



CM-P00065156

SPS/AC/DB/JG/EEK

24th June, 1980

IMPROVEMENT REPORT NO. 181

MEASUREMENTS OF THE SPS COUPLING IMPEDANCE

D. Boussard, J. Gareyte

I. INTRODUCTION

The coupling impedance of the beam with its surroundings is an essential ingredient in the calculation of the beam dynamics.

The recent  $p\bar{p}$  workshop has brought to light the fact that the SPS coupling impedances (longitudinal and transverse) were not known with a sufficient precision. As a matter of fact, no specific measurements had been done recently, mainly due to other pressing problems demanding the attention of all machine physicists. A longitudinal inductive coupling impedance of  $\frac{Z_{11}}{n} = 30\Omega$ , close to the one measured at the ISR <sup>1)</sup>, was found to explain reasonably well the observed thresholds for the longitudinal coherent modes of the dense single bunches <sup>2)</sup>. The model proposed at the workshop was that of a  $Q = 1$  resonator presenting a resistance of  $.9M\Omega$  at its resonant frequency of 1.3 GHz. The impedance of such a resonator at low frequencies is inductive and amounts to  $\frac{Z''}{n} = 30\Omega$  (see fig. 2).

The transverse coupling impedance  $Z_{\perp}$  was deduced from the longitudinal one by the formula:

$$Z_{\perp}(\omega) = \frac{2c}{b^2\omega_0} \frac{Z''}{n} \quad (1)$$

where  $c$  is the velocity of light,  $b$  the mean chamber half-height ( $b = 23$  mm in the SPS) and  $\omega_0$  the angular revolution frequency.

In the last few months it became possible to perform some systematic measurements, both transverse and longitudinal, which are reported here.

The real part of the transverse coupling impedance is deduced from growth-rate measurements of the head-tail instability of a dense single bunch on a magnetic flat-top at 270 GeV.

The longitudinal coupling impedance is calculated from growth-rate measurements of the microwave instability afflicting a dense bunch just after its injection in the machine at 15.8 GeV/c. Another estimate is given by the observed threshold for instability at this energy.

The low-frequency inductive impedance is calculated from the intensity dependent frequency shift of the longitudinal quadrupole mode, obtained by beam-transfer-function techniques.

## II. TRANSVERSE IMPEDANCE

The growth rate of head-tail mode  $m$  for a chromaticity  $\xi$  is given by

$$\frac{1}{\tau} = \frac{c^2}{2Q\omega_0} \frac{I}{E/e} \frac{1}{2\pi R} \frac{1}{m+1} \frac{1}{B} \frac{\sum R_e(Z_{\perp}(\omega)) h_m(\omega - \omega_{\xi})}{\sum h_m(\omega - \omega_{\xi})} \quad (2)$$

$B$  is the bunching factor

$$\omega_{\xi} = \frac{Q\xi\omega_0}{\eta} \text{ is the chromatic frequency}$$

For an impedance varying slowly with frequency like the wide band  $Q = 1$  resonator model replacing the sum on the r.h.s. by  $Z_{\perp}(\omega_{\xi})$  gives a good approximation for long bunches (narrow spectrum  $h_m(\omega)$ ). However, this approximation is not valid for the SPS (short bunches).

Growth rates of bunches containing  $2 \cdot 10^{10}$  particles and 1.5 ns long were measured on a 270 GeV flat top for varying vertical negative chromaticity (the horizontal chromaticity was kept at + . 2). The results are shown on fig. 1, together with the theoretical predictions obtained from equation (2) for a  $Q = 1$  resonator having a  $\frac{Z_{\perp}}{n} = 4.3 \Omega$  at its resonant frequency of 1.3 GHz (for mode  $m = 0$ ). This is comparable to the value found years ago in the CPS by the same method, namely  $\frac{Z_{\perp}}{n} = 6.7 \Omega$  at .8 GHz <sup>3)</sup>.

An interesting fact worth mentioning is that we were unable to detect any instability for a chromaticity of  $-1.37$ , whereas it appeared in full beauty at lower values of  $|\xi|$ . An interpretation in terms of the model used is easy to find. The real frequency shift of mode 0, due to the interaction of the bunch with the imaginary part of the coupling impedance, is maximum for zero chromaticity but falls off at high values of  $|\xi|$  (see fig. 2 where, in addition to the real and imaginary parts of  $\frac{Z}{n}$  for the resonator model, the bunch spectrum  $h_0(\omega)$  for  $|\xi| = 1.37$  is plotted.)

