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1. - Introduction

In this note we consider the possibility of measuring the proton-proton elastic scattering in the angular interval where the Coulomb scattering is relevant.

We shall discuss some experimental apparatus and a calibration system. The method of extracting information from the data will be illustrated by means of a Monte Carlo simulated experiment.

Assuming at the ISR energies the small angle parametrization of the differential cross section which is valid at the present machine energies, we write

$$\frac{1}{(p\hbar)^2}\frac{d\sigma}{d\Omega} = \left(\frac{2}{137}\right)^2 \frac{1}{t^2} + \left(\frac{\sigma_t}{4\pi\hbar^2}\right)^2 e^{-bt} \qquad (1)$$

where $t = 2p^2(1-\cos\vartheta) \simeq p^2\vartheta^2$. The first term represents the Coulomb scattering whose magnitude and angular distribution are both known. The second term represents the strong interaction scattering whose amplitude has been assumed to be purely imaginary.

In Fig. 1 the differential cross section computed from the previous expression with σ_t = 40 mb and b = 10 GeV/c^{-2} is shown for p = 15 GeV/c and 25 GeV/c. Coulomb scattering is expected to be larger than strong interaction scattering by a factor about 3 at very small angles, 1.5 mrad at 25 GeV/c and 2.5 mrad at 15 GeV/c. Because of the knowledge of Coulomb scattering, a measurement of the elastic scattering events rate in an angular range, which includes these very small angles, allows a determination of the strong interaction scattering. If the strong interaction amplitude is assumed to be purely imaginary, then, the total cross section is computed by applying the optical theorem. An estimate of the size of the real part can also be obtained.

This method of determining the strong interaction cross section relies on the use of the Coulomb scattering for the absolute normalization of the event rates and does not need a monitor of the luminosity.

The possibility of determining the ISR luminosity by measuring the elastic cross section at the angles where Coulomb scattering dominates was pointed out by Cocconi[1].

The major difficulties of a measurement at very small scattering angles lie, first, on the necessity of having detectors quite close to the circulating beams, and second, on the finite dimensions of the interaction region and the angular divergence of the beams which determine a condition of poor geometry.

On the other hand the large value of the small angles differential cross section results in a high counting rate, making the experiment still possible at values of the ISR luminosity much smaller than the design value.

2. - Choice of the experimental conditions

The choice of the experimental conditions is guided by the following considerations.

1) The beam dimension and angular divergence[2] are both larger in the horizontal than in the vertical plane.

2) A possible malfunctioning of the machine would result in the loss of the beam on the horizontal plane and consequent possible damage of the counter system.

3) Possible errors at the injection give larger effects on the horizontal than in the vertical plane.

4) The horizontal position of the beams might be affected by instability (of the order of a few tenths of mm peak to peak) due to the ripple of the magnet system.

5) The flux of background particles near the beam pipes might have an azimuthal dependence, being larger in the machine plane than in vertical plane.

We conclude that it is preferable to have detectors not lying in the machine plane and look at the protons scattered in the vertical plane.

The lower value of the measurable scattering angle is essentially fixed by the vertical dimension and divergence of the beams. It seems unlike that it can be less than about 1.5 mrad. The upper value should be at least 5 or 6 mrad.

From the actual estimates of the background flux(3), the identification of elastic scattering events at small angles seems perfectly feasible on the basis of the requirement of the directional correlation of the two protons, without need for magnetic analysis. We shall then consider experimental arrangements in which both protons scattered in the vertical planes are detected.

3. - Calibration system

It is essential for the present measurement to have a precise knowledge of the effective scattering angle ϑ and of the angular interval $\Delta\vartheta$ as seen by each counter of the apparatus.

A possible way of calibrating the apparatus is provided by the detection of protons scattered by the nuclei of an ion or atomic beam allowed to cross the proton beams in the interaction region. An ion beam, which can be easily focused, is preferable to an atomic beam, also in connection to the vacuum requirements. At present, ion currents as high as 100 mA with beam emittance of the order of 2 cm x degree can be obtained with the Duoplasmatron type of source(4).

The choice of the element to be used for the ion beam is dictated by the following facts: for a given ion current, the proton beam-ion beam luminosity is inversely proportional to the ion speed; the proton-nucleus differential cross section near to forward direction is approximately proportional to $A^{4/3}$, the angular distribution becomes steeper for increasing values of the mass number.

Interpolating between the proton-nucleus cross sections of Bellettini et al.(5), for an Argon beam of 100 mA with accelerating potential of 1 KV and ISR proton beam of 20 A we get, in the angular interval accepted by the apparatus, counting rates decreasing with the angle from $3.6 \cdot 10^4$ down to $4.0 \cdot 10^3$ for a solid angle of $4 \cdot 10^{-7}$ sterad.

With equally intense Krypton or Xenon beams, roughly the same rates are obtained using accelerating potentials of 10 KV and 20 KV respectively. Large accelerating potentials have to be preferred as far as the electric perturbance produced by the proton beams on the ion beam is concerned.

The flux of background particles produced in beam-gas interactions, calculated according to de Raad(3) and assuming a pressure of the residual gas (N_2) of 10^{-10} Torr, is about $1.6 \cdot 10^3$ part/cm$^2 \cdot$ s at the smallest angle and should decrease roughly as $1/\vartheta$.

Therefore the signal-to-noise ratio between proton-ion and proton-gas events comes out to vary from about 20 to about 8.

A test of the experimental apparatus can then be made using an ion source providing a beam which, lying on the horizontal plane, would cross both proton beams in the middle of the interaction region. The source would be pulsed for short time intervals in order to obtain calibration runs during the measurements.

An immediate output of the calibration runs is a check of the alignment of the detectors with respect to the beam lines.

Moreover, if the shape of the proton-nucleus differential cross section is accurately known, then the relative acceptance of the detectors can be obtained.

In Fig. 2 the differential cross sections for p-Argon, p-Krypton and p-Xenon scattering are plotted.

The characteristic S-shape of the proton-nucleus cross sections is very suitable for obtaining also the absolute value of the scattering angle.

We observe that the calculation of the background is pessimistic by a factor 10 at least, because it is expected that a pressure around $5 \cdot 10^{-11}$ Torr can be obtained, the residual gas being H_2. As a consequence the signal-to-noise ratio would be of the order of 100 or more.

4. - Possible experimental arrangements

The measurement of elastic scattering at very small angles seems to be impossible if no modification is made to the ISR vacuum chamber.

In any case a specially designed chamber has to replace a part of the standard one. We shall, however, consider two different kinds of arrangements.

1) Only a change of the vacuum chamber is required, without any modification to the magnetic structure of the ISR.

2) A change of the magnetic structure is also required.

Experimental arrangements of the first kind seem to be particularly appropriate for the first period of operation of the ISR. We start by considering an apparatus of the first cathegory to be placed in one of the even numbered intersection regions (where the c.m.s. moves radially inward). We observe that the vertical betatron amplitude function β at the entrance of the first F magnet downstream of the crossing point is about 19 m and at the exit is about 28 m [2]. The maximum betatron function anywhere, around the machine, is 51.4 m. By simple scaling (proportionally to the square root of the betatron function) one obtains that a vertical aperture of 31 mm at the entrance of the magnet, and 38 mm at the exit, is optically equivalent to the aperture of 50 mm at β = 51.4 m. It is then possible to reduce the vertical dimension of the vacuum chamber in the first F magnet to the values given above. This reduction does not represent an obstruction but a local matching of the aperture to the real beam dimensions. Obviously some clearance will be provided. In Fig. 3 the beam envelope in the vertical plane is shown against the distance from the interaction point. We have assumed that, at $\beta = \beta_{Max}$, the beam size is just equal to the vertical aperture of the ISR vacuum chamber; this actually corresponds to an overestimate of the beam size [6]. A sketch of a possible arrangement, not yet studied in detail, is shown in Fig. 4. The range of accepted scattering angles is from about 1.5 mrad up to 10 mrad. Very small scattering angles are seen by counters at the exit face of the F magnet. Larger angles are seen by counters placed in front of this magnet. A slight overlap is possible. A special section of vacuum chamber with two windows is needed. It is conceivable to think of a chamber having, inside the F magnet, two small movable parts which are left far apart during injection and possibly needed vertical displacements of the beams, and then brought nearer to the beam axis. In Fig. 5 a tracking of trajectories in the vertical plane is shown.

