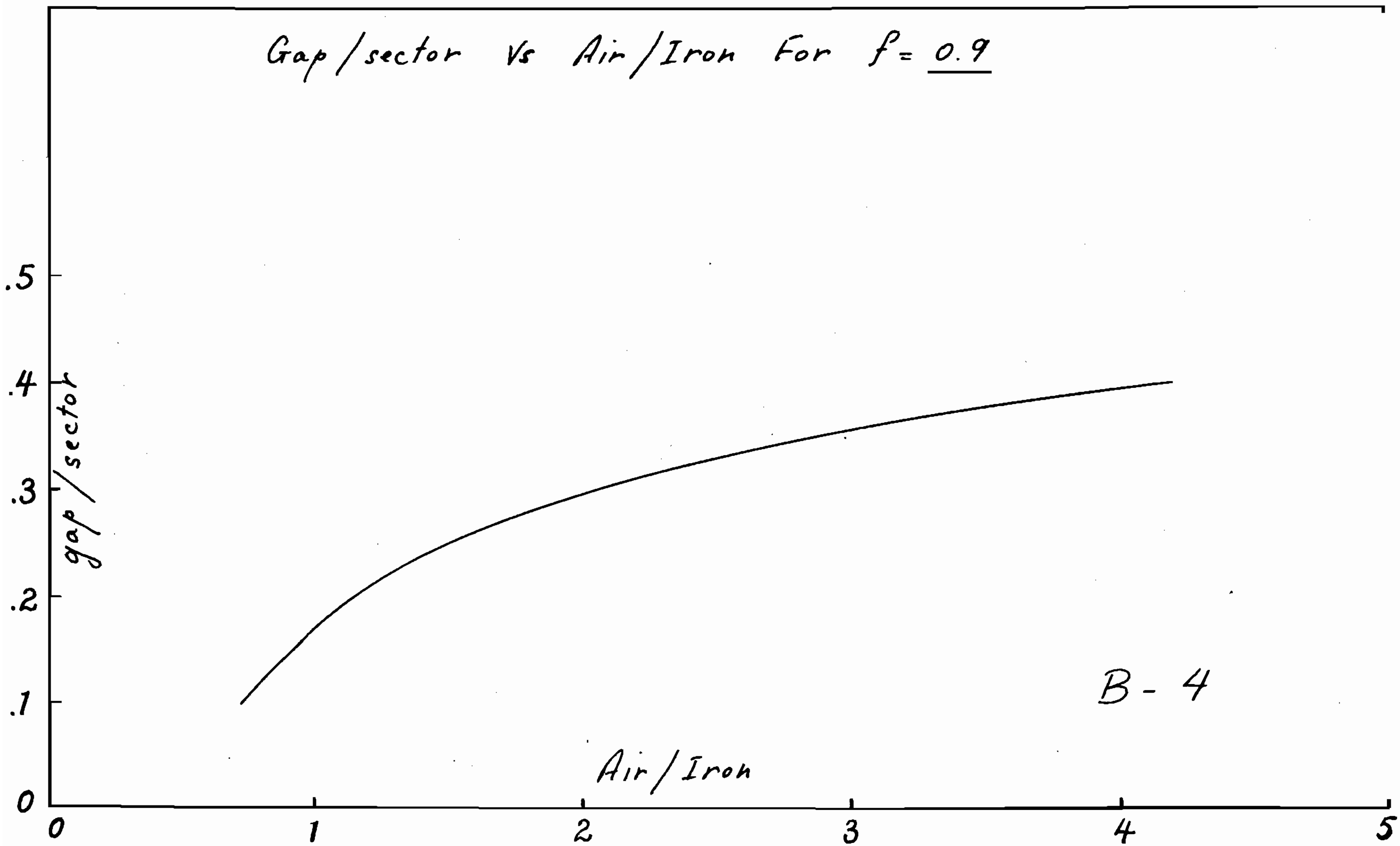


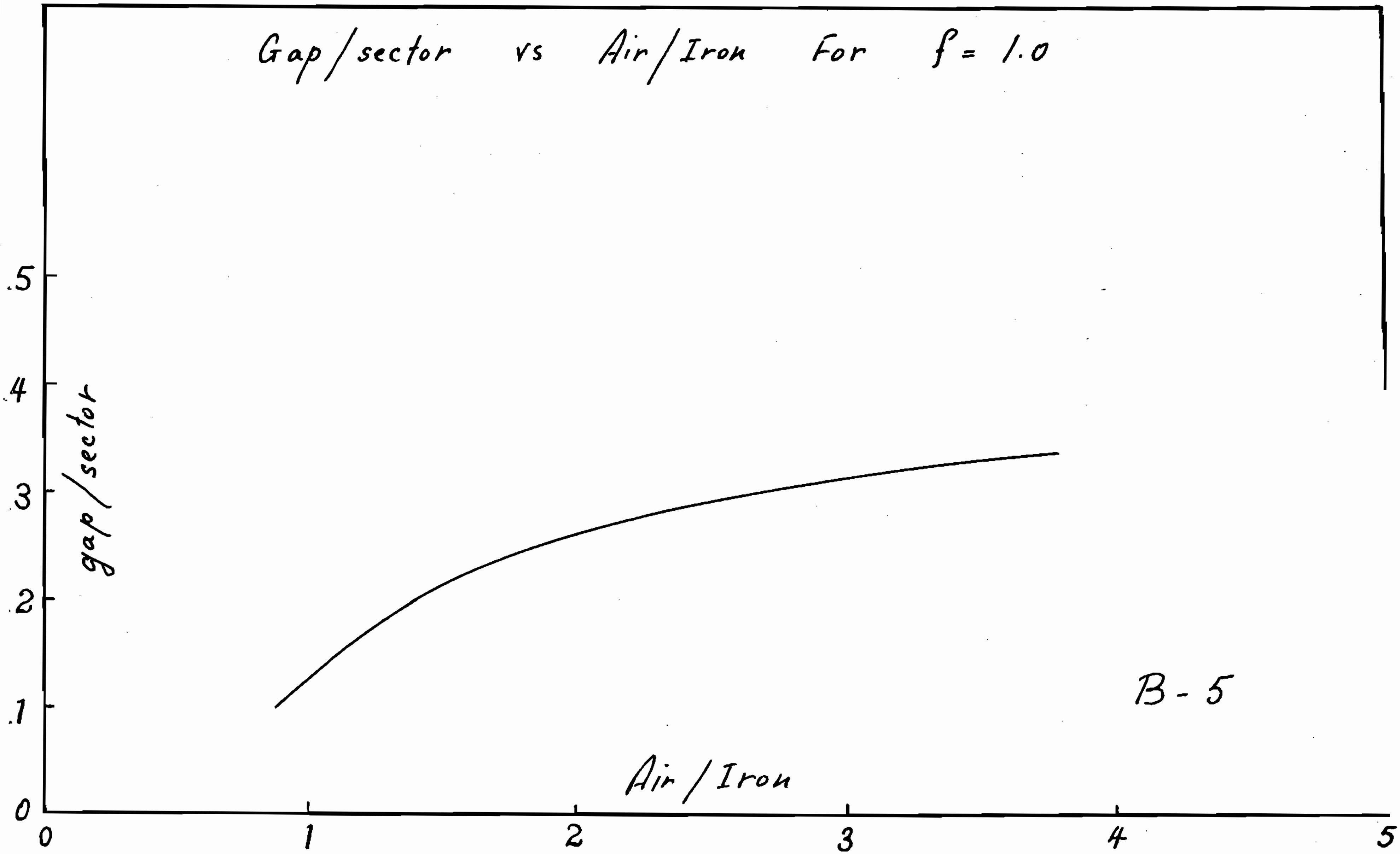
gap/sector vs Air/Iron Ratio for f=.6
gap/sector
·5
·4
·3
·2
·1
0
Air/Iron
0
1
2
3
4
5

B-1

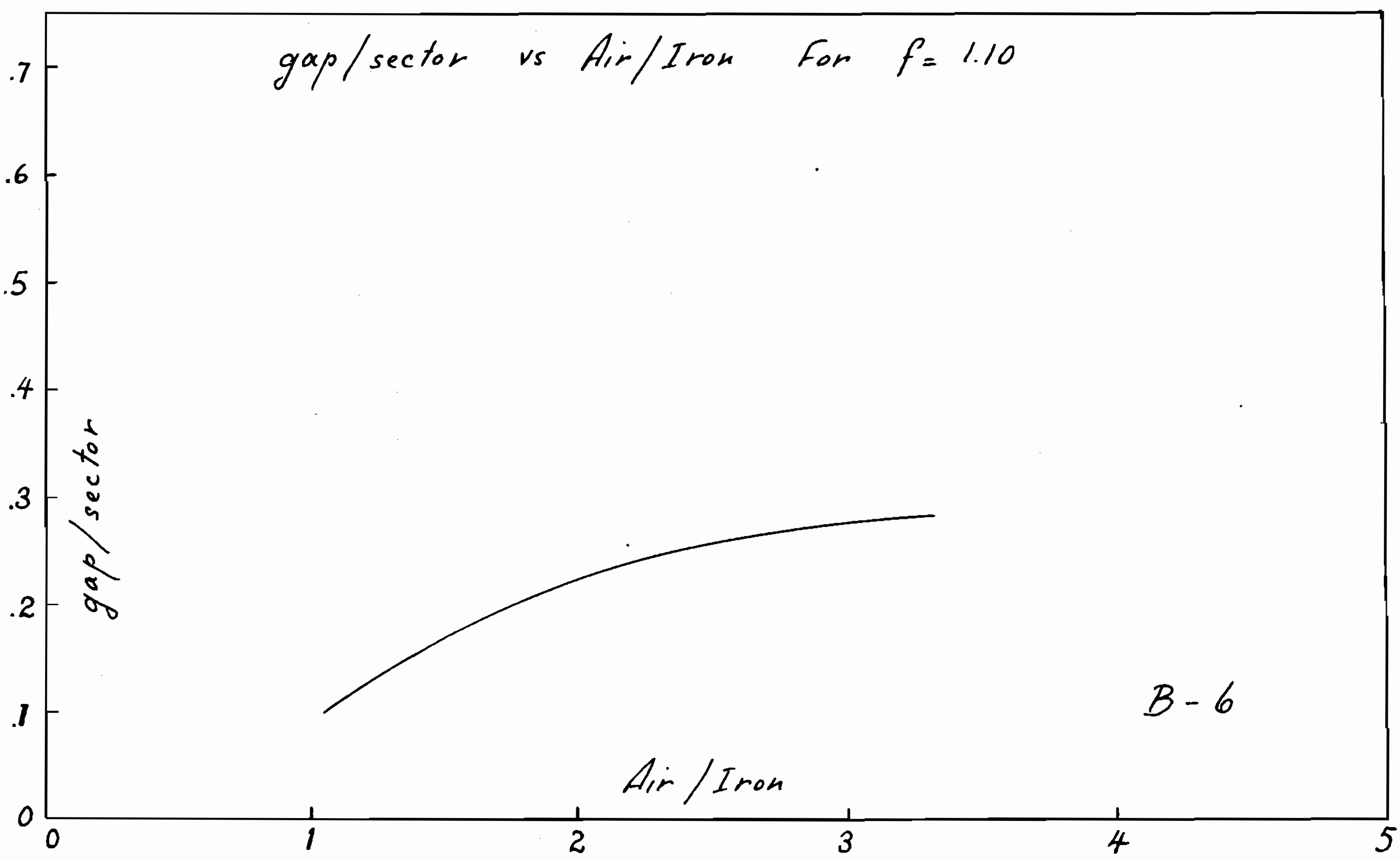

gap/sector vs Air/Iron for f = .70
gap/sector
.6
.5
.4
.3
.2
.1
0
0
1
2
3
4
5
Air/Iron
B-2