In the conditions of the experiment described above, and with a  $\frac{Z_{11}}{n} = 4.3\Omega$ , the model predicts a coherent Q shift

$$\Delta Q = - 5 \cdot 10^{-4}$$

for low values of  $\xi$ . It is readily seen on fig. 2 that for  $|\xi| = 1.37$ , this coherent Q shift is at least halved. The experimental findings then suggest that this value is no longer sufficient to overcome Landau damping by non linearities. A rough estimate of the non-linearities present in the machine has been made and the results are consistent with the presence of a Q spread of  $2 \cdot 10^{-4}$  in the beam.

### III. LONGITUDINAL IMPEDANCE

Longitudinal instabilities of debunched beams are described by well-established theories. So one could think that a dense debunched beam would be an ideal probe for the longitudinal impedance in an accelerator. However, we are interested in bunched beams for  $p\bar{p}$ , and an investigation with bunches is likely to give a better and more different feeling of the phenomena. On the other hand, with bunched beams one is on more shaky ground to interpret data in terms of a coupling impedance, theories not being so precisely understood.

#### 1. Measurements of microwave instabilities

These measurements were done to answer the following question :

what will be the maximum possible density of the p and  $\bar{p}$  bunches injected in the SPS at 26 GeV? The current  $p\bar{p}$  parameter list <sup>4)</sup> assumes a  $\frac{Z}{n}$  of

the order of  $25\Omega$ . A lower value would allow injection of denser beams which could be an advantage in some cases.

By injecting at the maximum energy allowed at present by the transfer line, namely 15.8 GeV/c, it turns out that one can probe coupling impedances ranging from a few ohms to about 50 ohms, with the spectrum of bunch densities the CPS is able to provide (at 10 GeV/c the beam is too stable in this respect).

So we injected without RF, a single bunch of  $10^{11}$  protons at 15.8 GeV, with different longitudinal emittances and different lengths, whose properties are summarized in Table 1.

TABLE I

No	Emittance mrad	Length ns	$\frac{\Delta p}{p}$ $10^{-3}$	$\left  \frac{Z}{n} \right $ threshold $\Omega$
1	400	6	2	52
2	200	5	1.2	16
3	200	10	.6	8
4	200	25	.24	3.2

The threshold impedances are calculated using the Keil-Schnell criterion applied to local  $\frac{\Delta p}{p}$  and local intensity :

$$\left| \frac{Z}{n} \right| = \frac{E_0}{e} \pi \gamma \frac{1}{\hat{I}} \left( \frac{\Delta p}{p} \right)_{FWHM}^2 \quad (3)$$

Assuming parabolic bunches we take :  $\hat{I} = \frac{1.5 I_0}{B}$

and  $\left. \frac{\Delta p}{p} \right)_{FWHM} = \sqrt{2} \frac{\Delta \hat{p}}{p}$ , the sign  $\hat{\quad}$  denoting maximum values.

The instability was monitored by looking at the high frequency signal from the wide band resistive pick up. The signal was sent through micro-

wave band-pass filters, amplified and detected by a diode whose response at different frequencies had been carefully measured. Fixed tuned wide-band filters (200 to 500 MHz width) were used, as well as a continuously tunable YIG filter (20 MHz width).

Bunch No. 1 was stable at injection : no signal growth could be seen from 1 to 3 GHz. Instead, the signal present at injection decreased rapidly, showing the debunching process.

Nevertheless, after a time ranging from 4 to 10 ms, a fast signal growth could be observed. As usual, we interpret this in the following way: during debunching, the bunch lengthens so that both  $\hat{\Delta p}$  and  $\hat{I}$  decrease linearly. Equation 3 then shows that the threshold impedance in these conditions also decreases linearly, and at some time the beam becomes unstable. So the time  $t$  after which fast signals appear can be used to estimate the machine impedance. In the case of bunch No. 1, taking  $t = 4$  ms, we find  $\frac{Z}{n} = 12.5\Omega$ .

Bunch No. 2 appeared to be just at the limit of instability at injection: the signals grew right from injection for  $10^{11}$  particles injected, but a small decrease of intensity was sufficient to render the beam stable for the first ms. This suggests that the effective coupling impedance in the GHz range for a bunch of 5 ns length is around  $16\Omega$ .

