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Link Loss Inference Algorithm with Minimal Cover Set and Compressive Sensing for Unicast Network Measurements

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Abstract. *To reduce the probe cost and improve the accuracy of the link loss inference, a novel algorithm under network tomography framework is proposed. The number of end-to-end paths is reduced by using minimal cover set measurements. Meanwhile, the accuracy of the link loss inference is improved by the implement of solving linear equations and compressive sensing techniques. Taking into account the constraints in compressive sensing theory, an approach for constructing a novel network tomography model that obeys the constraints of compressive sensing is developed. Simulation results show that this algorithm can obtain a higher accuracy with less end-to-end measurement paths.*
Keywords: Network tomography, Loss rates, Minimal cover set, Compressive sensing

1. **Introduction.** With the growing scale of the Internet, the instability of the network is also increasing. To manage and optimize the network, it is essential to obtain the network performance parameters (e.g. loss rate and delay) accurately and timely[1, 2]. Network tomography which can obtain the internal network parameters without the cooperation of internal nodes, has become one of the focused research technologies in the field of network measurement [3].

Under the network tomography framework, the number of end-to-end measurements is not sufficient to determine the unique state of each link. Therefore, this problem is generally under-determined. Many algorithms have been proposed to address above problem. CLINK [4] uses multiple measurements to learn the probabilities of network links congested. LIA [2] eliminates the least congested links (with the smallest variances) from the system equations, and obtains a full column rank system. Netscope proposed in [5] improves LIA by using l_1-norms minimization with non-negativity constraints. However, both LIA and Netscope use 10-100 snapshots in their methods to infer the link loss variances, which imposes too much additional traffic to the network. Moreover, the complexity of those algorithms grows exponentially with the increasing number of available end-to-end paths. ELIA [6] is composed of two phases. In the first phase, an approach is developed to find all links that can be determined directly. In the second phase, loss rates of rest links can be inferred with high accuracy by solving the utility maximization problem. NLPA [7] converts the link loss rate inference problem into the solution of non-linear programming issue. In contrast to LIA, NLPA has no extra deployment costs.

Selecting probing paths is the major issue of probe-based network link monitoring. Generally, there are two important considerations, minimizing probing cost and achieving identifiability. The probing cost is mainly defined as the number of selected probing paths [8, 9, 10, 11]. For an overlay network with n end hosts, all existing systems require $O(n^2)$ measurements. SPA which selectively monitors k linearly independent paths that can fully describe all the $O(n^2)$ paths is proposed in [8], and the value of k is equal to the rank of the routing matrix. The loss rates and latency of these k paths can be used to estimate the loss rates and latency of all other paths. In [11], all network links are classified into two types. If the performance parameter of a link can be uniquely inferred by a set of probes, this link is identifiable. Otherwise, the link is unidentifiable. Given a set of links to monitor, the objective is to select the minimum number of probing paths that can uniquely determine all identifiable links and cover all unidentifiable links.

In this paper, an efficient loss inference algorithm is developed. With the implement of minimal cover set measurement, the routing matrix which indicates both identifiable and unidentifiable links of the network is simplified, which means loss inference problem can be solved by utilizing less measurement paths. Meanwhile, link loss rates of the network can also be achieved accurately with the combination of the solution of linear equations and compressive sensing techniques.

The rest of this paper is organized as follows. Section 2 introduces the network model and inference problems of the unicast network measurement. Section 3 shows the path selection algorithm for end-to-end measurements with minimal cover set. Section 4 describes the process of link loss rate inference. Section 5 evaluates the performance of the proposed algorithm. Finally, section 6 concludes the paper.

2. Network Model & Inference Problems.

2.1. Network Model.

The network is modelled as a directed graph $G = (V, E)$, where the set of nodes $V = \{v_1, v_2, ..., v_{n_v}\}$ denotes the network routers/hosts and the set of edges $E = \{e_1, e_2, ..., e_{n_e}\}$ represents the communication links connecting them. The numbers of nodes and edges are denoted by $|V|$ and $|E|$, respectively. Let $P = \{p_1, p_2, ..., p_{n_p}\}$ be the set of all paths between the sources and the destinations, and $|P|$ is the number of paths. The routing matrix R of dimension $n_p \times n_e$ is defined as follows. The entry $R_{i,j} = 1$ if the path p_i contains the link e_j. Each row of R therefore corresponds to a path, whereas a column corresponds to a link.

In the example of Figure.1, each of the two sources S_1 and S_2 sends probes to four destinations D_1, D_2, D_3 and D_4. The routing topology contains 7 end-to-end paths and 10 directed links as shown in this figure. Any sequence of consecutive links without a branching point cannot be distinguished from each other using end-to-end measurements, so those links can be combined into one virtual link. As shown in Figure.1, links e_7 and e_{10} cannot be distinguished, they will be converted into a link e_7.

The routing matrix can be expressed by Eq.(1):

$$R = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

All paths in the network are listed in Table.1.