We shall now consider two arrangements of the second cathegory. The first one requires the replacement of the first F magnet unit with a similar unit having a larger gap width. The new magnet would have only the vertical aperture enlarged while the horizontal useful region can be the same as for the standard ISR magnets. Planes of counters would be placed at the exit face of the magnet. A gap width of 16 cm (60% more than the standard one) would allow detection of scattering angles up to about 6 mrad.

Another possible arrangement of the second kind would make use of a system similar to that one suggested by de Raad(7) for the analysis of protons elastically scattered on the horizontal plane. The first two, one short F and one short D, magnet units, downstream of the crossing point in one of the even-numbered intersection regions are replaced by two quadrupoles with a uniform field bending magnet in between. For the present purposes both quadrupoles can be of the normal design (not of the open type considered by de Raad in his scheme) with 20 cm diameter of the aperture. The gap width of the magnet should be at least 25 cm. A possible layout, which we have not yet studied in detail, is shown, very schematically, in Fig. 6. An identical system is placed on the other beam. A specially designed vacuum chamber having a window at the exit face of the bending magnet is required. Two sets of counters are placed beyond the bending magnet symmetrically with respect to the medium plane of the machine. For this system, the range of accepted scattering angles is from 1.5 to 6.0 mrad. The counters are placed in such a way to slightly overlap each other. In Fig. 7 the results of a computation of trajectories along this system are shown.

A reduction of background can be obtained by measuring coincidences between the hodoscope shown in Fig. 6 and another hodoscope placed in front of the next D magnet.

5. - Monte Carlo simulation of the experiment

It has already been observed that the difficulty of the present measurement lies mainly in the rather poor geometry conditions, while the expected rate is quite high. In fact, at the nominal ISR luminosity ($4 \cdot 10^{30}$ cm^{-2} s^{-1} for $\Delta p/p = 2\%$) the elastic scattering events rate, in the angular range which we have considered, is about 15 ev/s for $\Delta \Omega = 4 \cdot 10^{-7}$ sterad. We may then accept to make an even strong selection on the data if, as a counterpart, we can define more favorable geometrical conditions.

We have studied this problem at p = 25 GeV/c by means of the Monte Carlo method. Elastic scattering events have been generated according to the distribution given by eq. (1) with $\sigma_t = 40$ mb, where for simplicity we have put b = 0. A gaussian distribution has been taken for the divergence of the beams and for the beams density along the vertical direction. For the beams density along the transverse direction, a continuous distribution of gaussians superimposed in an interval Δx, determined by the momentum spread, has been assumed.

For the standard deviation of the gaussian distributions, we have used the numerical values of ref. 2 (Table II, pag. 25).

Displacements and divergences have been taken to be uncorrelated. This is a good approximation because the phase plane ellipses are nearly up-right at the intersection point.

The magnetic fields have not been included in the Monte Carlo, so that both protons are taken to travel on a straight line from the interaction region to the detectors. We have first taken a value of the stacking width $\Delta x = 4$ cm, which corresponds to $\Delta p/p \simeq 2\%$ and $L \simeq 4 \cdot 10^{30}$ cm^{-2}s^{-1}. We have assumed that each detector consists of 13 adjacent counters. Each counter covers an azimuthal angle $\Delta\varphi$ of $\pm$ 15° around the vertical plane and is seen from the center of the interaction region through an angle ϑ equal to 0.31 mrad (which corresponds to a counter 5 mm high at 16 m).

The minimum value of ϑ which is accepted is 1.5 mrad. The detected events are classified in a 13 x 13 matrix. Each matrix element represents the number of coincidences between a counter of one detector and a counter of the other detector. The diagonal elements represent coincidences between opposite corresponding counters.

In the case of ideal beams, a matrix is obtained which has only the diagonal elements different from zero. In the real case, however, the events are dispersed around the diagonal.

We have studied what information can be obtained by using the diagonal elements, that is, considering only the coincidences between corresponding counters. The angular resolution in ϑ of the counters for these events, as found with the Monte Carlo, is shown in Fig. 8. In the ideal beams case the resolution is determined only by the size of the counters and is sharp.

For real beams the resolution is gaussian-like with half-width at half-height equal to about 0. 2 mrad. Such a resolution is adeguate enough to measure the Coulomb scattering.

In Fig. 9 we have plotted the quantity $n/\Delta\Omega$ against the scattering angle ϑ , where n represents the coincidences rate for couples of corresponding counters normalized to the total number of coincidences.

The points represented as full circles refer to the ideal beams case and have been obtained from a sample of 300000 events. Each point has a statistical error of about 0.5%.

They follow very closely the full line, representing the assumed differential cross section, which checks the correctness of the computer program.

The points represented by crosses refer to a sample of 45000 events in the case of real beams. Each one has a statistical error of the order of 2%. They correspond to 35% of the total number of coincidences.

These points agree quite well with the given distribution showing that the selection of events made by taking only the diagonal elements does not give rise to a distortion of the distribution.

It is quite obvious, however, that at least the first point would need a correction which is of the same order as the statistical error of the data of the present sample. This small systematic effect can be managed, however, if an appropriate folding procedure is employed in fitting the data by using the effective (gaussian-like) angular aperture of the counters (Fig. 8).

We have made a least square fit with eq. (1) to both sets of data. The free parameters to be determined were the total cross section, the slope b, and a normalization constant. The results are given in Table I.

We remark that, assuming the real part to be negligible, a 2 ~ 3% estimate of the total cross section can be obtained in less than 10 min of running time at the maximum luminosity.

We have also studied in detail the case of not complete stacking, taking $\Delta x = 1$ cm which corresponds to $L \simeq 2.5 \cdot 10^{29}$ $cm^{-2}s^{-1}$. In this case we have considered detectors composed of 20 counters with $\Delta\varphi = \pm 10°$, $\Delta\vartheta = 0.31$ mrad and angular range from 1.5 to 7.5 mrad.

In Fig. 10 we have plotted the results of the Monte Carlo for both ideal beams and real beams cases and also the ratio of the normalized counting rates.

The sample of data obtained by means of the Monte Carlo has been fitted in different ways. We have made a three parameters fit with eq. (1) (σ_t, b and a normalization constant) and a four parameters fit, the fourth parameter being the ratio δ between the real and the imaginary part of the strong interaction amplitude in the forward direction. The results are collected in Table I, where it is also shown how the accuracy of the fitted parameters is affected if the point at the smallest angle is missing.

We observe that the matrix of the events contains more information than we have used, and can be used to check the alignment independently of the calibration and to estimate the size of the interaction region and the vertical beams divergence. In particular the elements of the matrix which lie on lines parallel to the diagonal can be used to increase the statistics.