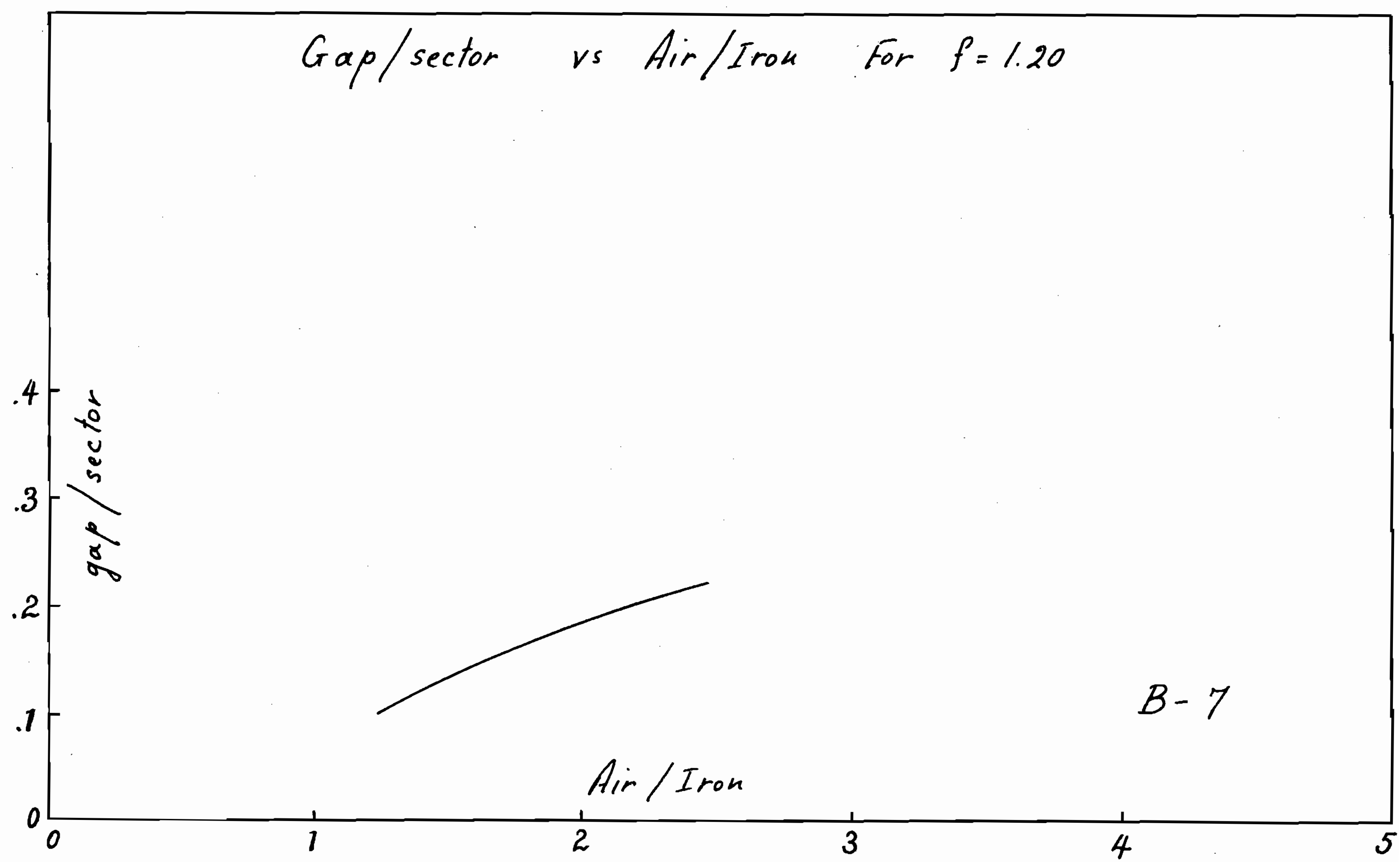

gap/sector Vs Air/Iron For f= .80
.5
.4
3
.2
.1
0
gap/sector
0
1
2
3
4
5
Air/Iron
B-3

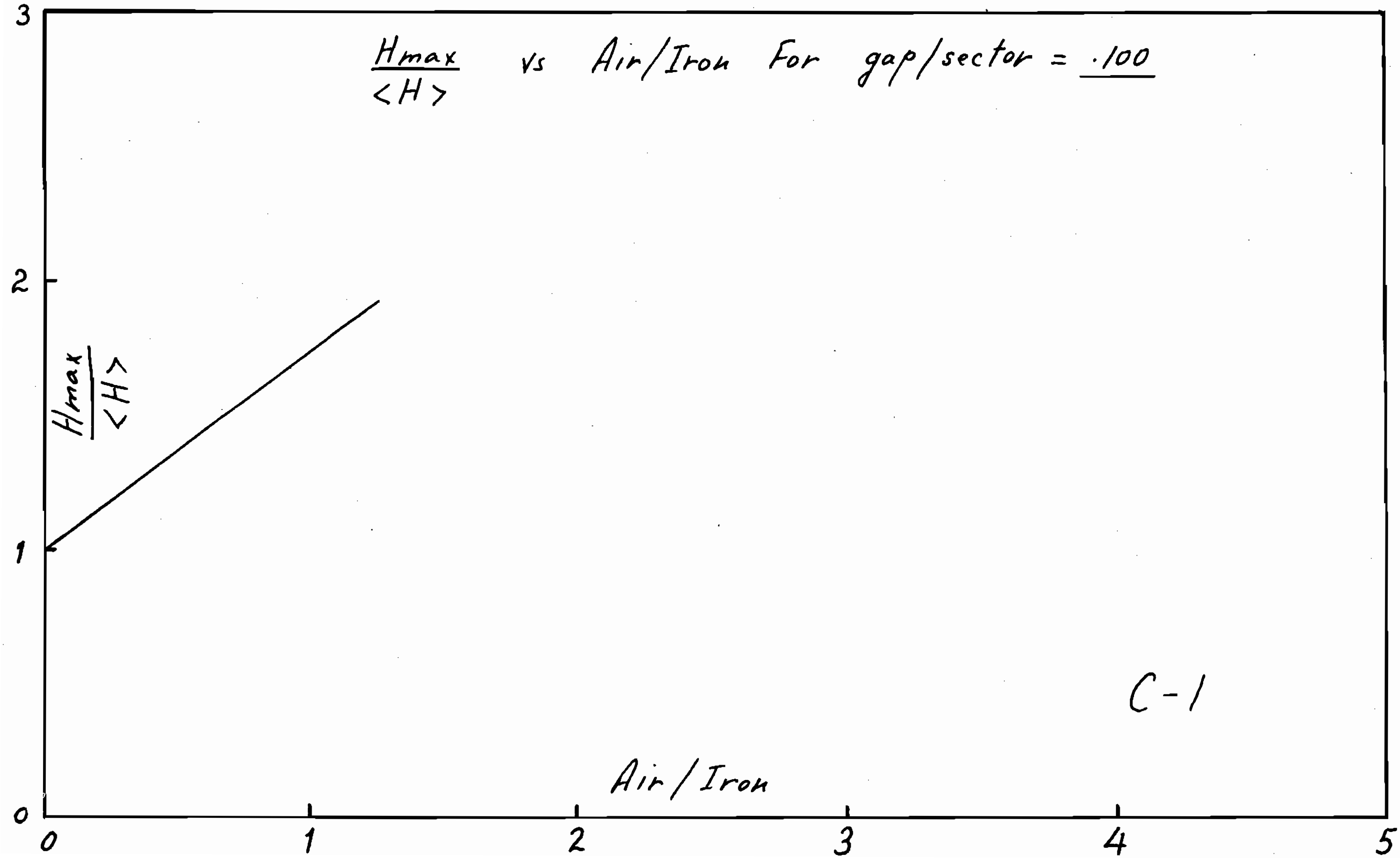

Gap/sector Vs Air/Iron For f = 0.9
gap/sector
.5
.4
.3
.2
.1
0
Air/Iron
0
1
2
3
4
5
B - 4

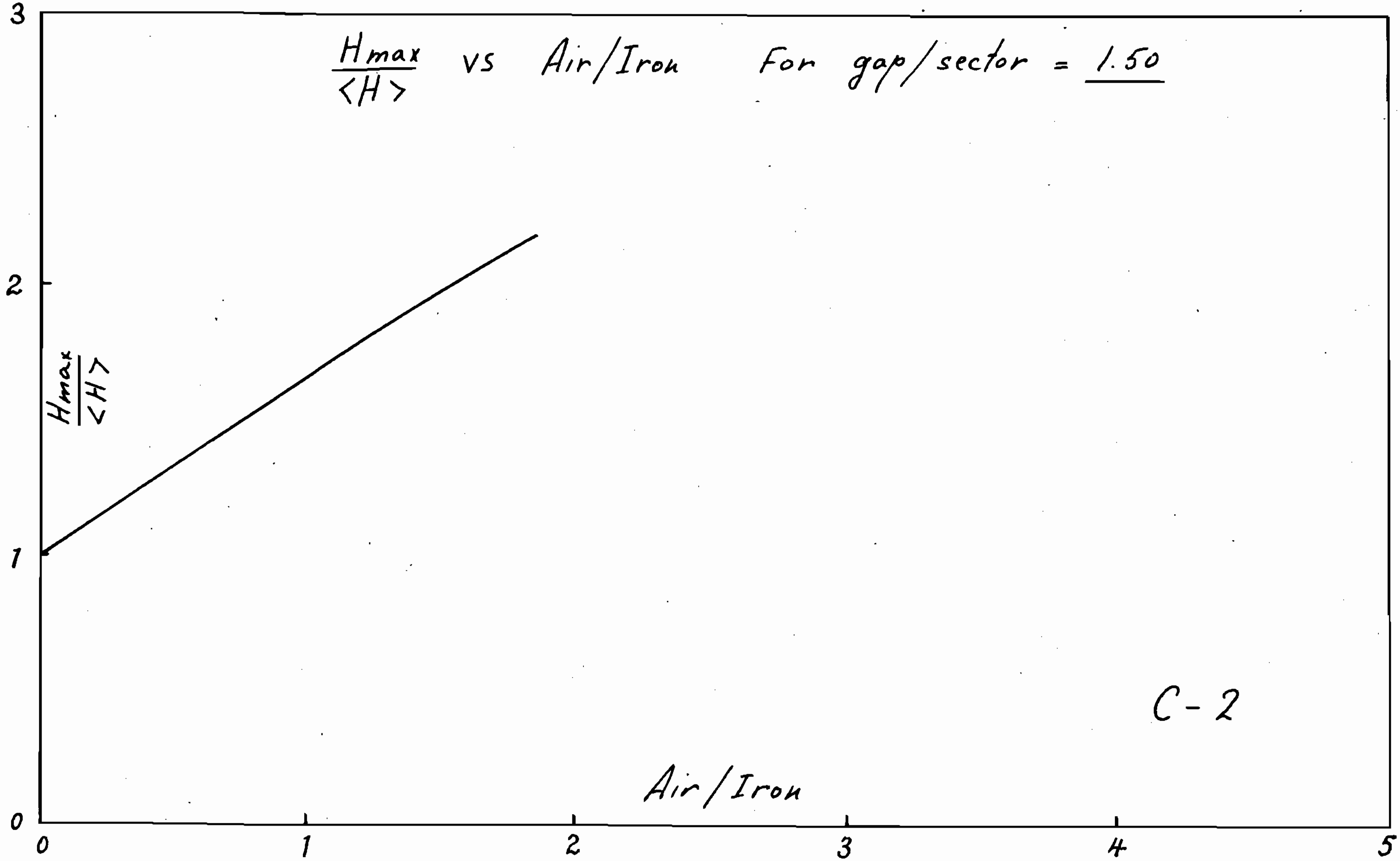


Gap/sector vs Air/Iron For f = 1.0
gap/sector
.5
.4
.3
.2
.1
0
0
1
2
3
4
5
Air/Iron
B-5