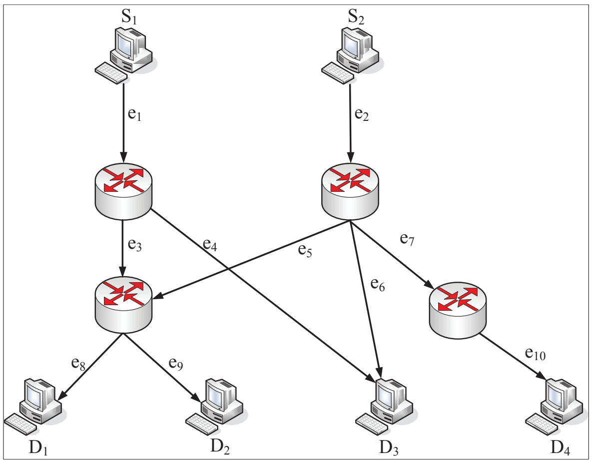

FIGURE 1. Network model

TABLE 1. Paths in the network shown in Figure.1

Paths	Path routes
p_1	$e_1 \to e_3 \to e_8$
p_2	$e_1 \to e_3 \to e_9$
p_3	$e_1 \to e_4$
p_4	$e_2 \to e_5 \to e_8$
p_5	$e_2 \to e_5 \to e_9$
p_6	$e_2 \to e_6$
p_7	$e_2 \to e_7 \to e_{10}$

2.2. **Assumptions.**

2.2.1. *Time-Invariant Routing.* The routing matrix R is assumed to be unchanged throughout the measurement period. This assumption can be violated in the Internet where routing changes can happen at any time-scale. To overcome the influence of routing changes, the network topologies should be measured frequently. However, this process is neglected because it requires a significant amount of repeated traceroute measurements, which is prohibitive in many networks.

2.2.2. *Link Independence.* The packet loss is assumed to be independent among links. Let $\widehat{\phi}_k$ be the random variable describing the fraction of probes traverses link e_k in the current round. The random variable $\widehat{\phi}_k$ is assumed to be independent. This assumption may also not apply to all links, however previous work [12] shows that the correlation of network links is weak and does not significantly affect the accuracy of their diagnosis.

2.2.3. *Identical Sampled Rates.* This assumption means the characteristics of links in the network are stable during the measurement process. For each path p_j, let $\widehat{\phi}_{jk}$ be the fraction of probes that traverses link e_k successfully, then we have $\widehat{\phi}_k = \widehat{\phi}_{jk}$.

2.3. **Loss Rate Inference.** For each path p_j, $\widehat{\Phi}_j$ is defined as the random variable describing the fraction of probe packets that arrives the destination of the path p_j. Its transmission rate is defined as $\Phi_j = \mathrm{E}(\widehat{\Phi}_j)$. Similarly, the transmission rate of link e_k is defined as $\phi_k = \mathrm{E}(\widehat{\phi}_k)$.

Let $L = \{e_1, e_2, ..., e_s\}, e_1, e_2, ..., e_s \in E$ be links that path p_j pass through, the relationship between the transmission rates of path p_j and links can be formulated as follows.

$$\Phi_j = \prod_{k=1}^{s} \phi_k \tag{2}$$

Let Ψ_j be the loss rate of path p_j, where $\Psi_j = 1 - \Phi_j$. Similarly, the loss rate of link e_k can be defined as $\varphi_k = 1 - \phi_k$, then Eq.(2) can be expressed as follows.

$$1 - \Psi_j = \prod_{k=1}^{s} (1 - \varphi_k) \tag{3}$$

Taking the logarithms on both sides of Eq.(3).

$$\log(1 - \Psi_j) = \sum_{k=1}^{s} \log(1 - \varphi_k) \tag{4}$$

The routing matrix R represents the relationship between paths and links, so Eq.(4) can be rewritten with R as follows.

$$\log(1 - \Psi_j) = \sum_{k=1}^{n_e} R_{jk} \log(1 - \varphi_k) \tag{5}$$

Let $x = [\log(1-\varphi_1), ..., \log(1-\varphi_{n_e})]^T$, $y = [\log(1-\Psi_1), ..., \log(1-\Psi_{n_p})]^T$. Then Eq.(5) can be rewritten as follows.

$$y = Rx \tag{6}$$

To identify loss rates of individual links, Eq.(6) has to be resolved. Normally, the number of rows in R is much larger than the number of columns. Unfortunately, R is still column-deficient in most cases. As a result, the unique solution cannot be obtained without additional information on the system.

3. Path Selection based on Minimal Cover Set.

3.1. Simplification of the Routing Matrix. In the process of link loss inference based on network tomography, the probing cost is mainly defined as the number of selected probing paths. For an overlay network with n end hosts, all existing systems require $O(n^2)$ measurements. Chen et al. [8] briefly proposed an algebraic approach that selectively monitors k linearly independent paths that can fully describe all the $O(n^2)$ paths, and the value of k is equal to the rank of the routing matrix. For a power-law networks, the minimal number of probing paths is $k = O(n \log n)$. To select k linearly independent paths from this network, they use the standard rank-revealing decomposition technique, which is a variant of the QR decomposition with column pivoting.