6. - Conclusions

From the examination of Table I we see that in a 7 hours running time with a luminosity equal to 1/16 of the nominal value, it should be possible to determine the total cross section within $2 \sim 3\%$, the slope b of the angular distribution with an error of the order of $\pm$ 0.5 $(GeV/c)^{-2}$, and the ratio of the real to the imaginary part within about $\pm$ 0.05.

If, however, the real part is assumed to be negligibly small a $\sim 1\%$ determination of the total cross section can be obtained. This can be done with a relatively simple apparatus.

In the Monte Carlo simulation we have assumed, for simplicity, that the detectors cover only a relatively small azimuthal angle.

It seems possible, however, to enlarge the range of φ angles up to about $\pm$ 30° around the vertical plane without a deterioration of the geometrical conditions.This amounts to an increase of the rate by a factor 2 in case 1) and by a factor 3 in case 2) of Table I.

Each detector would then consist of a mosaic of small counters.

We have always considered a momentum of 25 GeV/c. At lower values of the momentum the conditions are more favorable,because if the vertical size and divergence of the beam go roughly as $1/\sqrt{p}$, on the other hand, for a given value of the minimum momentum transfer t, the minimum angle of the detectors is inversely proportional to p.

A determination of the total cross section can also be obtained from the measurement of the total interaction rate[8].

The measurement of small angles elastic scattering, however, has the advantage of being self-monitoring through the Coulomb scattering. Moreover a determination of the slope of the angular distribution near the forward direction and a reasonably accurate estimate of the real part can be obtained, in the hypothesis of spin independence of the amplitude.

On the other side a measurement of the interaction rate is a more direct way of obtaining the total cross section in spite of the fact that it requires a sizeable extrapolation to a 4π geometry and a monitor of the luminosity. In conclusion one can say that the two measurements are complementary.

For the present experiment no monitor of machine luminosity is needed. However we remark that the ion beam could be sent along the vertical direction and bunched so to provide the "wire" requested by Darriulat and Rubbia in their proposal for beam monitoring(9). Another possibility consists in sending a vertically narrow ion beam in the horizontal plane. By sweeping it vertically and by recording the scattered protons in the two hodoscopes, the effective height of the overlap region can be obtained.

February 10, 1969
revised March 18, 1969

References

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L. di Lella, CERN internal report 68/ISRU/111

2) P. Strolin, CERN internal report ISR-TH/68-36

3) B. de Raad, CERN report PS/6487

4) H. Wroe, Nucl. Instr. and Meth. 52 (1967) 67
Ortec - Catalog Number 1001 and IS 500

5) G. Bellettini et al., Nucl. Phys. 79 (1966) 609

6) "The Design Study of Intersecting Storage Rings (ISR) for the CERN Proton Synchrotron" - AR/INT SG 64-9, pag. 57

7) B. de Raad, CERN report AR/Int. SG/62-12 and ISR report AR/Int. SG-64-9 pag. 89

8) G. Bellettini et al., "Measurement of the p-p total cross section" (Proposal for an experiment at the ISR), March 1969.

9) P. Darriulat and C. Rubbia, CERN report 68/ISRU/106

Table I

1) $\Delta x = 4$ cm, $L = 4 \cdot 10^{30} cm^{-2} s^{-1}$; 13 counters, angular range 1.5-5.0 mrad, $\Delta\vartheta = 0.31$ mrad, $\Delta\varphi = \pm 15°$, $\Delta\Omega_{av.} = 6 \cdot 10^{-7}$ sterad

	Number of events	N_{par}	σ_t (mb)	b(GeV/c^{-2})	Average rate	Running time
Ideal beams	300000	3	40.3 ± 0.4	0.56 ± 0.51		
Real beams	45000	3	39.4 ± 1.0	1.4 ± 1.3	8.5 ev/s	7 min

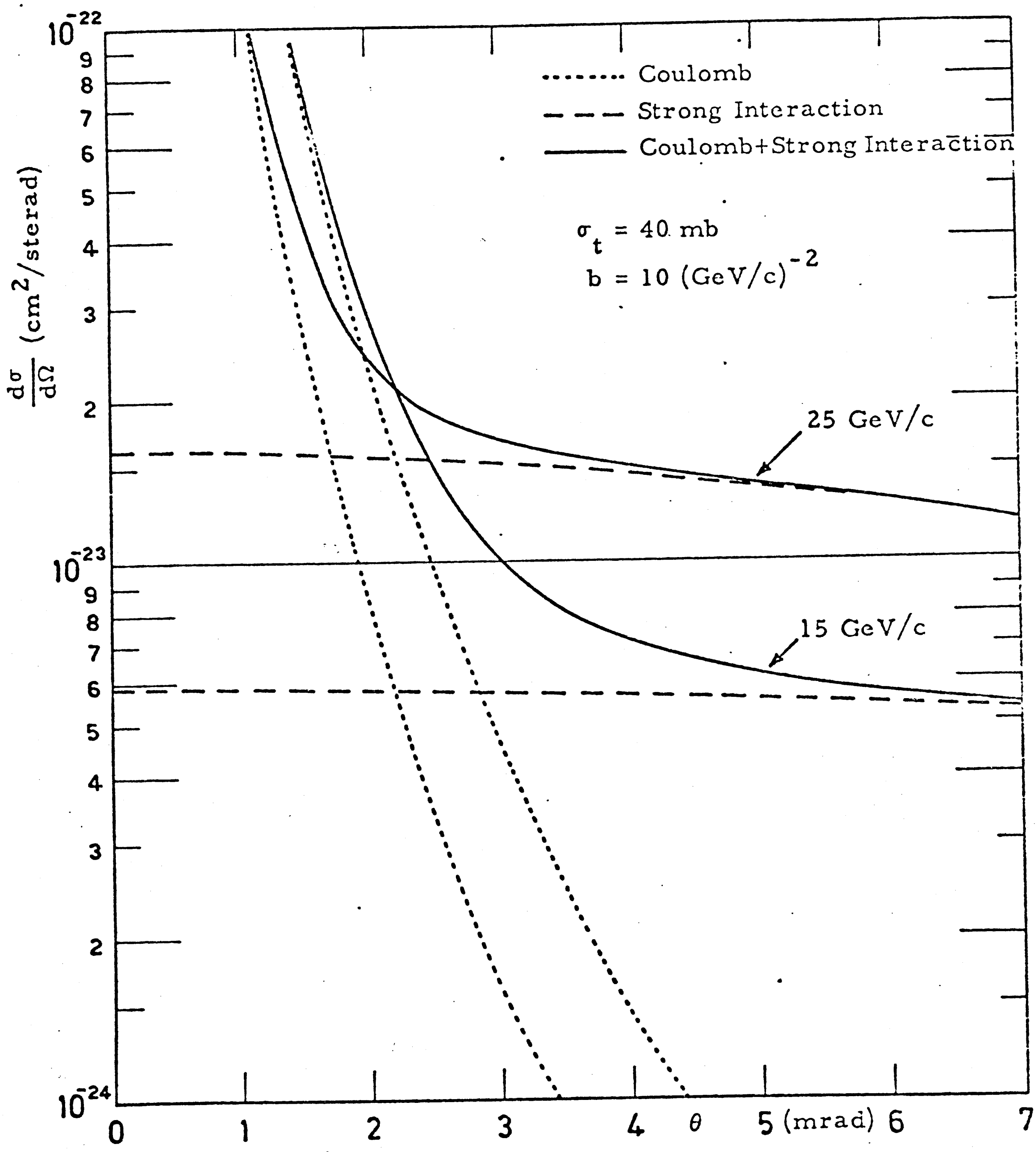
2) $\Delta x = 1$ cm, $L = 2.5 \cdot 10^{29} cm^{-2} s^{-1}$; 20 counters, angular range 1.5-7.5 mrad, $\Delta\vartheta = 0.31$ mrad, $\Delta\varphi = \pm 10°$, $\Delta\Omega_{av.} = 5.3 \cdot 10^{-7}$ sterad