Bunches Nos. 3 and 4 were clearly unstable. Fig. 3 shows the exponential growth of the signal received at 1.5 GHz. Their growth rates were measured for different frequencies using both the wide-band filters and the YIG filter. The unfolding of the coupling impedance from growth rates measurements is much more tricky than from the Keil-Schnell criterion. The formula giving the growth rate induced by a coupling impedance  $\frac{Z}{n}$  at a certain mode number  $n$ , is, above transition<sup>5)</sup> :

$$\frac{1}{\tau} = \text{Im} \left[ n \left( \frac{|\eta| \omega_0^2 I_0}{2 \pi E/e} \right)^{\frac{1}{2}} \left( -i \frac{Z}{n} \right)^{\frac{1}{2}} \right] \quad (4)$$

This is valid for a coasting beam of intensity  $I_0$ , far from the stability limit. What intensity should be considered for a bunched beam? Lacking

more elaborate theory, we will make the same hypothesis as for the local Keil-Schnell criterion, and replace  $I_0$  by  $\hat{I}$ , the peak intensity. If in addition we consider only real  $\frac{Z}{n}$ , (4) boils down to

$$\frac{Z}{n} = \frac{4\pi E/e}{n^2 \tau^2 |\eta| \omega_0^2 \hat{I}} \quad (5)$$

The  $\frac{1}{\tau^2}$  dependance makes this method very sensitive to measurement errors.

The results obtained with 25 ns long bunches are displayed on fig.4. Measurements with wide-band filters are represented by horizontal segments showing the width of the filters. Circles represent measurements done with the YIG filter. The dotted line shows the theoretical response of a  $Q = 1$  resonator peaked at 1.3 GHz, with  $\frac{Z}{n} = 16\Omega$ .

It seems clear from these measurements that the coupling impedance is made up of a number of sharp spikes at different frequencies, some of them being detected when using the 20 MHz bandwidth YIG filter. Data obtained with wide band filters is much smoother, and not inconsistent with a  $\frac{Z}{n}$  of  $16\Omega$  in the GHz range. Notice that 5ns long bunches have mode spectra of 400 MHz width, comparable to the width of the filters used.

Three main spikes appear at respectively 630 MHz, 930 MHz, 1240 MHz. The first corresponds to the well-known second mode of the RF cavities. Have the other two something to do with the higher modes? Clearly it would have been interesting to scan the frequency range in more detail.

The cluster of RF cavity modes centered around 628 MHz can be approximated by a single resonator with a central  $\frac{Z}{n}$  of  $60\Omega$ , and a  $Q$  of  $5 \cdot 10^2$ . It should not be efficient in driving instabilities in a bunch 25 ns long, because the bunch is much shorter than the decay time of the wake-field (in other words, a small part of the bunch mode spectral lines are excited by the resonator). Using Sacherer's arguments <sup>(6)</sup> we anticipate an effective impedance ten times smaller than the peak

impedance, that is  $6\Omega$ . We measure experimentally  $37\Omega$ , but in a bandwidth of 20 MHz, over which other resonators in the ring probably contribute. Measurements made with 10 ns long bunches give results consistently smaller than with 25ns bunches. The explanation could be that they are not far enough from the stability limit, and growth rates are reduced by Landau-damping .

## 2. Quadrupole mode transfer function measurements

The spectrum of a single bunch ( $\ell \approx c \times 2\text{ns}$ ) is essentially located below the equivalent resonator central frequency (1.3 GHz) where the wall impedance is mainly inductive. The effect of the small inductance is to shift downwards the incoherent frequency of the centre particle  $f_s$ . The magnitude of the frequency shift is given by <sup>(1)</sup>.

$$\frac{f_{s_0} - f_s}{f_{s_0}} = \frac{\Delta f_s}{f_{s_0}} = \frac{3 I}{2\pi^2 h V_0} \left( \frac{2\pi R}{\ell} \right)^3 \frac{Z}{n}$$

where  $f_{s_0}$  is the unperturbed frequency,  $I$  is the DC beam current,  $h$  the harmonic number,  $V_0$  the RF voltage (stationary buckets),  $R$  the machine radius and  $\frac{Z}{n}$  the inductive part of the wall impedance. By measuring the synchrotron frequency shift, one can estimate the average value of  $\frac{Z}{n}$  over the bunch spectrum frequencies, <sup>(1,7)</sup>.

The quadrupole mode transfer function of the single bunch can be easily measured by modulating the RF voltage amplitude and detecting the quadrupole oscillation of the bunch via the wide band electrode (peak detected). The dipole mode is not easy to measure because it is strongly damped by the phase loop.

