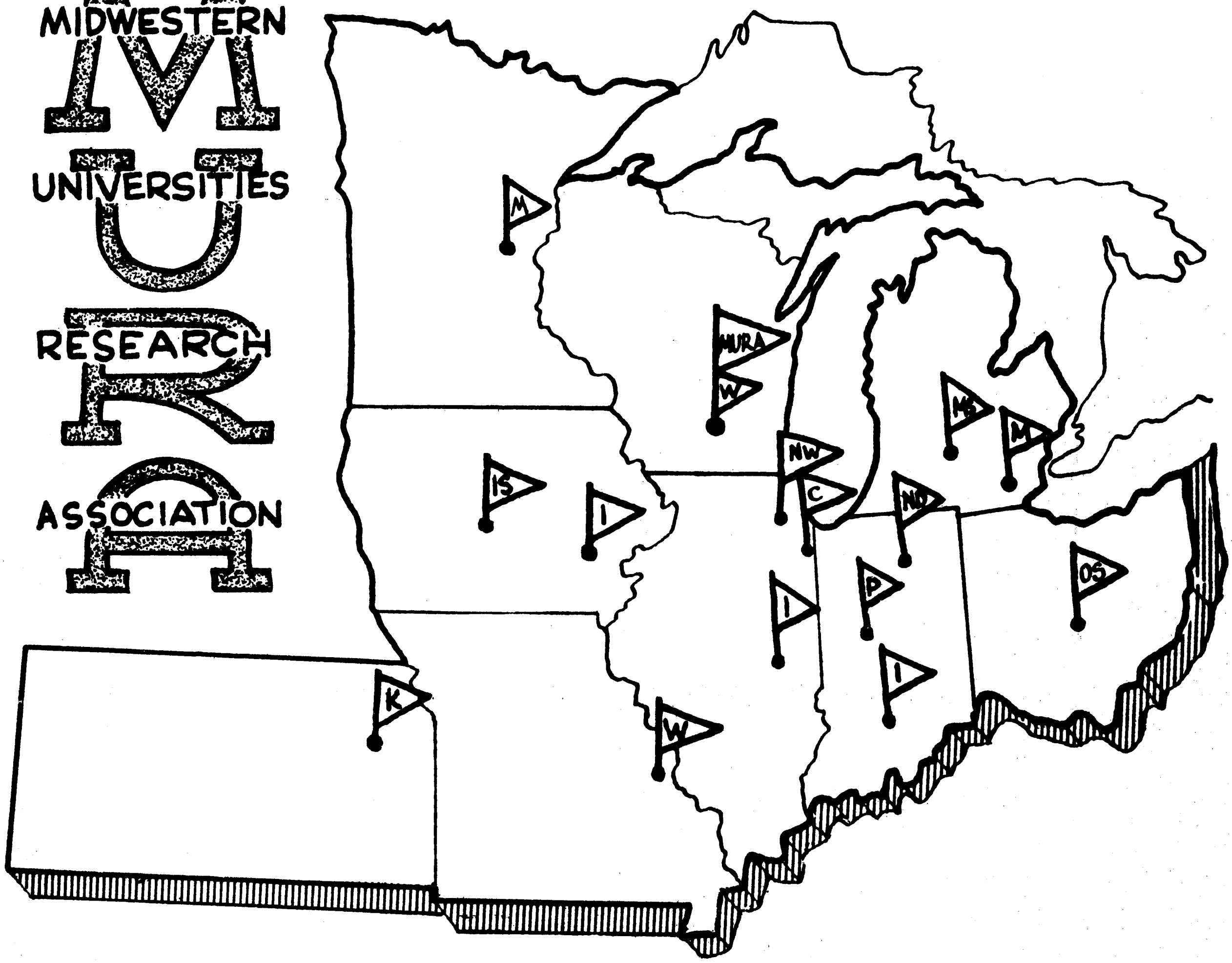
MIDWESTERN UNIVERSITIES RESEARCH ASSOCIATION



EFFECTS OF RADIAL STRAIGHT SECTIONS ON THE BETATRON OSCILLATION FREQUENCIES IN A SPIRAL SECTOR FFAG ACCELERATOR

R. A. Dory and P. L. Morton

REPORT NUMBER 514

Printed in USA. Price $0.50. Available from the

Office of Technical Services
U. S. Department of Commerce
Washington 25, D. C.

