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Connected bipancyclic isomorphic m-factorizations of the Cartesian product of graphs

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Abstract

An *m-factorization* of a graph is a decomposition of its edge set into edge-disjoint m-regular spanning subgraphs (or factors). In this paper, we prove the existence of an isomorphic m-factorization of the Cartesian product of graphs each of which is decomposable into Hamiltonian even cycles. Moreover, each factor in the m-factorization is m-connected, and bipancyclic for $m \geq 4$ and nearly bipancyclic for $m = 3$.

1 Introduction

All graphs considered here are simple and undirected. The *Cartesian product* $G_1 \square G_2$ of two graphs G_1 and G_2 is a graph with the vertex set $V(G_1) \times V(G_2)$, where (u_1, u_2) is adjacent to (v_1, v_2) in $G_1 \square G_2$ if and only if either $u_1 = v_1$ and u_2 is adjacent to v_2 in G_2, or $u_2 = v_2$ and u_1 is adjacent to v_1 in G_1. In what follows by a product we mean the Cartesian product.

The n-dimensional hypercube Q_n is the product of n copies of K_2. The hypercube is a popular interconnection network in parallel computing [15]. A *factorization* of the graph G is a decomposition of its edge set into edge-disjoint spanning subgraphs (or factors). An *isomorphic factorization* of G is a factorization in which all of the factors are isomorphic with each other. A factorization is an *m-factorization* if each factor is m-regular. A *Hamiltonian decomposition* of G is a decomposition of its edge set into Hamiltonian cycles. Therefore, a Hamiltonian decomposition of a graph is a particular isomorphic 2-factorization.

Factorizations of graphs are well studied in the literature (see [1, 8, 12, 19, 21, 22]). Harary et al. [12] studied isomorphic factorizations of complete graphs. Bass and Sudborough [6] obtained an isomorphic $(n/2)$-factorization of the hypercube Q_n, for even n, where each factor has diameter $n+2$. As pointed out in [6], m-factorizations of Q_n have potential applications in the area of fault-tolerant computing and can be used in the construction of adaptive routing algorithms. For regular graphs, 2-factorizations have been studied for long time. In 1891, Petersen [18] proved that a

$2k$-regular graph has a 2-factorization. Kotzig [14], in 1973, proved that the product of two cycles is decomposable into Hamiltonian cycles while Foregger [10] considered such a decomposition of the product of three cycles. These results are generalized by Aubert and Schneider [5] as follows.

Theorem 1.1 *Let G be a 4-regular graph that is decomposable into two Hamiltonian cycles and let Z be a cycle. Then $G \Box Z$ can be decomposed into three Hamiltonian cycles.*

Alspach et al. [2] obtained the following important consequences of Theorem 1.1.

Corollary 1.2 *For $n \geq 1$, the product of n cycles has a Hamiltonian decomposition.*

Corollary 1.3 *For even n, the hypercube Q_n has a Hamiltonian decomposition.*

Further, using Corollary 1.2, they settled a conjecture of Kotzig [14] by proving that the graph $G_1 \Box G_2 \Box \ldots \Box G_n$ has a Hamiltonian decomposition if each G_i is decomposable into p Hamiltonian cycles. El-Zanati and Eynden [22] proved the existence of an isomorphic factorization of the product of cycles each with length a power of 2 such that all components of each factor are cycles of same length.

In this paper, we consider the problem of determining the existence of isomorphic m-factorizations of the product of graphs of even orders each of which has a Hamiltonian decomposition, where the factors are m-connected and satisfy an additional property of bipancyclicity. We generalize Theorem 1.1 and its consequences for m-factorizations.

A graph G with even number of vertices is *bipancyclic* if G is either a cycle or contains a cycle of every even length from 4 to $|V(G)|$ (see [16]). Some authors use the term"even pancyclic" for "bipancyclic" (see [4]). We say that a 3-regular graph G with even number vertices is *nearly bipancyclic* if it contains a cycle of every even length from 4 to $|V(G)|$, except possibly 4 and 8. Bipancyclicity of a given network is an important factor in determining whether the network topology can simulate rings of various lengths. Connectivity is one of the fundamental properties for interconnection networks. These properties for hypercube networks are well studied in the literature (see [9, 13, 16]).

