Direction Assignment in Wireless Networks

Boaz Ben-Moshe * Paz Carmi ‡ Lilach Chaitman‡ Matthew J. Katz‡ Gila Morgenstern‡
Yael Stein‡

Abstract

In this paper we consider a wireless network, where each transceiver is equipped with a directional antenna, and study two *direction assignment* problems, determined by the type of antennas employed. Given a set S of transceivers with directional antennas, located in the plane. We investigate two types of directional antennas — *quadrant* antennas and *half-strip* antennas, and show how to assign a direction to each antenna, such that the resulting communication graph is connected.

1 Introduction

Wireless ad hoc networks have received much attention in the last decade due to their role in civilian and military applications [3, 4, 6, 7]. A wireless network consists of numerous devices that are equipped with processing, memory and wireless communication capabilities, and are linked via short-range ad hoc radio connections. Each node in such a network has a limited energy resource (battery), and each node operates unattended. Consequently, energy efficiency is an important design consideration for these networks. One way to conserve energy is to use directional antennas, whose coverage area is often modeled by a sector of a given angle and radius. A *direction assignment* is the task of aiming each directional antenna in a certain direction, so that the induced communication graph has some desired properties, such as connectivity. The communication graph G has an edge between two transmitters p and q if and only if p lies in the region covered by q and q lies in the region covered by p.

In this paper we consider two types of directional antennas, *quadrant* and *half-strip* antennas. For the first type (quadrant), we give a two approximation on the required range to ensure connectivity. That is, if there exists a direction assignment with range r, we find a direction assignment with range at most $2r$. For the second type (half-strip), we give a direction assignment to the antennas that induces a connected communication graph, if one exists. At first glance it is not clear why we consider the second type of directional antennas (i.e., half-strip antennas). However, consider Figure 1 where the region covered by a narrow RF antenna is depicted, see also [9] and [2, 5, 8]. We believe that a half-strip is a good approximation for the covered region in this case. Due to space considerations, we restrict our attention to the *decision* version, where r is given and we only need to assign a direction to each antenna, such that the resulting communication graph is connected. Notice that if r is not given, we can find it using binary search on the $O(|S|^2)$ potential ranges. Moreover, this can be done efficiently using parametric search [1].

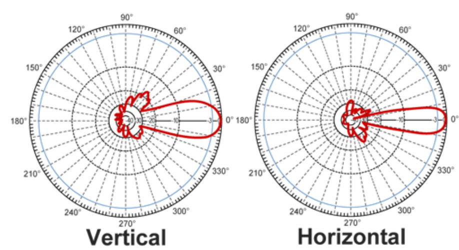

Figure 1: An example of patterns of narrow antennas. Considering a free space pathloss propagation model, the coverage region of a directional antenna can be approximated using a narrow rectangle.

1.1 Notation and problem definitions

Notation 1 *Let $H_r(p)$ denote a vertical strip of width $2r$, such that the vertical line that passes through the point p splits the strip into two equal strips of width r.*

Notation 2 *Let $H_r^+(p)$ denote the upper half-strip of strip $H_r(p)$, having p on its boundary. See Figure 2.*

Notation 3 *Let $H_r^-(p)$ denote the lower half-strip of strip $H_r(p)$, having p on its boundary. See Figure 2.*

In this paper we consider the following problems.

Problem 1 *Given a set S of directional antennas in $\Re^2$ and a range r. The goal is to direct each antenna to one of the four quadrants, such that the resulting communication graph is connected, where there is an edge between p and q, if and only if the following conditions hold.*

*Department of Computer Science, Ariel University Center.
‡Department of Computer Science, Ben-Gurion University, Beer-Sheva, Israel. Partially supported by the Lynn and William Frankel Center for Computer Sciences.

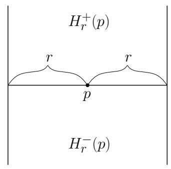

Figure 2: Strip $H_r(p)$ and the two half strips $H_r^+(p)$ and $H_r^-(p)$.

(1) antenna q lies in the quadrant that p is directed to,
(2) antenna p lies in the quadrant that q is directed to,
(3) $|pq| \leq r$

Problem 2 *Given a set S of directional antennas in $\Re^2$ and a range r. The goal is to direct each antenna either upwards or downwards, such that the resulting communication graph is connected, where there is an edge between p and q, if and only if one of the following conditions holds.*

(1) antenna p is directed up, q is directed down, and $q \in H_r^+(p)$.

