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NONLINEAR LENS EXPERIMENTS IN THE SPS

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1. Introduction

The SPS p-$\bar{p}$ collider will be the first proton storage ring with bunched beams and head-on collisions. In this respect it is more similar to electron machines than to the ISR, but without the important radiation damping mechanism. It is well known that in electron machines the strongly nonlinear forces experienced by the beams at the crossing points give rise to phenomena which limit the maximum luminosity. In particular, there is a fairly sharp limit to the beam-beam tune shift (which is a measure of the strength of the nonlinear forces) above which the luminosity no longer increases with intensity. It is not known how to extrapolate results of electron machines to the SPS but at present it is felt that the beam-beam tune shift per intersection should be kept below 3×10^{-3}.

Some time ago, a nonlinear lens[1)] was constructed and installed in the ISR to simulate the beam-beam effect to values of the beam-beam tune shift higher than that obtainable with proton beams. It consists of a pair of current-carrying copper bars which can be positioned above and below the beam to produce a very nonlinear magnetic field. Although the field configuration cannot be the same as for the beam-beam effect, the strength and symmetry of the forces can be made comparable by a suitable choice of lens and beam parameters. This lens has been installed in the SPS. Preliminary experiments have shown that the beam lifetime is strongly dependent on the lens strength and the tune of the machine. In particular, resonances at least up to 10th order must be avoided.

2. Mechanical Construction

The lens consists of a pair of cylindrical copper conductors 867 mm long and 15 mm diameter, which can be accurately positioned above and below the beam. Each bar can be displaced parallel to the beam axis both vertically and radially by a set of 4 D.C. motors driven in pairs. These motors replaced the stepping motors used in the ISR in order to be easily integrated into the SPS control system. A pair of metallic scrapers accurately aligned with the bars allow the lens to be precisely centred on the beam by scraping a predetermined fraction of the beam from all sides (fig.1).

The lens has been installed in LSS5 downstream of quadrupole QD 519 where the mean amplitude functions of the unperturbed machine are $\beta_x = 30.1$, $\beta_y = 75.5$. The maximum current available is 320 amps. Both bars are powered in the same direction to avoid dipole field components.

3. The nonlinear lens field

The magnetic field components can be obtained from Ampere's law. With the nomenclature of figure 1 they are

$$B_x(x,y) = \frac{\mu_o I y}{\pi} \; \frac{x^2 + y^2 - h^2}{\{x^2 + (h+y)^2\}\{x^2 + (h-y)^2\}} \qquad 1)$$

$$B_y(x,y) = \frac{-\mu_o I x}{\pi} \; \frac{x^2 + y^2 + h^2}{\{x^2 + (h+y)^2\}\{x^2 + (h-y)^2\}}$$

The quadrupole gradient for small amplitude is then

$$\frac{\partial Bx}{\partial y} = - \frac{\partial By}{\partial x} = \frac{\mu_o I}{\pi h^2}$$

and the linear tune shift $\Delta Q_{x,y}$ is

$$\Delta Q_{x,y} = \frac{\beta_{x,y}}{4\pi} \cdot \frac{L}{B\rho} \cdot \frac{\mu_o I}{\pi h^2} \qquad 2)$$

where L is the length of the bars.

The higher multipoles of the lens field drive even order nonlinear resonances. The width of a resonance of order $N = n_x + n_y$ is given by [2]:

$$\Delta e_N = \frac{n_x^2 \varepsilon_y + n_y^2 \varepsilon_x}{2^{N-2} n_x! n_y!} \cdot \frac{R}{B\rho} \cdot \varepsilon_x^{\frac{n_x}{2}-1} \varepsilon_y^{\frac{n_y}{2}-1} |dp| \qquad 3a)$$

where $|dp|$ is the driving term of the resonance, given by

$$|dp| = \frac{1}{2\pi} \beta_x^{\frac{n_x}{2}} \beta_y^{\frac{n_y}{2}} \cdot \frac{L}{R} \cdot \left. \frac{\partial^{N-1} B_y}{\partial x^{N-1}} \right|_{x = y = o} \qquad 3b)$$

The nonlinear lens field can be expanded as a power series [3]. The n^{th} multipole coefficient is

$$\frac{\partial^{n-1} B_y}{\partial x^{n-1}} = \frac{(n-1)! \mu_o I}{\pi h^n} \qquad 4)$$

Therefore, for a one-dimensional resonance of order n, the resonance width is :

$$\Delta Q_e = \frac{\Delta e}{2n} = \frac{\varepsilon_o^{\frac{n}{2}-1} \beta^{\frac{n}{2}}}{(2h)^n} \left[\frac{L \mu_o I}{\pi^2 B\rho} \right] = \frac{\Delta Q_{lin} (\varepsilon_o \beta)^{\frac{n}{2}-1}}{(2h)^{n-2}} \qquad 5)$$