Report No. MURA-514
UC-28, Particle Accelerators
and High Voltage Machines
TID-4500 (15th Edition)

MIDWESTERN UNIVERSITIES RESEARCH ASSOCIATION*

2203 University Avenue, Madison, Wisconsin

EFFECTS OF RADIAL STRAIGHT SECTIONS ON THE BETATRON OSCILLATION FREQUENCIES IN A SPIRAL SECTOR FFAG ACCELERATOR

R. A. Dory** and Phil. L. Morton***

September 4, 1959

ABSTRACT

This report contains results obtained from the IBM-704 digital computer on the introduction of radial straight sections into spiral sector FFAG accelerators. The effect of radial straight sections on the magnetic field in the median plane is studied first. Then the variation of the betatron oscillation frequencies with energy is determined for various lengths of radial straight sections and for various combinations of the number of straight sections and spirals. These results bear out the theorem in MURA report 434, which describes a method of introducing the radial straight sections in such a way as to keep the betatron oscillation frequencies independent of energy.

*AEC Research and Development Report. Research supported by the Atomic Energy Commission, Contract No. AT(11-1)-384.

**Student from the University of Wisconsin, Madison, Wisconsin

***Student from The Ohio State University, Columbus, Ohio.

I. INTRODUCTION

An analytical treatment of radial straight sections in a spiral sector accelerator is given in MURA report 434, and the results of this treatment have stimulated us to investigate the problem with the aid of the IBM-704 digital computer. The purpose of the present report is to test the following theorem, proved in MURA-434; if the number q of radial straight sections per period of the magnetic field is greater than twice the maximum number, n_{max}, of Fourier harmonics of the magnetic field without radial straight sections, then the betatron oscillation frequencies ν_x and ν_y are independent of energy or, equivalently, radius. It is found below that in all the cases examined the theorem is valid.

The effect of radial straight sections on the magnetic field is represented by a straight section function S(θ), and the "FOROCYL"[1] program is used to compute the field (see Section II). In Section III, the variations of radial and vertical betatron oscillation frequencies with energy are investigated by means of the program "SPIRIT."[2] Several different values for the length of straight sections and also for the parameter q are used in these calculations.

II. DIGITAL FIELD COMPUTATIONS

It is assumed that the effect of radial straight sections on the field of a spiral sector FFAG accelerator may be represented by a modulation S(θ) of the spiral field

$$B_z = S(\theta) \cdot \left\{ B_z \right\} \tag{1}$$

where $\{B_z\}$ represents (with $u = 1$) the unmodulated field,[3] and

$$S(\theta) = \sum_{n=0}^{\infty} a_n \cos n q N' [\theta + \tau]$$

$$\{B_z\} = - B_o (1 + x)^k \sum_{m=0}^{\infty} \left\{ g_m \cos m u \left[N' \theta - \frac{1}{u w} \ln (1 + x)\right] + \right. \\ \left. + f_m \sin m u \left[N' \theta - \frac{1}{u w} \ln (H x)\right] \right\}$$

$$x = \frac{r - r_o}{r_o}$$

N' = number of supersectors

u = number of spiral sectors per supersector

q = number of straight sections per supersector

$1/w$ = spiraling parameter

and

τ = phase of the straight sections relative to the spirals. Notice that a change in radius, i.e., x, corresponds to a change in the phase τ, and in order to study the particle dynamics at different radii, τ can be given several values between τ_o and $\tau_o + \frac{2\pi}{u q N'}$.

To obtain an idea of the magnitude of the Fourier coefficients, a_n, of the modulation, the IBM-704 program "FOROCYL"[1] is used to solve the two-dimensional Laplace equation and find the median plane field for the magnet configurations shown in Fig. 1.

The ratio of the modified to the unmodified field is found as a function of θ, and by use of the IBM program "FORANAL"[4] this function is Fourier analyzed. A graph of such a function is shown in Fig. 2. The quantities δH

and $\delta\theta$ are measures of the relative change in the field (due to the straight sections), and of the approximate angle in which this change is significant, respectively.

