

## Contour Retrieval and Matching by Affine Invariant Fourier Descriptors

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### Abstract

*In the image plane, a 3 Dimensional movement of planar rigid object can be represented satisfactory by an affine motion of his boundary (the exterior contour) which we assume to be closed. The object movement can be described by a special affine transformation of the plane  $\mathbb{R}^2$ . The extraction of invariant features under affinities is important in such procedure. Recently we have proposed a new shape representation invariant to affine transformation based on Fourier descriptors [3]. The performances of the new family was analysed with experiments on the numerical invariance, the robustness of the shape features and the invariance to parameterization [3][4]. In this paper, we present two applications of the affine descriptors for contour-based retrieval and curve matching. Experimental results on a curve database subject to perspective distortions show that these features are very robust against such distortions. A comparative study with other affine invariant descriptors will be presented in the context of contour retrieval.*

### 1. Introduction

A fundamental problem in computer vision is to deal with image observations under different geometric transformations. Sensitivity of the object's appearance to the pose of the observer makes this a common problem in almost all computer vision applications. The hardness of the problem depends on the nature of the geometric transformation involved. It is much more difficult to recognize objects which have been subject to arbitrary perspective distortions than objects where the only transformation is shifting along the image plane. For near planar objects, these deformations can be modelled approximately by affine transformation, if the viewpoint is sufficiently far away. In the literature, there are many papers on the shape recognition invariant under affine transformation. Some examples are affine-invariant Fourier descriptors [1], affine invariant moments [5][13] and affine curvature scale space [9]. In these works, the underlying idea is to use an affine parameterization, usually the affine-length parameter [1][5][13].

Some of the main ideas of the approach followed in this paper have been introduced in [3][4]. In these works, a complete and stable set of Affine Invariant Fourier Descriptors (AIFD) have been proposed. Experiments have shown that these descriptors are robust under affine transformations. In this paper, we propose to validate the robustness of the proposed descriptors in the context of shape-based retrieval system and curve matching. We will show that the AIFD are very robust against affine transformations and much more for strong distortions such perspective distortions.

The rest of the paper is organized as follows:

In Section 2, we remind the formulation of the Affine Invariant Fourier Descriptors. The performance of the AIFD in the context of curve retrieval as well as a comparative study with other well known affine invariants will be presented in Section 3. A curve Database with shapes related by perspective distortions will be used for experiments. In section 4, we will present a new approach for matching two curves related by an affine transformation.

### 2. Affine Invariant Fourier Descriptors (AIFD):

In [3] we have proposed a complete and stable set of Affine-Invariant Fourier Descriptors. These invariants are computed on a normalized affine arc length re-parameterization. In the following we describe the main steps for the definition of the AIFD. For more details the reader can consult [3][4].

For a parametric contour  $\gamma$ , given by its Cartesian coordinates  $x$  and  $y$  (formally,  $\gamma(t) = (x(t), y(t))$  where  $t$  represent the associated parameter). We re-parameterized the contour using the affine-length parameter:

$$s(t) = \frac{1}{L} \int_a^t \|\gamma'(t) \wedge \gamma''(t)\|^{2/3} dt \quad (1)$$

where  $L$  denotes the total equi-affine length of the considered contour.

Let  $\alpha$  and  $\beta$  positives real, and  $k_0, k_1, k_2$  and  $k_3$  four positives integers. Let's notice  $c_n^x$  and  $c_n^y$ , respectively, the complex Fourier coefficients of the coordinates  $x$  and  $y$ ,  $\Delta$  denotes the determinant and

$$\Delta_n^m = \Delta \begin{pmatrix} c_n^x & c_m^x \\ c_n^y & c_m^y \end{pmatrix},$$

The both families of descriptors  $I$  and  $J$  are respectively given by Eq.2 and Eq.3. For more details, derivations, proofs, the reader is referred to [6].

$$I: \begin{cases} I_{k_1}(C) = |\Delta_{k_1}^{k_0}| \\ I_{k_2}(C) = |\Delta_{k_2}^{k_0}| \\ I_k(C) = \left(\Delta_{k_k}^{k_0}\right)^{k_1-k_2} \left(\Delta_{k_1}^{k_0}\right)^{k_2-k} \left(\Delta_{k_2}^{k_0}\right)^{k-k_1} \\ \quad \left|\Delta_{k_1}^{k_0}\right|^{k-k_2+\alpha} \left|\Delta_{k_2}^{k_0}\right|^{k_1-k+\beta} \end{cases} \quad (2)$$

for all  $k \in \mathbb{N}^* - \{k_0, k_1, k_2\}$ .

$$J: \begin{cases} J_{k_1}(C) = |\Delta_{k_1}^{k_3}| \\ J_{k_2}(C) = |\Delta_{k_2}^{k_3}| \\ J_k(C) = \left(\Delta_{k_k}^{k_3}\right)^{k_1-k_2} \left(\Delta_{k_1}^{k_3}\right)^{k_2-k} \left(\Delta_{k_2}^{k_3}\right)^{k-k_1} \\ \quad \left|\Delta_{k_1}^{k_3}\right|^{k-k_2+\alpha} \left|\Delta_{k_2}^{k_3}\right|^{k_1-k+\beta} \end{cases} \quad (3)$$

for all  $k \in \mathbb{N}^* - \{k_1, k_2, k_3\}$ .