4. The beam-beam field

We wish to calculate resonance widths due to the beam-beam force. We consider only a very simplified model by taking a round Gaussian beam of rms width σ and N particles per unit length. The electric and magnetic fields can be found by Gauss law and Amperes law respectively, and can be combined to give an equivalent magnetic field B_{eff} given by

$$B_{eff}(r) = \frac{Ne}{\pi \varepsilon_o \beta c r} \left(1 - e^{-\frac{r^2}{2\sigma^2}}\right) \qquad 6)$$

Expanding the exponential we get for the n^{th} multipole coefficient

$$\frac{\partial^{n-1} B_{eff}}{\partial r^{n-1}} = \frac{Ne}{2\pi\varepsilon_o \beta c \sigma^2} \cdot \frac{(n-1)!}{(2\sigma^2)^{\frac{n}{2}-1} (\frac{n}{2})!}$$

7)

$$= \frac{8\pi \Delta Q_{lin}}{\beta^*} \frac{B\rho}{L} \frac{(n-1)!}{(2\sigma^2)^{\frac{n}{2}-1} (\frac{n}{2})!}$$

where Q_{lin} is the linear beam-beam tune shift given by

$$\Delta Q_{lin} = \frac{\beta^*}{8\pi} \cdot \frac{L}{B\rho} \cdot \frac{Ne}{2\pi \varepsilon_o \beta c \sigma^2} \qquad 8)$$

The stopband width can then be obtained from 3)

$$\Delta Q_e = \frac{\Delta Q_{lin} (\varepsilon_o \beta^*)^{\frac{n}{2}-1}}{2^{3(\frac{n}{2}-1)} \sigma^{n-2} (\frac{n}{2})!} \qquad 9)$$

5. Comparison of NLL and beam-beam resonances

The experimental method is to measure the lifetime of aperture-limited beams as a function of tune and of lens strength when the lens itself forms the aperture limitation. The stopband width at the beam edge is then

$$\Delta Q_e = \frac{g^{n-2} \beta}{2^{n-1} (g + r)^n} \frac{L \mu_o I}{2\pi^2 B\rho} \qquad 10)$$

where $g + r = h$

and $L = 0.867$ M

$I = 320$ amps maximum

$r = 7.5$ mm

$\beta = 75.5$

The resonance width at 2σ for the beam beam force is simply

$$\Delta Q_e = \frac{\Delta Q_{1in}}{2^{(\frac{n}{2} - 1)} \; (\frac{n}{2})!} \qquad 11)$$

Figure 2 shows the resonance width as a function of order for this simplified beam-beam model with $\Delta Q_{1in} = 3 \times 10^{-3}$.

Initial experiments were performed at 150 GeV/c. The gap was chosen to give a lifetime of a few hours without the lens powered. Even though the linear tune shift was 3 times higher than for the beam-beam effect ($\Delta Q_V = 0.009$ for h = 12 mm, I = 300 amps), it proved to be a very poor simulation for higher multipoles (figure 2), the 8th order resonance being more than an order of magnitude weaker than the beam-beam resonance.

A much closer simulation can be obtained by increasing the gap and reducing the energy. A second experiment was performed at 40 GeV/c with the gap opened to ± 12.5 mm (2π acceptance). The stopband width for these parameters is also plotted in figure 2.

6. Experimental results

6.1 Experiments at 150 GeV/c

The first experiment was performed at 150 GeV/c. The beam centre was first defined by scraping the beam from both sides, the first scraper removing a fairly large fraction of the beam (∿ 20%) and the second removing as little as possible (typically 2%). In this way the beam centre could be defined with a precision of ± 0.1 mm.

The chromaticity was measured to be $\xi_H = 0.19$, $\xi_V = 0.16$. The RF was set at full voltage (5MV) to give the longest possible bunched beam lifetime. Since these experiments were done with a continuous beam, the low noise loops could not be used, so the bucket quickly became full due to RF noise. The bucket height was $\Delta p/p = \pm 1.6 \times 10^{-3}$. The Q-values were continuously monitored with Schottky scans (fig. 3).

