10-418/10-618 Machine Learning for Structured Data

Machine Learning Department
School of Computer Science
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# Neural Potentials + MBR Decoding

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Lecture 10
Oct. 3, 2022

# Reminders

- **Homework 2: Learning to Search for RNNs**
  - Out: Sun, Sep 18
  - Written (except for Empirical Questions)
    - Due: Thu, Sep 29 at 11:59pm
  - Programming + Empirical Questions
    - Due: Mon, Oct 24 at 9:00am
- **Homework 3: General Graph CRF Module**
  - Out: Thu, Sep 29
  - Due: Mon, Oct 10 at 11:59pm

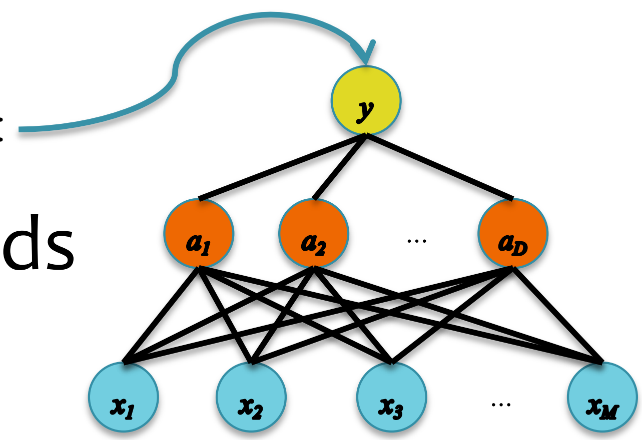
# MRF AND CRF LEARNING (LOG-LINEAR PARAMETERIZATION)

# Options for MLE of MRFs

- **Setting I:** $\psi_C(\boldsymbol{x}_C) = \theta_{C,\boldsymbol{x}_C}$
  A. MLE by inspection (Decomposable Models)
  B. Iterative Proportional Fitting (IPF)

- **Setting II:** $\psi_C(\boldsymbol{x}_C) = \exp(\boldsymbol{\theta} \cdot \boldsymbol{f}(\boldsymbol{x}_C))$
  C. Generalized Iterative Scaling
  D. Gradient-based Methods

- **Setting III:** $\psi_C(\boldsymbol{x}_C) =$
  E. Gradient-based Methods



# MRF and CRF Learning

***Whiteboard***

- log-linear MRF model (i.e. with feature based potentials)
- log-linear MRF derivatives
- log-linear MRF training with SGD
- log-linear CRF model (i.e. with feature based potentials)
- log-linear CRF derivatives
- log-linear CRF training with SGD

# Recipe for Gradient-based Learning

1. Write down the objective function
2. Compute the partial derivatives of the objective (i.e. gradient, and maybe Hessian)
3. Feed objective function and derivatives into black box


4. Retrieve optimal parameters from black box

# Optimization Algorithms

**What is the black box?**


- Newton’s method
- Hessian-free / Quasi-Newton methods
  - Conjugate gradient
  - L-BFGS
- Stochastic gradient methods
  - Stochastic gradient descent (SGD)
  - SGD with momentum
  - Adam

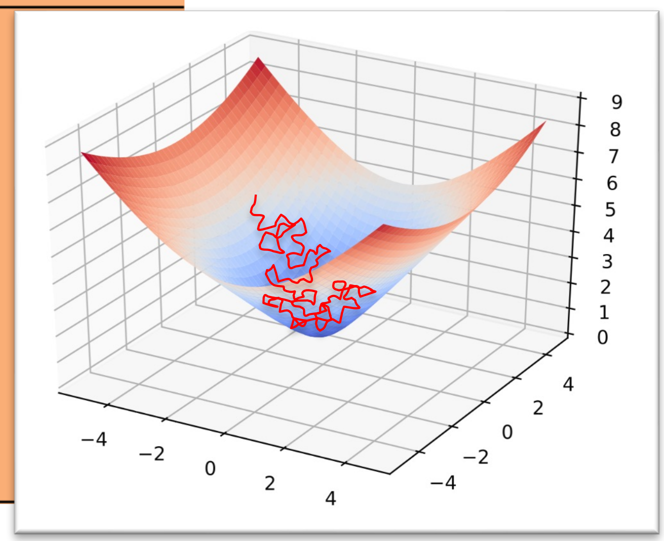
# Stochastic Gradient Descent (SGD)