The following result is the main theorem of the paper.

Theorem 1.4 *Let G be a 4-regular graph with even order that is decomposable into two Hamiltonian cycles and let Z be an even cycle. Then $G \Box Z$ has a 3-factorization, where each factor is 3-connected. Moreover, if G is bipartite, then the factors are isomorphic and nearly bipancyclic.*

This result is analogous to Theorem 1.1. We obtain several consequences of Theorem 1.4 for m-factorizations. The following result is analogous to Corollary 1.3 for m-factorizations.

Theorem 1.5 *Let $n \geq 2$ be even and $m \geq 2$ divide n. Then Q_n has an isomorphic m-factorization, where each factor is m-connected, and bipancyclic for $m \neq 3$ and nearly bipancyclic for $m = 3$.*

A related result which states that Q_n, for $n = n_1 + n_2$ with $n_i \geq 2$, has a decomposition into two spanning bipancyclic subgraphs H_1 and H_2 such that H_i is n_i-regular and n_i-connected is obtained in [7]. Theorem 1.5 can be compared with the following problem posed by Bass and Sudborough [6].

Open Problem 1.6 *Determine the existence of an isomorphic m-factorization of Q_n, where m divides n, $m < n/2$ and the diameter of the factors is n.*

We prove Theorem 1.4 in Section 2 and its consequences in Section 3.

2 Proof of Theorem 1.4

Alspach et al. [4] proved that a connected Cayley graph of degree at least 3 on an abelian group is bipancyclic. Since the product of two cycles is a connected Cayley graph of degree 4 on an abelian group, it is bipancyclic. Therefore, we get the following lemma, which also follows from a result of Mane and Waphare [17].

Lemma 2.1 *If G_1 and G_2 are Hamiltonian graphs, then $G_1 \square G_2$ is bipancyclic.*

The following lemma is a consequence of a result from [20].

Lemma 2.2 *Let G_i be an m_i-regular, m_i-connected graph for $i = 1, 2$. Then the graph $G_1 \square G_2$ is $(m_1 + m_2)$-regular, $(m_1 + m_2)$-connected.*

The next lemma follows from the definition of the product of graphs.

Lemma 2.3 *Suppose G_1 and G_2 are two graphs such that G_1 is decomposable into spanning subgraphs $H_1, H_2, \ldots, H_r$, and G_2 is decomposable into spanning subgraphs $F_1, F_2, \ldots, F_r$. Then the graph $G_1 \square G_2$ is decomposable into spanning subgraphs $H_1 \square F_1, H_2 \square F_2, \ldots, H_r \square F_r$.*

For $n \geq 1$, let $[n] = \{1, 2, \ldots, n\}$. We now prove Theorem 1.4.

Proof: By definition of the product, $G \square Z$ is obtained by replacing each vertex of Z by a copy of G and replacing each edge of Z by a matching between two copies of G corresponding to the end vertices of that edge. Let $|V(G)| = s$ and $|V(Z)| = r$. Then r and s are even and further, $s \geq 6$ and $r \geq 4$ as G and Z are simple. Let Z have vertices $\{1, 2, \ldots, r\}$, where j is adjacent to $j + 1$ modulo r. Suppose G decomposes into two Hamiltonian cycles C and D. Then $G = C \cup D$. Label the vertices of G with $v_1, v_2, \ldots, v_s$ so that v_p is adjacent to $v_{p+1(\text{modulo } s)}$ in C. For compactness, let v_p^j denote the vertex (v_p, j) of $G \square Z$; superscripts are computed modulo r with representative in $[r]$ and subscripts are modulo s with representative in $[s]$. For $j \in [r]$, let G^j be the copy of G induced by the set $\{v_p^j \mid p \in [s]\}$ and let C^j be the copy of C in G^j. For convenience, we will denote $j + 1$ modulo r by $j + 1$. Let F be the set of edges of $G \square Z$ between the graphs G^j. Then $F = \{v_p^j v_p^{j+1} \mid p \in [s]; j \in [r]\}$ and $G \square Z = G^1 \cup G^2 \cdots \cup G^r \cup F$. Partition the set F into sets F_1 and $F_2 = F \setminus F_1$, where $F_1 = \{v_p^j v_p^{j+1} \mid j = 1, 3, 5, 7, \dots, r-1;\ p = 1, 3, 5, 7, \dots, s-1\} \cup \{v_p^j v_p^{j+1}\} \mid j = 2, 4, 6, \dots, r;\ p = 2, 4, 6, \dots, s\}$.