With the conclusion of [8], the number of selected probing paths is limited by the rank of the routing matrix, so it can be reduced by the simplification of the routing matrix. All network links are classified into two types [11], if the performance of a link can be uniquely inferred, this link is identifiable. Otherwise, the link is unidentifiable. If a path is normal (which means the loss rate of this path is low), it is obvious that all links passed through by this path are normal (uncongested links).

$$\Psi_j = 0 \Rightarrow \varphi_k = 0, \forall R_{jk} = 1 \tag{7}$$

Therefore, more identifiable links can be achieved when all columns that corresponding to normal links are removed from the routing matrix. As the network shown in Figure.1, Table.2 lists loss rates of end-to-end paths.

TABLE 2. Loss rates of paths in Table.1

Paths	Path routes	Loss rates
p_1	$e_1 \to e_3 \to e_8$	0.71
p_2	$e_1 \to e_3 \to e_9$	0.48
p_3	$e_1 \to e_4$	0.56
p_4	$e_2 \to e_5 \to e_8$	0.44
p_5	$e_2 \to e_5 \to e_9$	0.0
p_6	$e_2 \to e_6$	0.05
p_7	$e_2 \to e_7 \to e_{10}$	0.08

In this network, links e_7 and e_{10} cannot be distinguished, they will be converted into a link e_7. As the loss rates shown in Table.2, $p_5 : e_2 \to e_5 \to e_9$ is a normal path. Thus, links e_2, e_5 and e_9 are all normal links which can be removed from the routing matrix R. At the same time, links e_6, e_7 and e_8 come to be identifiable, that means loss rates of them can be calculated uniquely. Through repeatedly removing above links, a simplification of routing matrix can be achieved. After removing repeated and all zeros rows, the simplest routing matrix which is composed of columns corresponding to unidentifiable links is determined. The process of the simplification of the routing matrix is shown in Figure.2.

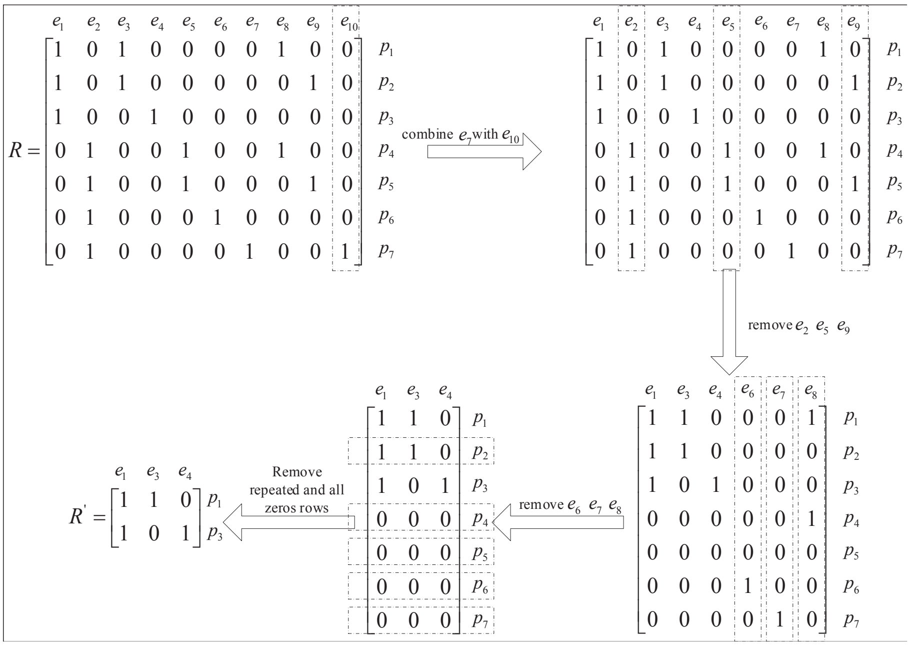

FIGURE 2. The simplification of the routing matrix

Therefore, more identifiable links can be figured out if normal links are achieved as many as possible. In addition, the rank of the routing matrix decreases greatly if more columns (which corresponding to normal links and identifiable links) are removed. As the conclusion of [8], the number of the probing paths is equal to the rank of the simple routing matrix R' which is much smaller than the original routing matrix R. Therefore, the key issue for reducing the number of probing paths is to obtain more normal links.

3.2. **Minimal Cover Set Measurements.** To achieve more normal links, a useful method is to cover more links of a network. As is known to all, to cover each link with a minimum number of measurements is the well-known "minimum set cover" problem. This problem can be resolved by using group testing methods [13], and a greedy algorithm for the minimal cover set problem will be described as follows.

Definition.1 (Minimal Cover Set) Given a universal set U of n elements, denote $S = \{S_1, S_2, ..., S_k\}$ and $c : S \to Q^+$ as the collection of subsets of U and the cost function, respectively. Minimal cover set is defines as the sub-collection of S that covers all elements of U and has the minimum cost.