Assume we have an objective that decomposes additively:

$$\text{Let } J(\boldsymbol{\theta}) = \sum_{i=1}^{N} J^{(i)}(\boldsymbol{\theta})$$

**Algorithm 2** Stochastic Gradient Descent (SGD)

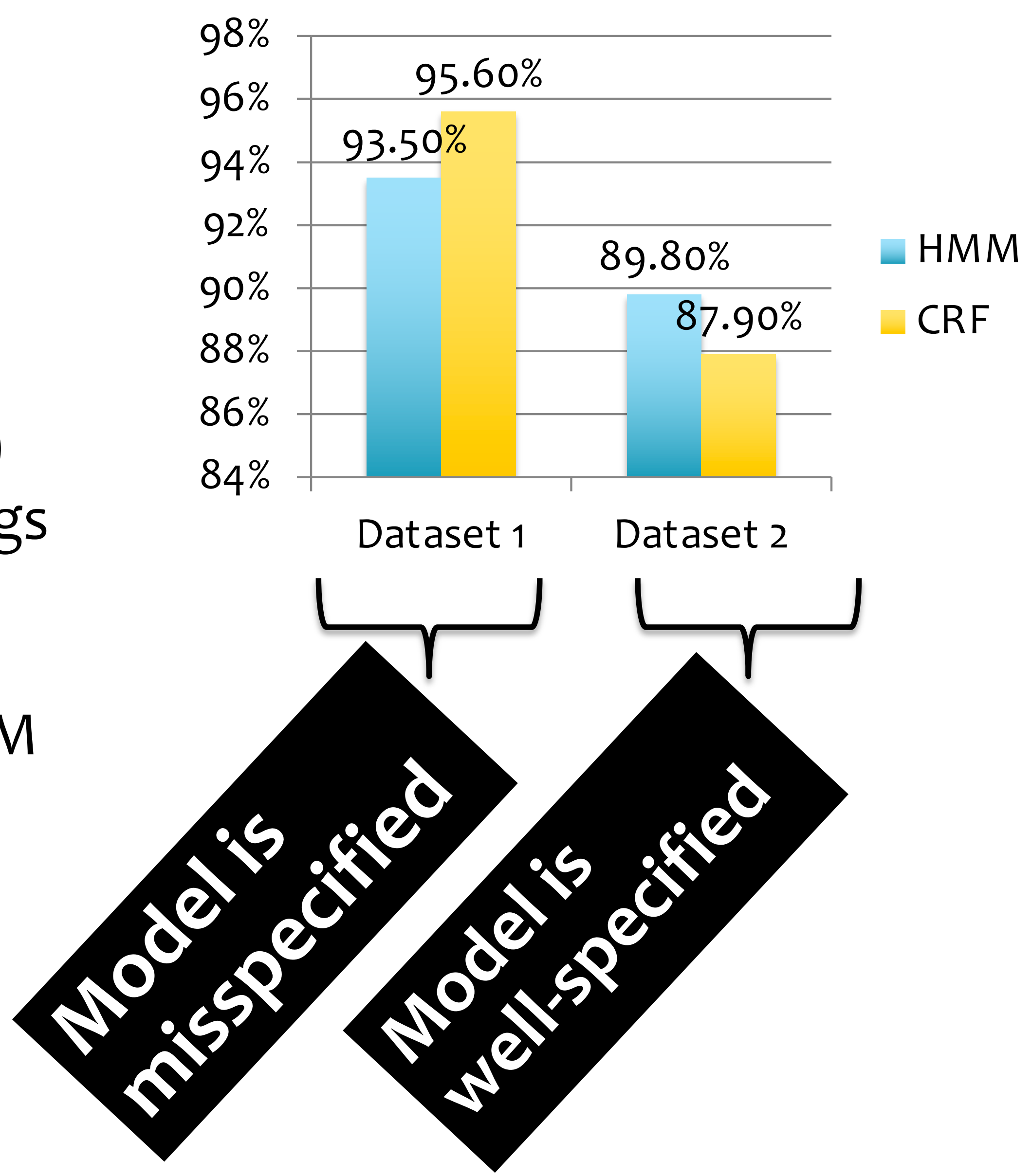
1: **procedure** SGD($\mathcal{D}, \boldsymbol{\theta}^{(0)}$)
2: $\quad \boldsymbol{\theta} \leftarrow \boldsymbol{\theta}^{(0)}$
3: $\quad$ **while** not converged **do**
4: $\quad\quad i \sim$ Uniform($\{1, 2, \ldots, N\}$)
5: $\quad\quad \boldsymbol{\theta} \leftarrow \boldsymbol{\theta} - \gamma \nabla_{\boldsymbol{\theta}} J^{(i)}(\boldsymbol{\theta})$
6: $\quad$ **return** $\boldsymbol{\theta}$