- a) Low intensity case. At low intensity and with long bunch (this is typically the situation after the beam has been excited for some time) the beam is strongly Landau-damped and shows a very broad amplitude response. However, the phase response exhibits a singularity at the centre incoherent frequency (fig.5a). In this case the incoherent centre frequency  $f_s$  is easily measured and using (6) one can directly calculate  $\frac{Z}{n}$ .

- b) High intensity case. At the beginning of a coast the bunch is short and intense and Landau damping is lost. The transfer function exhibits a pole at the so-called collective (or coherent frequency)  $f_c$  which is well observed on the amplitude response (Fig. 5b). The frequency shift of the coherent frequency is smaller than that of  $f_s$ . In the limit of a zero frequency spread ( $S$ ), one finds, for the quadrupole mode (1)

$$\Delta f_c = \frac{1}{4} \Delta f_s$$

A more elaborate calculation is given in ref. (8) from which we can plot the following graph (Fig. 6).  $\delta f$  is the frequency shift of either the incoherent center frequency ( $\delta f/s < 1.25$ ) or the coherent frequency ( $\delta f/S > 1.25$ ) for the quadrupole mode of oscillation of a parabolic bunch. These considerations are developed in more detail in reference 9.

- c) Experimental results. At the end of a 270 GeV/c coast we are in case a) (long bunch, low intensity). Unfortunately the bunch is no longer parabolic (long tails) and it is difficult to estimate properly the equivalent bunch length. The results span from  $\frac{Z}{n} \approx 10\Omega$  to  $\frac{Z}{n} \approx 18\Omega$  depending on the assumed bunch length.

To evaluate the impedance by using measurements at the beginning and at the end of a coast we must fit the experimental data to the curve in Fig. 6. The precision is better at the beginning of the coast because the bunch length is well defined ( $\tau = 1.5$  ns, parabolic bunch). These measurements give  $\frac{Z}{n} \approx 10\Omega$ .

Remark: The assumption we have made that the bunch mode spectrum lies mainly in the region where the reactive part of  $\frac{Z}{n}$  is constant is not so valid for such short bunches. A glance at fig. 2 shows that a reduced effect has to be expected, which means that the result  $\frac{Z}{n} = 10\Omega$  might be underestimated.



#### IV. CONCLUSIONS

The effective coupling impedance for single bunches in the SPS has been measured by different means. Longitudinal impedance measurements indicate a  $\frac{Z_{||}}{n}$  of 10 to 16  $\Omega$  whereas transverse measurements give  $Z_{\perp} = 18M\Omega \text{ m}^{-1}$ .

The longitudinal impedance one would infer from the transverse measurements using the usual formula (1) is only 4.3  $\Omega$ , consistently smaller than the longitudinal value. A similar discrepancy is found in the CPS<sup>3,9</sup>. An explanation could be that a non-negligible part of the resonating objects (for instance RF cavities in the SPS) are placed where the vacuum chamber aperture is large, and so contribute little to the transverse impedance.

#### Acknowledgements

These measurements were performed during storage experiments in which the whole SPS machine physics team contributed. T. Linnecar's expertise in microwave hardware was essential, as well as E. Brouzet and J.P.Delahaye's ability to produce whatever CPS bunches were needed.

#### References

- 1) S.Hansen et al. IEEE transactions NS - 22, p.1381 (1975).
- 2) D. Boussard et al. IEE transactions on Nuclear Science NS 26, June 79 p.3484.
- 3) J. Gareyte PS/DL/Note 76-10.
- 4) SPS  $p\bar{p}$  study team. CERN SPS/AC/79-18.
- 5) V.K. Neil, A.M. Sessler, Rev. Scient. Instr. 36, 4, 429 (1965)
- 6) F. Sacherer IEEE transactions on Nuclear Science NS-24 p.1393, June 77.
- 7) J. Gareyte. Landau damping of the longitudinal quadrupole mode in SPEAR II. SPEAR 207, August 77.
- 8) G. Besnier. Thesis Université de Rennes (France).
- 9) E. Boussard et al. PS OP/LR Note 80-16.