	Number of events	N_{par}	σ_t (mb)	b(GeV/c^{-2})	δ	Average rate	Running time
Ideal beams	300000	3	40.2 ± 0.4	0.27 ± 0.21	/		
	"	4	40.1 ± 1.0	0.35 ± 0.27	0.017 ± 0.041		
Real beams	63000	3	41.3 ± 1.1	1.2 ± 0.5	/	0.46 ev/s	1.9 h
	"	4	36.7 ± 4.8	2.6 ± 1.1	0.27 ± 0.22	"	"
	220000	3	40.4 ± 0.5	0.56 ± 0.25	/	"	6.7 h
	"	4	39.8 ± 1.2	0.81 ± 0.34	0.049 ± 0.056	"	"
	"	3	40.5 ± 0.9	b = 0 (fixed)	-0.035 ± 0.043	"	"
	"	4	40.6 ± 2.9	1.0 ± 0.6	0.06 ± 0.13	(without the first point)	
	"	3	41.7 ± 1.3	0.77 ± 0.33	/	("	")

Number of events = Total number of coincidences between opposite counters.

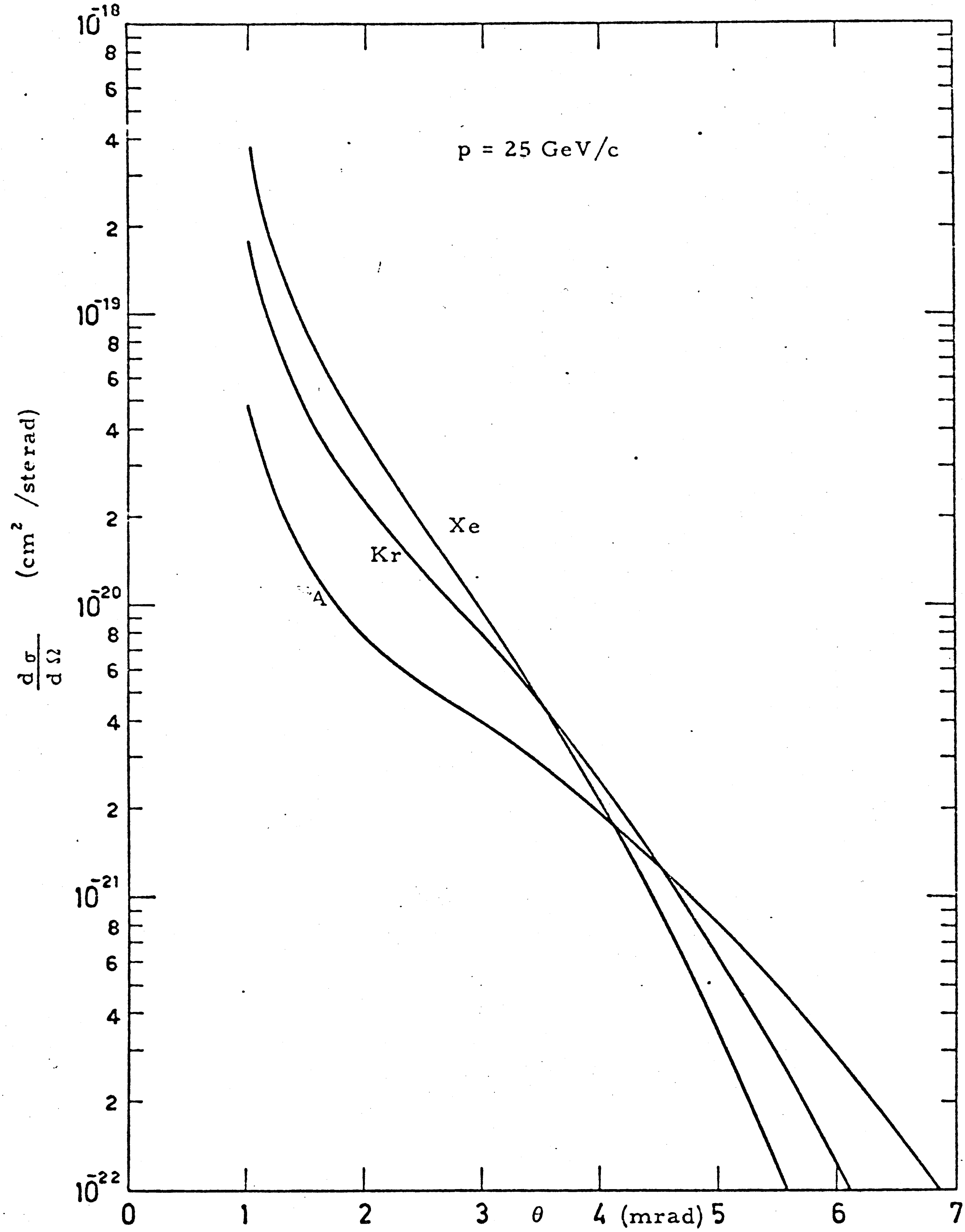
N_{par} = Number of free parameters determined in each fit.

Average rate = Average coincidence rate for a couple of opposite counters.



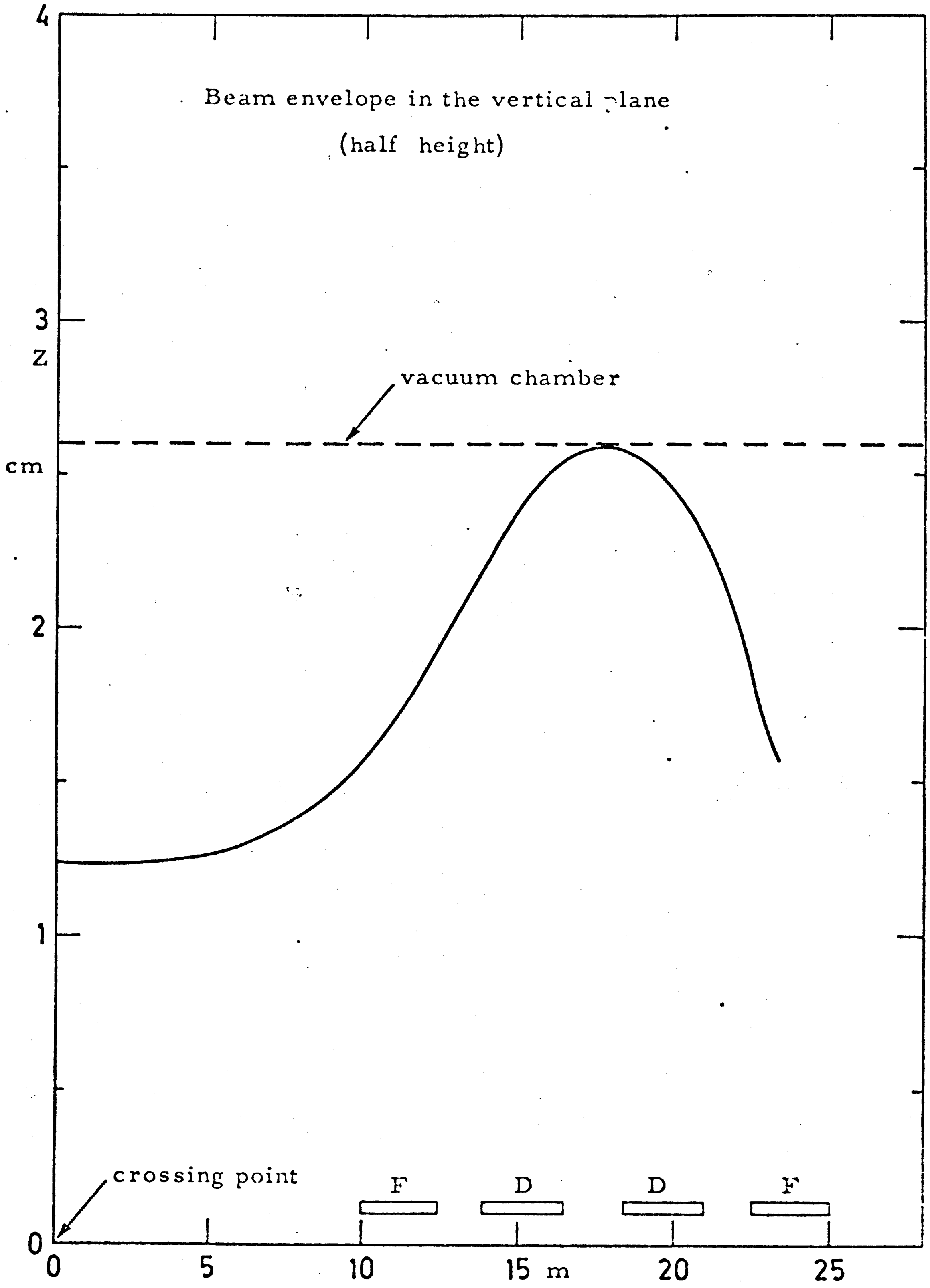
10^{-22}
9
8
7
6
5
4
3
2
10^{-23}
9
8
7
6
5
4
3
2
10^{-24}
$\frac{d\sigma}{d\Omega}$ (cm^2/sterad)
Coulomb
Strong Interaction
Coulomb+Strong Interaction
σ_t = 40 mb
b = 10 (GeV/c)$^{-2}$
25 GeV/c
15 GeV/c
0
1
2
3
4
θ
5 (mrad) 6
7

