Matching Extensions with Prescribed and Forbidden Edges

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Abstract

Suppose G is a connected graph on p vertices that contains a perfect matching. Then G is said to have property $E(m,n)$ if $p \geq 2(m+n+1)$ and if for each pair of disjoint independent sets $M, N \subseteq E(G)$ of sizes m, n respectively, there exists a perfect matching P in G such that $M \subseteq P$ and $N \cap P = \emptyset$. We discuss the circumstances under which $E(m,n) \Rightarrow E(x,y)$, and prove that (surprisingly) in general $E(m,n)$ does not imply $E(m,n-1)$.

1 Introduction

In this paper we consider finite, simple graphs. An *m-matching* is a set of m independent edges of a graph. For a graph G with $2n$ vertices, a *perfect matching* is an n-matching in G. Let G be a connected graph on p vertices which contains a perfect matching. G is said to be *n-extendable*, for positive integer n, if $p \geq 2(n+1)$ and every n-matching can be extended to (i.e., is contained in) a perfect matching in G.

Plummer first introduced the concept of n-extendable graphs [4]. Since then there has been a large amount of study into relationships between n-extendability and other graph parameters, such as connectivity, degree and genus. For an excellent survey we refer the reader to [7].

Generalizing this idea of n-extendability, Liu and Yu introduced the concept of *(m,n)-extendability* [3]. For a graph G, let M be an m-matching of G and $U = \{u_1, \ldots, u_n\}$ be a set of n distinct vertices of G such that u_i $(1 \leq i \leq n)$ is not incident with any edge of M. A *matching extension* of (M, U) is a perfect matching M^* in G such that $M \subseteq M^*$ and $u_iu_j \notin M^*$ for any $u_i, u_j \in U$. A graph G is called *(m,n)-extendable* if it contains a perfect matching and for arbitrary M and U described above, there exists a matching extension of (M, U). Liu and Yu studied the properties of (m,n)-extendable graphs and, in particular, relationships between (m,n)-extendability and graph products.

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It is clear from the definition of (m, n)-extendability that every matching extension of given (M, U) must necessarily exclude any edge $e \in E(\langle U \rangle)$, a potentially large number of edges as n becomes large ($|E(\langle U \rangle)| \leq |E(K_n)| = n(n-1)/2$). In this paper we present an alternative generalization of n-extendability that avoids this restriction, with what we call the $E(m, n)$ property and study the properties of $E(m, n)$ graphs.

Motivation for this alternative comes from analogous study of path and cycle properties in graphs. Wilson, Hemminger and Plummer [9] introduced the $P(m, n)$ property of graphs. A graph on p vertices is said to be $P(m, n)$ (originally (m^+, n^-)-connected) if $p \geq m + n, m \geq 2, n \geq 0$, and for any given set $\{u_1, \ldots, u_m, v_1, \ldots, v_n\} \subseteq V(G)$ there is a path in G containing $u_1, \ldots, u_m$, and avoiding $v_1, \ldots, v_n$. Holton and Plummer [2] presented a similar definition in the cycle case as an extension to Dirac's Theorem. A graph G on p vertices is said to be $C(m, n)$ if $p \geq m + n$ and given arbitrary disjoint sets $M, N \subseteq V(G)$ of sizes m, n respectively, there exists a cycle C^* in G that contains each vertex $u \in M$ and avoids each vertex $v \in N$.

In each of these properties the structure of the avoided elements/set is the same as that of the included set. With this in mind, we define the $E(m, n)$ property of graphs. Let G be a connected graph on p vertices that contains a perfect matching. Then G is said to have property $E(m, n)$ if $p \geq 2(m+n+1)$ and given arbitrary disjoint independent sets $M, N \subseteq E(G)$ of sizes m, n respectively, there exists a perfect matching P in G such that $M \subseteq P$ and $N \cap P = \emptyset$. We say that P is an (M, N)-*extension.* In the case $M = \{e\}$ we refer to an (e, N)-extension and similarly for the case $|N| = 1$.