Let $H_1 = C^1 \cup C^2 \cup \cdots \cup C^r \cup F_1$ (see Figure 1) and let $H_2 = G\Box Z - E(H_1)$. Then $H_2 = D^1 \cup D^2 \cup \cdots \cup D^r \cup F_2$, where D^j is the copy of the cycle D in G^j. Obviously, H_1 and H_2 are 3-regular and spanning edge-disjoint subgraphs of $G\Box Z$.

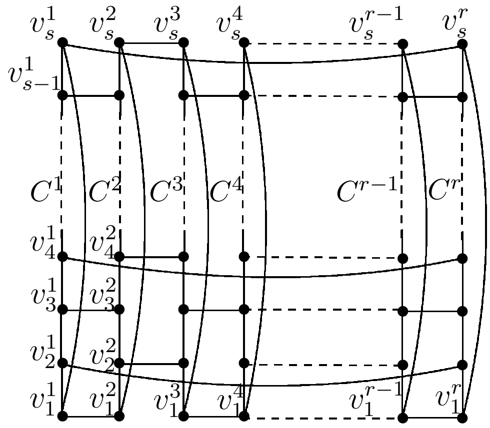


Figure 1: The graph H_1

Claim 1. H_1 and H_2 are 3-connected.

First, we prove that H_2 is 3-connected. For $j \in [r]$, half of the vertices of D^j have distinct neighbours in D^{j-1} and the remaining half have distinct neighbours in D^{j+1} along the edges of F_2. Let x and y be two vertices of H_2. Then $x \in V(D^j)$ and $y \in V(D^k)$ for some $j, k \in [r]$. Suppose $j = k$. Then $x, y \in V(D^j)$. Clearly, $D^j - \{x, y\}$ has at most two components each of which is joined to D^{j-1} or D^{j+1}. Already, $H_2 - V(D^j)$ is connected and contains D^{j-1} and D^{j+1}. Therefore, $H_2 - \{x, y\}$ is connected. Suppose $j \neq k$. Then $D^j - \{x\}$ and $D^k - \{y\}$ are connected. If $k \in \{j-1, j+1\}$, then $H_2 - V(D^j \cup D^k)$ is connected and further, each of $D^j - \{x\}$ and $D^k - \{y\}$ has a neighbour in $H_1 - V(D^j \cup D^k)$. Therefore, $H_1 - \{x, y\}$ is connected. Suppose $k \notin \{j-1, j+1\}$. Then $H_2 - V(D^j \cup D^k)$ has two components one of them contains D^{j-1}, D^{k+1}, and the other contains D^{j+1}, D^{k-1}. Therefore, each component of $H_2 - V(D^j \cup D^k)$ contains a neighbour of each of $D^j - \{x\}$ and $D^k - \{y\}$. Hence, $H_2 - \{x, y\}$ is connected. Thus H_2 is 3-connected.

With similar arguments, one can prove that H_1 is 3-connected. However, 3-connectedness of H_1 also follows from known results. By a result of Alspach and Dean [3], being the honeycomb toroidal graph HTG$(r, s, 0)$, H_1 is a Cayley graph. Hence, H_1 is a vertex-transitive connected graph of degree 3. It is known that a vertex-transitive connected graph of degree d has connectivity at least $2(d+1)/3$ (see Theorem 3.4.2, [11]). It follows that the connectivity of H_1 is 3 and hence, it is 3-connected. Note that these arguments does not help with H_2.

Thus H_1 and H_2 are 3-connected.

Therefore, $G\Box Z$ has a 3-factorization with 3-connected factors H_1 and H_2.

Suppose G is a bipartite graph. We claim that H_1 and H_2 are isomorphic and nearly bipancyclic.

Claim 2. H_1 and H_2 are isomorphic.