The object of the first experiment was to probe the resonance-free region between 8th and 3rd order resonances around $Q_H = 26.64$, $Q_V = 26.63$. Figure 4 shows a map of bunched beam lifetime as a function of working point on an enlarged scale both with and without the lens powered. When the lens was off it still acted as an aperture limitation at $\pm$ 4.5 mm. It can be seen that the lifetime is very critically dependent on the working point both with and without the lens powered, and is strongly correlated to the distance from the main diagonal and from the 3rd integer resonance. The maximum lifetime measured $\sim$1.7 - 2 hours corresponds to a mean vacuum pressure for multiple scattering of $\sim 2 \times 10^{-8}$ torr. This corresponded reasonably well with the mean pressure measured at the pumps. There were some slight vacuum leaks in the ring at the time. The lifetimes measured with the lens powered are consistently lower than without the lens, except in the bottom left corner, far from the 3rd integer stopband.

The effect of the main diagonal was further investigated by measuring the lifetime of a debunched beam away from the 3rd integer resonance using the lens only as an aperture limitation. Figure 5 shows the measured lifetime as a function of $Q_H - Q_V$, with a pronounced step at about $Q_H - Q_V = 0.02$.

The most probable explanation of this phenomenon is that as the beam moves closer to the coupling resonance $Q_H - Q_V = 0$, the effective aperture changes from a flat rectangle defined by the scrapers to a round aperture. In other words the beam blowup due to multiple Coulomb scattering in the radial plane is transmitted into the vertical plane through the coupling, with a resultant increase in the rate of beam loss on the scrapers. The expected difference in lifetime between completely coupled and decoupled cases is just a factor of 2, in reasonable agreement with the measurement. The vacuum pressure in the ring was much worse in this experiment due to an intervention to change a main dipole, and this is reflected in the reduced lifetime.

It is of interest to digress for a moment to investigate the cause of the strong coupling in the SPS. It is known [4] that there is a systematic vertical displacement of the coils of the main dipoles by about 0.1 mm from the median plane due to the weight of the coil. This results in a skew quadrupole field with a calculated gradient B' of $\sim 5.4 \times 10^{-4}\ B_o$ T/M [5]. The stopband width due to this zeroth harmonic skew quadrupole is then energy independent and is given by

$$\Delta Q = \frac{1}{4\ B\rho} \cdot \sqrt{\beta_H \beta_V}\ B'\ N\ell$$

N =744 magnets, $\ell = 6.2$m, $B_o/B\rho = 1.35 \times 10^{-2}$
giving $\Delta Q = 0.037$ full width.

This is in very good agreement with the value obtained from figure 5.

There remains to be explained the consistently lower lifetime near to the 3rd integer stopband with the lens powered. A perfectly centred lens would drive only even order resonances. However, a small radial misalignment δ would also drive odd order resonances. The width of the resonances $3Q_H = 80$ driven by such a displacement is

$$\Delta Q_e = 3\pi\sqrt{\varepsilon_o \beta_x} \cdot \frac{\delta}{h^2} \cdot \Delta Q_{1in}$$

Putting $\sqrt{\varepsilon_o \beta} = 3 \times 10^{-3}$m

$\Delta Q_{1in} = 8.3 \times 10^{-3}$

Then $\Delta Q_e \sim 2 \times 10^{-4}$ for $\delta = 0.1$ mm

This is to be compared with the stopband generated by random sextupole errors in the main dipoles, which is estimated to be $\sim 1 \times 10^{-4}$ [6]. It is therefore reasonable to postulate that the increased decay rate near the 3rd integer stopband with the lens powered is due to the enhancement of the strength of the resonance and its concomitent synchrotron sidebands by a small displacement error of the conductors.

The main conclusion of the experiments at 150 GeV/c is then that the effect observed can be explained by the strong linear coupling and by small misalignments of the conductors with respect to the beam centre. The lens has no effect sufficiently far from the main diagonal in a region of the working diagram which contains the family of 8th order resonances. This is a gratifying result, since we know that the stopbands generated by the lens in this region are more than an order of magnitude weaker than those generated by the beam-beam forces.

6.2 Experiments at 40 GeV/c

The experiments at 40 GeV/c were designed to simulate more closely the high order beam-beam resonances. After finding the beam centre, the lens was placed at $\pm$ 12.5 mm, defining an acceptance of 2π in the vertical phase plane. The beam was blown up to fill the acceptance by excitation of the old vertical damper with a tracking generator sweeping through one of the betatron lines. The conditions of the experiment were as follows :

$$\begin{aligned} p &= 40 \text{ GeV/c} \\ \xi_H &= 0.19 \\ \xi_V &= 0.08 \\ \Delta Q_{lin} &= 1.2 \times 10^{-2} \text{ at 320 amps} \\ V_{RF} &= 5 \text{ MV} \\ (\Delta p/p)_{max} &= \pm 3.6 \times 10^{-3} \end{aligned}$$

One of the biggest problems at this energy was the stability of the main power supplies, which gave a jitter in Q of $\pm 3 \times 10^{-3}$. Another problem was the low lifetime of the bunched beam even when only a single bunch was stored with the low noise phase loop. This was certainly at least partly due to the power supply instability. It made it very difficult to obtain meaningful results with a bunched beam.