Note the similarities between the definitions for $C(m, n)$ and $E(m, n)$. Also we see that the properties $E(m, 0)$ and $(m, 0)$-extendability are both equivalent to m-extendability. As examples of these properties, we have the Petersen graph which has property $E(1, 1)$ but not $E(1, 2)$ or $E(2, 0)$, and C_{2n} which has property $E(1, 0)$ but not $E(1, 1)$.

The following preliminary result due to Plummer combines theorems from [4] and [5]. Notice that part (a) induces a hierarchy upon the set of graphs admitting a perfect matching. As we shall see, the $E(m, n)$ property further refines this hierarchy.

Theorem 1.1 *Let G be a graph on p vertices that contains a perfect matching, and let n be such that $p \geq 2(n + 1)$ and let $e \in E(G)$. If G is n-extendable, then*

(a) G is $(n - 1)$-extendable, for $n \geq 2$,

(b) G is $(n + 1)$-connected, for $n \geq 1$.

(c) if $n = 1$, $G - e$ has a perfect matching.

(d) if $n \geq 2$, $G - e$ is $(n - 1)$-extendable. ■

It is interesting to note that whilst it is trivially clear that $C(m,n) \Rightarrow C(m,n-1)$ and similarly for the $P(m,n)$ property, it is not true that a graph which has property $E(m,n)$ necessarily has property $E(m,n-1)$. We present the graph $G_1 = 5K_2 + \overline{K_8}$ in Figure 1 as an example of this surprising result (for proof, see Lemma 2.3).

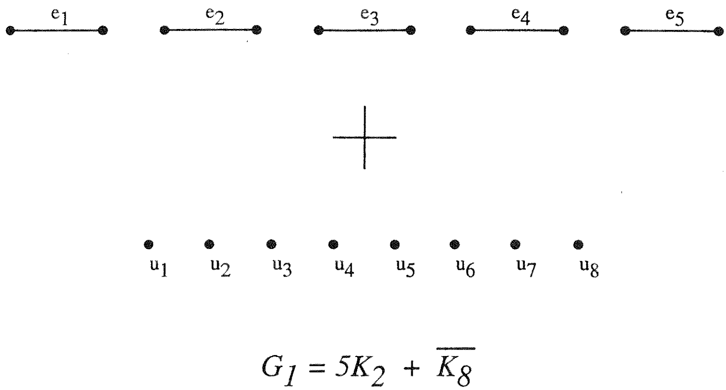

FIGURE 1: G_1 has property $E(1,7)$ but not $E(1,6)$

2 $E(m,n)$ does not imply $E(m,n-1)$

We have seen that the property $E(m,0)$ is just m-extendability. Our first theorem says that in general, the $E(m,n)$ property is a reasonable generalization of n-extendability.

Theorem 2.1 *If a graph G has property $E(m,n)$, then G has property $E(m,0)$.*

Proof: Suppose a graph G on p vertices has property $E(m,n)$. Then there exists a perfect matching P in G and $p \geq 2(m+n+1)$. Thus $|P| \geq m+n+1$. Now consider an arbitrary independent set $M \subseteq E(G)$, of size m. Then certainly $|P \cap (E(G) \backslash M)| \geq n+1$. So there exists a set $N \subseteq P \cap (E(G) \backslash M)$ of size n. Then M, N are independent disjoint sets and so, since G has property $E(m,n)$, there exists an (M,N)-extension P, a perfect matching containing M. It follows that G is m-extendable and thus has property $E(m,0)$. ∎

Notice that we require the vertex bound of $p \geq 2(m+n+1)$ in the definition of $E(m,n)$ for this proof. When $n=0$ we see that this restricton collapses to that of Plummer in Theorem 1.1. It is clear also that (if we relaxed the restriction on p) no graph on $p = 2(m+1)$ vertices would have property $E(m, 1)$ since for any perfect matching P we could take disjoint $M, N \subseteq E(G)$ of sizes m and 1 such that $M \cup N = P$ thus disallowing an (M, N)-extension.