Let X and Y be the bipartite sets of G, and for $j \in [r]$ let X^j and Y^j be the copies of X and Y in G^j, respectively. Clearly, the vertices of both C^j and D^j are alternately in X^j and Y^j. We may assume that $v_p^j \in X^j$ if and only if p is odd. In H_2, relabel the vertices of G^j with the labels $u_1^j, u_2^j, \ldots, u_s^j$ so that $u_1^j = v_1^j$, and u_p^j is adjacent to u_{p+1}^j for all $p \in [s]$. Then $u_p^j \in X^j$ if and only if p is odd. Note that $F_2 = \{u_p^j u_p^{j+1} \mid j \text{ odd};\ u_p^j \in Y^j\} \cup \{u_p^j u_p^{j+1} \mid j \text{ even};\ u_p^j \in X^j\}$.

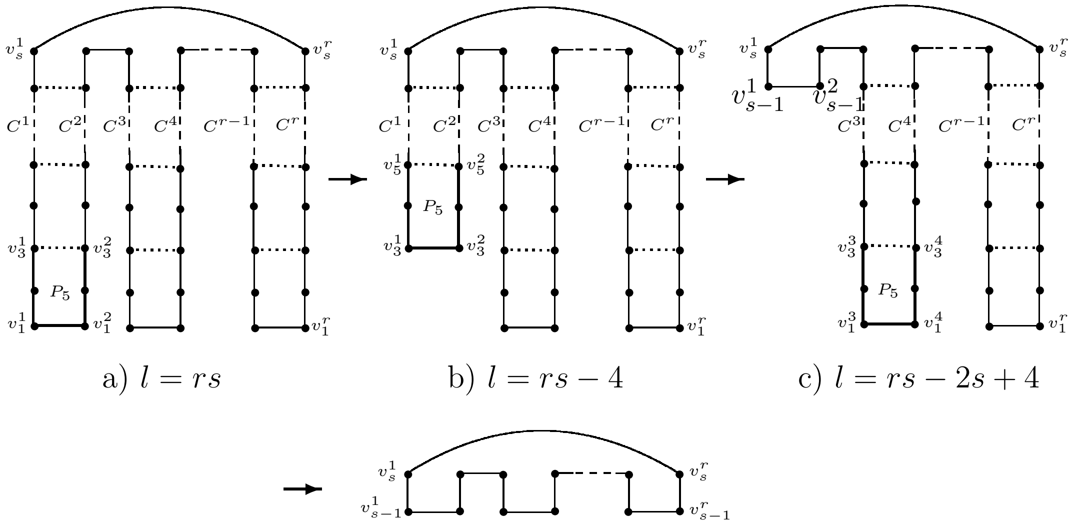
Now, define a map $f : V(H_1) \to V(H_2)$ by $f(v_p^j) = u_p^{j+1}$ for $j \in [r]$. Clearly, f is bijective. It is easy to see that f maps the cycle C^j onto the cycle D^{j+1} for $j \in [r]$ and also it maps F_1 onto F_2. This implies that f is an isomorphism between the graphs H_1 and H_2.

Claim 3. *H_1 and H_2 are nearly bipancyclic.*

It suffices to prove the claim for the graph H_1 as H_2 is isomorphic to H_1. Let l be an *even* integer such that $6 \le l \le rs$. We prove the claim by constructing a cycle of length l in H_1 except possibly for $l = 8$.

Case (i). l is a multiple of 4.

A Hamiltonian cycle, that is, a cycle of length rs in H_1 is shown in Figure 2(a). In this cycle, replacing the path P_5 of length 5 consisting of the vertices v_1^1, v_2^1, v_3^1 of C^1, and v_1^2, v_2^2, v_3^2 of C^2, by the chord $v_3^1 v_3^2$ produces a new cycle of length $rs - 4$ which is given in Figure 2(b). Now, replacing a P_5, as before, by the chord joining the end points of P_5 in the new cycle creates a cycle of length $rs - 8$. Continuing this process of obtaining a new cycle from the previous cycle by replacing a P_5 with a chord, as shown in Figure 2, we get cycles of length l for every l, a multiple of 4 with $2r \le l \le rs$.