Fig. 1



p = 25 GeV/c
Xe
Kr
A
$\frac{d\sigma}{d\Omega}$ (cm² /sterad)
10^{-18}
10^{-19}
10^{-20}
10^{-21}
10^{-22}
0
1
2
3
4
5
6
7
θ (mrad)

Fig. 2



Beam envelope in the vertical plane
(half height)
4
3
Z
cm
2
1
0
vacuum chamber
crossing point
F
D
D
F
0
5
10
15 m
20
25

Fig. 3

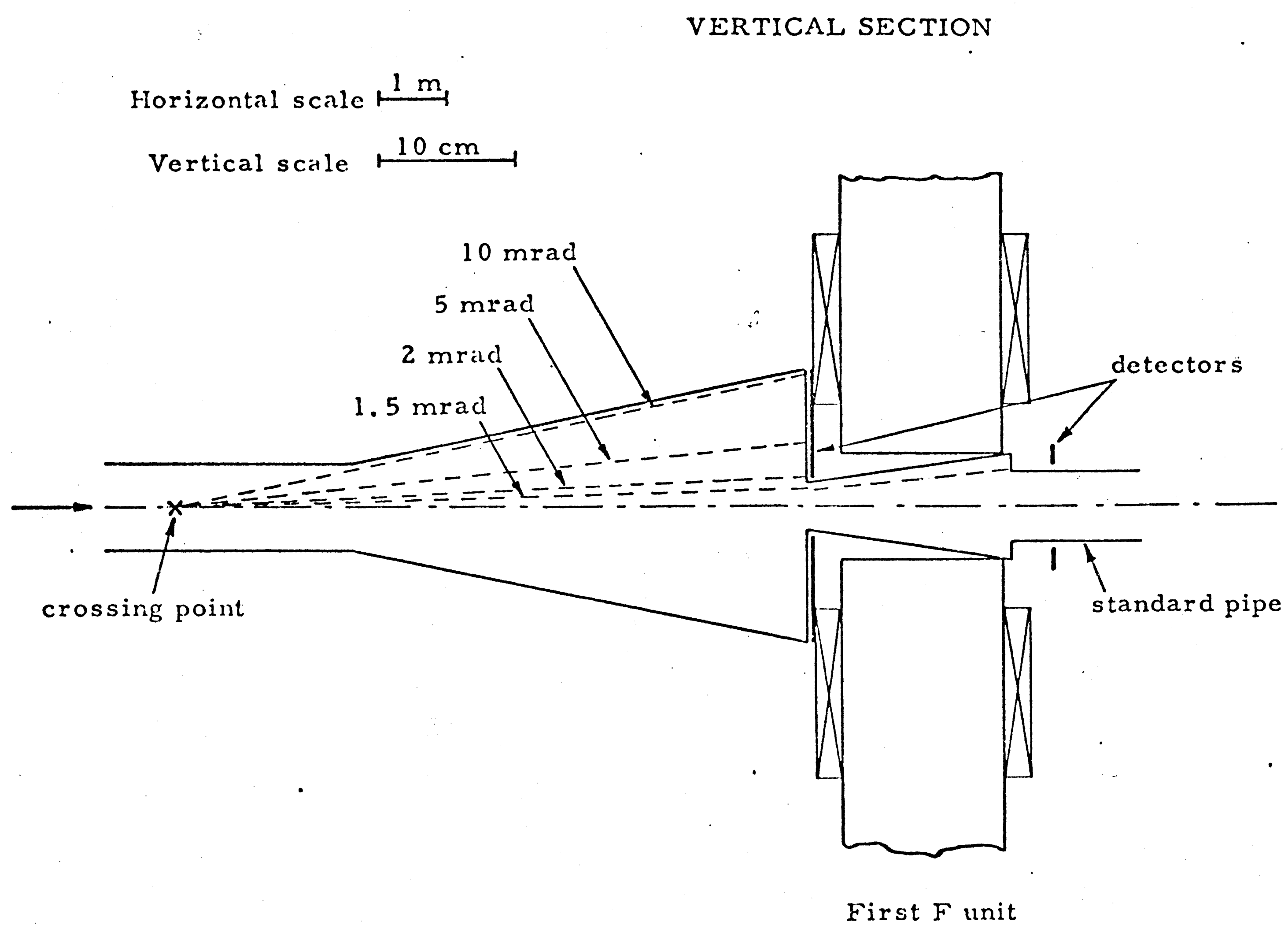

VERTICAL SECTION
Horizontal scale
1 m
Vertical scale
10 cm
10 mrad
5 mrad
2 mrad
1.5 mrad
detectors
crossing point
standard pipe
First F unit