From Theorems 1.1 and 2.1 we obtain the following corollary.

Corollary 2.2 *If a graph G has property $E(m, n)$, then*

(a) G has property $E(q, 0)$ for all $0 \leq q \leq m$; and

(b) G is $(m+1)$-connected.

Proof: Suppose G has property $E(m, n)$. By Theorem 2.1, G has property $E(m, 0)$ and thus m-extendable, and so by Theorem 1.1

(a) G has property $E(q, 0)$ for all $0 \leq q \leq m$; and

(b) G is $(m+1)$-connected. ■

We have seen in the introduction that although Theorem 2.1 holds, it is not true that $E(m, n) \Rightarrow E(m, n-1)$, as one might initially expect. We generalize graph G_1 to show that for $m \geq 1$ and $n \geq 6m+1$, the graph $G_2 = (n-2m)K_2 + \overline{K_{2(n-3m)}}$ in Figure 2 has property $E(m, n)$ but not $E(m, n-1)$.

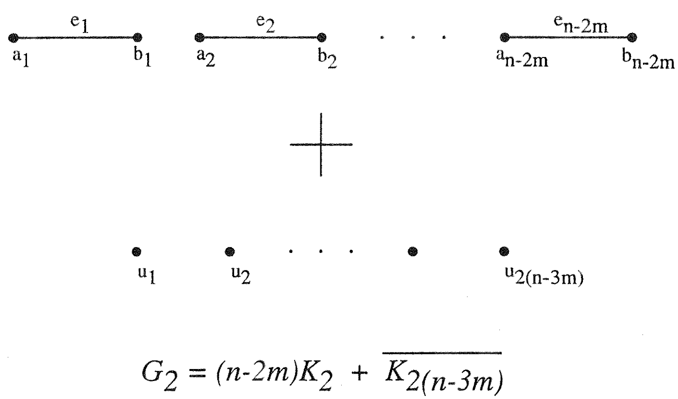

FIGURE 2: G_2 has property $E(m, n)$ but not $E(m, n-1)$, for $m \geq 1, n-1 \geq 6m$

Lemma 2.3 *For $m \geq 1$ and $n \geq 6m+1$, there exists a graph G that has property $E(m,n)$ but not $E(m,n-1)$.*

Proof: Let $m \geq 1$ and $n \geq 6m+1$. We will show that the graph $G_2 = (n-2m)K_2 + \overline{K_{2(n-3m)}}$ in Figure 2 is $E(m,n)$ but is not $E(m,n-1)$. Let $E = \{e_1, \ldots, e_{n-2m}\}$, $A = \{a_1, \ldots, a_{n-2m}\}$, $B = \{b_1, \ldots, b_{n-2m}\}$, $U = \{u_1, \ldots, u_{n-3m}\}$, $V = \{u_{n-3m+1}, \ldots, u_{2(n-3m)}\}$ and let $F = E(G_2)\backslash E$ (the edges between $A \cup B$ and $U \cup V$).

Firstly note that G_2 contains $p = 2(n-2m)+2(n-3m) = 4n-10m$ vertices, so any perfect matching in G_2 contains $2n-5m = 2(n-3m)+m$ edges. Therefore the perfect matching must include exactly m of the $(n-2m)$ edges in E (and $2(n-3m)$ edges in F).

Claim: G_2 does not have property $E(m,n)$.

We have $p = 4n-10m = 2(m+n+1)+2(n-(6m+1)) \geq 2(m+n+1)$. Now consider arbitrary disjoint independent edgesets $M, N \subseteq E(G_2)$ of sizes m, n respectively. Now $|N \cap E| \leq (n-2m)-2m = n-4m$, leaving at least $2m$ edges in $E\backslash N$. Suppose that $|M \cap (E\backslash N)| = q$, where $0 \leq q \leq m$. Since $|M| = m$, there are at least m edges in $E\backslash N$ independent of M, so choosing $m-q$ of these and an arbitrary matching between the remaining M-uncovered vertices in $A \cup B$ and vertices in $U \cup V$ (of which there are $2(n-3m)-(m-q)$ of each) we have an (M,N)-extension.