In [3] the performances of this new family with experiments on the numerical invariance, the robustness of the shape features and the invariance to re-parameterization have been analysed. Experiments have shown that the AIFD are robust under affine transformations and, therefore, were very useful in many computer vision applications.

In the following, we present the application of the AIFD for contour-based retrieval systems and curve matching.

### 3. Application to Shape-Based Retrieval:

An important problem in object retrieval is the fact that an object can be seen from different viewpoints, resulting in different images. Consequently, the invariance to viewpoints is a desirable property in many shape recognition systems. To evaluate the performance of the AIFD under perspective distortions we use the Multiview Curve Dataset (MCD)[14].

In the following we outline the shape retrieval algorithm. The retrieval process consists in finding the most similar model represented by a curve in a database. The database contains the AIFD of curve models. Given a query curve we first extract the AIFD next we look for similar AIFD in the database using the Euclidean distance. The search for similar descriptors in the database is sequential. Indeed, the performance of the sequential search is sufficient for the evaluation of the AIFD in the context of image retrieval, which is the purpose of this part of the work.

#### 3.1 Experimental results on a curve database subject to perspective distortions

The experiments for shapes with perspective changes were carried out on the Multiview Curve Dataset. This latter

comprises 40 shape categories, each corresponding to a shape drawn from an MPEG-7 shape category [2]. Each category in the new dataset contains 7 curve samples that correspond to different perspective distortions of the original shape. The original MPEG-7 shapes were printed on white paper and 7 samples were taken using a digital camera from various angles (Figure 1). The contours were extracted from the iso-intensity level set decomposition of the images [7].

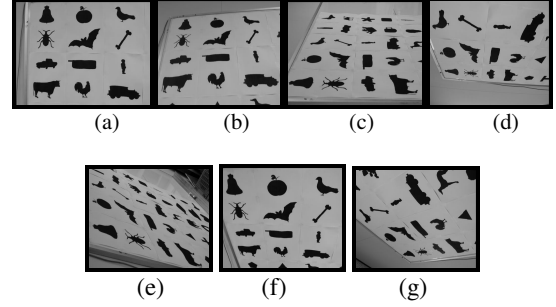


Figure 1. Some Examples of Images from the MCD database acquired from different viewpoints; (a) : Central (b) Bottom (c) Left (d) Right, (e) Top (f) Top-left, (g) Bottom- Right

In Figure 2 we display the retrieval results obtained using a random query from the MCD (“Insect”).

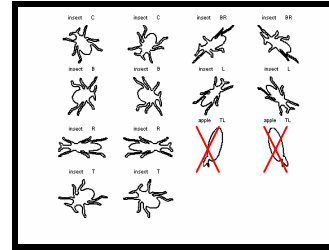


Figure 2. Retrieval result obtained using a random query from the MCD. The crosses mark retrievals that do not belong to the correct category.

The retrieval experiments show very accurate results. We can deduce that the AIFD are very robust even in presence of strong distortions such as perspective transformations.

#### 3.2 A comparative study with other affine invariants

In this section we will test the performance of the AIFD with other well known affine invariants such as the affine curvature and the affine invariant Fourier descriptors defined by Arbter [1]. The analytic formulation of affine curvature is given by :

$$KA = \frac{4}{3} \Delta(\gamma', \gamma'')^{\frac{5}{3}} \Delta(\gamma', \gamma^{(3)}) - \frac{5}{9} \Delta(\gamma', \gamma'')^{\frac{8}{3}} \Delta(\gamma', \gamma^{(3)})^2 + \frac{1}{3} \Delta(\gamma', \gamma'')^{\frac{5}{3}} \Delta(\gamma', \gamma^{(4)}) \quad (4)$$

where  $\Delta$  denotes the determinant and  $\gamma^{(p)}$  is  $p^{th}$  derivative of the contour  $\gamma$ . The evaluation of this local invariant requires therefore high order numerical derivatives. This is well known to be source of instability and of errors. Weiss propose to use a zero-moment filter [12]. Some studies [8][9] have shown that the affine curvature, calculated by Weiss filter, is very sensitive to noise. So it's problematic to use it in applications on real objects. We propose here to use B-spline. Indeed, it's well known that these functions have good smoothing quality and are robust relatively to multiple derivatives and rounding errors.