The purpose of the experiment was to measure lifetimes in the same region of the working diagram as in the previous experiment.

The estimated lifetime assuming an equivalent vacuum pressure for multiple scattering of 6×10^{-9} torr was 3.5 hours. This agreed well with the measured lifetime of a debunched beam, but the lifetime of a bunched beam was almost an order of magnitude lower.

Figure 6 shows the measured lifetime as a function of lens current at two working points. The first measurement was made with both bunched and debunched beams in the region of the 8th order resonance, $Q_H = 26.637$, $Q_V = 26.617$. In contrast with the previous experiment in the same region, the lifetime clearly depends on the lens strength.

The beam profile was measured with the wire scanner. Even though the beam had been originally blown up to fill the 2π acceptance, the equilibrium emittance with the lens at full current (320 amps) was found to be only $\sim \pi$ mm mrad. Evidently, large amplitude particles are rapidly lost.

The working point was then moved to above the 8th order resonances $Q_H = 26.652$, $Q_V = 26.635$ and the lifetime of a debunched beam was measured. This time the lifetime was found to be practically independent of the lens current.

7. Discussion

The considerations of the previous chapters have been based entirely on a simple classical model of isolated nonlinear resonances. It is supposed that the beam sweeps repeatedly through the resonances either through synchrotron motion coupled with non-zero chromaticity or, as in the last experiment, through power supply ripple. According to the Guignard-Schoch theory [2] the emittance growth for repeated crossings with random phase is given by

$$\left[\frac{1 - (r_o/r)^{n-2}}{n-2}\right]^2 = \frac{\pi^2 \, \Delta Q_e^2 \, f_{rev}}{n\Delta Q} \, t$$

Where ΔQ_e is the stopband width and ΔQ is the Q-oscillation amplitude of the particles. This equation says that the emittance blowup is independent of the rate of crossing - faster crossing gives smaller kicks but more frequently. Therefore one would expect that the effect of synchrotron motion and power supply ripple would be about equivalent, since the amplitude of modulation were roughly the same. It is instructive to compute the time for a particle's amplitude to increase by say 10% at the amplitude of the scraper, where the stopband width is a maximum. This is summarized in the following table, for maximum lens strength and supposing $\Delta Q = \xi Q \, \Delta p/p = 5 \times 10^{-3}$.

Order	ΔQ_e	t(sec)
8	1.1×10^{-5}	4.7
10	1.1×10^{-6}	490
12	1×10^{-7}	6×10^{4}

Although it must be emphasised that this model is very rough and certainly wildly inaccurate, it is not surprising from the above figures that we find no particles with large emittance when the beam is sitting on 8th order resonances. Of course, the emittance growth rate or diffusion coefficient is a strong function of amplitude. To find the equilibrium lifetime and distribution of the whole beam would require a much more sophisticated approach.

8. Conclusions

Under certain conditions the nonlinear lens can drive stopbands of the same order of magnitude as the beam-beam effect. The results obtained up to now are in at least qualitative agreement with a model based on repeated crossings of isolated high order resonances. 8th order resonances are catastrophic to beam survival. It even looks as if 10th order resonances should be avoided although no experimental evidence is available.