Fig.4

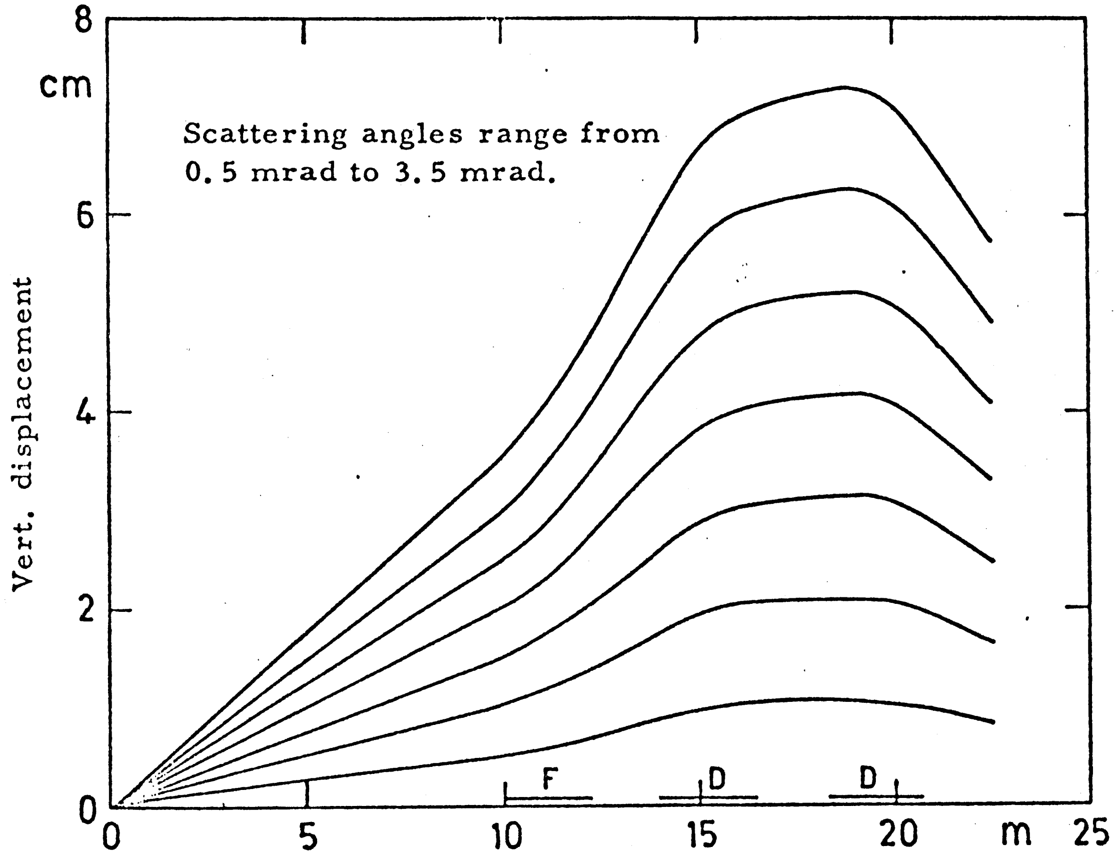

Scattering angles range from
0.5 mrad to 3.5 mrad.
Vert. displacement
cm
0
2
4
6
8
0
5
10
15
20
m
25
F
D
D