# Generative vs. Discriminative

Liang & Jordan (ICML 2008) compares **HMM** and **CRF** with **identical features**

- Dataset 1: (Real)
  - WSJ Penn Treebank (38K train, 5.5K test)
  - 45 part-of-speech tags
- Dataset 2: (Artificial)
  - Synthetic data generated from HMM learned on Dataset 1 (1K train, 1K test)
- Evaluation Metric: Accuracy



# NEURAL PARAMETERIZATION OF CONDITIONAL RANDOM FIELD

# Options for MLE of MRFs

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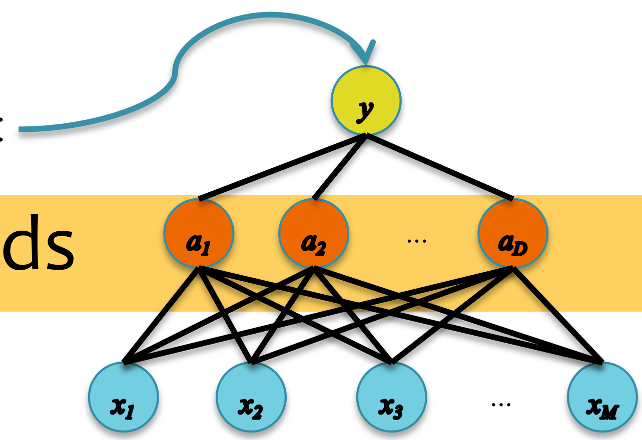
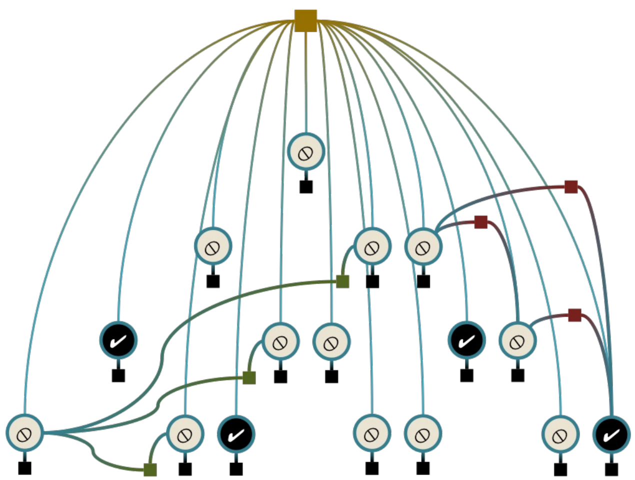


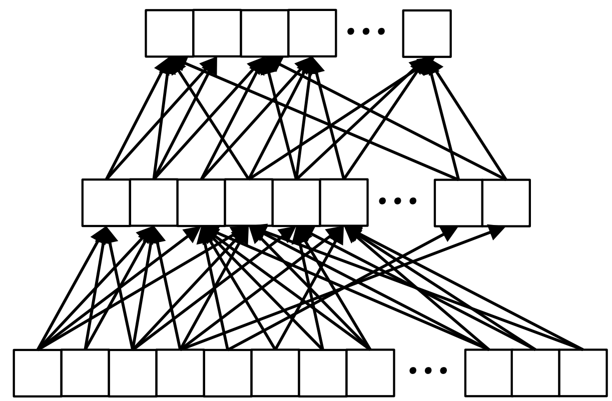
# Motivation:

## Hybrid Models

**Graphical models** let you encode domain knowledge



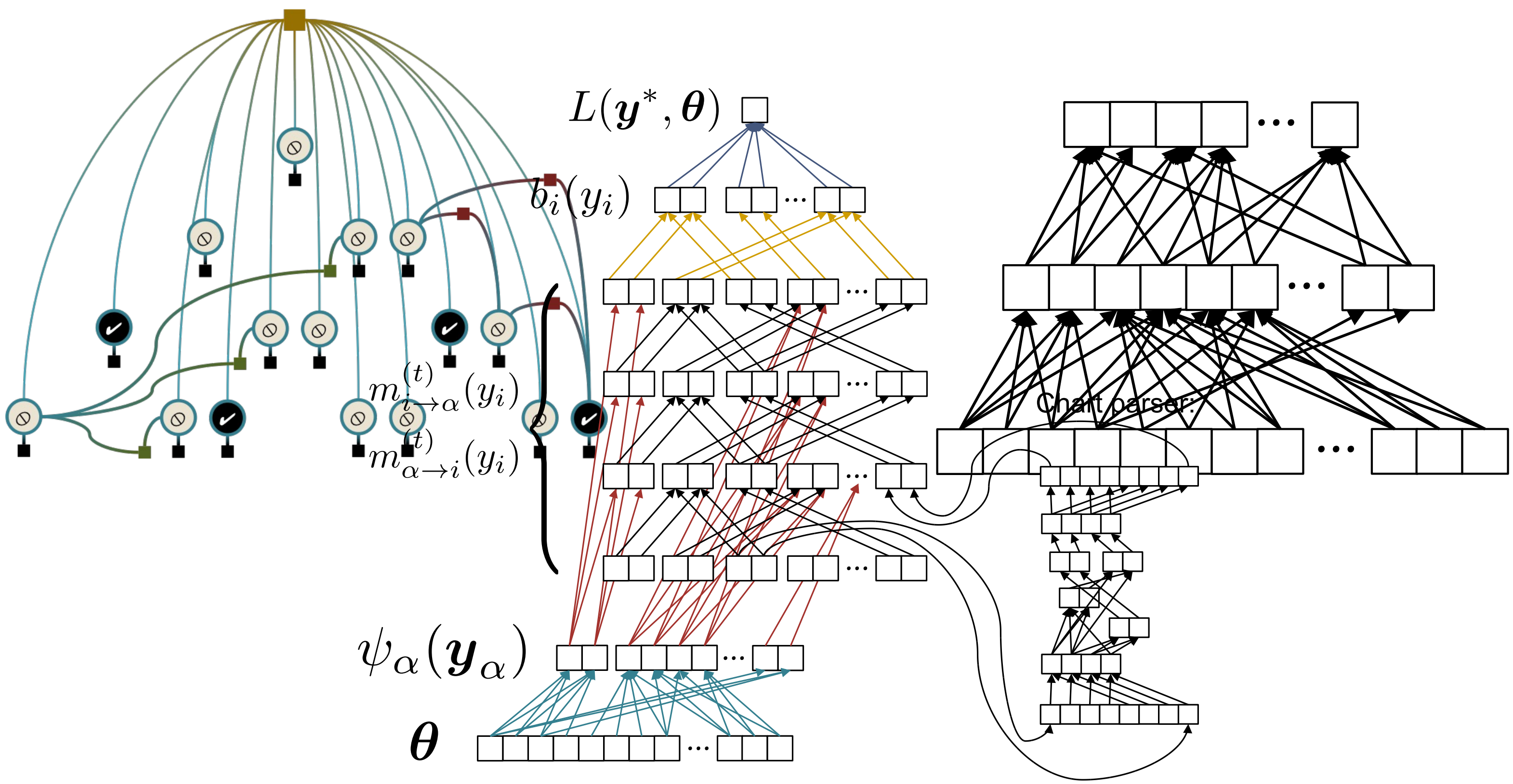
**Neural nets** are really good at fitting the data discriminatively to make good predictions



**Could we define a neural net that incorporates domain knowledge?**

# Motivation:
## Hybrid Models

*Key idea:* Use a NN to learn features for a GM, then train the entire model by backprop