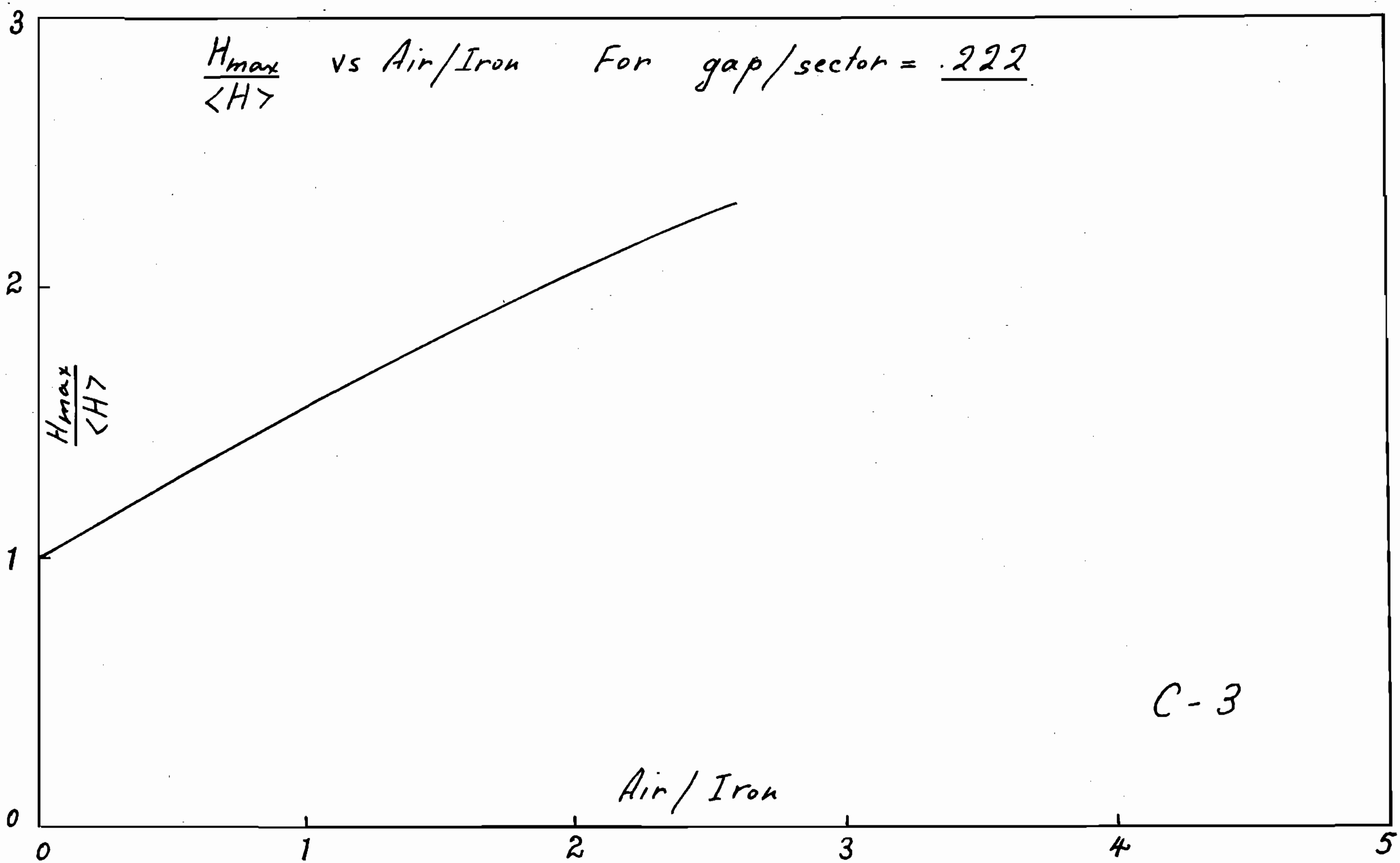

gap/sector vs Air/Iron For f= 1.10
gap/sector
Air/Iron
B-6
.7
.6
.5
.4
.3
.2
.1
0
0
1
2
3
4
5

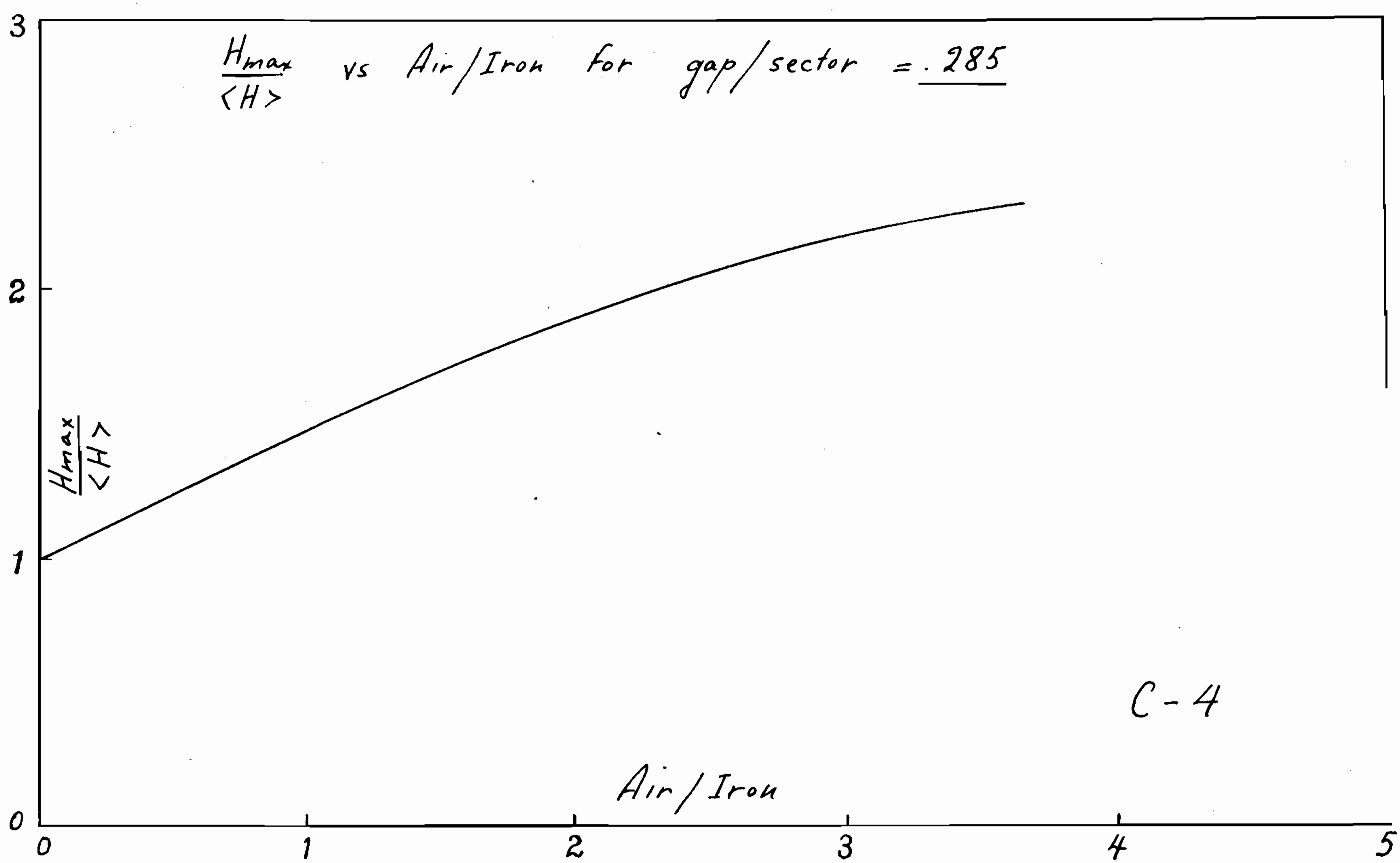

Gap/sector vs Air/Iron For f = 1.20
gap / sector
.4
.3
.2
.1
0
Air / Iron
0
1
2
3
4
5
B-7


Hmax/<H> vs Air/Iron For gap/sector = .100
3
2
1
0
Hmax/<H>
0
1
2
3
4
5
Air/Iron
C-1

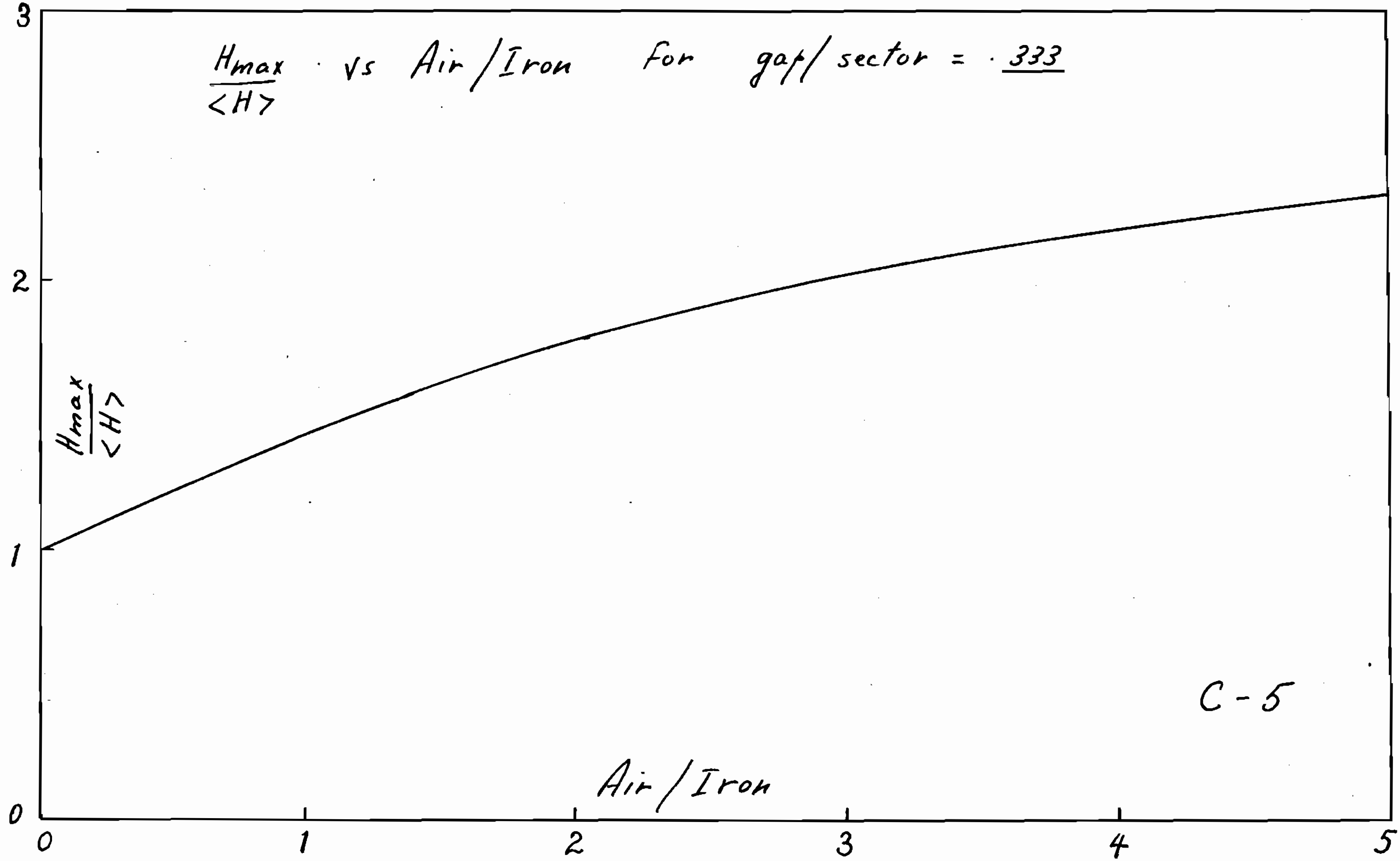


$\frac{H_{max}}{\langle H \rangle}$ vs Air/Iron For gap/sector = 1.50
3
2
1
0
$\frac{H_{max}}{\langle H \rangle}$
0
1
2
3
4
5
Air/Iron
C-2