Fig. 5

VERTICAL SECTION

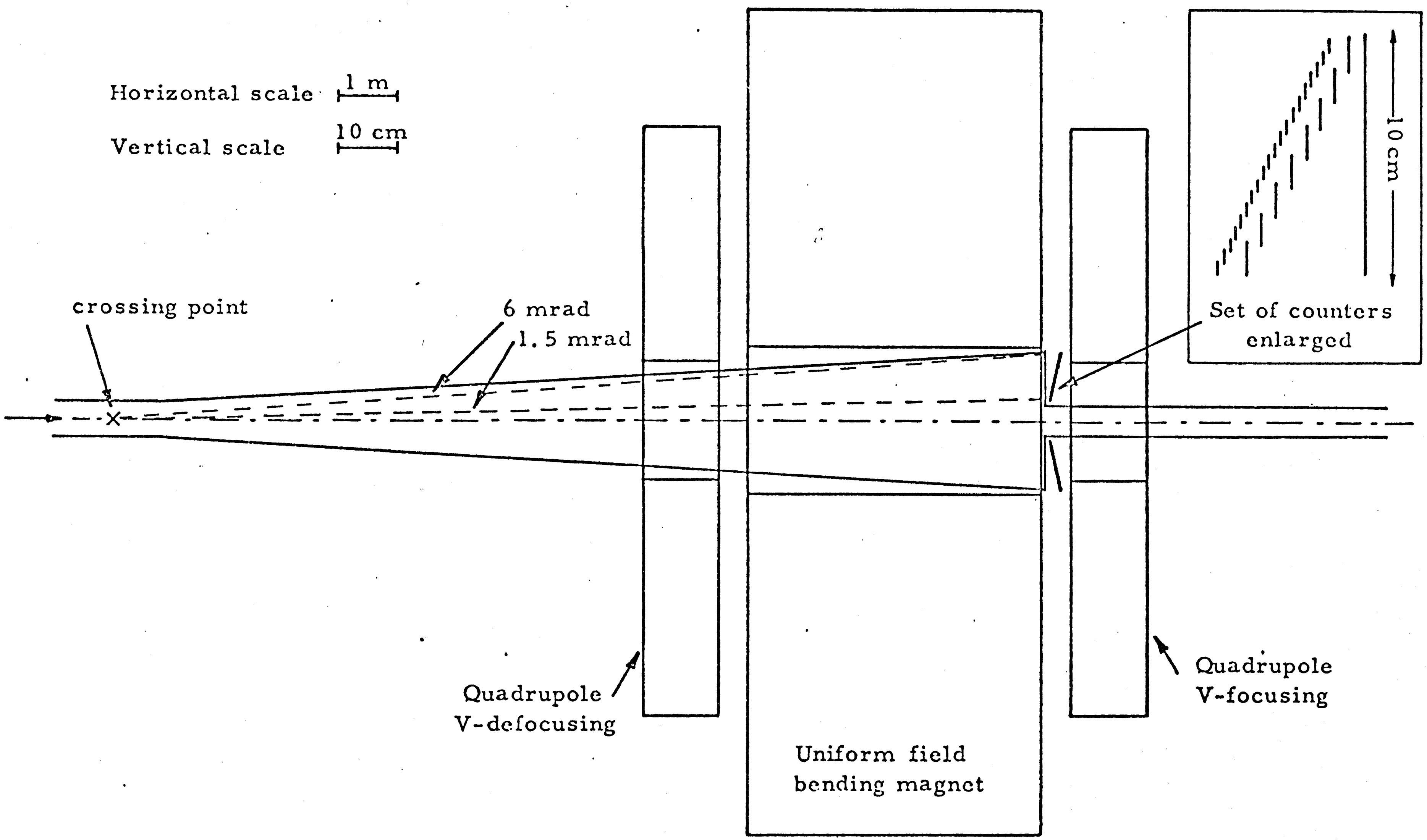


Fig. 6

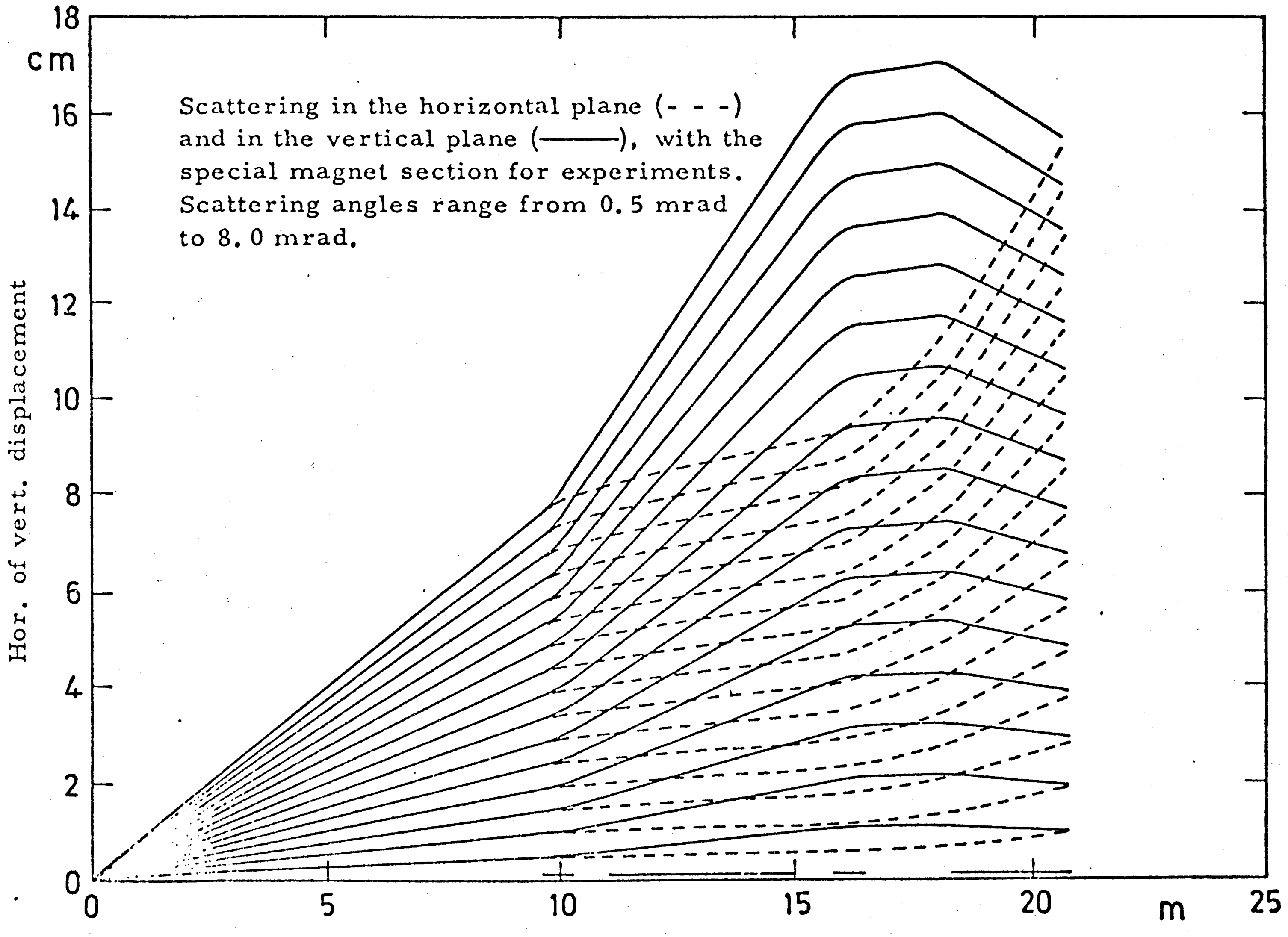

Scattering in the horizontal plane (- - -)
and in the vertical plane (———), with the
special magnet section for experiments.
Scattering angles range from 0.5 mrad
to 8.0 mrad.
Hor. of vert. displacement
cm
18
16
14
12
10
8
6
4
2
0
0
5
10
15
20
m
25

Fig. 7

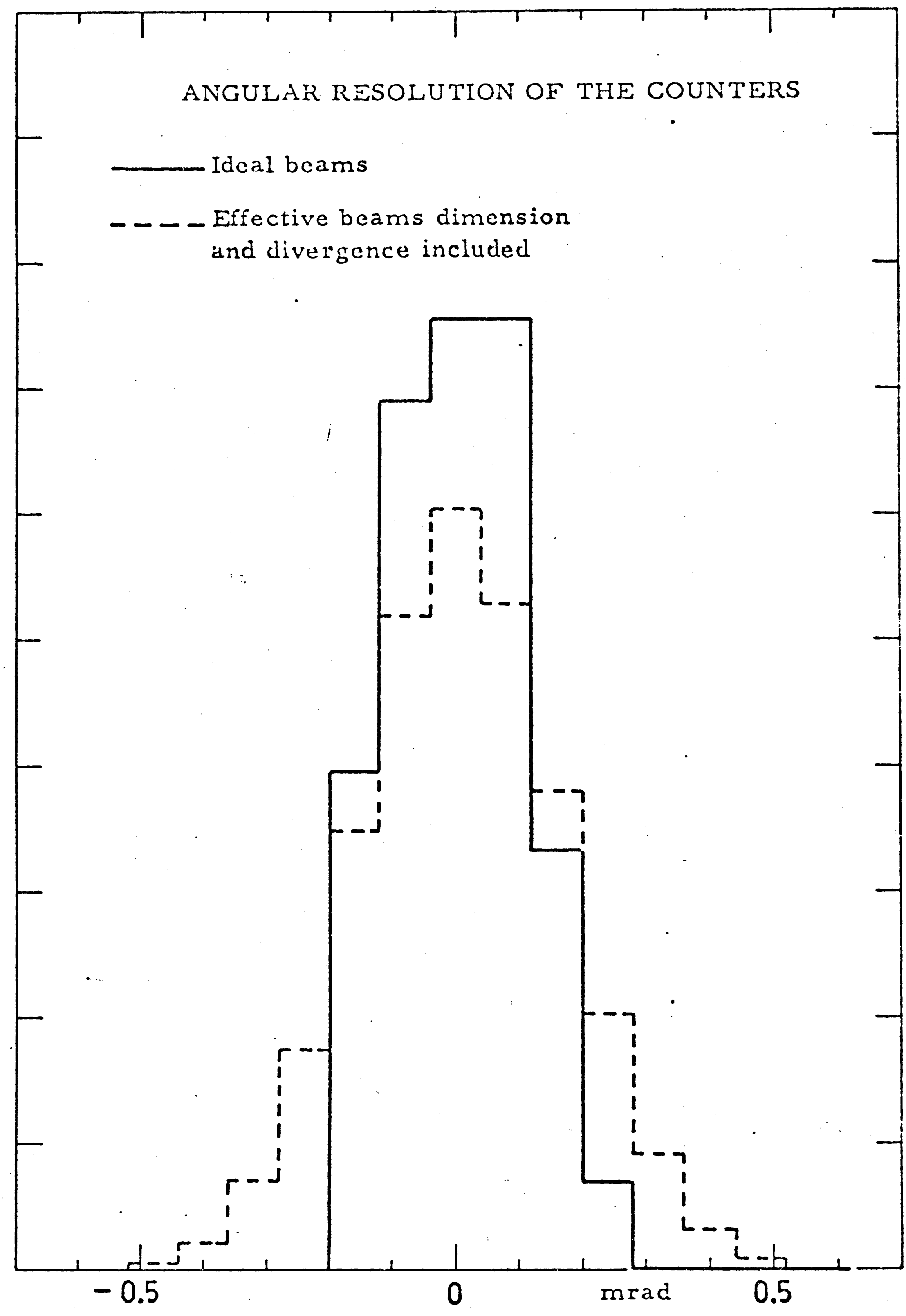
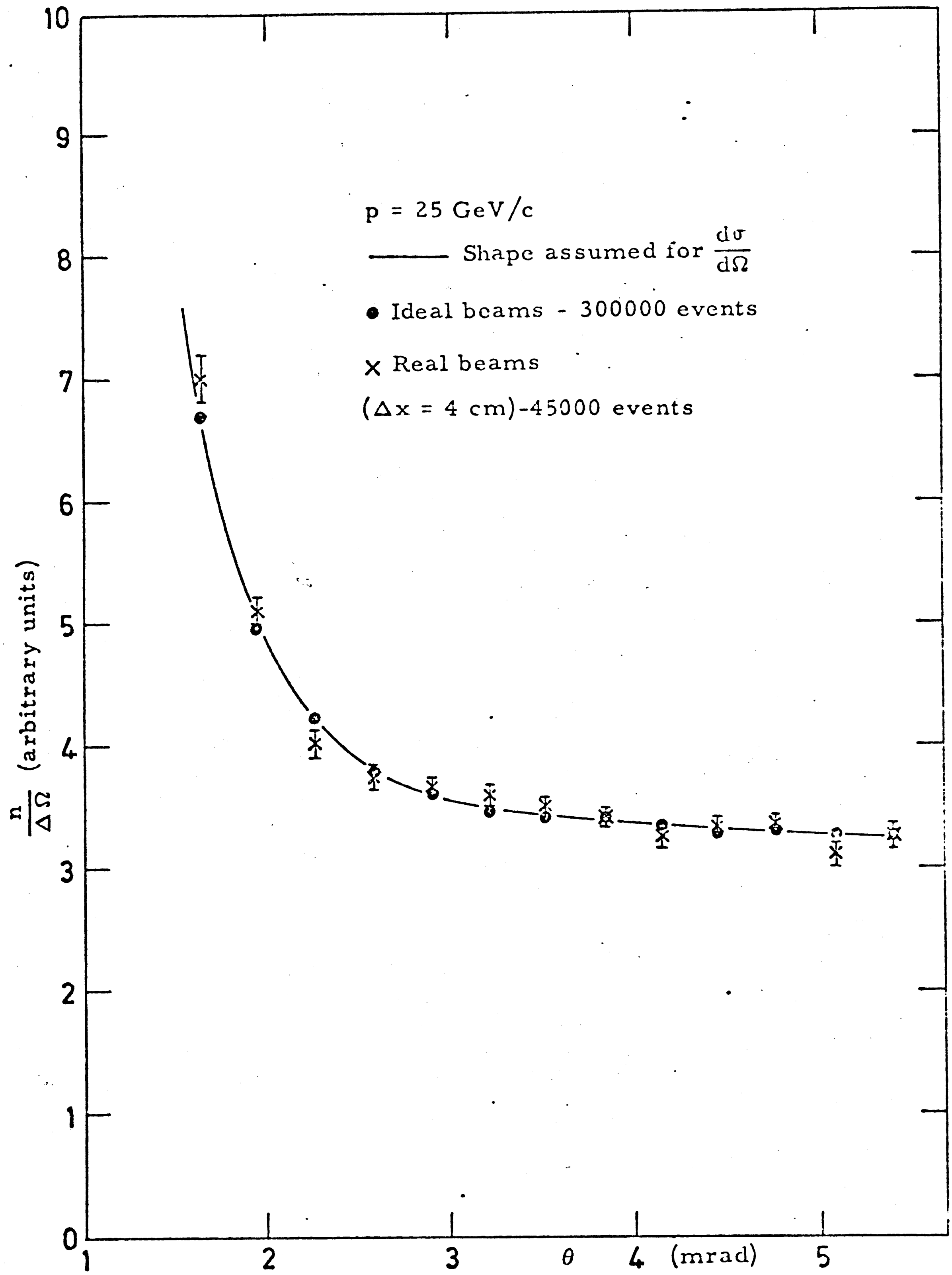