For network tomography, the set U corresponds to the set of all links and the collection of subsets S are all possible end-to-end measurement paths. The cost function is defined as $c : S \to Q^+$ is $c(S_i) = 1, \forall i \in \{1, 2, ..., k\}$, which denotes the number of probes sent per path. The objective is to seek the sub-collection of S that covers all elements of U and has the minimum number of elements.

The greedy algorithm is very suitable for solving above problem [14]. Firstly, it searches for a path that covers the largest number of links. Then all links covered by the selected path are removed from the routing matrix. The algorithm iterates by searching for next path that covers the largest number of uncovered links until all links in the network have been covered. This set of all selected paths is the minimal cover set.

According to the condition of the research in this field [2, 4, 5, 6, 7], there are no more than 20% congested links (links that have high loss rates) in a network. So it is possible to acquire more normal links by minimal cover set measurements, the number of paths need for end-to-end measurements can be reduced by the simplification of the routing matrix R'. After minimal cover set measurements, all normal links are removed and a simplification of routing matrix is achieved. Therefore, the number of paths for end-to-end measurements can be reduced because of the decrease of the rank of the routing matrix. According to the method proposed in [8], a set of $rank(R')$ paths is selected to deal with end-to-end measurements.

4. Inference of Link Loss Rates based on Compressive Sensing.

4.1. Inference of Loss Rates for Identifiable Links.

When the end-to-end measurement data of $rank(R')$ paths are obtained, the problem of link loss rate inference can be rewritten as follows.

$$\widetilde{Y} = R' \times \widetilde{X} \tag{8}$$

Here, $\widetilde{Y}$ and $\widetilde{X}$ are reduced vectors of Y and X, R' is simplification of routing matrix by using minimal cover set measurements. Eq.(8) is a non-homogeneous linear equation, and its solution $x = [x_1, x_2, ..., x_{n_e}]$ can be expressed by the combination of the general solution and the particular solution.

$$x = W \times r + x_0 = \begin{bmatrix} W_{1,1} & W_{1,2} & \cdots & W_{1,n_e - r_{\mathbf{R}'}} \\ W_{2,1} & W_{2,2} & \cdots & W_{2,n_e - r_{\mathbf{R}'}} \\ \cdots & \cdots & \cdots & \cdots \\ W_{n_e,1} & W_{n_e,2} & \cdots & W_{n_e,n_e - r_{\mathbf{R}'}} \end{bmatrix} \times \begin{bmatrix} r_1 \\ r_2 \\ \cdots \\ r_{n_e - r_{\widetilde{\mathbf{R}}}} \end{bmatrix} + \begin{bmatrix} x_{0_1} \\ x_{0_2} \\ \cdots \\ x_{0_{n_e}} \end{bmatrix} \tag{9}$$

Here, W is a basis for the null space of the routing matrix R', x_0 is a particular solution to Eq.(8). n_e is the number of the links which should be calculated (both identifiable and unidentifiable links), and $r_{R'}$ is the rank of the routing matrix R'.

It is known that the value of x_k depends on the particular solution when $W_{k,i} = 0, \forall i, 1 \leq i \leq n_e - r_{\mathbf{R}'}$ for $1 \leq k \leq n_e$. Therefore, the link corresponding to x_k is an identifiable link, otherwise, it is an unidentifiable link. Therefore, the value of x_k corresponding to each identifiable link can be calculated directly with the particular solution.

4.2. **Inference of Loss Rates for Unidentifiable Links.** After all the columns that corresponding to identifiable links are removed from the routing matrix, Eq.(8) can be rewritten as follows.

$$\overline{\overline{y}} = R^{''}\overline{\overline{x}} \tag{10}$$

Here, $\overline{\overline{Y}}$ is achieved by subtracting the value of x_k corresponding to the unidentifiable links from $\widetilde{Y}$, $\overline{\overline{X}}$ is composed of x_k corresponding to the unidentifiable links. $R^{''}$ is the simplification of $R^{'}$ after all columns that corresponding to identifiable links are removed.

It is well-known there are a few congested links (with high loss rate) in a network. Most values of x_k ($x_k = \log(1 - \varphi_k)$) in $\overline{\overline{X}}$ are zeros except those corresponding to congested links, which means the vector $\overline{\overline{X}}$ is sparse.

Compressive sensing techniques are regarded as efficient and precise means to figure out the single solution from an ill-posed and under-constrained inference problem with data sparsity. However, the measurement matrix $\overline{\overline{X}}$ in Eq.(10) is a deterministic matrix, it is difficult to make sure that it obeys RIP of compressive sensing theory. To construct a measurement matrix meets RIP, an approach proposed in [15] is adopted to construct the measurement matrix as follows.

$$\mathrm{M} = G\Sigma(\lambda)R^{''} \tag{11}$$

where G is a Gaussian random matrix whose entries are i.i.d asymptotically normally distributed. $\Sigma(\lambda)$ is a diagonal matrix whose entries are 1 or 0, and the number of zero entries is λ. $\Sigma(\lambda)$ is a sampling matrix in practice, it is used to delete λ rows of routing matrix $R^{''}$ with λ-largest l_1-norms. Eq.(10) can be converted into

$$L = G\Sigma(\lambda)\overline{\overline{y}} = G\Sigma(\lambda)R^{''}\overline{\overline{x}} = \mathrm{M}\overline{\overline{x}} \tag{12}$$