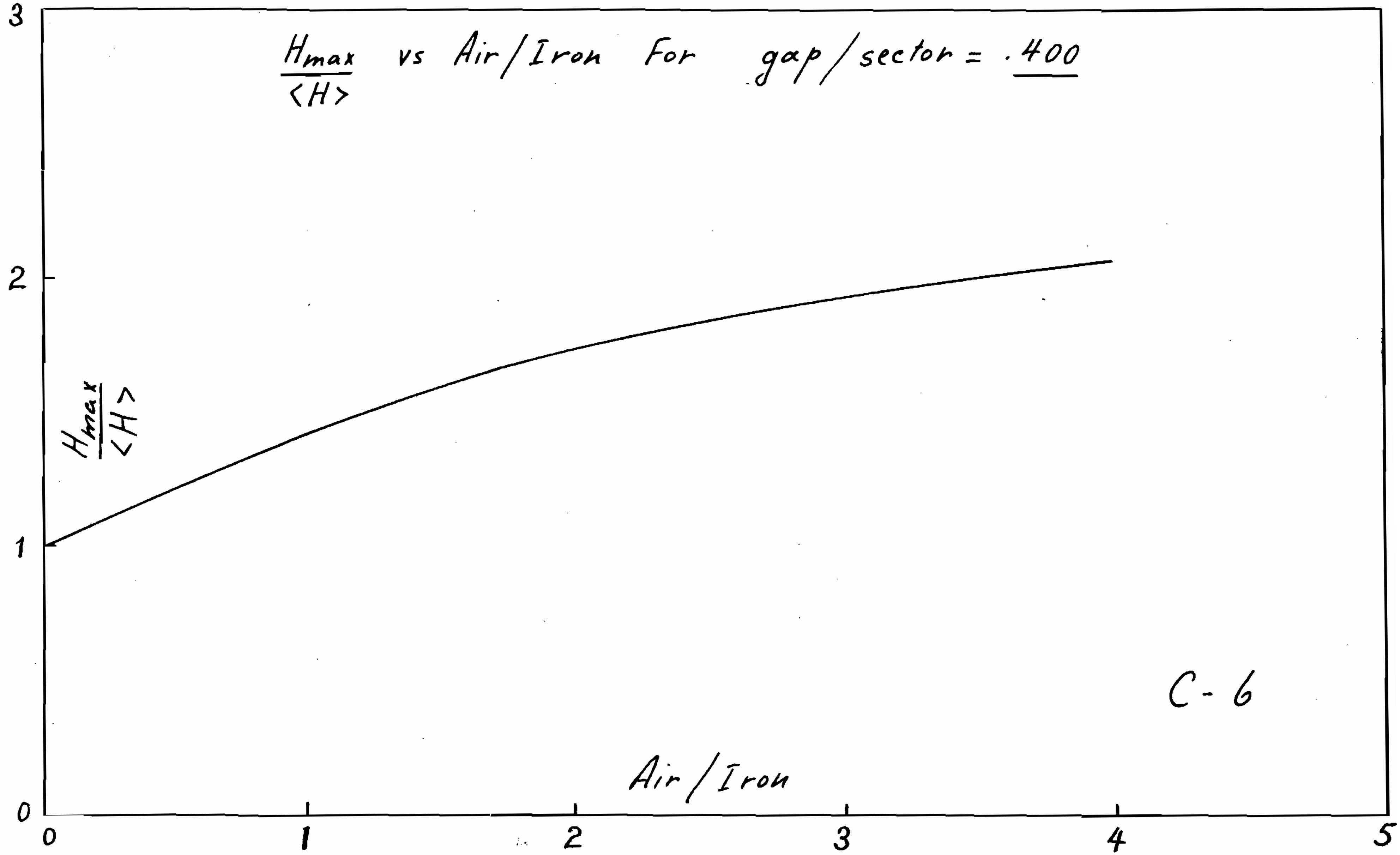

Hmax/⟨H⟩ vs Air/Iron For gap/sector = .222
Hmax/⟨H⟩
Air/Iron
0
1
2
3
4
5
C-3



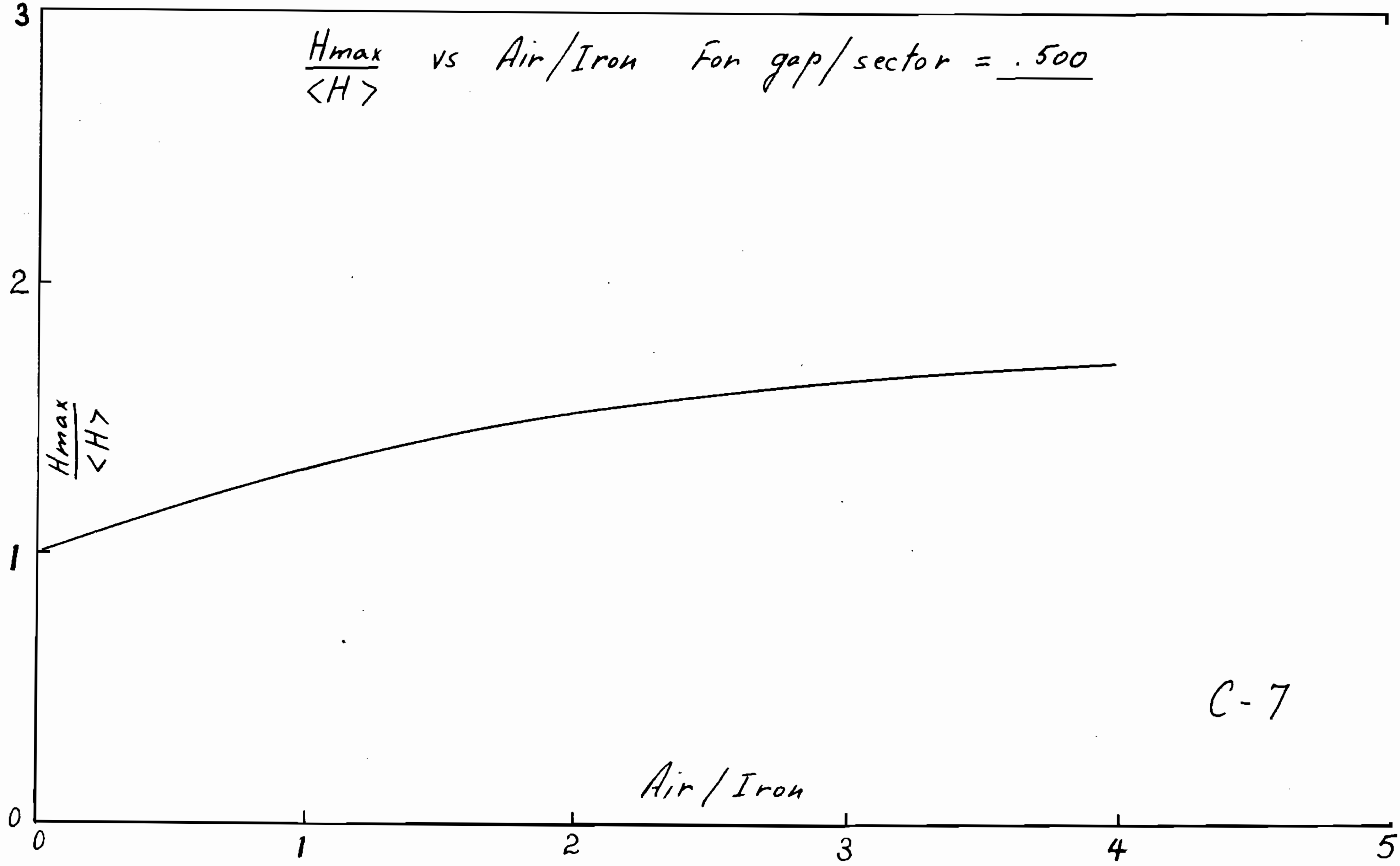
$\frac{H_{max}}{\langle H \rangle}$ vs Air/Iron for gap/sector = .285
3
2
1
0
$\frac{H_{max}}{\langle H \rangle}$
0
1
2
3
4
5
Air/Iron
C-4



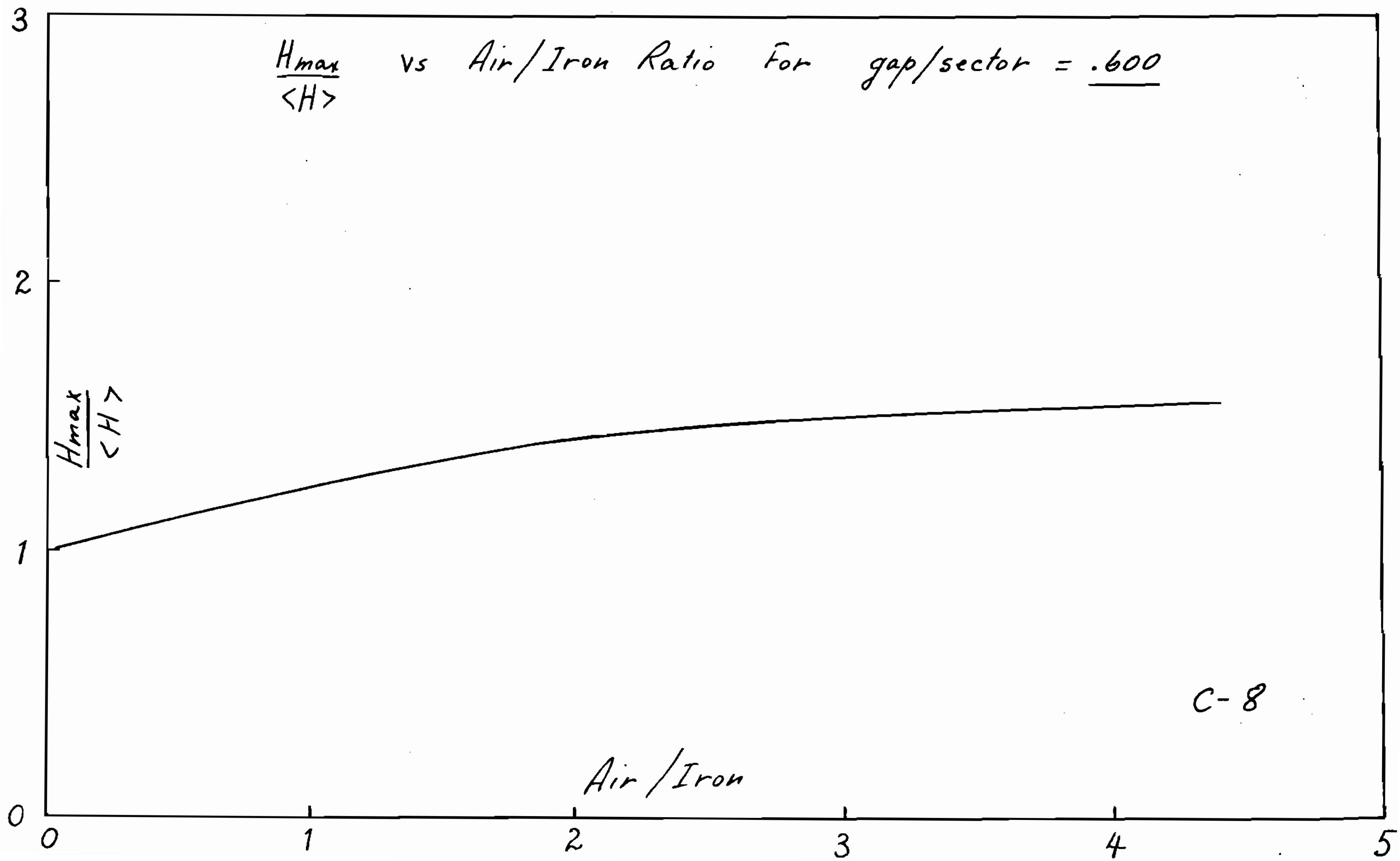
Hmax / <H> vs Air/Iron for gap/sector = .333
Hmax / <H>
Air/Iron
0
1
2
3
4
5
0
1
2
3
C-5