Claim: G_2 is not $E(m,n-1)$.

Consider the following disjoint independent sets $M, N \subseteq E(G_2)$. Let N contain $n-4m+1 = (n-2m)-(2m-1)$ edges in E, and $4m-2 = 2(2m-1)$ edges in F, adjacent to the remaining N-uncovered vertices in $A \cup B$. Let M contain $m = (2m-1)-(m-1)$ edges in F such that each M-edge is incident with a distinct edge in $E\backslash N$. Clearly $|N| = n-1$ and $|M| = m$. Further, it is clear that any (M,N)-extension P must avoid $[(n-2m)-(2m-1)]+[(2m-1)-(m-1)] = (n-2m)-(m-1)$ edges in E, i.e., all but $m-1$ edges in E. Thus P can contain at most $m-1$ edges in E, a contradiction as P is a perfect matching. It follows that there does not exist an (M,N)-extension and thus G_2 does not have property $E(m,n-1)$. ∎

In contrast to the above, the following theorem states that when n is small enough with respect to m, the implication $E(m,n) \Rightarrow E(m,n-1)$ does hold.

Theorem 2.4 *If a graph G has property $E(m,n)$ then G has property $E(m,n-1)$, for $n \leq 2m+2$.*

Proof: Suppose a graph G on p vertices has property $E(m,n)$ with $n \leq 2m+2$. Assume that G is not $E(m,n-1)$ and let $M, N \subseteq E(G)$ be disjoint independent sets of sizes $m, n-1$ respectively such that there does not exist an (M,N)-extension. Let $M = \{m_1, m_2, \ldots, m_m\}$ and let $N = \{n_1, n_2, \ldots, n_{n-1}\}$. Let $I = V(G)\backslash V(\langle M \cup N\rangle)$ be the set of vertices in G not incident with an edge in $M \cup N$.

Now since G has property $E(m,n)$ we know that no edge $e \in (E(G)\backslash(M \cup N))$ is independent of N, else there exists an $(M, N \cup \{e\})$-extension which is also an (M, N)-extension. Thus every edge uv with $u \in I$ has vertex v incident with an edge in N.

We now consider the two cases:

(a) Suppose $n - 1 \leq m$.

As $p \geq 2(m+n+1) = 2m+2(n-1)+4$, we have $|I| \geq 4$. By Corollary 2.2, G has property $E(n-1, 0)$ since $n - 1 \leq m$, and so there exists a perfect matching P in G containing N. However, this isolates each vertex in I, a contradiction.

(b) Let $m + 1 \leq n - 1 \leq 2m + 1$.

Let $q = (n-1) - m$, so that $1 \leq q \leq m+1$. Let x be the number of edges in M independent of N and let c be the number of vertices incident with both an edge in M and an edge in N so that $c \geq m - x$. As $p \geq 2(m+n+1) = (2m-c)+2(n-1)+(4+c)$, then certainly $|I| \geq 4+c > m-x+2$. By Theorem 2.1, G has property $E(m, 0)$, and so there exists a perfect matching P' in G containing the independent set of edges $\{n_{q+1}, \dots, n_{q+m}\}$. Thus P' must match every vertex in I by independent edges to vertices incident with edges $n_1, \dots, n_q$, so in particular there exists $m - x + 2$ independent edges, $e_1, \dots, e_{m-x+2}$ say, between I and edges $n_1, \dots, n_q$. Now consider disjoint independent sets $M_1, N_1 \subseteq E(G)$ given by $M_1 = \{n_1, \dots, n_m\}$ and $N_1 = \{n_{m+2}, \dots, n_{q+m}, m_1, \dots, m_x, e_1, \dots, e_{m-x+2}\}$. Now $|M_1| = m$ and $|N_1| = (q-1) + x + (m - x + 2) = n$. Thus there exists an (M_1, N_1)-extension P_1 in G. P_1 must match every vertex $v \in V(G)\backslash V(\langle N \rangle)$ to a vertex $u \in \{n_{m+1}, \dots, n_{q+m}\}$, that is (at least) $2(m+2)$ vertices to $2q$ vertices, a contradiction as $q \leq m+1$. This completes the proof. ■