Generally speaking, the orthogonal match pursuit algorithm (OMP) can be adopted to solve Eq.(12). It is known that the link loss rate ranges from 0 to 1, so the value of $x_k : x_k = \log(1-\varphi_k)$ in vector $\overline{\overline{x}}$ ranges in (-∞, 0]. However, the l_1-minimization problem is unable to make sure the solutions $\overline{\overline{x}}$ are negative. To solve this problem, the iterative proportional fitting algorithm [16] is utilized to revise the estimation results by setting all non-negative entries to zero.

5. **Simulations Results and Analysis.** To validate the performance of the proposed link loss inference algorithm with minimal cover set and compressive sensing (MCSA for short), a series of experiments are conducted. SPA [8] and ELIA algorithms [6] are used to compare with the proposed MCSA algorithm.

5.1. **Experiment Setup.**

5.1.1. *Network Topologies.* The network topology influences the correlations among end-to-end paths. There are two typical network topologies: classical random networks and power-law networks [17]. The random network was popular in the past several years, and it was considered as a baseline model. However, in recent years, more and more attention has been paid to power-law networks because many real-life networks, both artificial (Internet, world wide web, peer-to-peer systems, etc.) and natural (ecological, biological, social, etc.) are revealed to follow power-law degree distributions [18]. In this paper, power-law networks used in each experiment are generated by the BRITE generator [19].

5.1.2. *Loss Rates.* In each experiment, each link is set to be congested with a probability p. Since p affects the diagnostic accuracy, its value varies in a range to evaluate the performance of the proposed algorithm under different congestion levels. Moreover, two different loss rates models LLRD1 and LLRD2 of [20] are adopted for assigning loss rates to links. In the LLRD1 model, congested links have loss rates uniformly distributed in [0.05, 0.2], and normal links have the loss rates in [0, 0.002]. In LLRD2 model, the loss rate ranges for congested and normal links are [0.002, 1] and [0, 0.002], respectively. Since the results of those two models are not very different, we only present our results for the LLRD1 model. After assigning a loss rate for each link, the actual loss on each link follows a Gilbert process. The link in the Gilbert model fluctuates between normal and congested states. Links do not drop any packet when in a normal state, and drop all packets when in a congested state.

5.1.3. *End-to-end Measurements.* Leaf-nodes in the network topologies are defined as the end hosts, which make up about 30% of the network nodes. Each end host sends probe packets to the rest of the hosts to estimate the loss rates of paths. All paths between any two hosts are generated following the shortest-path rule in order to simulate the QoS mechanism.

5.2. **Metrics.** To illustrate the experiment results more intuitively, the following well-known metrics are considered for the evaluation of the proposed MCSA algorithm.

(1) Detection rate (DR)

$$DR = \frac{|F \cap X|}{|F|} \tag{13}$$

(2) False positive detection rate (FPR)

$$FPR = \frac{|X \backslash F|}{|X|} \tag{14}$$

where F denotes the set of the actual congested links, and X denotes the set of links identified as congested by the inference algorithms.

Suppose q is the real loss rate on a link and $\widehat{q}$ is the inferred loss rate of that link. Given some error margin $\delta > 0$, the error factor is defined as follows.

(3) Error factor (EF)

$$f_\delta(q, \widehat{q}) = \max\left\{\frac{q(\delta)}{\widehat{q}(\delta)}, \frac{\widehat{q}(\delta)}{q(\delta)}\right\} \tag{15}$$

where $q(\delta) = \max\{q, \delta\}$ and $\widehat{q}(\delta) = \max\{\widehat{q}, \delta\}$. The value of δ is set to 10^{-3} for all experiments.

5.3. **Simulation Results.**

5.3.1. *Number of Paths for Measurements.* Figure.3 and Figure.4 show the number of paths required by SPA and MCSA algorithms in different scales of networks for Waxman and BA topologies. The percentage of congested links is fixed to 10%. As shown, the number of paths required by MCSA algorithm is less than SPA algorithm under the same network size. As the network size grows, the gap between those two curves becomes even larger.

For Waxman topologies in Figure.3, the number of paths required by MCSA algorithm (1261.6) is 64.4% percent of SPA algorithm (1959.1) when the number of the links is about 2000. For BA topologies in Figure.4, the number of paths required by MCSA algorithm (1219.2) is 64.7% percent of SPA algorithm (1885.2) at the same size of the network.