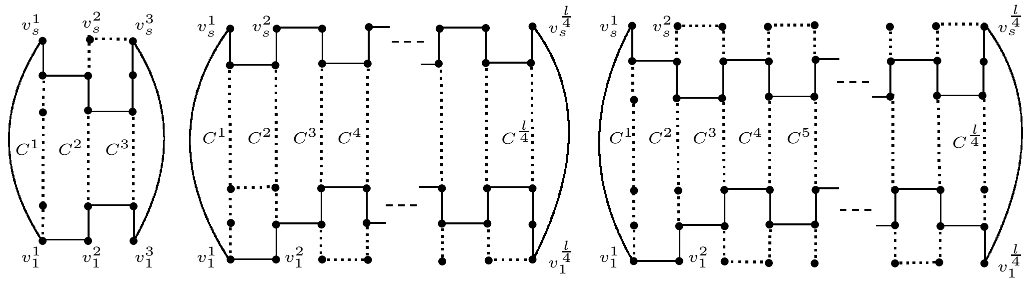


a) $l = rs$ b) $l = rs - 4$ c) $l = rs - 2s + 4$

d) $l = 2r$

Figure 2. Cycle of length l, where l is a multiple of 4 and $2r \leq l \leq rs$

Recall that r, s are even and $r \geq 4$, $s \geq 6$. Note that if $r = 4$, then we get a cycle in H_1 of length 8 from Figure 2(d). Suppose $r \geq 6$. Figure 3(a) depicts a cycle of length 12 in H_1. For $16 \leq l \leq 2r - 4$, the cycles of length l are constructed from the cycles $C^1, C^2, \ldots, C^{l/4}$ of H_1 as shown in Figure 3(b) and (c) by considering two cases depending on whether $l/4$ is even or odd.



a) $l = 12$ b) $l/4$ is even c) $l/4$ is odd

Figure 3: Cycle of length l, where l is a multiple of 4 and $12 \leq l \leq 2r - 4$

Case (ii). l is not a multiple of 4.

Obviously, $6 \leq l \leq rs - 2$. A cycle of length $rs - 2$ is given in Figure 4(a). In this cycle, we replace a P_5 consisting of vertices $v_1^1, v_2^1, v_3^1, v_1^2, v_2^2, v_3^2$ with the chord $v_3^1 v_3^2$ to produce a new cycle of length $rs - 6$ which is given in Figure 4(b). Recursively, we construct the cycles of length l, as in Case (i), shown in Figure 4. This proves the claim.