This function calculated in the center of a magnet was used for $S(\theta)$. This method does not give exactly the field across the straight section as the position of the straight section relative to the magnets is changed,[5] but it is believed to be fairly accurate.

Two cases are treated: for the first, a magnet extending through one-third of a sector is used; for the second, the magnet length is doubled, while the magnet gap is decreased to keep the field flutter in the median plane constant (≈ 1).

In Table I the values of δH and $\delta\theta$, corresponding to a straight section whose width is Δ% of a spiral sector, are tabulated.

TABLE I

Short Magnet		Large Gap
Δ(%)	δH (%)	$\delta\theta$ (%)
3	1.72	50
5	4.97	53
10	20.91	67

Long Magnet		Small Gap
Δ(%)	δH (%)	$\delta\theta$ (%)
10	43 72	33

For a given Δ, the product $\delta H \cdot \delta\theta$ remains constant while the magnet length and gap width vary, $\delta\theta$ increasing and δH decreasing with increasing gap width. This is in accord with intuition.

III. DIGITAL COMPUTATIONS OF THE TUNE

Orbits are integrated numerically on the IBM-704 through enough periods to find the equilibrium orbit and to measure ν_x and ν_y, by means of the "SPIRIT"[2] program developed for this purpose. This program multiplies the scaling field by the straight section function, $S(\theta)$, to find the field, and integrates the exact equations of the two-dimensional motion by the Runge-Kutta method.

Tune calculations are made for the short magnet, large gap accelerator described in Section II. The machine parameters used are $N = 30$, $k = 53$, $1/w = 280$, $g_o = g_1 = 1$, $f_m = 0$ for all m. When higher unmodulated field harmonics are desired, g_2 and g_3 are both taken as 0.2.

The Fourier coefficients, a_n, of $S(\theta)$ which are used are found by the methods of Section II. They are tabulated in Table II, for various values of Δ. The case $\Delta = 10\ \%$ corresponds roughly to a one-meter straight section in a 10 Gev accelerator.

In Table III are tabulated the relative tune changes $\frac{\Delta\nu}{\langle\nu\rangle} = \frac{\nu_{max} - \nu_{min}}{\langle\nu\rangle}$ caused by varying the phase τ from 0 to $\frac{2\pi}{u\,q\,N'}$, for various lengths of straight sections. For this table, q, (the number of straight sections per supersector), u (the number of spirals per supersector), and n_{max} (the maximum number of scaling field harmonics) are taken to be unity. The number of supersectors, N', is 30.

TABLE II

Fourier Coefficients of the Straight Section Function

$\Delta(\%)$ =	3.33	5	10
a_0	1	1	1
a_1	0.00471	0.01391	0.06210
a_2	0.00360	0.01053	0.04547
a_3	0.00250	0.00726	0.02997
a_4	0.00157	0.00441	0.01744
a_5	0.00094	0.00260	0.00954
a_6	0.00054	0.00193	0.00483
a_7	0.00030	0.00077	0.00233
a_8	0.00016	0.00040	0.00105
a_9	0.00009	0.00021	0.00
a_{10}	0.00005	0.00010	0.00

TABLE III

Digital Computation Results on Effects of Radial Straight Section

$\Delta\%$	$\langle\nu_x\rangle$	$\langle\nu_y\rangle$	$\frac{\Delta\nu_x}{\langle\nu_x\rangle}$ (%)	$\frac{\Delta\nu_y}{\langle\nu_y\rangle}$ (%)
3.33	8.221	5.819	1.53	2.83
5	8.197	5.819	5.16	8.36
10	8.060	5.788	24.50	38.02

The results obtained by varying q and u for the case $\Delta = 10\%$ are tabulated in Table IV. In order to keep the total number of spiral sectors constant, N' is varied to keep $uN' = 30$. The spaces marked "U" are unstable because of the stop-band near $\nu = N'/2$ when N' is 15 ($u = 2$). This stop-band occurs only when q and u are related by: $mq + nu = 1$, where m, n are positive or negative integers and $|n| < n_{max}$. Thus for $q = 3$, $u = 2$, $N' = 15$, instability is encountered for all n_{max}, while for $q = 5$, $u = 2$, $N' = 16$, the motion is stable for $n_{max} = 1$ but unstable for $n_{max} > 1$.