ANGULAR RESOLUTION OF THE COUNTERS
Ideal beams
Effective beams dimension and divergence included
− 0.5
0
mrad
0.5

Fig. 8



p = 25 GeV/c
Shape assumed for $\frac{d\sigma}{d\Omega}$
Ideal beams - 300000 events
Real beams
(Δx = 4 cm)-45000 events
$\frac{n}{\Delta\Omega}$ (arbitrary units)
θ (mrad)
0
1
2
3
4
5
6
7
8
9
10
1
2
3
4
5

Fig. 9

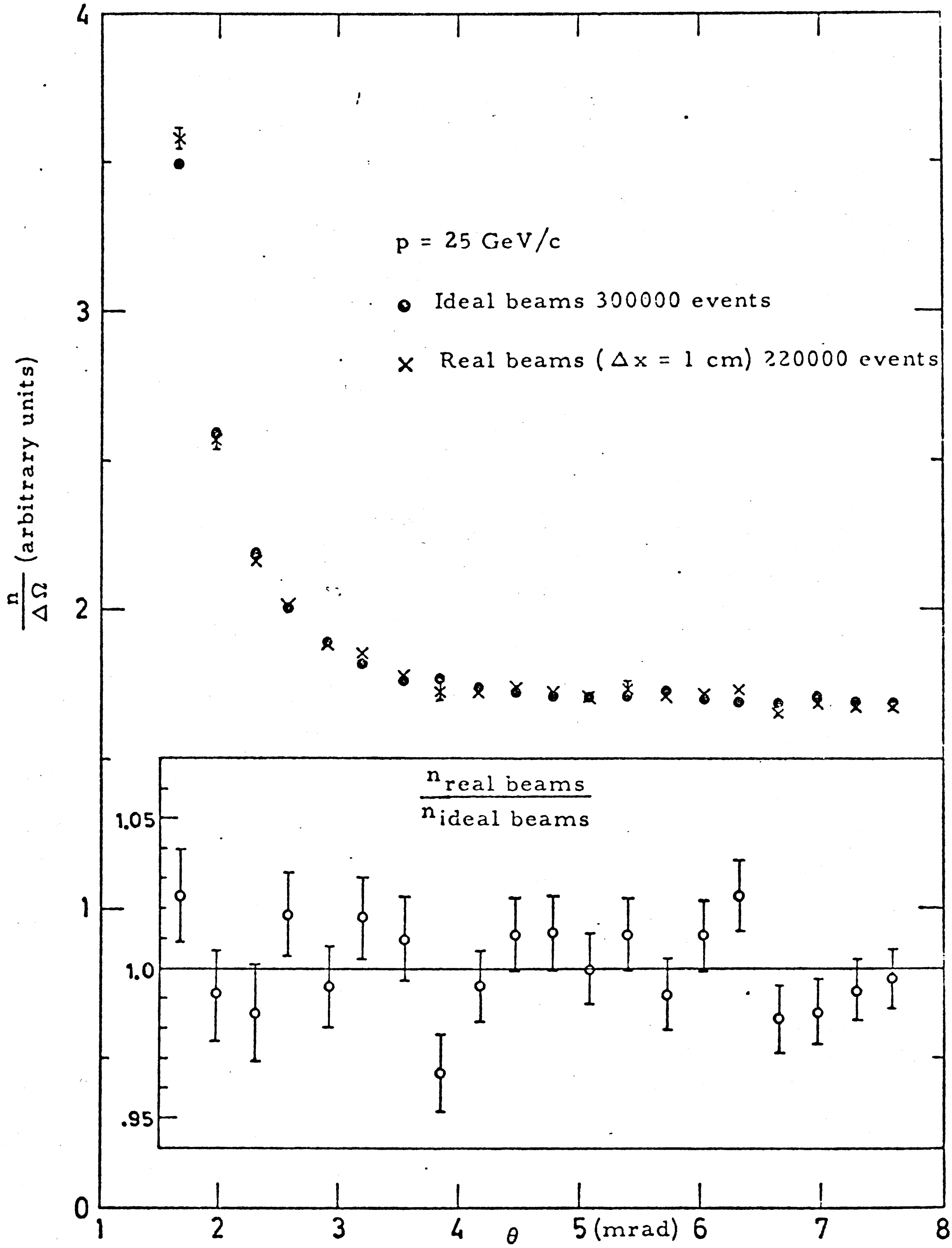

p = 25 GeV/c
Ideal beams 300000 events
Real beams (Δ x = 1 cm) 220000 events
$\frac{n}{\Delta\Omega}$ (arbitrary units)
$\frac{n_{real\ beams}}{n_{ideal\ beams}}$
4
3
2
1
0
1.05
1
1.0
.95
1
2
3
4
θ
5 (mrad)
6
7
8

Fig. 10