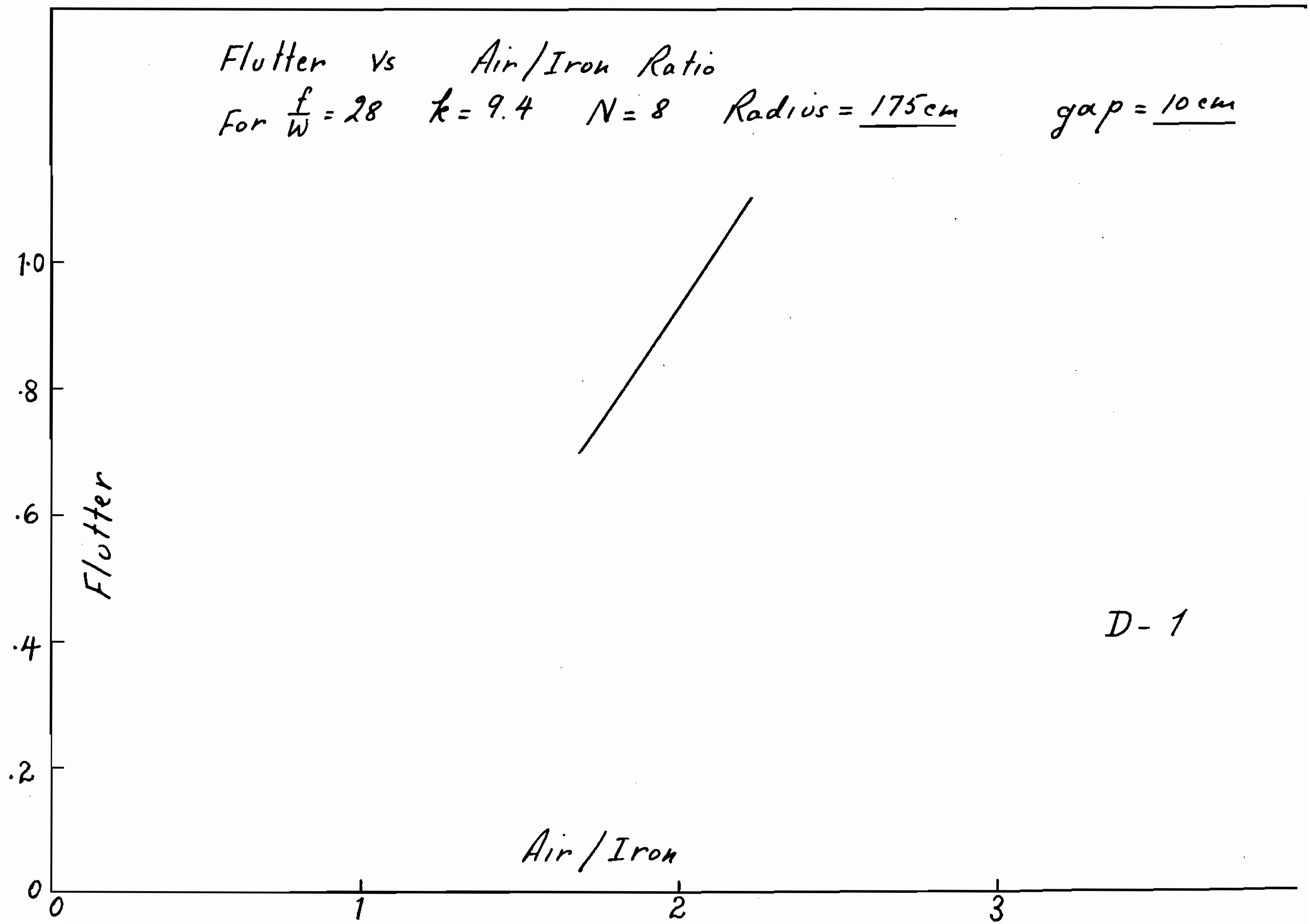
Hmax / <H> vs Air/Iron For gap/sector = .400
3
2
1
0
Hmax / <H>
0
1
2
3
4
5
Air/Iron
C-6