9. Acknowledgements

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References

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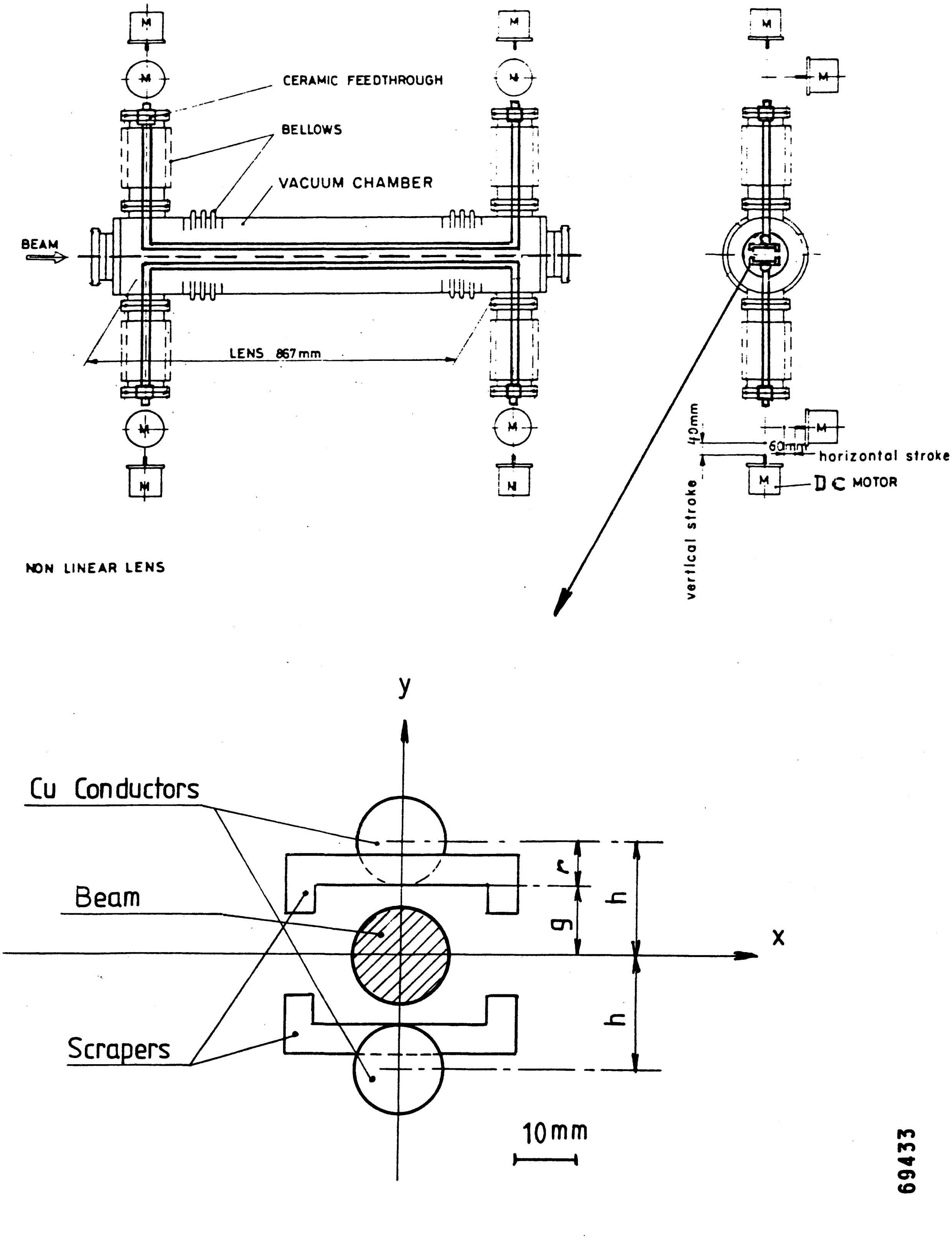