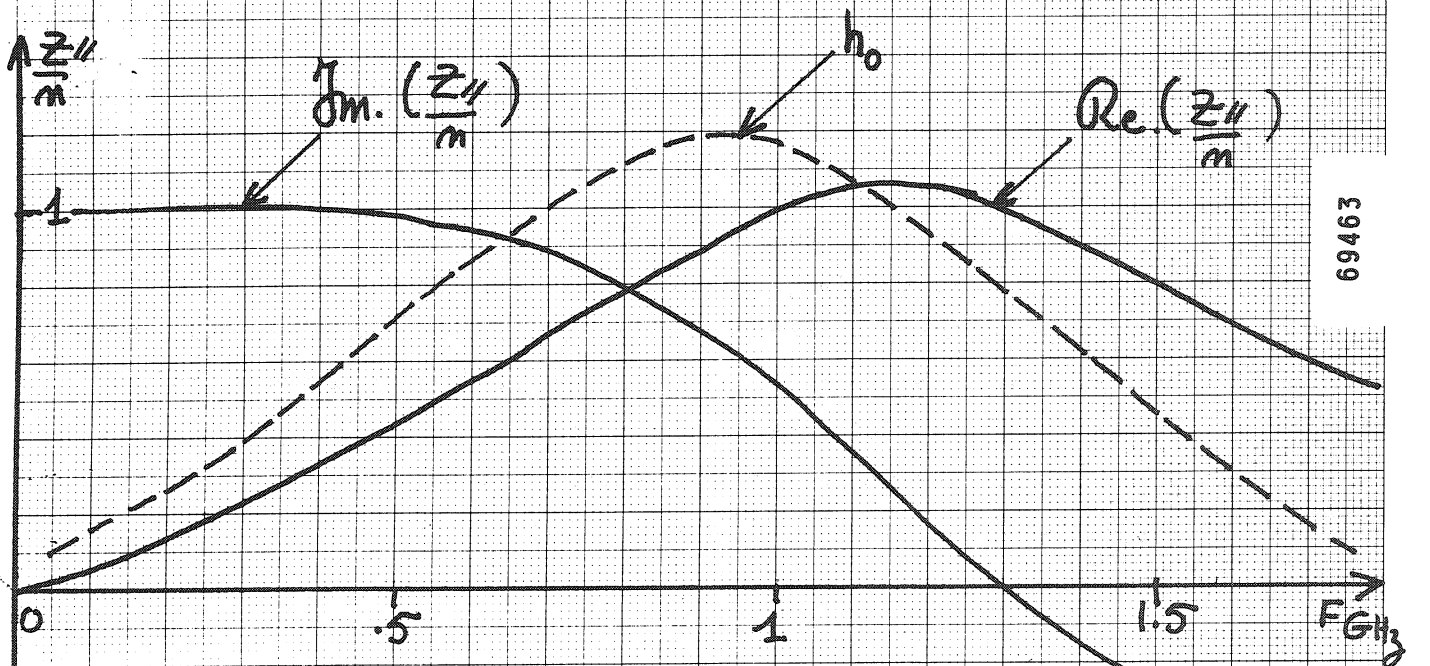
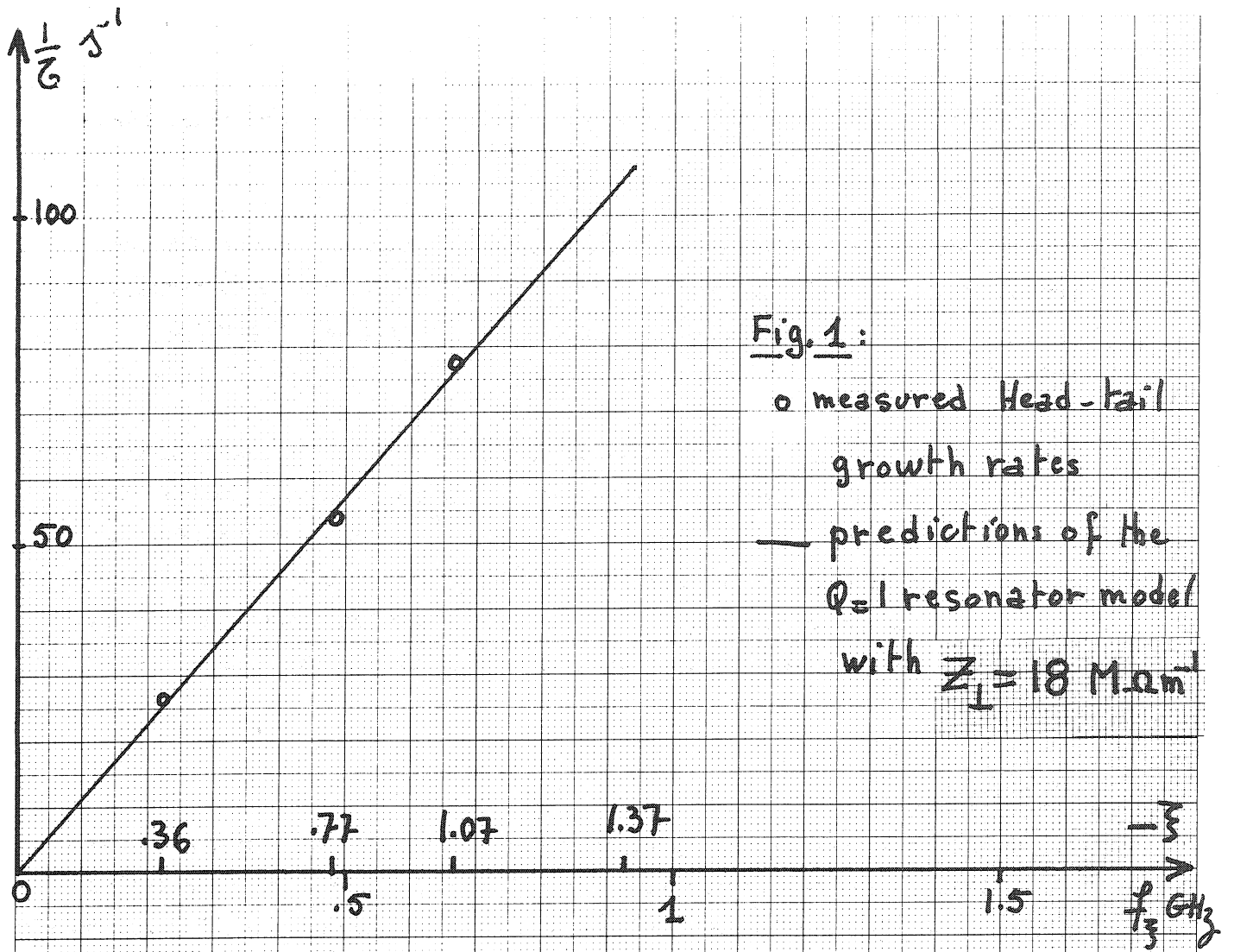