This gives us the following immediate corollary.

Corollary 2.5 *If a graph G has property $E(m,n)$ with $n \leq 2m+2$ then it has property $E(m,p)$ for all $p \leq n$.* ■

Given $m \geq 1$, there is clearly a large gap between $n \geq 6m+1$ for which Lemma 2.3 shows us that G has property $E(m,n) \not\Rightarrow G$ has property $E(m, n-1)$, and $n \leq 2m+2$ for which Theorem 2.4 shows us that G has property $E(m,n) \Rightarrow G$ has property $E(m, n-1)$. The following theorem resolves this, showing that Theorem 2.4 is in fact a best possible result. (The proof is several pages so is omitted here - we refer the reader to [8] for full details.)

Theorem 2.6 *For $m \geq 1$ and $n \geq 2m+3$, there exists a graph G that has property $E(m,n)$ but not $E(m, n-1)$.*

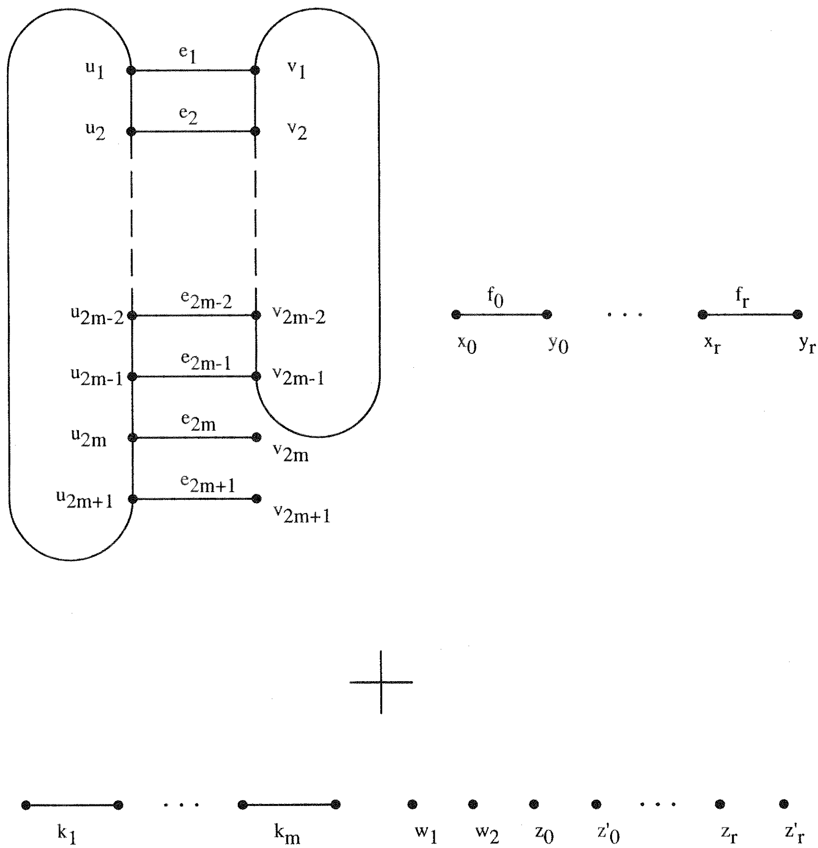

FIGURE 3: G_3 has property $E(m, 2m+3+r)$ but not $E(m, 2m+2+r)$ for $m \geq 1$ and $r \geq 0$

Outline of proof: The proof is of similar nature to that of Theorem 2.3, showing that the graph G_3 in Figure 3 has property $E(m, 2m+3+r)$ but not $E(m, 2m+2+r)$ for $m \geq 1$ and $r \geq 0$. Define $A = \{u_i, v_i, x_j, y_j : 1 \leq i \leq 2m+1, 0 \leq j \leq r\}$ and $B = V(G_3)\backslash A$.