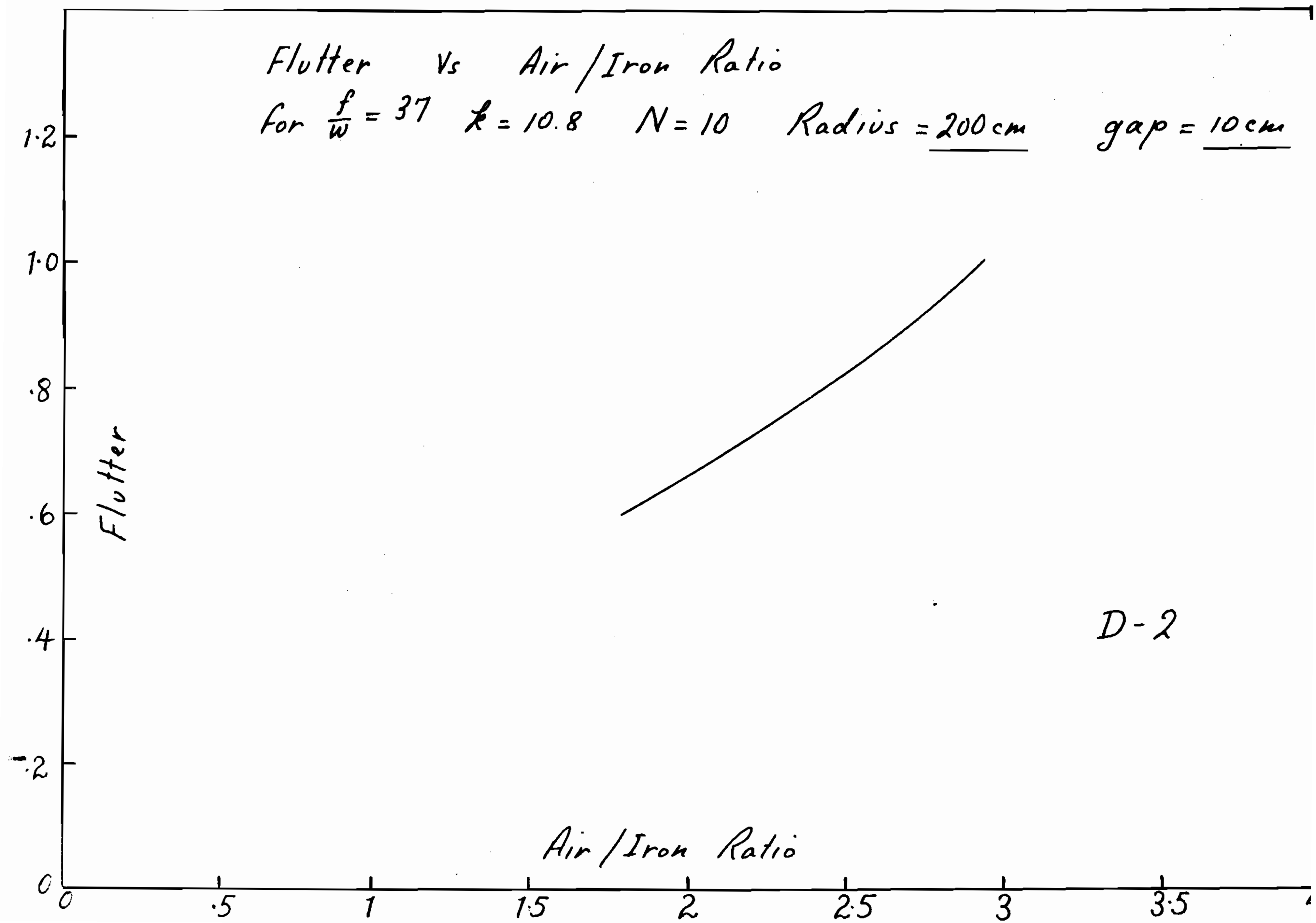


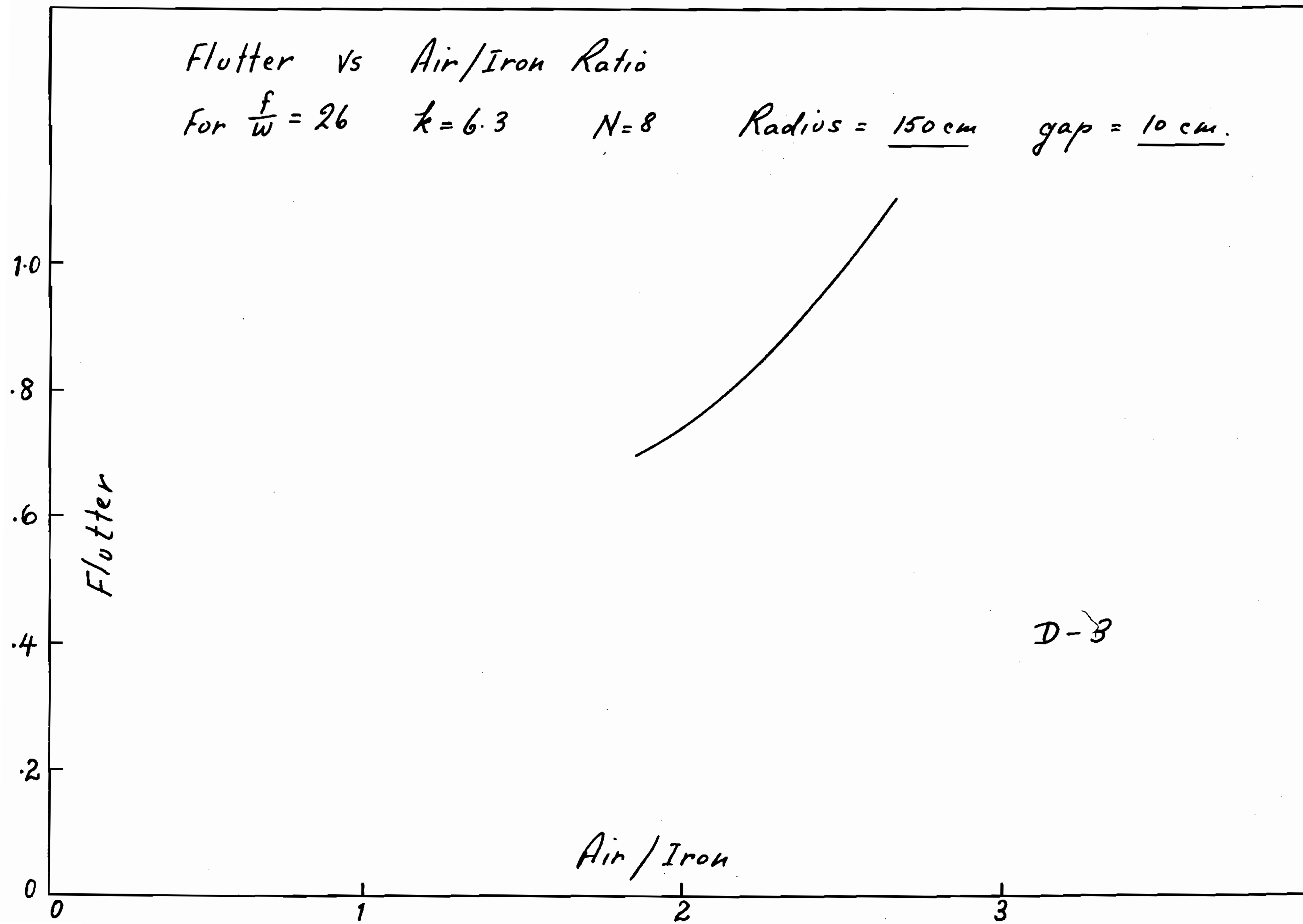
C-7



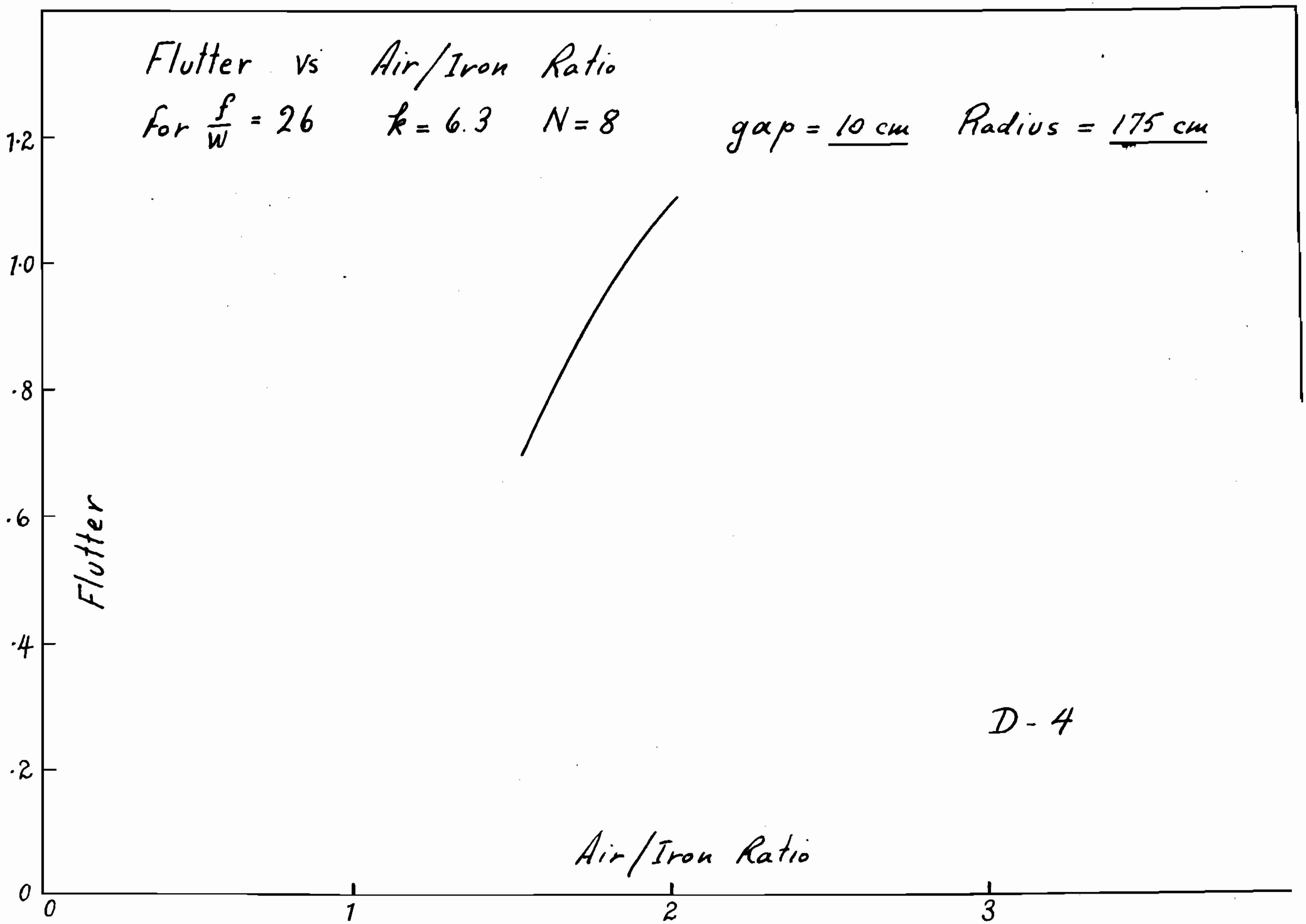
Hmax / <H> vs Air/Iron Ratio For gap/sector = .600
Hmax / <H>
Air/Iron
0
1
2
3
4
5
C-8


Flutter vs Air/Iron Ratio
For f/W = 28 k = 9.4 N = 8 Radius = 175cm gap = 10cm
1.0
.8
.6
.4
.2
0
Flutter
0
1
2
3
Air/Iron
D-1

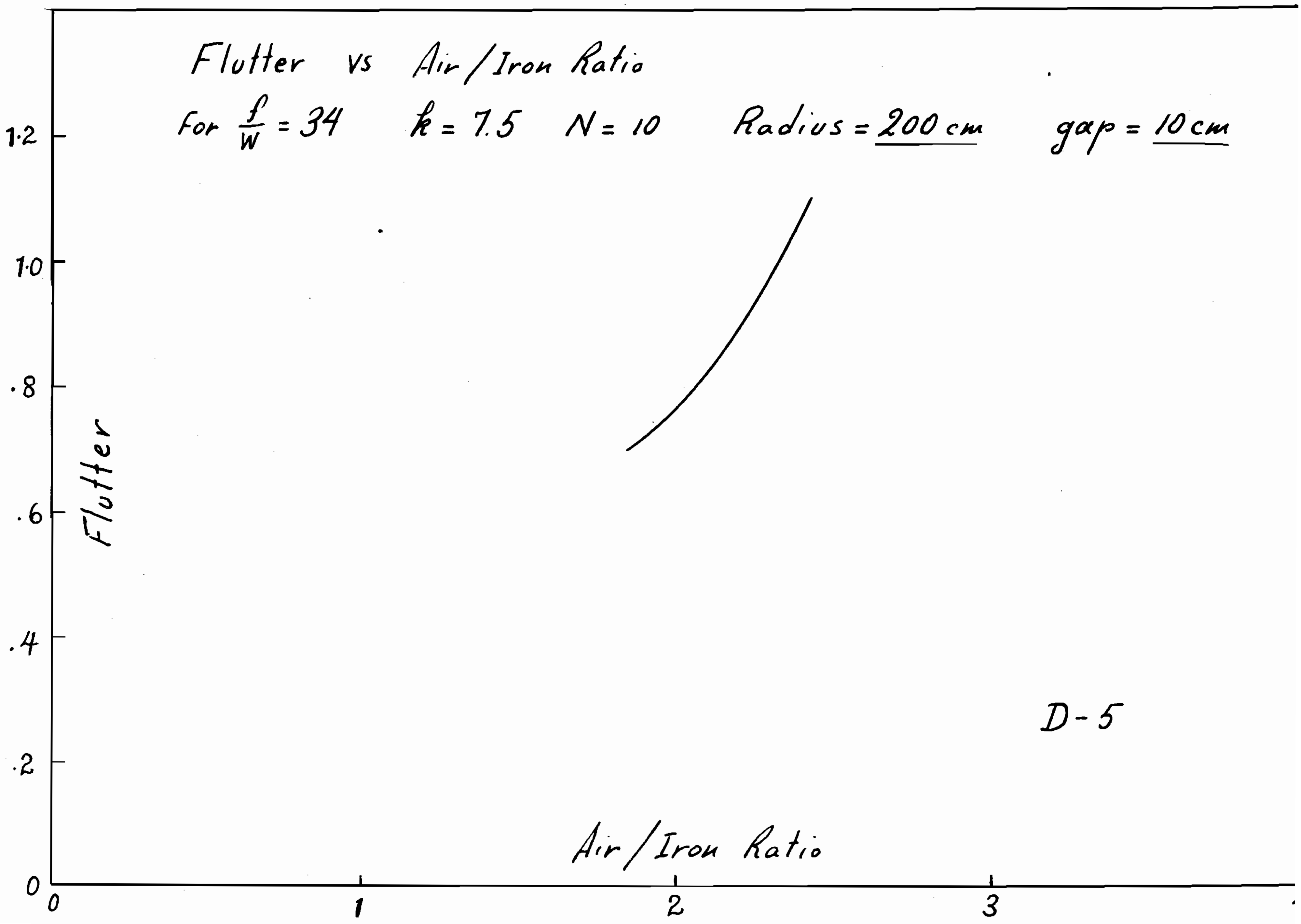

Flutter Vs Air/Iron Ratio
for $\frac{f}{w}$ = 37 k = 10.8 N = 10 Radius = 200 cm gap = 10 cm
Flutter
1·2
1·0
·8
·6
·4
·2
0
Air/Iron Ratio
0
·5
1
1·5
2
2·5
3
3·5
D-2