Fig. 2:  $Q=1$  resonator model for the coupling impedance

dashed curve: spectrum of Head-tail mode 0 for a bunch of length 1.5 ns and  $|S|=1.37$  at 2706 eV/c

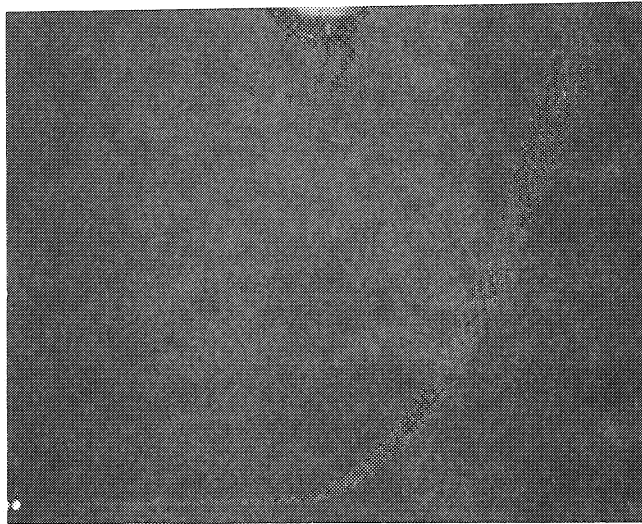


Fig. 3 Microwave instability at 15.8 GeV/c

trig.: injection

.2 ms/div; .1 V/div

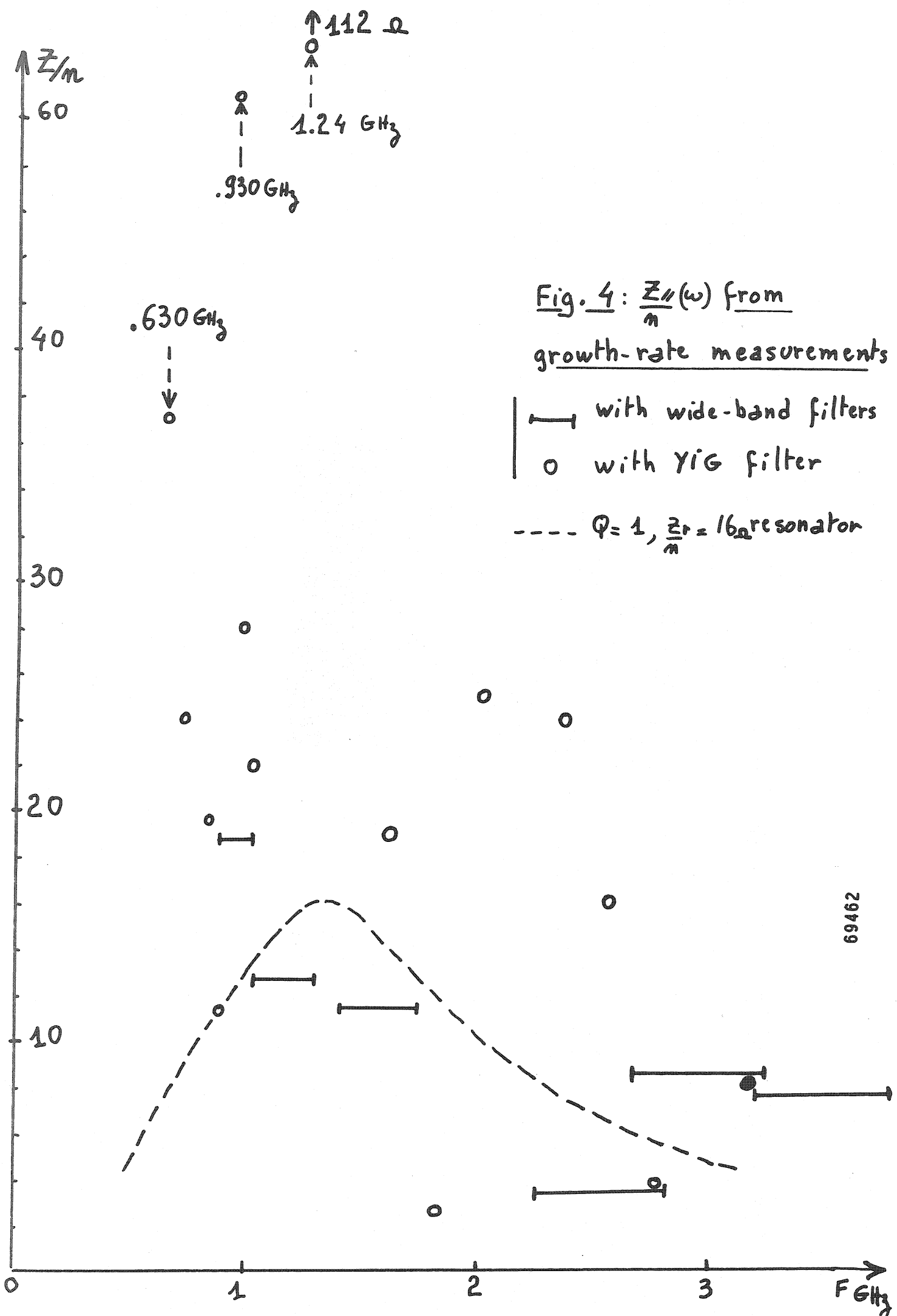