It is fairly easy to see that it is not $E(m, 2m+2+r)$, by considering the sets $M = \{k_1, \ldots, k_m\}$ and $N = \{e_1, \ldots, e_{2m+1}, f_0, \ldots, f_r\}$. Any (M, N)-extension requires exactly $2m$ edges between vertices in set A (with the remaining edges outside of M between the sets A and B). This is clearly not possible whilst avoiding set N.

The length of the proof arises as a result of the need to show that G_3 has property $E(m, 2m+3+r)$, i.e. showing that for *every* pair of disjoint independent sets M, N of sizes m, n there exists an (M, N)-extension. ■

We have seen that the $E(m, n)$ property behaves unexpectedly for some graphs with $n \geq 2m+3$. The next two results however show that in some senses this behaviour is a minor aberration. The first result improves on Theorem 2.1 for $E(m, n)$ graphs where n is large with respect to m. The second shows that for any given m, n, the graphs which have property $E(m, n)$ but not $E(m, n-1)$ are restricted in size. Therefore for any given m, n there are a finite number of such graphs.

Theorem 2.7 *Let a graph G have property $E(m, n)$, with $n > m$. Then G has property $E(m, p)$ for all $p \leq m$.*

Proof: Let a graph G on p vertices have property $E(m, n)$ with $n > m$, so that $p \geq 2(m+n+1)$. We first note that by Corollary 2.5 we need only show that G has property $E(m, m)$. Consider then arbitrary disjoint independent edgesets M and N each of size m. By Theorem 2.1 G has property $E(m, 0)$ so certainly set N extends to a perfect matching P. This matches all the remaining vertices, so there must be exist at least $(m+n+1) - m = n+1$ edges in G independent of N.

Since $|M| = m$, there are at least $n+1-m$ edges independent of N that are not in M. Choose $n-m$ of these together with the m edges in N to form an independent edgeset N_1 of size n that is disjoint of M. It follows that there exists an (M, N_1)-extension which must also be an (M, N)-extension. Thus G has property $E(m, m)$. ■

Theorem 2.8 *If a graph G has property $E(m, n)$ but not $E(m, n-1)$ then $|V(G)| \leq 4(n-1) - 2m$, where $m \geq 1$.*

Proof: Consider a graph G on p vertices that has property $E(m, n)$ but not $E(m, n-1)$. Take disjoint independent edgesets M and N of sizes $m, n-1$ respectively such that there exists no (M, N)-extension in G. From Theorems 2.4

and 2.7 we have $n \geq 2m+3$ and G has property $E(m,m)$ so we may choose $M_1 \subseteq N$ to be of size m and let $N_1 = M$ so that there exists an (M_1, N_1)-extension P_1.

Since G has property $E(m,n)$ but not $E(m,n-1)$ it follows that there are no edges independent of set N outside of M. Therefore the matching P_1 which avoids the set $N_1 = M$ must match every vertex that is not N-covered to a distinct vertex in the graph induced by the edges $e \in N \backslash M_1$. Thus we have $|V(G)| \leq 2(n-1) + 2(n-1-m) = 4(n-1) - 2m$. ■

3 Other $E(m,n)$ implications

We now present some other relationships that hold amongst the $E(m,n)$ graphs. Firstly we state a corollary to Theorem 1.1(c) and (d).