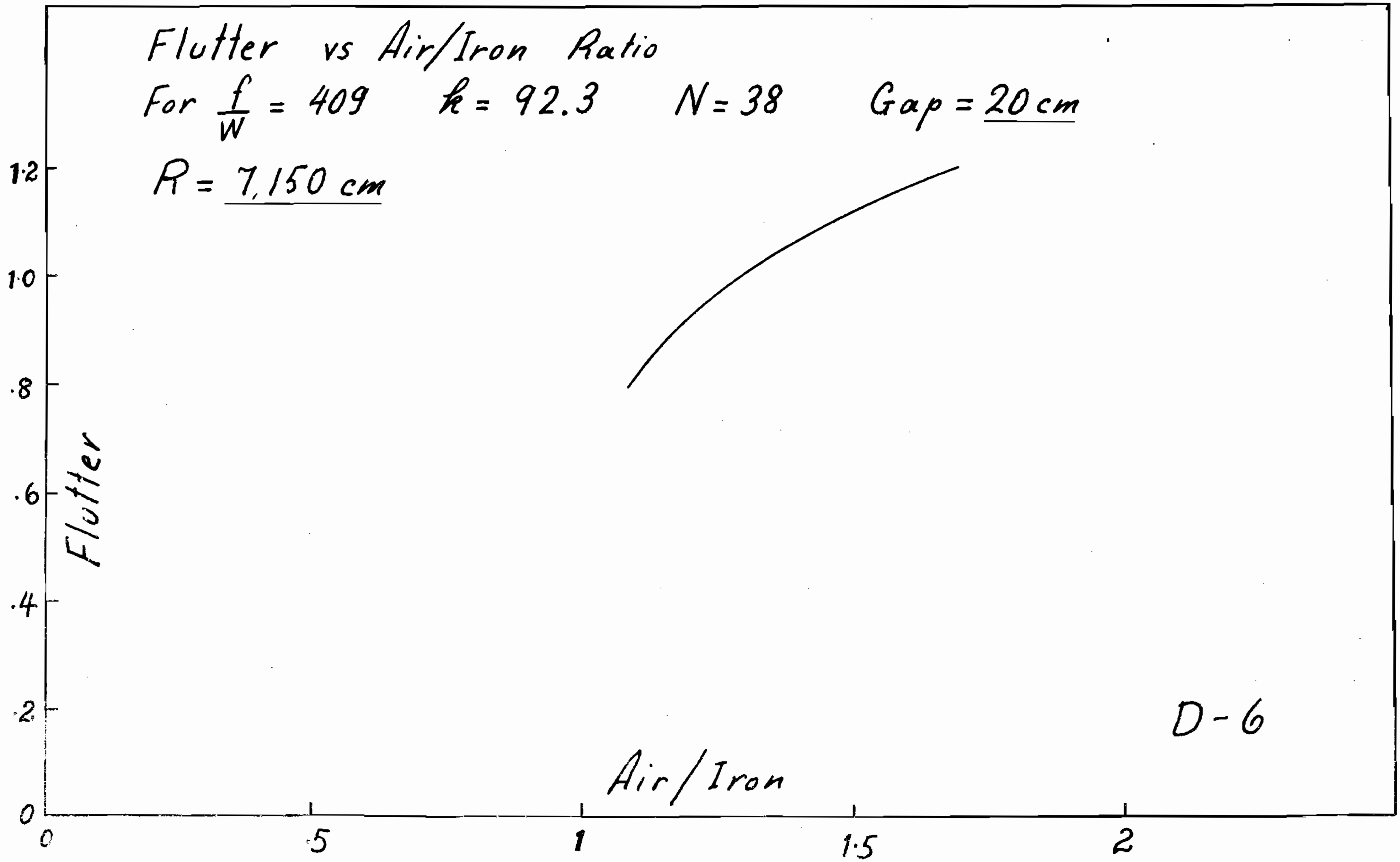


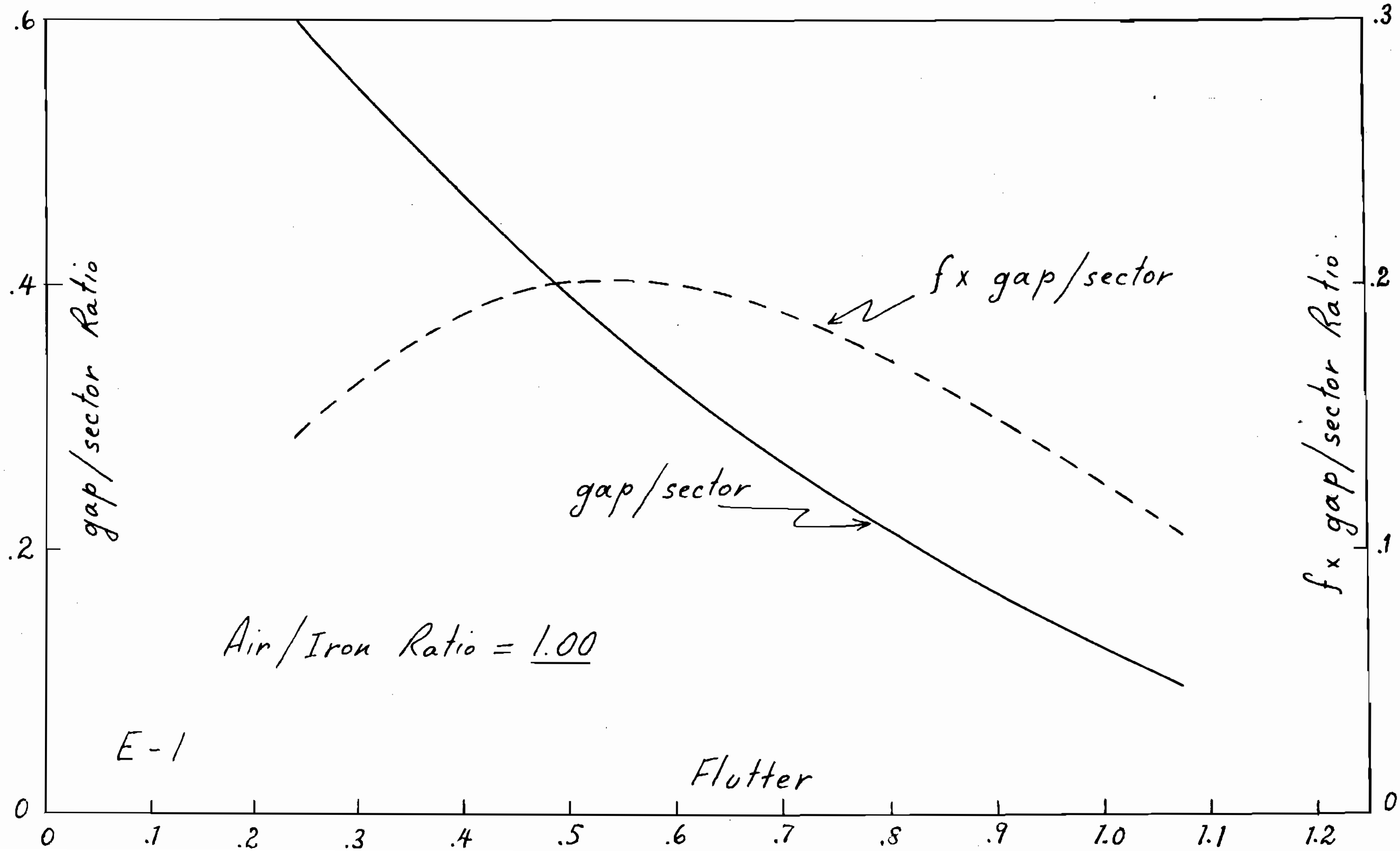
Flutter vs Air/Iron Ratio
For f/w = 26 k = 6.3 N = 8 Radius = 150 cm gap = 10 cm.
D-3
Flutter
1.0
.8
.6
.4
.2
0
Air / Iron
0
1
2
3



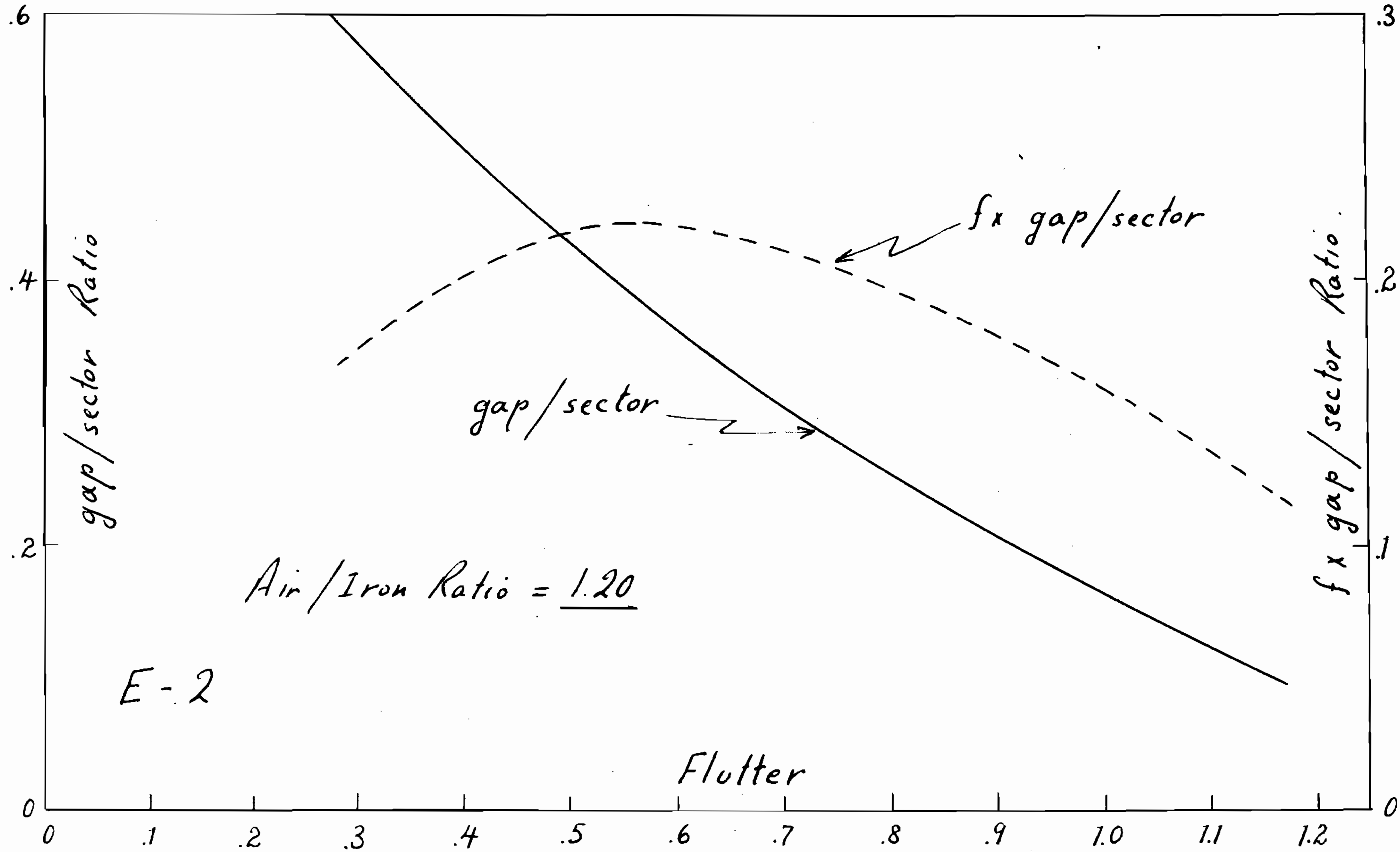
Flutter vs Air/Iron Ratio
for $\frac{f}{w}$ = 26 k = 6.3 N = 8 gap = 10 cm Radius = 175 cm
1.2
1.0
.8
.6
.4
.2
0
Flutter
0
1
2
3
Air/Iron Ratio
D-4


Flutter vs Air/Iron Ratio
For f/W = 34   k = 7.5   N = 10   Radius = 200 cm   gap = 10 cm
Flutter
Air/Iron Ratio
1.2
1.0
.8
.6
.4
.2
0
0
1
2
3
D-5

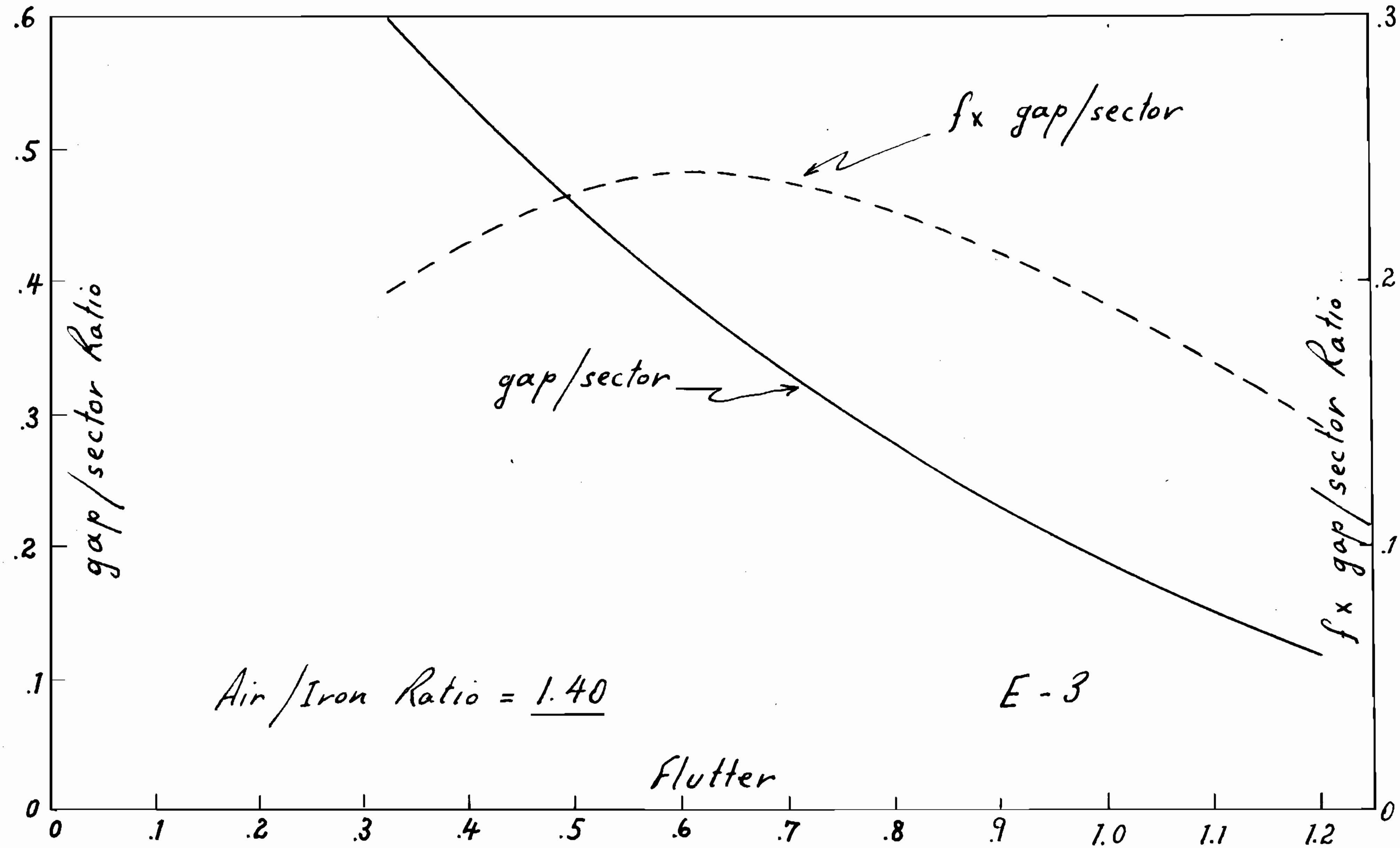

Flutter vs Air/Iron Ratio
For $\frac{f}{W}$ = 409 k = 92.3   N = 38   Gap = 20 cm
R = 7,150 cm
1·2
1·0
·8
·6
·4
·2
0
Flutter
Air/Iron
0
·5
1
1·5
2
D-6