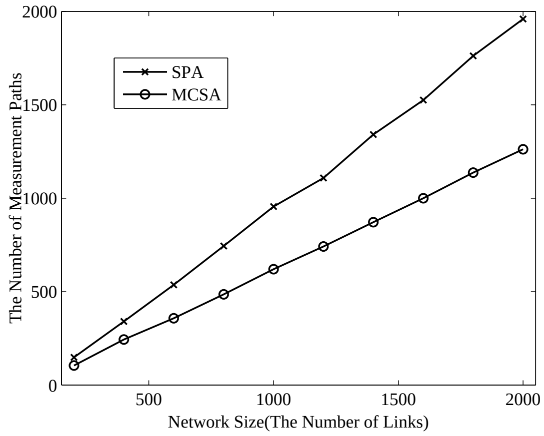

Figure 3. The number of measurement paths under the Waxman topology model versus the number of links

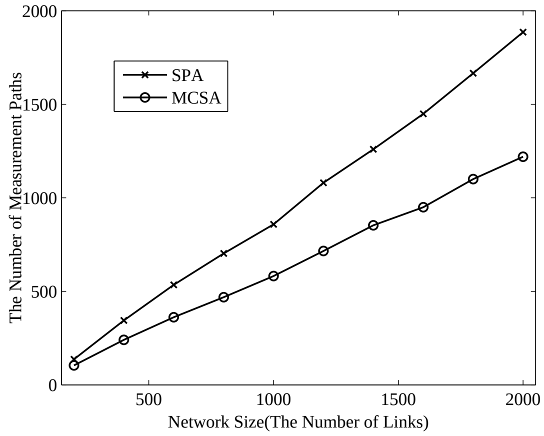

Figure 4. The number of measurement paths under the BA topology model versus the number of links

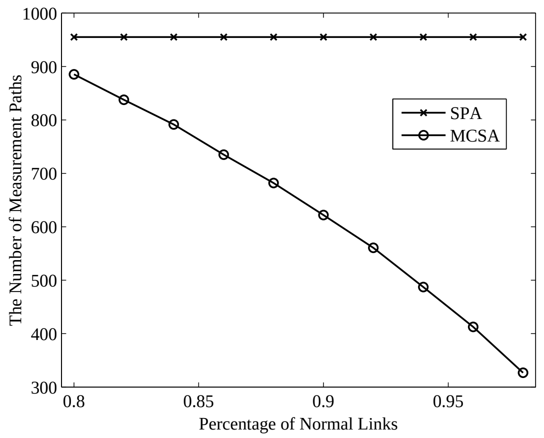

Figure 5. The number of measurement paths under the Waxman topology model versus the percentage of normal links

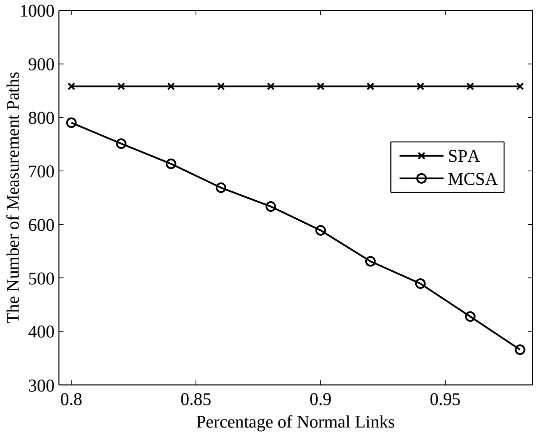

FIGURE 6. The number of measurement paths under the BA topology model versus the percentage of normal links

Figure.5 and Figure.6 show the number of paths required by SPA and MCSA algorithms under different congestion probabilities for Waxman and BA topologies. The network size is set to 1,000 links. As shown, the number of paths required by MCSA algorithm is less than SPA algorithm at the same congestion probability. As the congestion probability decreases, the gap between those two curves becomes even larger.

For Waxman topologies in Figure.5, the number of paths required by MCSA algorithm (326.9) is 34.2% percent of SPA algorithm (955.2) when the congestion probability is 0.02. For BA topologies in Figure.6, the number of paths required by MCSA algorithm (356.8) is 42.6% percent of SPA algorithm (858.6) at the same congestion probability.

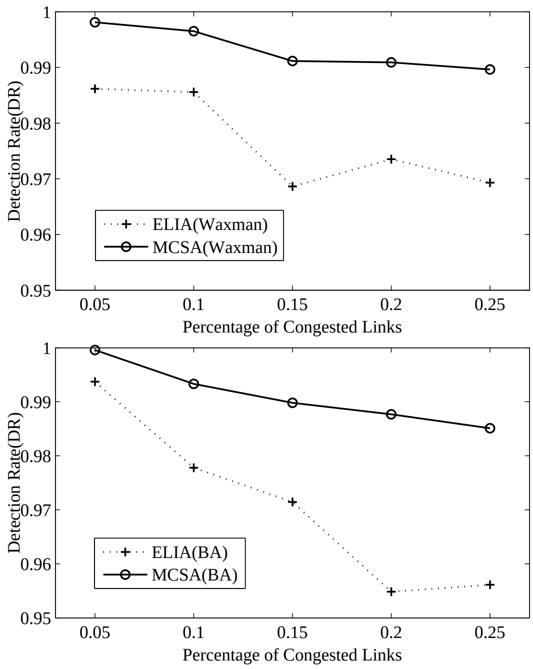

FIGURE 7. Detection rate versus the percentage of congested links

In the network with the low congestion probability, MCSA algorithm can find normal paths and normal links by using minimal cover set measurements. When columns corresponding to normal links are removed, the rank of the routing matrix which is equal to the number of paths required by end-to-end measurements will decrease greatly.