filter [1.4-1.73 GHz]

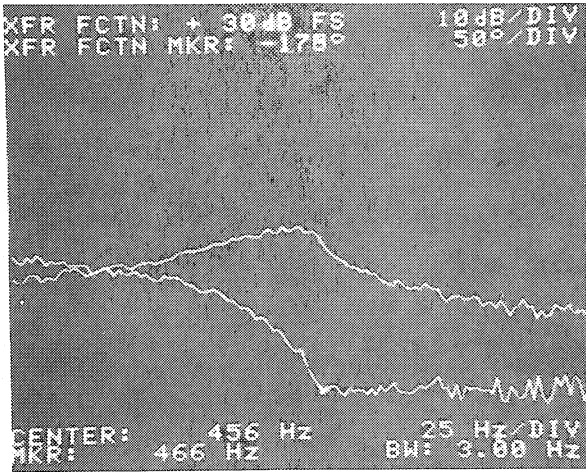
$N = 10^{11}$  p/bunch

bunch length: 25 ns

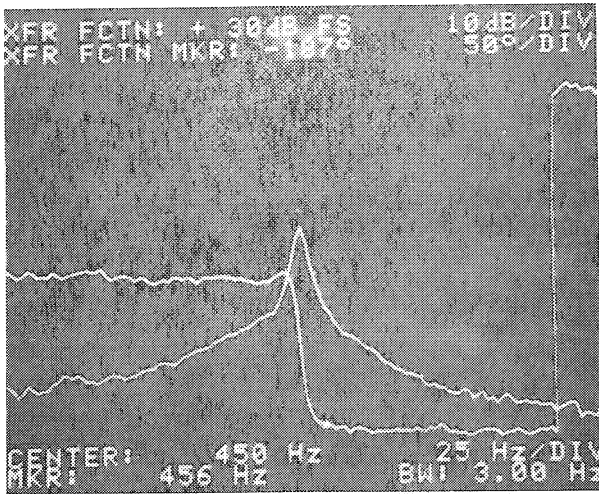
bunch emittance: 200 mrad

69459





- a) Low intensity case.  
The incoherent centre frequency is well defined: discontinuity on the phase curve.