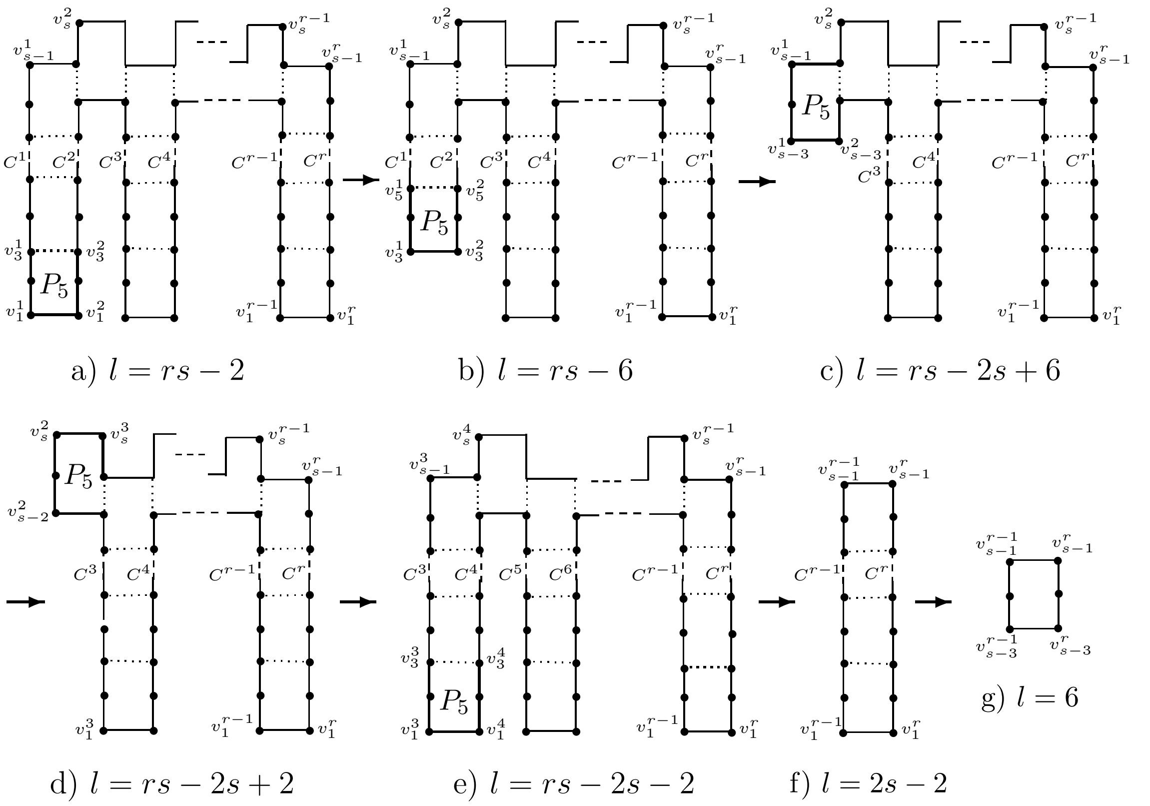


Figure 4: Cycle of length l, where l is not a multiple of 4 and $6 \leq l \leq rs - 2$

By Claims 1, 2 and 3, H_1 and H_2 give a 3-factorization of the graph $G\Box Z$, as desired. □

3 Consequences of Theorem 1.4

We get the following result that is more general than Theorem 1.4.

Theorem 3.1 *Let G_1 and G_2 be graphs with even orders that are decomposable into $2n$ and n Hamiltonian cycles, respectively. Then $G_1\Box G_2$ has a 3-factorization, where each factor is 3-connected. Moreover, if G_1 is bipartite, then the factors are isomorphic and nearly bipancyclic.*

Proof: Suppose G_1 is decomposable into Hamiltonian cycles $C_1, C_2, \ldots, C_{2n}$, and G_2 is decomposable into Hamiltonian cycles $Z_1, Z_2, \ldots, Z_n$. Then $G_1 = C_1 \cup C_2 \cup \ldots \cup C_{2n}$ and $G_2 = Z_1 \cup Z_2 \cup \ldots \cup Z_n$. Suppose $|V(G_1)| = s$ and $|V(G_2)| = r$. For $i \in [n]$, let $W_i = (C_{2i-1} \cup C_{2i})\Box Z_i$. Then W_i is a spanning 6-regular subgraph of $G_1\Box G_2$. By Lemma 2.3, $G_1\Box G_2$ has a 6-factorization into n factors $W_1, W_2, \ldots, W_n$. As in the proof of Theorem 1.4, we get a 3-factorization of each W_i into two 3-connected factors W_i' and W_i''. Suppose G_1 is bipartite. Then, for every $i \in [n]$, the factors W_i' and W_i'' are nearly bipancyclic and each is isomorphic to the graph shown in Figure 1. Thus $W_1', W_2', \ldots, W_n'$ and $W_1'', W_2'', \ldots, W_n''$ give a desired 3-factorization of $G_1\Box G_2$. □

Theorem 3.2 *Let $m \geq 2$ divide n and let $C_1, C_2, \ldots, C_n$ be even cycles. Then the product $C_1 \Box C_2 \Box \ldots \Box C_n$ has an isomorphic m-factorization, where each factor is m-connected, and bipancyclic if $m \neq 3$ and nearly bipancyclic if $m = 3$.*

Proof: We prove the result by the induction on m. For $m = 2$, the result follows from Corollary 1.2. Suppose $m = 3$. Then $n = 3k$ for some k. Let $G_1 = C_1 \Box C_2 \Box \ldots \Box C_{2k}$ and let $G_2 = C_{2k+1} \Box C_{2k+2} \Box \ldots \Box C_{3k}$. By Corollary 1.2, G_1 can be decomposed into $2k$ Hamiltonian cycles and G_2 can be decomposed into k Hamiltonian cycles. Since the cycles C_i, $1 \leq i \leq 3k$ have even length, G_1 and G_2 are bipartite and so Hamiltonian cycles of G_1 and G_2 are even. By Theorem 3.1, $G_1 \Box G_2 = C_1 \Box C_2 \Box \ldots \Box C_{3k}$ has an isomorphic 3-factorization, where each factor is 3-connected and nearly bipancyclic.