# A Recipe for Neural Networks

1. Given training data:

$$\{\boldsymbol{x}_i, \boldsymbol{y}_i\}_{i=1}^N$$

*Face* *Face* *Not a face*



2. Choose each of these:
   - Decision function

$$\hat{\boldsymbol{y}} = f_{\boldsymbol{\theta}}(\boldsymbol{x}_i)$$

**Examples:** Linear regression, Logistic regression, Neural Network

   - Loss function

$$\ell(\hat{\boldsymbol{y}}, \boldsymbol{y}_i) \in \mathbb{R}$$

**Examples:** Mean-squared error, Cross Entropy

# MRF AND CRF LEARNING (NEURAL PARAMETERIZATION)

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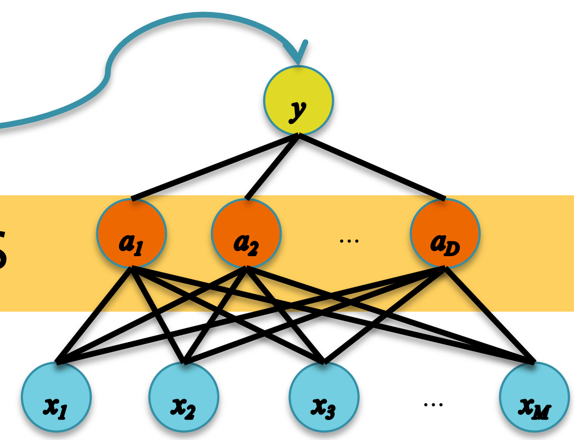

***Whiteboard:***

- CRF w/LSTM potentials
- Gradient of MRF/CRF log-likelihood with respect to log potentials
- Gradient of MRF/CRF log-likelihood with respect to potentials
- Backprop with MRF/CRF log-likelihood as a loss function

# Factor Derivatives

Log-probability:

$$\log p(\mathbf{y}) = \left[\sum_{\alpha} \log \psi_\alpha(\mathbf{y}_\alpha)\right] - \log \sum_{\mathbf{y}' \in \mathcal{Y}} \prod_{\alpha} \psi_\alpha(\mathbf{y}'_\alpha) \tag{1}$$

Derivatives:

$$\frac{\partial \log p(\mathbf{y})}{\partial \log \psi_\alpha(\mathbf{y}'_\alpha)} = \mathbb{1}(\mathbf{y}_\alpha = \mathbf{y}'_\alpha) - p(\mathbf{y}'_\alpha) \tag{2}$$

$$\frac{\partial \log p(\mathbf{y})}{\partial \psi_\alpha(\mathbf{y}'_\alpha)} = \frac{\mathbb{1}(\mathbf{y}_\alpha = \mathbf{y}'_\alpha) - p(\mathbf{y}'_\alpha)}{\psi_\alpha(\mathbf{y}'_\alpha)} \tag{3}$$

# HYBRIDS OF NEURAL NETWORKS WITH GRAPHICAL MODELS

# Outline of Examples

- **Hybrid NN + HMM**
  - Model: neural net for emissions
  - Learning: backprop for end-to-end training
  - Experiments: phoneme recognition (Bengio et al., 1992)
- **Hybrid RNN + HMM**
  - Model: neural net for emissions
  - Experiments: phoneme recognition (Graves et al., 2013)
- **Hybrid CNN + CRF**
  - Model: neural net for factors
  - Experiments: natural language tasks (Collobert & Weston, 2011)
  - Experiments: pose estimation
- **Tricks of the Trade**

# HYBRID:
# NEURAL NETWORK + HMM

# Markov Random Field (MRF)

Recall…

Joint distribution over tags $Y_i$ <u>and</u> words $X_i$

The individual factors aren't *necessarily* probabilities.

$$p(\text{n, v, p, d, n, time, flies, like, an, arrow}) = \frac{1}{Z}(4 * 8 * 5 * 3 * \ldots)$$

| | v | n | p | d |
|---|---|---|---|---|
| v | 1 | 6 | 3 | 4 |
| n | 8 | 4 | 2 | 0.1 |
| p | 1 | 3 | 1 | 3 |
| d | 0.1 | 8 | 0 | 0 |

| | v | n | p | d |
|---|---|---|---|---|
| v | 1 | 6 | 3 | 4 |
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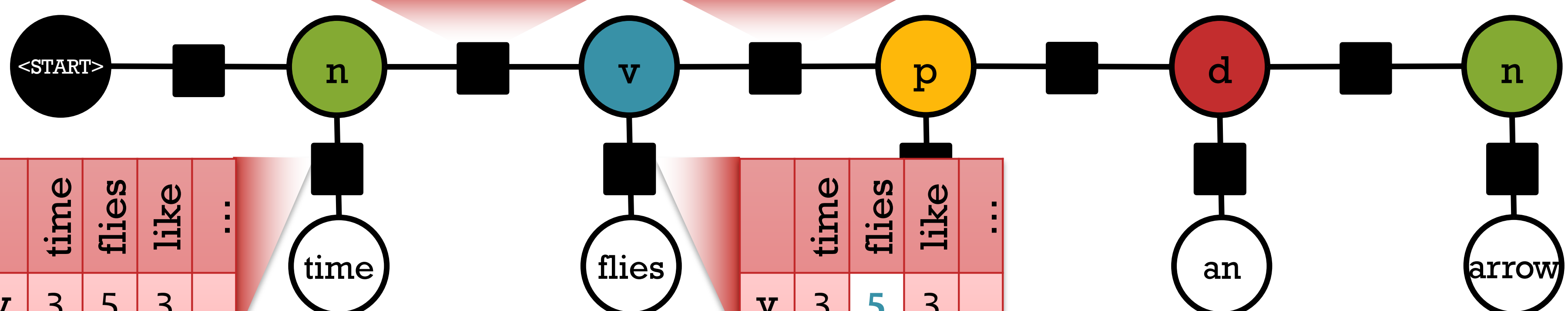