- b) High intensity case.  
The coherent frequency is well defined: peak on the amplitude curve.

Fig. 5 Quadrupole mode transfer function

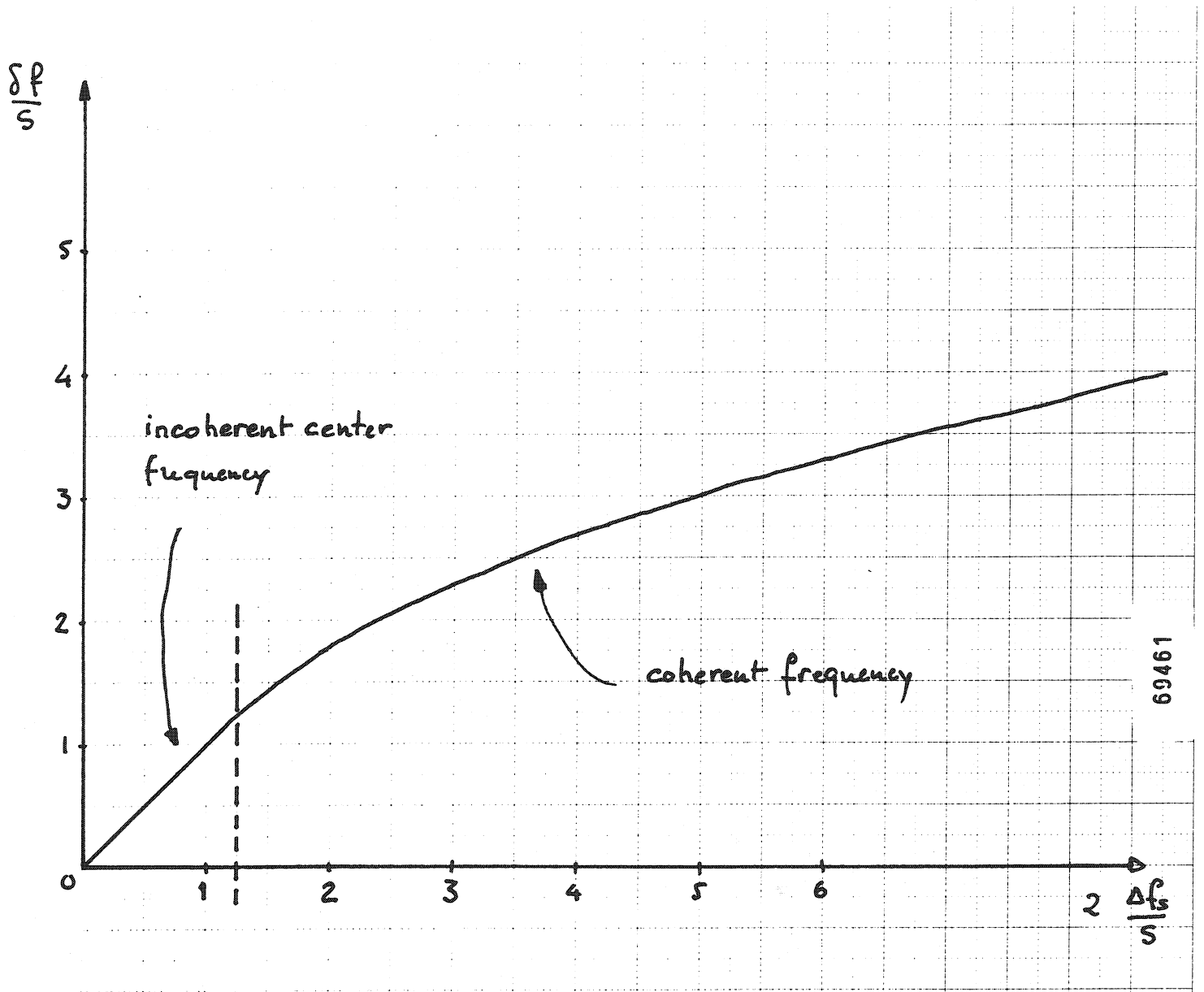


Fig. 6