5.3.2. *Diagnostic Ability.* Figure.7 and Figure.8 show the network fault diagnostic ability of SPA and MCSA algorithms under different congestion probabilities for Waxman and BA topologies. The network size here is set to 1000 links. As shown, the proposed MCSA algorithm performs well for all topologies, while the accuracy of ELIA drops as the percentage of congested links goes up. For the reason that, in the network with high congestion probability, Gauss model cannot describe the transmission rates of links in a network accurately.

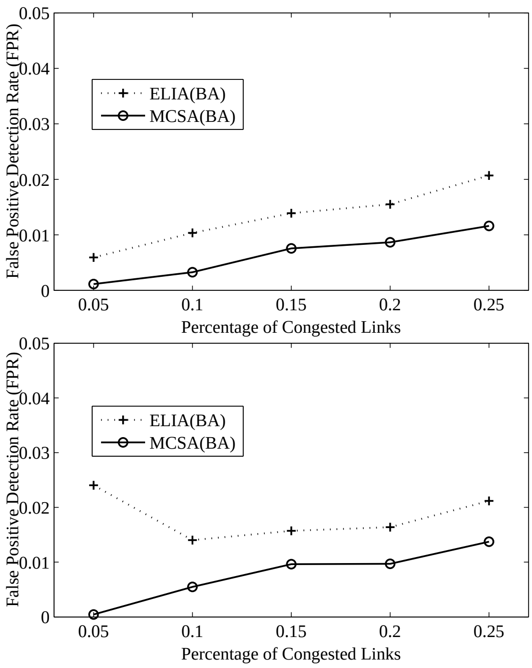

Figure 8. False positive detection rate versus the percentage of congested links

In Figure.7, MCSA algorithm's DR (99.2%) is 2.0% higher than ELIA algorithm (97.2%) for Waxman topologies when the congestion probability is 0.25. For BA topologies, MCSA algorithm's DR (98.8%) is 3.3% higher than ELIA algorithm (95.5%) at the same congestion probability.

5.3.3. *Accuracy of Link Loss Inference.* Figure.9 and Figure.10 compare mean absolute errors and mean error factors of SPA and MCSA algorithms for Waxman and BA topologies. The network size here is set to 1,000 links. As shown, both errors of those two algorithms are small. With the increase of the congestion probability, the gap between those two curves becomes even larger. However, the result of MCSA algorithm is 1/3 percent of that of ELIA algorithm when the congestion probability is 0.25, so MCSA algorithm can identify most of the link loss rates precisely.

Figure.11-Figure.14 show the cumulative distribution function of absolute errors (the difference between the actual and the inferred loss rates) and error factors for the lossy links for the particular case where 15% of links in the network are lossy. For MCSA algorithm, 80% of the lossy links have an absolute error of less than 0.02, which means

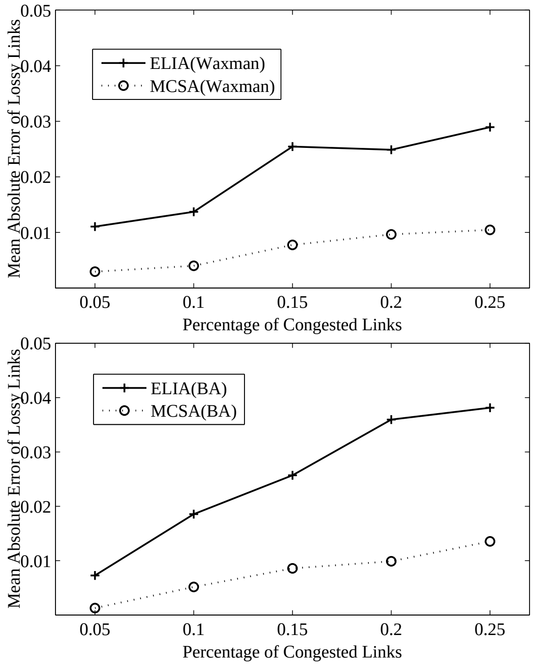

FIGURE 9. Absolute errors of lossy links versus the percentage of congested links