TABLE IV

Digital Computation Results on Effects of Radial Straight Sections

q	u	N'	n_{max}	$\langle \nu_x \rangle$	$\langle \nu_y \rangle$	$\frac{\Delta \nu_x}{\langle \nu_x \rangle}$ (%)	$\frac{\Delta \nu_y}{\langle \nu_y \rangle}$ (%)
0	1	30	1	8.266	5.818	0	0
			2	8.261	6.379	0	0
			3	8.226	6.144	0	0
1	1	30	1	8.060	5.788	24.50	38.02
			2	8.060	6.283	29.89	33.20
2	1	30	1	8.204	5.832	12.24	4.04
			2	8.231	6.399	15.69	13.14
			3	8.284	6.109	20.45	13.15
3	1	30	1	8.220	5.821	3.19	0.83
			2	8.256	6.383	7.90	1.64
			3	8.192	6.144	9.87	13.38
3	2	15	1	U			
			2	U			
			3	U			
4	1	30	1	8.223	5.820	0.58	0.13
			2	8.258	6.380	2.66	0.33
5	2	15	1	8.044	5.830	1.86	0.26
			2	U			
			3	U			
5	3	10	1	8.069	5.617	0.43	4.45
			2	8.027	6.294	0.94	4.45
7	2	15	3	U			

To find the relative tune changes for some of the q and u combinations which are unstable, the values of 1/w and k are changed to 210 and 30 (other parameters remain unchanged) so that ν is shifted away from the value N'/2. Table V lists the results for these combinations.

TABLE V

Digital Computation Results on Effects of Radial Straight Sections

q	u	N'	n_{max}	$\langle \nu_x \rangle$	$\langle \nu_y \rangle$	$\frac{\Delta \nu_x}{\langle \nu_x \rangle}$ (%)	$\frac{\Delta \nu_y}{\langle \nu_y \rangle}$ (%)
0	1	30	1	5.891	4.391	0	0
1	1	30	1	5.703	4.338	27.02	42.46
2	1	30	1	5.886	4.399	10.40	2.86
3	2	15	1	5.916	4.397	0.92	1.06

These results appear to bear out the theorem that the tune remains constant with varying energy (i.e., τ) if $q > 2\, n_{max}$.[6]

Apparently this condition on q may be relaxed, in the case of vertical oscillations, and replaced by the weaker condition that q be greater than n_{max} (rather than $2\, n_{max}$), in order to keep the tune constant (as a function of energy). This effect is presumably due to detailed cancellation of terms in ν_y, but is not yet understood.

To demonstrate that the effects outlined above are independent of the length, Δ, of the individual straight sections, a few runs are made, as before, with Δ 5 %. In these runs the theorem is again confirmed.

ACKNOWLEDGEMENTS

The authors would like to thank most sincerely F. T. Cole for his inspiration and constructive suggestions, G. Parzen for most informative discussions and Mrs. E. Z. Chapman who wrote the "SPIRIT" program for the MURA IBM-704. It is also a pleasure to thank R. O'Connell for doing some of the computational work, as well as M. R. Storm, P. R. Marty and Mrs. A. C. Frazier for their work with the IBM-704 computer.

REFERENCES

1. J. N. Snyder and L. J. Laslett, FOROCYL (IBM Program 13), MURA-221 (February-March, 1957).
2. E. Z. Chapman, SPIRIT (IBM Program 245), MURA-527 (September, 1959),
3. Symon, Kerst, Jones, Laslett and Terwilliger, Phys. Rev. 103, 1837-1859 (1956).
4. J. N. Snyder, FORANAL (IBM Program 52), MURA-228 (February-March, 1957).
5. A three-dimensional version called TRIXIAC currently being prepared by J. C. Anderson and R. S. Christian will allow this provision.
6. This theorem is proved in P. L. Morton, MURA Report 434, (October 21, 1958).