Corollary 3.1 *If a graph G has property $E(m,0)$, then G has property $E(m-1,1)$ for $m \geq 1$.* ■

Theorem 3.2 *If a graph G has property $E(m,n)$, then G has property $E(m-1,n+1)$ for $m \geq 1$.*

Proof: If $n=0$ then the theorem follows from Corollary 3.1, so we may suppose that $n \geq 1$.

Let G be a graph on p vertices that has property $E(m,n)$. Now consider arbitrary disjoint independent sets $M, N \subseteq E(G)$ of sizes $m-1, n+1$ respectively. By Corollary 2.2, G has property $E(m-1,0)$, and so there exists a perfect matching P in G containing M. Further since $p \geq 2(m+n+1)$, we have $|P| \geq m+n+1$. Then $|P \cap (E(G) \backslash (M \cup N))| \geq 1$, so we may choose $e \in P \cap (E(G) \backslash (M \cup N))$. Then e is independent of M as it belongs to the same perfect matching P, so let $M_1 = M \cup \{e\}$.

Now consider two possibilities:

(a) There exists an edge $f \in N$ adjacent to M_1.

Taking $N_1 = N \backslash \{f\}$ then there exists an (M_1, N_1)-extension P_1 in G. Further, $f \notin P_1$ as it is adjacent to an M_1 edge. Thus P_1 is an (M,N)-extension.

(b) Otherwise N is independent of M_1.

For any edge $g \in N$, let $N_2 = N \backslash \{g\}$. There exists an (M_1, N_2)-extension P_2 in G. If P_2 avoids g also then P_2 is an (M,N)-extension. Else since $|N_2| = n \geq 1$, there exists an edge $h \in N_2$. This edge is avoided by P_2 but since P_2 is a perfect matching h must be adjacent to some $q \in P_2$. Further, since $h \in N_2 \subseteq N$, a set independent of M, it follows that $q \notin M$ and so q is independent of M. Now let $M_2 = M \cup \{q\}$ and $N_3 = N \backslash \{h\}$. Then there exists an (M_2, N_3)-extension P_3 with $h \notin P_3$. Thus P_3 is an (M,N)-extension. ■

The following corollary to Theorem 3.2 ensures that if $\mathcal{E}(m,n) = \{G : G$ *is an* $E(m,n)$ *graph*$\}$ we have

$$\mathcal{E}(m+n,0) \subseteq \mathcal{E}(m,n) \subseteq \mathcal{E}(m,0)$$

and thus relates numbers of $E(m,n)$ graphs to numbers of k-extendable graphs.

Corollary 3.3 *If a graph G has property $E(m+n,0)$ then G has property $E(m,n)$.*

Proof: The result follows from repeated applications of Theorem 3.2 to G. ■

Our final theorem in this section confirms what we might expect.

Theorem 3.4 *If a graph G has property $E(m,n)$, then G has property $E(m-1,n)$.*

Proof: Suppose a graph G on p vertices has property $E(m,n)$. Now consider arbitrary disjoint independent sets $M, N \subseteq E(G)$ of sizes $m-1, n$ respectively. Now, by Corollary 2.2, G has property $E(m-1,0)$. So there exists a perfect matching P in G that contains M. Further since $p \geq 2(m+n+1)$, we have $|P| \geq m+n+1$. Then $|P \cap (E(G) \backslash (M \cup N))| \geq 2$, so we may choose $e \in P \cap (E(G) \backslash (M \cup N))$. The edge e is independent of M as it belongs to the same perfect matching P, so with $M_1 = M \cup \{e\}$ then there exists an (M_1, N)-extension P_1 in G. Clearly P_1 is also an (M,N)-extension and the result follows. ■

We present the graph $G_4 = (2m+n)K_2 + \overline{K_{2(m+n)}}$ in Figure 4 to show that the results of Theorems 3.2 and 3.4 are in some senses best possible. It can be easily seen using arguments similar to those in the proof of Lemma 2.3 that G_4 has property $E(m,n)$ but not $E(m+1,n)$, $E(m,n+1)$ or $E(m-1,n+2)$.