The Affine invariant defined by Arbter [1] is given by :

$$Q_k = \frac{\Delta_k}{\Delta_p} = \frac{\det[U_k, U_p^*]}{\det[U_p, U_p^*]} = \frac{X_k Y_p^* - Y_k X_p^*}{X_p Y_p^* - Y_p X_p^*}, \Delta_p \neq 0, k = \pm 1, \pm 2, \dots \quad (5)$$

where  $U_k = (X_k, Y_k)^T$  and  $X_k, Y_k$  represents the Fourier coefficients of  $x(t), y(t)$  respectively.  $P$  is a constant not null. All of the contours are re-parameterized by affine arclength.

Each shape from the MCD was used as a query, and the number of similar shapes (which belong to the same category with different view point) was counted in the top 16 matches. Figure 3 illustrate the retrieval results obtained for AIFD (Figure 3.a), Arbter's affine invariants (Figure 3.b) and affine curvature (Figure 3.c) using a random query from the MCD ("Bottle"). It is interesting to note that the four mismatched curves that are retrieved by AIFD share a similar shape (they belong to the category bottle). Table 1 contains the retrieval rates for 10 random query shapes. As can be seen, the AIFD significantly outperform the other two descriptors, and are, therefore the most useful to search for similar shapes obtained by perspective distortions. The low rate retrieval of the affine curvature invariants indicates that they are not robust under such distortions.

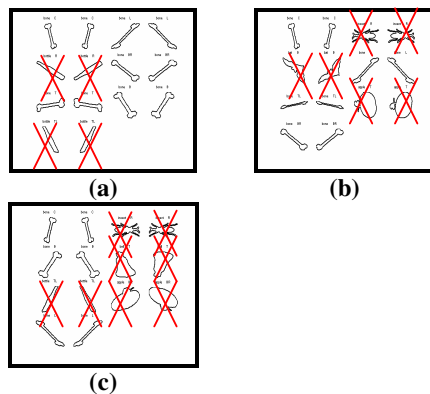


Figure 3. Retrieval results for object *bone\_I\_1*. Comparative study for AIFD (a), Arbter's invariants (b) and affine curvature (c). the curve scale has been normalized for visualization purposes.

Table1. Retrieval rates for 10 random query shapes.

Shape	AIFD	Arbter's Invariants	Affine Curvature
Bat	100%	28%	14%
Bell	86%	57%	0%
Butterfly	86%	43%	14%
Insect	86%	43%	14%
Bone	72%	72%	28%
Camel	72%	43%	28%
Bird	72%	14%	28%
Apple	72%	72%	14%
Bottle	57%	28%	14%
Brick	57%	14%	14%

#### 4. Application to Curve Matching:

In this section, we propose a robust method which can be used to match planar curves extracted from image pairs related by an affine transformation. In the following developments, we will assume that the object of interest has been extracted from the background. The shapes were restricted to simple pre-segmented shapes defined by their outer closed contours.

Figure 4 describe the proposed matching process of two curves related by an affine transformation of the group  $SA(2)$ .  $I_1$  and  $I_2$  are the affine invariant descriptors corresponding to each curve.  $KA$  denotes the affine curvature of re-parameterized curves  $\gamma_1$  and  $\gamma_2$ . In the following we outline the curve matching algorithm.

The matching process is subdivided in two stages:

- **Global Matching:** This consists on determining the corresponding curve pairs by evaluating the Euclidean distance  $D$  between their AIFDs ( $I_1(\gamma_1)$  and  $I_2(\gamma_2)$ ). In this step we provide the best matched curves without defining the internal points which can be matched. We deal about a global matching.
- **Local Matching:** Based on the results of the first global matching, this step provides the matched points in each curve. We follow the classical matching algorithm. We determine point-to-point correspondences using the cross correlation measure defined between High Curvature Points (HCP) of each curve. Using only HCP in shape matching can reduce the size of features from the original data points to just small number of the representation (HCP).