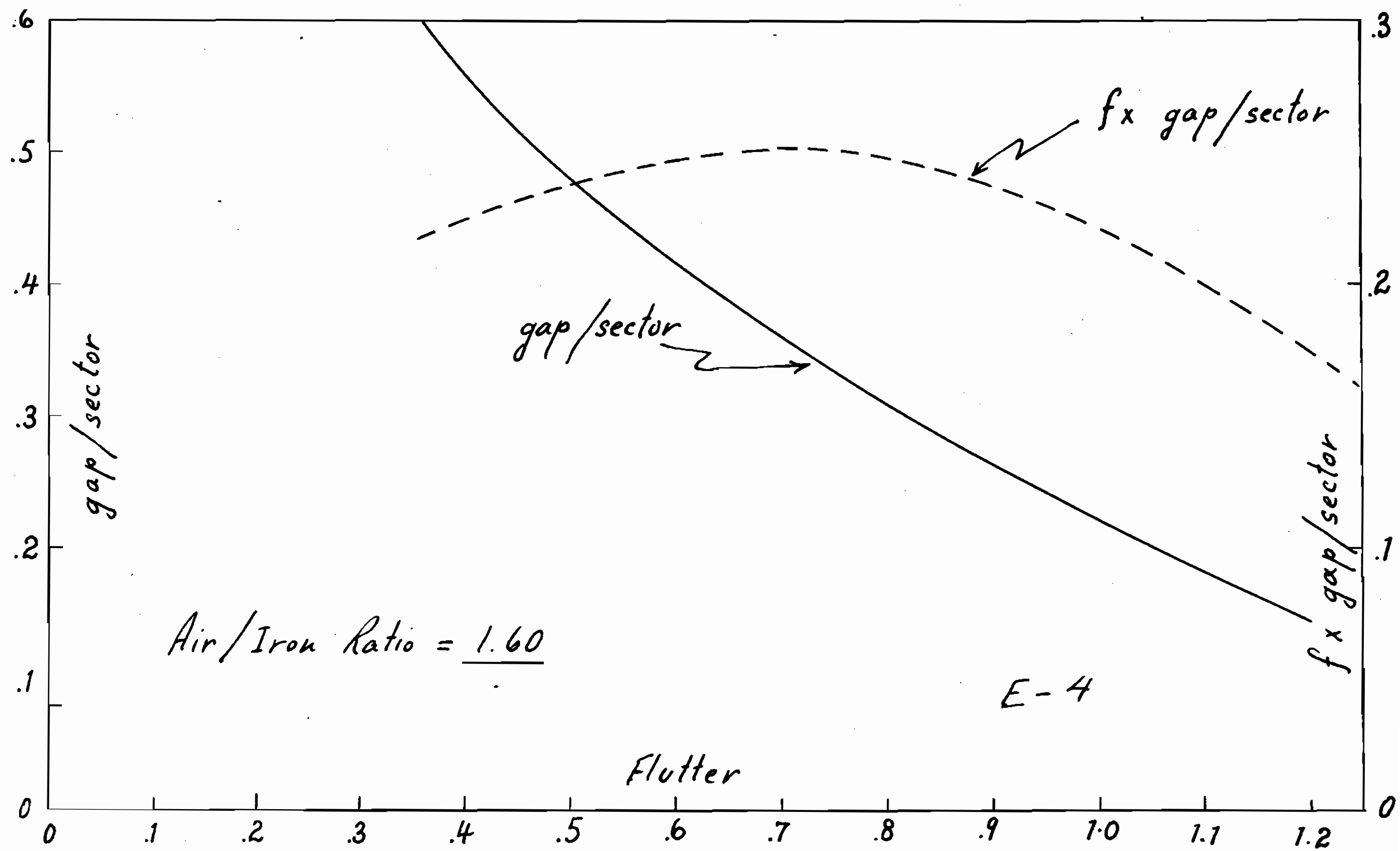
gap/sector Ratio
f x gap/sector Ratio
f x gap/sector
gap/sector
Air/Iron Ratio = 1.00
E-1
Flutter
.6
.4
.2
0
.3
.2
.1
0
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
1.1
1.2



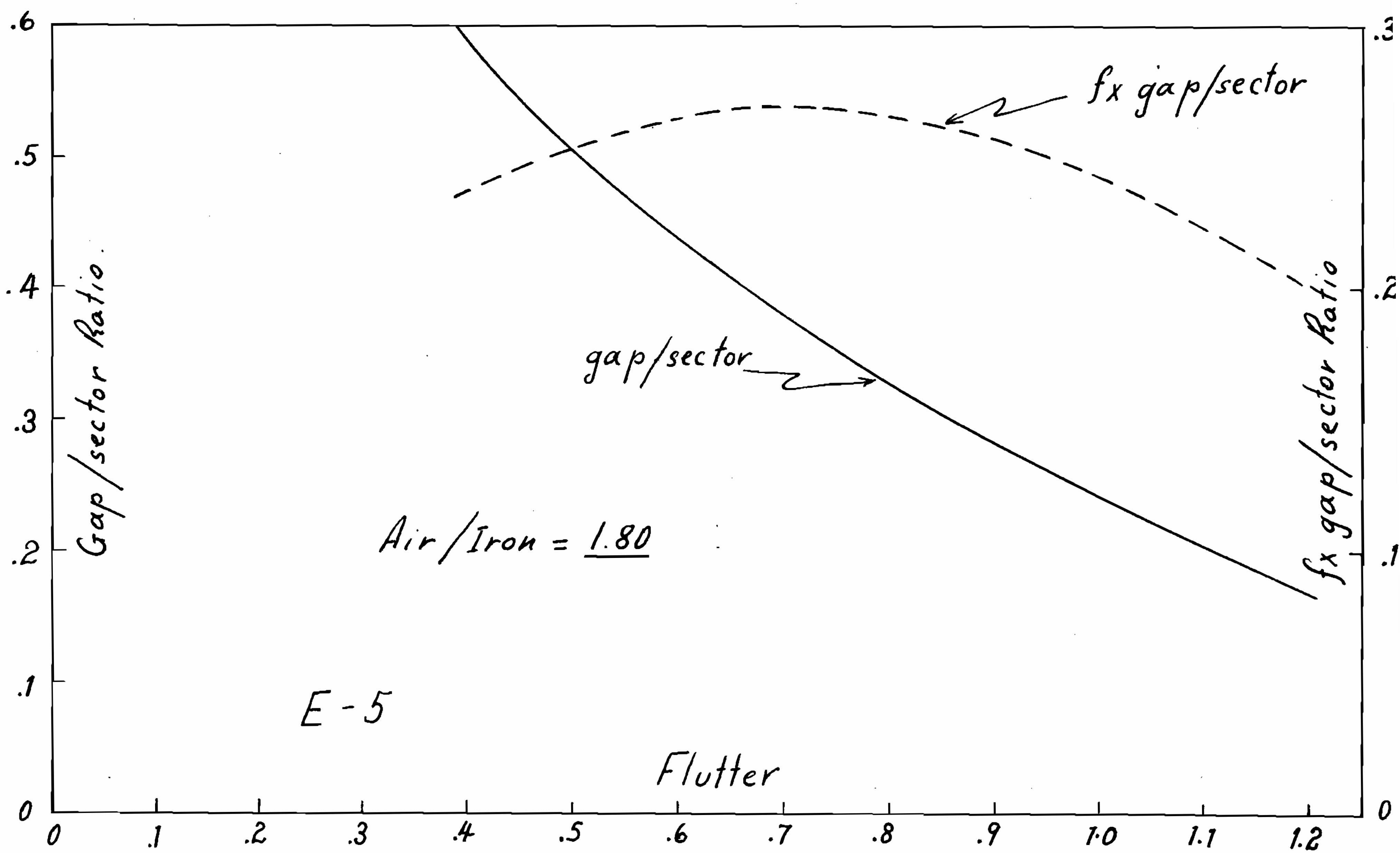
E-2
Air/Iron Ratio = 1.20
gap/sector Ratio
f x gap/sector Ratio
f x gap/sector
gap/sector
Flutter
.6
.4
.2
0
.3
.2
.1
0
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
1.1
1.2



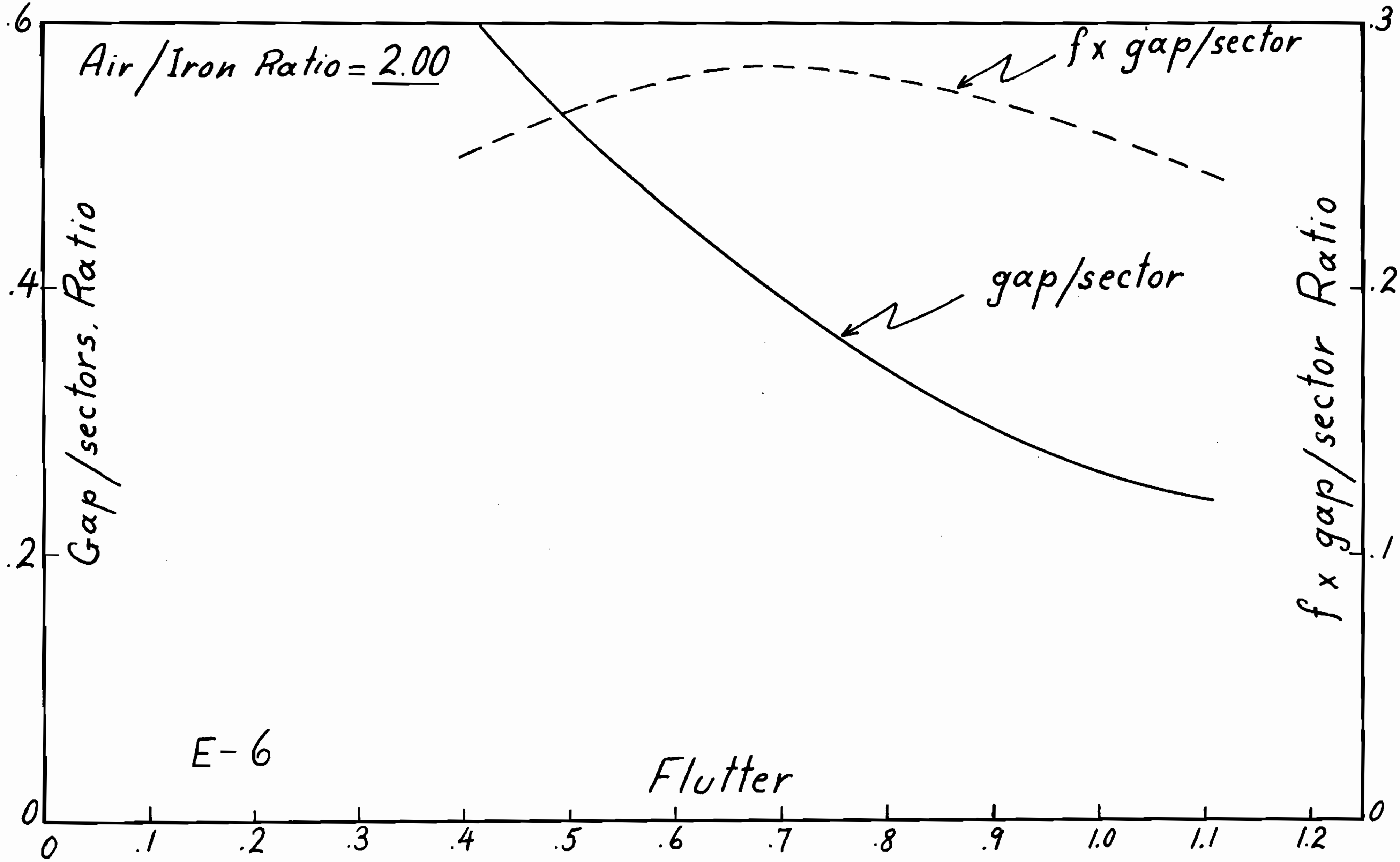
f x gap/sector
gap/sector
Air/Iron Ratio = 1.40
E - 3
Flutter
gap/sector Ratio
f x gap/sector Ratio
.6
.5
.4
.3
.2
.1
0
.3
.2
.1
0
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
1.1
1.2



.6
.5
.4
.3
.2
.1
0
gap/sector
.3
.2
.1
0
f x gap/sector
f x gap/sector
gap/sector
Air/Iron Ratio = 1.60
E-4
Flutter
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
1.1
1.2



Gap/sector Ratio
fx gap/sector Ratio
fx gap/sector
gap/sector
Air/Iron = 1.80
E-5
Flutter
.6
.5
.4
.3
.2
.1
0
.3
.2
.1
0
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
1.1
1.2



Air/Iron Ratio = 2.00
f x gap/sector
gap/sector
Gap/sectors, Ratio
f x gap/sector Ratio
Flutter
E-6
.6
.4
.2
0
.3
.2
.1
0
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
1.1
1.2