Suppose $m \geq 4$. Then $m - 2 \geq 2$. Let $G_1 = C_1 \Box C_2 \Box \ldots \Box C_{(m-2)k}$ and let $G_2 = C_{(m-2)k+1} \Box C_{(m-2)k+2} \Box \ldots \Box C_{mk}$. Then G_1 and G_2 are bipartite. By the induction, G_1 has an isomorphic $(m-2)$-factorization, where factors say $W_1, W_2, \ldots, W_{2k}$ are $(m-2)$-connected and bipancyclic or nearly bipancyclic. Therefore, each W_i contains a Hamiltonian cycle. Since G_2 is a product of $2k$ cycles, by Corollary 1.2, it can be decomposed into Hamiltonian cycles $Z_1, Z_2, \ldots, Z_{2k}$. Let $H_i = W_i \Box Z_i$ for $i = 1, 2, \ldots, 2k$. By Lemmas 2.1 and 2.2, each H_i is m-regular, m-connected and bipancyclic. By Lemma 2.3, the graphs H_i, $1 \leq i \leq 2k$ are spanning edge-disjoint subgraphs of $G_1 \Box G_2$ such that $G_1 \Box G_2 = H_1 \cup H_2 \ldots \cup H_{2k}$. Moreover, for $i \neq j$, H_i is isomorphic to H_j because W_i is isomorphic to W_j, and Z_i is isomorphic to Z_j. □

The following result is a consequence of Theorem 3.2 and Lemma 2.3.

Corollary 3.3 *Let $m \geq 2$ divide n and let $G_1, G_2, \ldots, G_n$ be graphs of even orders each of which is decomposable into p Hamiltonian cycles. Then $G_1 \Box G_2 \Box \ldots \Box G_n$ has an isomorphic m-factorization, where each factor is m-connected, and bipancyclic if $m \neq 3$ and nearly bipancyclic if $m = 3$.*

We now prove Theorem 1.5 which is restated here.

Theorem 3.4 *Let $n \geq 2$ be even and $m \geq 2$ divide n. Then Q_n has an isomorphic m-factorization, where each factor is m-connected, and bipancyclic for $m \neq 3$ and nearly bipancyclic for $m = 3$.*

Proof: Let $n = mk$ for some k. As n is even, k is even or m is even. Further, Q_n is the product of $n/2$ cycles of length four each. If k is even, then m divides $n/2$ and hence, the result follows from Theorem 3.2.

Suppose m is even. We prove the result by the induction on m. The result holds for $m = 2$ as, by Corollary 1.2, Q_{2k} can be decomposed into k Hamiltonian cycles say $Z_1, Z_2, \ldots, Z_k$. Suppose $m \geq 4$. Then $m - 2 \geq 2$ is even. By the induction, $Q_{(m-2)k}$ has an isomorphic $(m-2)$-factorization with factors $W_1, W_2, \ldots, W_k$ such that each W_i is $(m-2)$-connected and bipancyclic. Write Q_n as $Q_n = Q_{(m-2)k} \Box Q_{2k}$. Let $H_i = W_i \Box Z_i$ for $i = 1, 2, \ldots, k$. By Lemma 2.3, the subgraphs H_i, for $1 \leq i \leq k$, are spanning edge-disjoint subgraphs of Q_n such that $Q_n = H_1 \cup H_2 \cup \ldots \cup H_k$. By Lemmas 2.1 and 2.2, each H_i is m-regular, m-connected and bipancyclic. Further, the subgraphs H_i are isomorphic, as the subgraphs W_i are isomorphic. Thus Q_n has an isomorphic m-factorization into the m-connected bipancyclic factors $H_1, H_2, \ldots, H_k$. □

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