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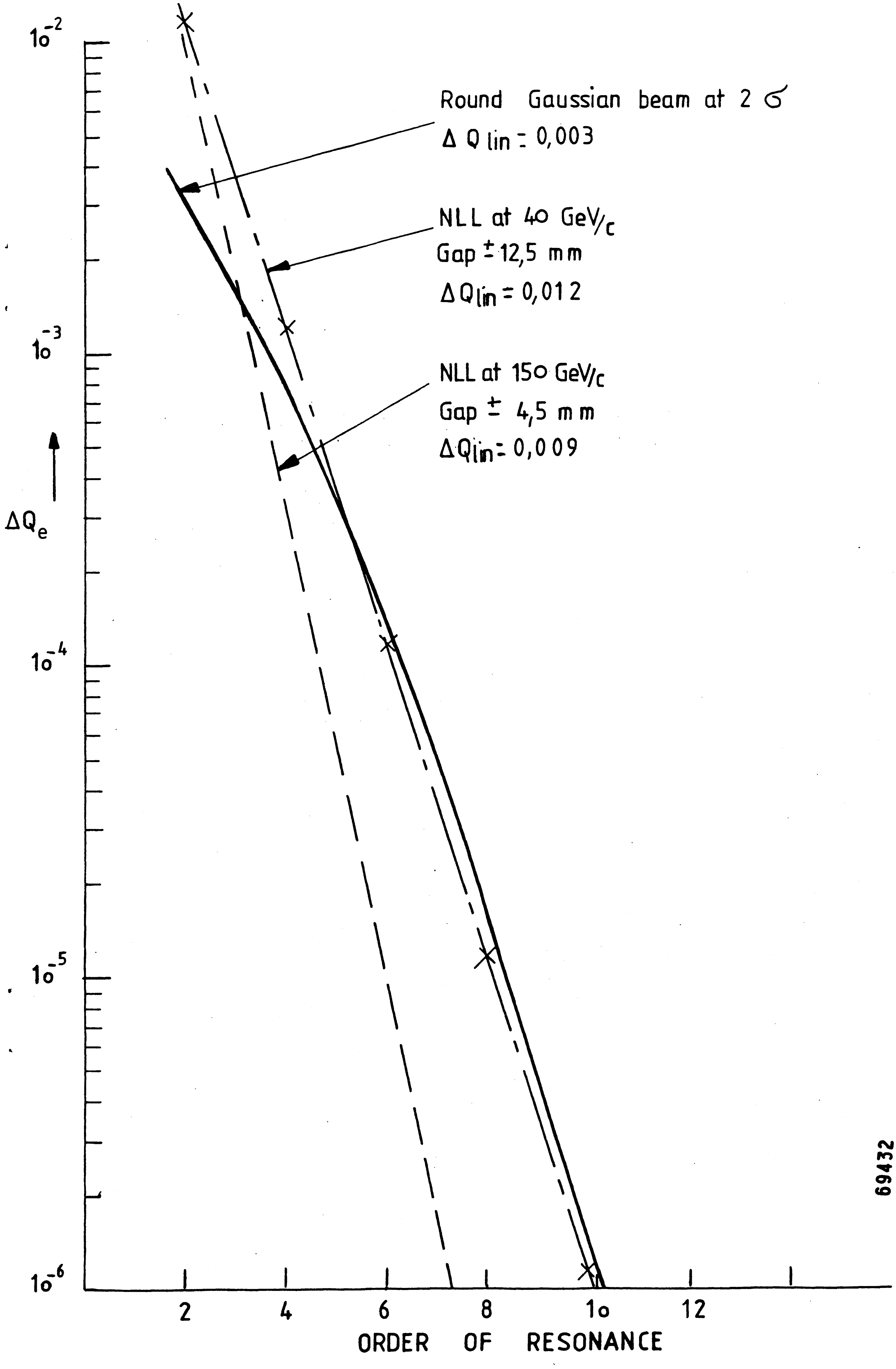
Fig. 1 View of nonlinear lens



Round Gaussian beam at 2 σ
ΔQ lin = 0,003
NLL at 40 GeV/c
Gap ± 12,5 mm
ΔQlin = 0,012
NLL at 150 GeV/c
Gap ± 4,5 mm
ΔQlin = 0,009
ΔQe
1o-2
1o-3
1o-4
1o-5
1o-6
2
4
6
8
1o
12
ORDER OF RESONANCE

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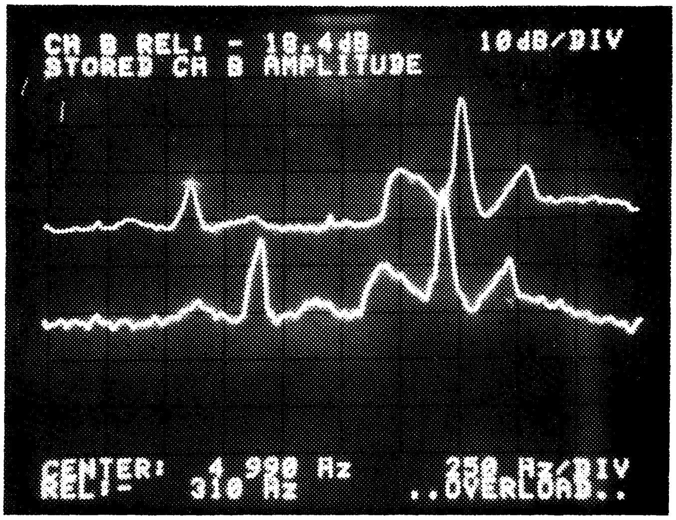

Figure 3

Bunched-beam Schottky signal with and without lens

	Measured	Expected
ΔQ_H	$= + 1.7 \times 10^{-3}$	$+ 2.4 \times 10^{-3}$
ΔQ_V	$= - 7 \times 10^{-3}$	$- 6.2 \times 10^{-3}$

The synchrotron frequency can also be measured from the sideband of the radial Schottky line giving $f_s \sim 290$ Hz.