(2) antenna p is directed down, q is directed up, and $q \in H_r^-(p)$.

2 Quadrant antennas

In this section we consider Problem 1. Let S be a set of n transceivers of transmission range r, each equipped with a quadrant antenna. A quadrant antenna can assume one of four directions: north-east (position I), north-west (II), south-west (III), and south-east (IV). Thus, the coverage area of a transceiver is a quarter disk of radius r that coincides with one of the four quadrants. The goal is to assign one of the four directions (positions) to each of the transceivers, such that the resulting communication graph is connected, where there is an edge between two transceivers, if and only if each transceiver lies in the assigned quadrant of the other and the distance between the two transceivers is at most r. We assume that the transceivers are in general position; i.e., no two transceivers have the same x-coordinate or the same y-coordinate.

Lemma 4 *In a solution to our problem, if one of the antennas is in position I (alternatively, position III), then all antennas are in position I or III.*

Proof. Assume by contradiction that there exist two antennas p and q, such that p is in position I (alternatively, III) and q is in position II or IV. Consider a path P between p and q. Then, there must be two consecutive antennas $p', q' \in P$, such that p' is in position I or III and q' is in position II or IV, and there is an edge between p' and q'. However, since the antennas are in general position this cannot occur — contradiction. □

Corollary 5 *We may assume w.l.o.g. that all antennas are in positions I or III.*

Notation 6 *Let $Arc_r^i(p)$ denote the region covered by transceiver p of range r when its antenna is in position i, where i is either I or III.*

Assuming there exists a direction assignment with range r, such that the resulting communication graph is connected, we show how to find such a direction assignment with range at most $2r$. I.e., we present a 2-approximation algorithm (on the range).

Lemma 7 *Let SOL_r denote a solution with range r. Let p and q be two antennas in S, such that $p \in Arc_r^{III}(q)$ and $q \in Arc_r^I(p)$. Then, there exists a solution SOL_{2r} with range $2r$, in which p is in position I and q is in position III.*

Proof. In order to show this, it is enough to show that any antenna that is connected in SOL_r to p or to q, can be connected to either p or q when p and q are in positions I and III, respectively (and the range is $2r$). Notice that there are exactly four different ways to position two antennas. Thus, antennas p and q are positioned in one of the following four ways in SOL_r.

- **Both p and q are in position I**. Then, all antennas that are connected to q in SOL_r, are connected to p in SOL_{2r}, and antenna p is positioned as in SOL_r.
- **p is in position I and q is in position III**. In this case, antennas p and q are positioned as in SOL_r.
- **Both p and q are in position III**. Symmetric to case 1.
- **p is in position III and q is in position I**. Then, all antennas that are connected to q in SOL_r, are connected to p in SOL_{2r}, and all antennas that are connected to p in SOL_r, are connected to q in SOL_{2r}. □

Lemma 8 *Let $p, q \in S$ be two antennas with direction assignment i, j respectively, such that $Arc_r^i(p) \cap Arc_r^j(q) \neq \emptyset$. Let $s \in Arc_r^i(p) \cap Arc_r^j(q)$. Then, directing s to either p or q will produce a connection between p, q, and s (allowing range $2r$).*

Proof. There are two cases to consider.
Case 1: Antennas p and q are in the same position, w.l.o.g. both are in position I. Then assigning s position III will clearly connect s to both p and q.
Case 2: Antennas p and q are in different positions, w.l.o.g. p is in position I and q is in position III. Then, $|pq| \leq |ps| + |sq| \leq 2r$ (and $p \in Arc_{2r}^{III}(q)$ and $q \in Arc_{2r}^I(p)$). Therefore, there is a connection between p and q, and we can assign s either position I or III to obtain connectivity. □

Corollary 9 *From the above lemmas it follows that Algorithm 1 finds a valid direction assignment with range $2r$, if there exists a direction assignment with range r.*

Algorithm 1 Quadrant assignment

Input: A set P of n quadrant antennas.

Output: A direction to each antenna of P, such that the resulting communication graph (with range $2r$) is connected.