| | time | flies | like | … |
|---|---|---|---|---|
| v | 3 | 5 | 3 | |
| n | 4 | 5 | 2 | |
| p | 0.1 | 0.1 | 3 | |
| d | 0.1 | 0.2 | 0.1 | |

| | time | flies | like | … |
|---|---|---|---|---|
| v | 3 | 5 | 3 | |
| n | 4 | 5 | 2 | |
| p | 0.1 | 0.1 | 3 | |
| d | 0.1 | 0.2 | 0.1 | |

Recall…

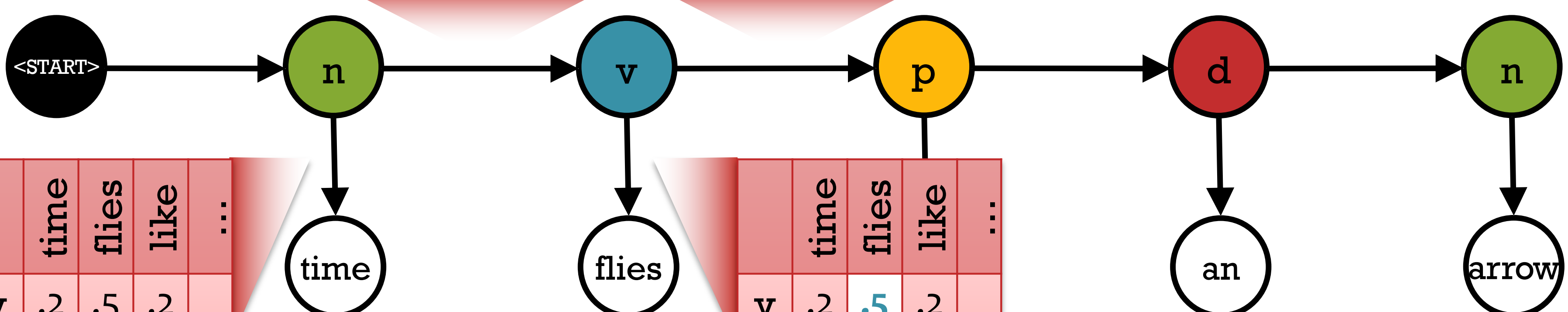
# Hidden Markov Model

But sometimes we *choose* to make them probabilities.
Constrain each row of a factor to sum to one. Now $Z = 1$.

$$p(\text{n, v, p, d, n, time, flies, like, an, arrow}) = \cancel{\frac{1}{Z}}(.3 * .8 * .2 * .5 * \ldots)$$

| | v | n | p | d |
|---|---|---|---|---|
| v | .1 | .4 | .2 | .3 |
| n | .8 | .1 | .1 | 0 |
| p | .2 | .3 | .2 | .3 |
| d | .2 | .8 | 0 | 0 |

| | v | n | p | d |
|---|---|---|---|---|
| v | .1 | .4 | .2 | .3 |
| n | .8 | .1 | .1 | 0 |
| p | .2 | .3 | .2 | .3 |
| d | .2 | .8 | 0 | 0 |




| | time | flies | like | … |
|---|---|---|---|---|
| v | .2 | .5 | .2 | |
| n | .3 | .4 | .2 | |
| p | .1 | .1 | .3 | |
| d | .1 | .2 | .1 | |

| | time | flies | like | … |
|---|---|---|---|---|
| v | .2 | .5 | .2 | |
| n | .3 | .4 | .2 | |
| p | .1 | .1 | .3 | |
| d | .1 | .2 | .1 | |

# Hybrid: NN + HMM

(Bengio et al., 1992)