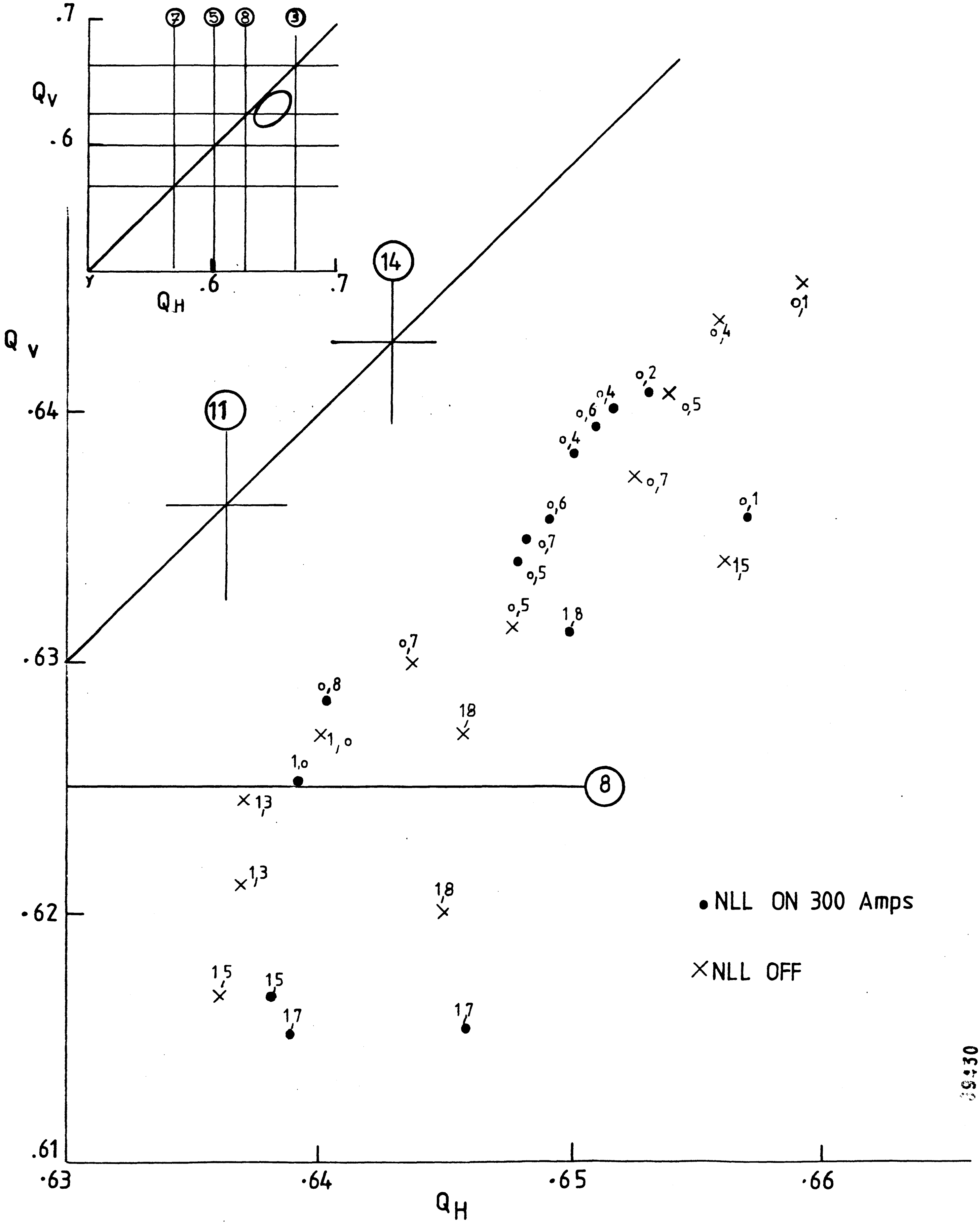

Figure: 4

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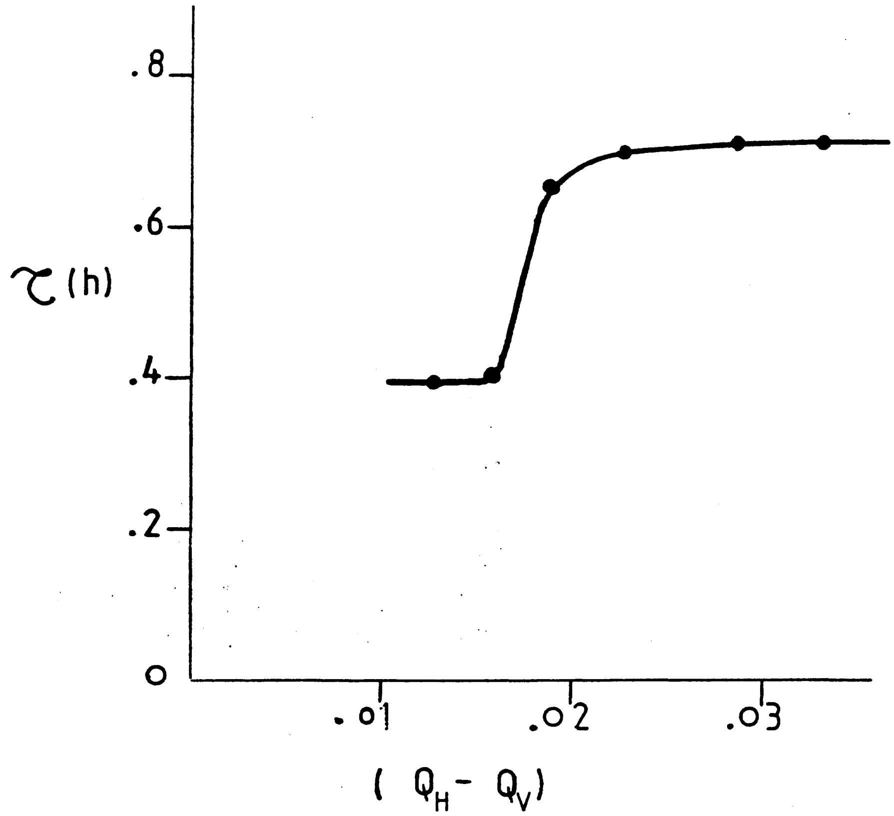