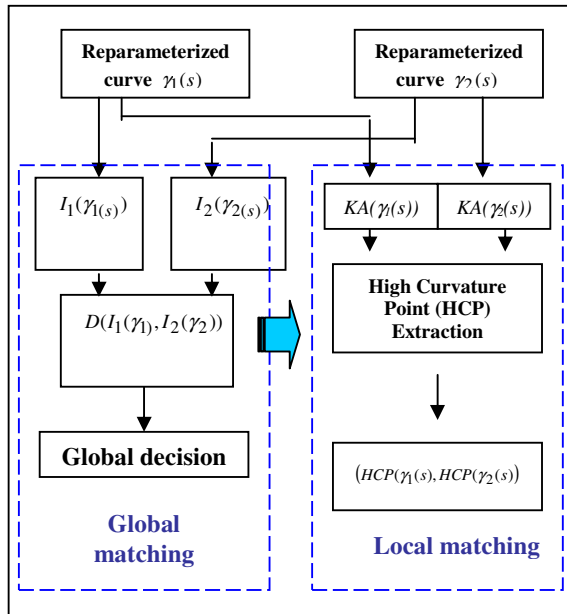


Figure 4. Block diagram of the proposed matching process

#### 4.1 Experimental results for matching:

In this section we present the matching results for the approach described in the previous section. Experiments for shapes with perspective changes were carried out on the Multiview Curve Dataset [14]. Figure 5 illustrate two reparameterized samples from the category “camel” which correspond to Central and Top Left view, respectively.

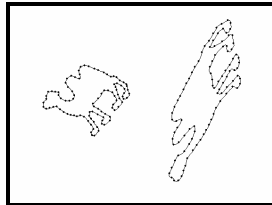


Figure 5. Central and Top Left View of two reparameterized samples curves from the category camel

In Figure 6 we display the final results of matching process. Red points correspond to matched ones in original curve.

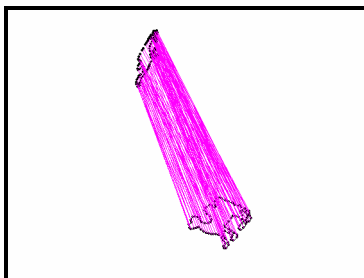


Figure 6. Final matching result for shapes in Figure 5.

## 5. Conclusion and perspectives:

In [3] we have proposed a new Affine Invariant Fourier Descriptors (AIFD). Experiments have shown that these descriptors are robust and can be used in many computer vision applications.

A first application of AIFD in the context of contour retrieval has been carried in this work. The experimental results on the MCD show that the AIFD has good retrieval accuracy and if applied to the shape of a curve it can deal with affine transformed curves. The comparative study has shown that the AIFD are more efficient than affine curvature and Arbter’s invariants.

Finally, we have demonstrated that AIFD are effective in matching curves despite perspective distortion.

In the future, we intend to compare our method with the Curvature Scale Space (CSS) approach in terms of both performance under perspective distortion and complexity.

## 6. References

- [1] K. Arbter, W.E. Snyder, G. Hirzinger and H. Burkhardt, “Application of affine-invariant fourier descriptors to recognition of 3-d objects”, *IEEE trans. Pattern Analysis and Machine Intelligence*, vol. 12, no. 7, pp. 640-646, 1990.
- [2] M. Bober, F. Preteux and W. Y. Kim, Introduction to MPEG7: Multimedia Content Description Interface, chapter 15, pp. 234-238, John Wiley & Sons Ltd., 2002.
- [3] F.Chaker, F. Ghorbel and M.T.Bannour. A complete and stable set of affine invariant Fourier descriptors, ICIAP’03, Mantova, Italy, September 2003.
- [4] F. Chaker and F. Ghorbel. Stereo matching by affine Invariant Fourier descriptors, ICIP’03, Barcelone, Spain, September 2003.
- [5] J. Flusser and T. Suk, “Pattern recognition by affine moment invariants”, *Pattern recognition*, vol. 26, no. 1, pp. 167-174, 1993.
- [6] F. Ghorbel. Towards a unitary formulation for invariant images description : application to images coding, *Annales des Télécommunications* ; France 1998.
- [7] J. L. Lisani, shape Based Automatic Images Comparison, PhD thesis, Université Paris IX- Dauphine, 2001.
- [8] S. Matusiak.,M. Daoudi., F. Ghorbel. Indexation d’une base de données d’images par une description de formes invariante aux affinités. CORESA 97, Issy-les Moulineaux 26-27 Mars 1997.
- [9] F. Mokhtarian and S. Abbasi, “Affine curvature scale space with affine length parameterization”, *Pattern Analysis & Applications*, vol. 4, pp. 1-8, 2001.
- [10] S. Sapiro and A. Tannenbaum, “Affine invariant scale space”, *International Journal of Computer Vision*, vol. 11, no. 1, pp. 25-44, 1993.
- [12] I. Weiss. Projective Invariants of shapes, *In Proc. Darpa, IU Workshop*, 1989, pp. 1125.
- [13] C. T. Zhang and R. Z. Roskies, Fourier Descriptors for plane closed Curves, *IEEE Trans. On Computers*, Vol. 21, No. 3, pp. 269-281, March 1972.
- [14] M.Zuliani, S. Bhagavathy, B. S. Manjunath and C. S. Kenney. Affine-Invariant Curve matching.