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P' ← P,  Q ← ∅.
while there are two points p, q ∈ P', such that q ∈ Arc_r^I(p) and p ∈ Arc_r^III(q) do
    Choose such p and q
    Q ← Q ∪ {p, q}.
    Assign p position I.
    Assign q position III.
    P' ← P' \ {v ∈ P' | v ∈ Arc_r^I(p) ∪ Arc_r^III(q)}.
S ← P \ Q.
while S ≠ ∅ do
    Choose s ∈ S and p ∈ Q such that p is in position i and s ∈ Arc_r^i(p).
    Direct s to p.
    S ← S \ {s}
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3 Half-strip antennas

In this section we consider Problem 2. Let S be a set of n transmitters in general position (i.e., no two have the same x-coordinate or same y-coordinate), and let r be a real number. Each transmitter $p \in S$ is equipped with a half-strip antenna that can transmit either upwards, covering the region $H_r^+(p)$, or downwards, covering $H_r^-(p)$. The goal is to assign a direction either up or down to each of the antennas, such that the resulting communication graph is connected. Where there is an edge between transmitters p and q, if and only if one of the following cases holds.

• p is directed up, q is directed down, and $q \in H_r^+(p)$.

• p is directed down, q is directed up, and $q \in H_r^-(p)$.

Lemma 10 *If there exists a direction assignment to the antennas of S, such that in the resulting communication graph the leftmost and rightmost antennas are connected, then there exists a solution to our problem (i.e., a direction assignment, such that the resulting communication graph is connected).*

Proof. Let p_0 be the leftmost antenna, p_n the rightmost antenna, and $P = (p_0, p_1, \ldots, p_n)$ a path connecting p_0 and p_n. In order to show that there exists a solution to our problem, it is enough to show that each antenna $q \in S \backslash P$ can be connected to an antenna in the path P, without changing the directions assigned to the antennas of P. q is either above or bellow an edge $(p_i, p_{i+1}) \in P$; w.l.o.g. assume q is above (p_i, p_{i+1}). Moreover, exactly one of the two antennas p_i and p_{i+1} is directed upwards; assume w.l.o.g. that antenna p_i is directed upwards. Then, since q_x between p_{i_x} and p_{i+1_x}, q is already covered by p_i, and by directing q downwards we establish the edge (q, p_i), without changing the directions assigned to the antennas along the path. $\square$

Among all solutions, consider a solution in which the shortest path between the two x-extreme points is shortest (in terms of number of hops), and let OP be such a shortest path.

Lemma 11 *Let* $\mathrm{OP} = (p_0, \ldots, p_n)$*, then for any two antennas $p_i, p_j \in \mathrm{OP}$, $i < j$, that are directed upwards (alternatively, downwards), it holds that p_i is to the left of p_j.*

Proof. Assume to the contrary that there exist two antennas $p_i, p_j \in \mathrm{OP}$, $i < j$, that are both directed upwards and $p_{j_x} < p_{i_x}$. Consider the first such pair of antennas p_i, p_j, $i < j$ (determined by the lexicographic order over $\{(i,j)\}$). We distinguish between two cases.

Case 1: $p_{j_y} < p_{i-1_y}$

Notice that both p_i and p_j are to the right of p_{i-2}. Since, if one of them is leftward to p_{i-2}, then (observing that p_{i-2} is directed upwards) we reach a contradiction; the pair p_{i-2}, p_j already violates the claim. It follows that $p_{i-2_x} < p_{j_x} < p_{i_x}$. But, if so, p_{i-1} also captures p_j (since $p_{j_y} < p_{i-1_y}$), and we can shorten the path OP — contradiction.

Case 2: $p_{j_y} > p_{i-1_y}$

Let $j' \geq j$ be the maximal index such that $p_{j'}$ is directed upwards and is to the left of p_i. Then $p_{j'_y} > p_{i-1_y}$, since, by Case 1, the opposite is impossible. We also know that $p_{i_x} < p_{j'+2_x}$, by the maximality of j'. Now, because $p_{i_y} < p_{i-1_y} < p_{j'_y} < p_{j'+1_y}$, and because $p_{j'_x} < p_{i_x} < p_{j'+2_x}$, we obtain that there is a connection between p_i and $p_{j'+1}$ and thus we can shorten the path OP — contradiction. $\square$

Notation 12 *An edge* $e = (u, v) \in \mathrm{OP}$ *is* good, *where* OP is as in Lemma 11, if for each $s \in S$ such that $u_x < s_x < v_x$, s is not in OP.