Figure : 5

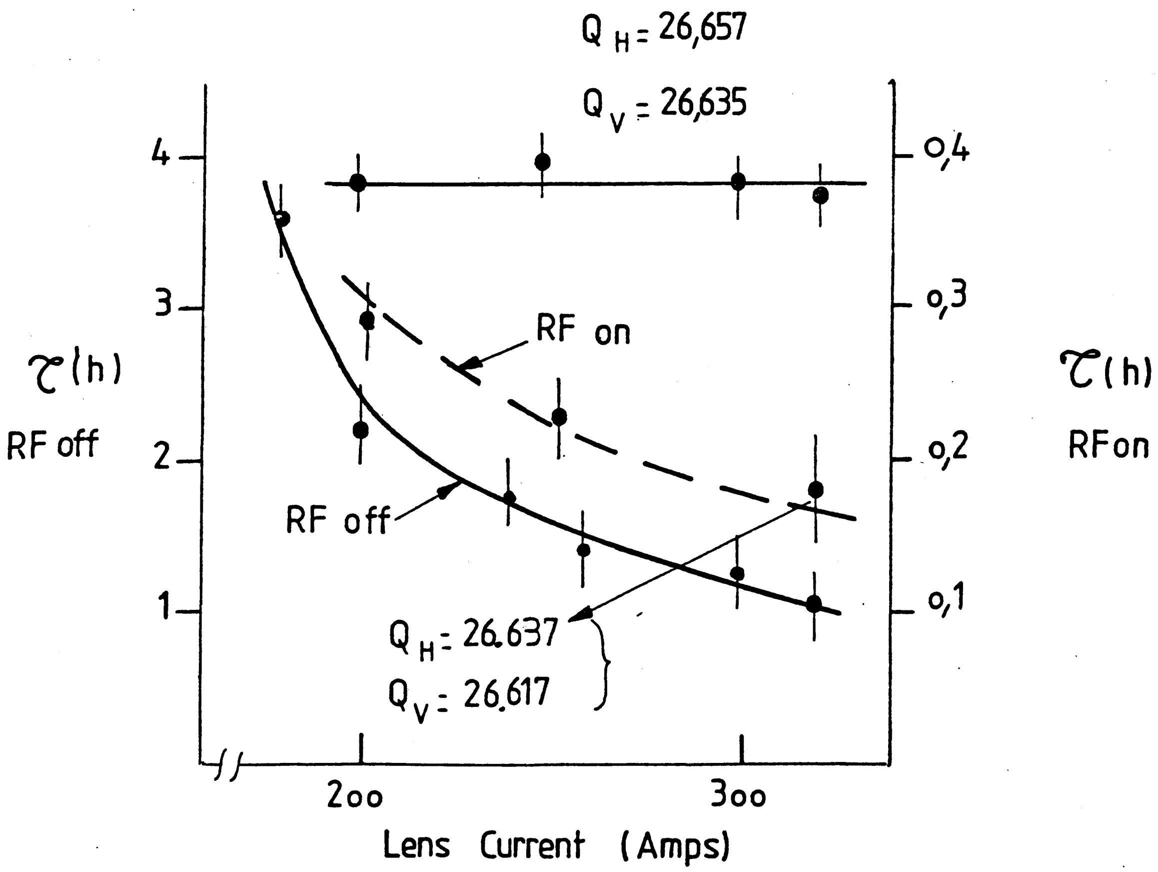

Figure : 6

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