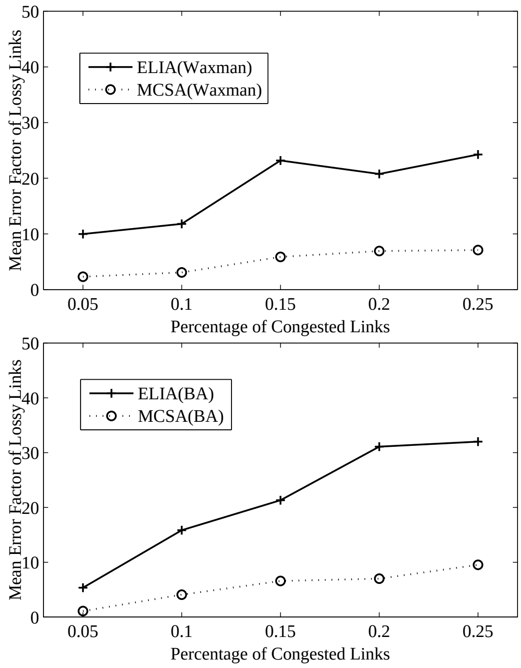

FIGURE 10. Error factors of lossy links versus the percentage of congested links

that, if a link has $X\%$ loss, we infer that it has a loss in the range $X \pm 2\%$, while ELIA algorithm infers that it has a loss in the range $X \pm 4\%$, so it is 2% worse in identifying the actual link loss rates. Similarly, for 80% of the lossy links, MCSA has an error factor of less than 10, while ELIA algorithm achieves an error factor of less than 30.

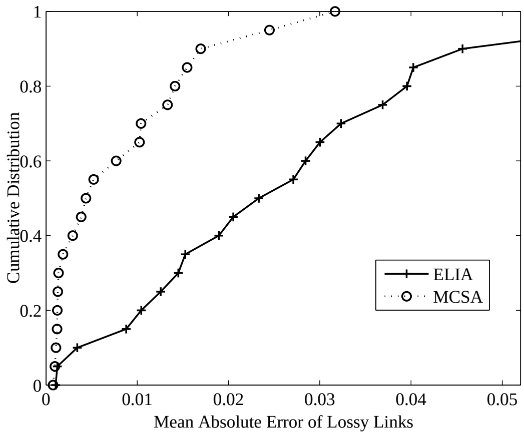

FIGURE 11. Cumulative distribution of absolute errors for lossy links under the Waxman topology model

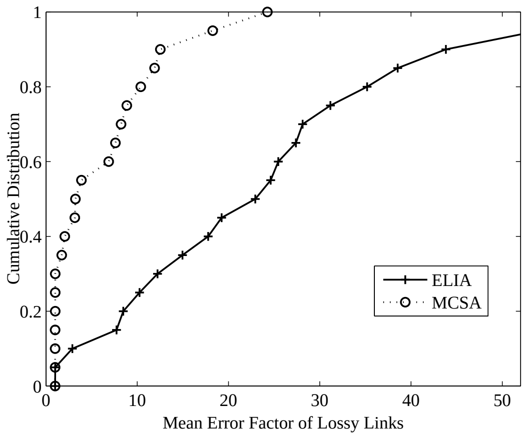

FIGURE 12. Cumulative distribution of absolute errors for lossy links under the BA topology model

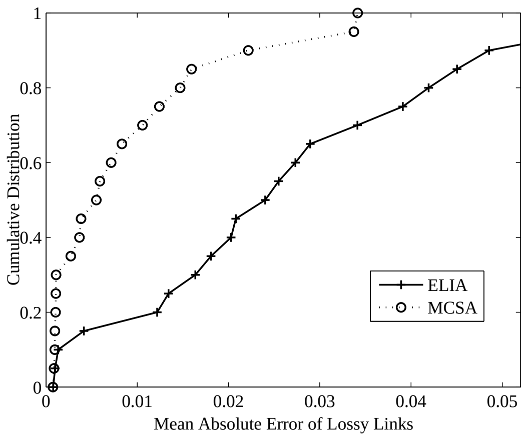

FIGURE 13. Cumulative distribution of error factors for lossy links under the Waxman topology model

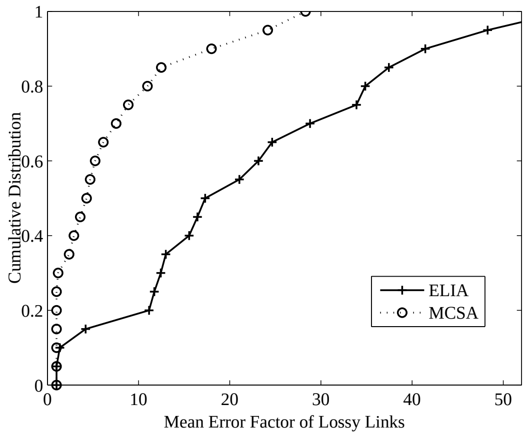

Figure 14. Cumulative distribution of error factors for lossy links under the BA topology model

6. **Conclusions.** In this paper, a link loss inference algorithm named MCSA is proposed to make an improvement on the probing cost and accuracy. To reduce the number of paths required for end-to-end measurements, the minimal cover set of grouping test is employed to achieve more normal links, and simply the routing matrix by removing the columns corresponding to normal links. Then, all network links are classified into identifiable links and unidentifiable links, and loss rates of identifiable links are estimated using the particular solution of linear equations. Finally, loss rates of unidentifiable links are achieved by compressive sensing techniques. The proposed MCSA algorithm is evaluated by experiments, the results show that it has good performance on both accuracy and efficiency.

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