Fig. 1a

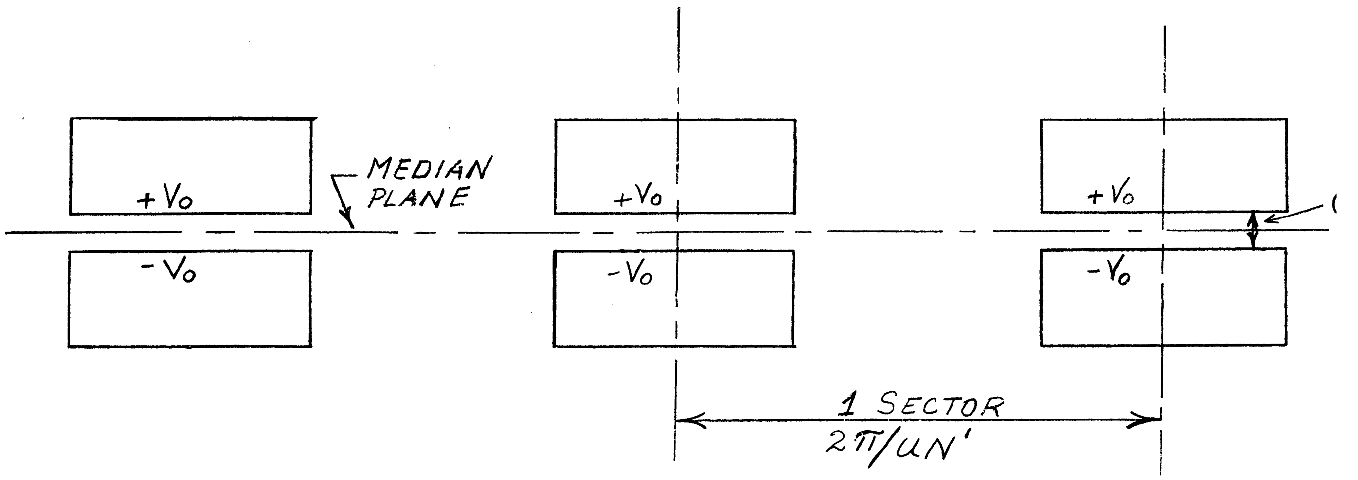


Fig. 1b

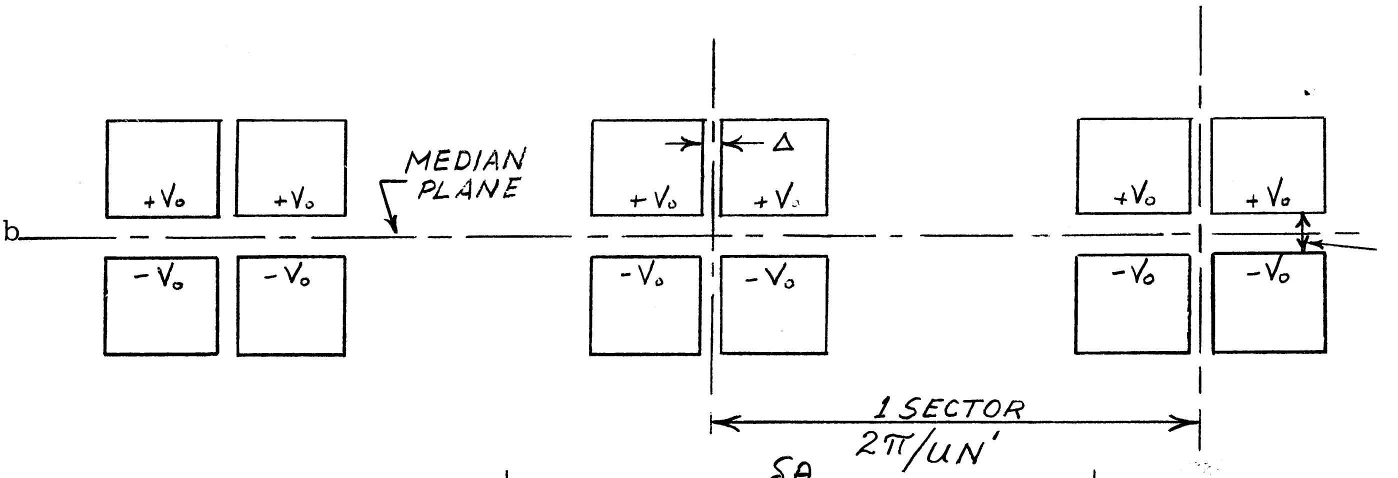


Fig. 2