As a summary of the relationships between the $E(m,n)$ properties we obtain the following corollary which extends to the result in Corollary 3.6 in the special case when $n \leq 2m+2$.

Corollary 3.5 *If a graph G has property $E(m,n)$ then G has property $E(r,s)$, for $r+n \leq r+s \leq m+n$.*

Proof:
$$\begin{aligned}
E(m,n) &\Rightarrow E(m-1,n+1) \qquad \{\text{from Theorem 3.2}\} \\
&\dots \Rightarrow E(m-(s-n), n+(s-n)) = E((m+n)-s, s) \\
&\qquad\qquad \{\text{from Theorem 3.2}\} \\
&\Rightarrow E((m+n)-s-1, s) \quad \{\text{from Theorem 3.4}\} \\
&\dots \Rightarrow E(r,s), \text{ since } r \leq m+n-s \\
&\qquad\qquad \{\text{from Theorem 3.4}\}.
\end{aligned}$$
■

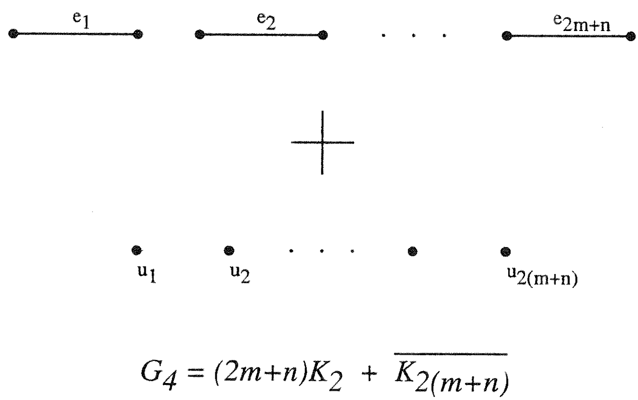

FIGURE 4: G_4 has property $E(m,n)$ but not $E(m+1,n)$, $E(m,n+1)$ or $E(m-1,n+2)$

Corollary 3.6 *Suppose $n \leq 2m+2$. Then if a graph G has property $E(m,n)$ then G has property $E(r,s)$, for $0 \leq r \leq m$ and $0 \leq r+s \leq m+n$.*

Proof: $E(m,n) \Rightarrow E(m-1,n+1)$ {from Theorem 3.2}
$\ldots \Rightarrow E(m-(m-r), n+(m-r)) = E(r, m+n-r)$
{from Theorem 3.2}
$\Rightarrow E(r, m+n-r-1)$ {from Theorem 2.4}
$\ldots \Rightarrow E(r,s)$, since $s \leq m+n-r$
{from Theorem 2.4}. ∎

4 Two More Results

We have seen that the property of $E(m,n)$ is basically a refinement of the widely studied property of n-extendability. There has already been some progress made using this to improve results obtained during the study of n-extendability in graphs. The last two results presented here give the flavour of this research.

In [6] Plummer showed that if G is a $(2n+1)$-connected even claw-free graph with $n \geq 0$ then G is n-extendable. This has been improved with the following theorem, which in particular tells us more about claw-free graphs with even connectivity.

Theorem 4.1 [1] *If a graph G is $(2m+n+1)$-connected even and claw-free with $|V(G)| \geq 2m+2n+2$, then G has property $E(m, n)$.* ∎

Corollary 4.2 [1] *Let G be an even, claw-free graph. If G is*

(a) *$2n$-connected, then it has property $E(n-1,1)$ unless $G \cong K_{2n}$.*

(b) *$(2n+1)$-connected, then it has property $E(n,0)$.* ■

The final result given here concerns matching extension in bipartite graphs.

Theorem 4.3 [1] *If G is a regular $(2n+1)$-edge-connected bipartite graph with $|V(G)| \geq 2(n+1)+2$, then G has property $E(1,n)$.* ■

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