Lemma 13 *Let (p_i, p_{i+1}) and (p_{i+1}, p_{i+2}) be two consecutive edges in* OP*, such that $p_{i_x} < p_{i+2_x} < p_{i+1_x}$, then (p_{i+1}, p_{i+2}) is good.*

Proof. Assume by contradiction that edge (p_{i+1}, p_{i+2}) is not good. Assume w.l.o.g. that point p_i is directed up and point p_{i+1} is directed down. Let p_j be a point that lies between p_{i+2} and p_{i+1} (i.e., $p_{i+2_x} < p_{j_x} < p_{i+1_x}$). If $j < i$ then by Lemma 11 p_j is directed down, and if $j > i$ then p_j is directed up. Let us consider these two cases.

Case 1: $j < i$, p_j is directed down.

In this case $p_{j_y} < p_{i+2_y}$, since otherwise there would be a shortcut in the path OP between p_j and p_{i+2}, contradicting the minimality of OP. Consider a direction assignment as in OP with two modifications: p_j is directed up and p_{i+2} down. Since $p_{j-1_x} < p_{i_x}$ and

$p_{j-1_y} < p_{j_y} < p_{i+2_y}$, we have a connection between p_{j-1} and p_{i+2}. Moreover, we have a connection between p_j and p_{i+3}, and between p_j and p_{i+2}, contradicting the minimality of OP.

Case 2: $j > i$, p_j is directed up.
In this case $p_{j_y} > p_{i+1_y}$, since otherwise there would be a shortcut in the path OP between p_j and p_{i+1}, contradicting the minimality of OP. Consider a direction assignment as in OP with two modifications: p_j is directed down and p_{i+1} up. Since $p_{j+1_x} > p_{i+1_x}$ and $p_{j_y} > p_{i+1_y} > p_{i+2_y}$, we have a connection between p_{i+1} and p_{j+1}. Moreover, we have a connection between p_j and p_i, and between p_j and p_{i+1}, contradicting the minimality of OP. □

Corollary 14 *Let $e = (p_i, p_{i+1})$ and $e' = (p_{i+1}, p_{i+2})$ be two consecutive edges in* OP*, such that $p_{i+1_x} > p_{i_x}, p_{i+2_x}$ or $p_{i+1_x} < p_{i_x}, p_{i+2_x}$, then by Observation 13 either edge e or edge e' is good.*

Lemma 15 *Consider any three consecutive edges in* OP*, then at least one of them is good.*

Proof. Assume by contradiction that there are three consecutive edges (p_{i-1}, p_i), (p_i, p_{i+1}), (p_{i+1}, p_{i+2}), such that none of them is good. Then, by Corollary 14 $p_{i-1_x} < p_{i_x} < p_{i+1_x} < p_{i+2_x}$. If there exists a point p_j such that $p_{i_x} < p_{j_x} < p_{i+1_x}$, then by Lemma 11 it cannot be directed upwards nor downwards; thus, there cannot be such a point. We conclude that (p_i, p_{i+1}) is good. □

Observation 16 *Consider the ordered sequence of good edges in* OP *(by x-coordinate), then it follows from Lemma 15 that any two consecutive edges in this sequence are connected either directly or via a single point that lies between them (with respect to the x-axis).*

Corollary 17 *If there exists a solution (with range r), then there also exists a shortest path* OP $= (p_0, p_1, \ldots, p_n)$ *as above. Moreover, from Lemma 15 and Observation 16 it follows that we can apply dynamic programing to find this path, since there always exists an edge that splits the problem into two independent problems. Therefore, Algorithm 2 below finds a valid direction assignment if such exists.*

Algorithm 2 Half-strip assignment

Input: A set S of n transmitters.
Output: A direction assignment to each antenna in S, such that the resulting communication graph is connected.

Let L be the set of potential good edges, sorted by x-coordinate of the left endpoint.
Construct a table for each pair of edges $e_i, e_j \in L$.
Initialize each entry as false.
for each e_i and e_j in L **do**
  if e_i and e_j can be connected directly, or via a point that lies between them (with respect to the x-axis) **then**
    Set entry (e_i, e_j) as true. **else**
    if there exists e' in L between e_i and e_j, such that the entries (e_i, e') and (e', e_j) are both true **then**
      Set entry (e_i, e_j) as true
if (there exists an entry (e_i, e_j) marked true) and (p_0 can be connected to e_i) and (p_n can be connected to e_j) **then**
  Return the direction assignment induced from the table entries.
  Connect each remaining transmitter according to Lemma 10.

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