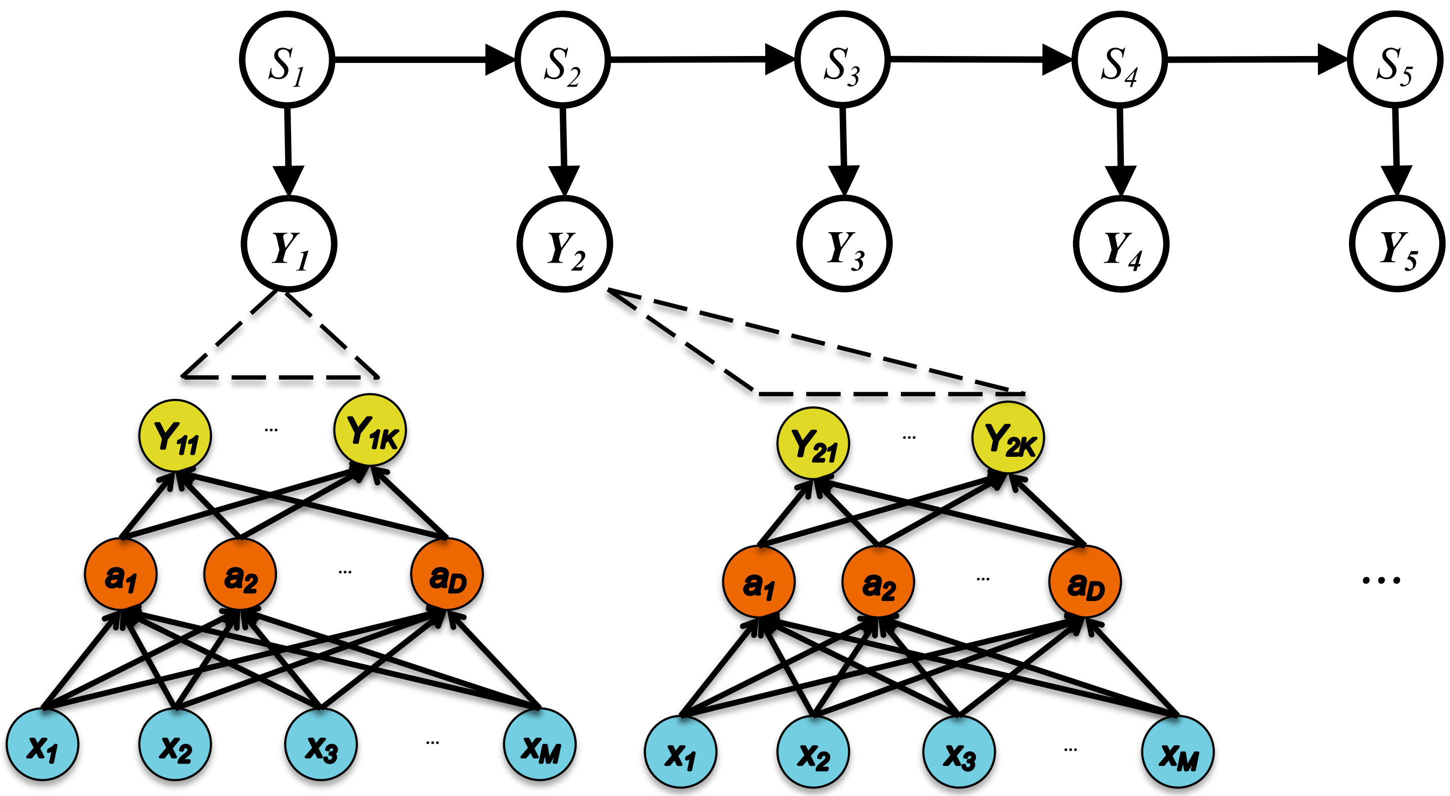
Discrete HMM state: $S_t \in \{/p/, /t/, /k/, /b/, /d/, \ldots, /g/\}$

Continuous HMM emission: $Y_t \in \mathcal{R}^K$

HMM: $p(\mathbf{Y}, \mathbf{S}) = \prod_{t=1}^{T} p(Y_t|S_t) p(S_t|S_{t-1})$

Gaussian emission:

$$p(Y_t|S_t = i) = b_{i,t} = \sum_k \frac{Z_k}{((2\pi)^n \mid \Sigma_k \mid)^{1/2}} \exp(-\frac{1}{2}(Y_t - \mu_k)\Sigma_k^{-1}(Y_t - \mu_k)^T)$$



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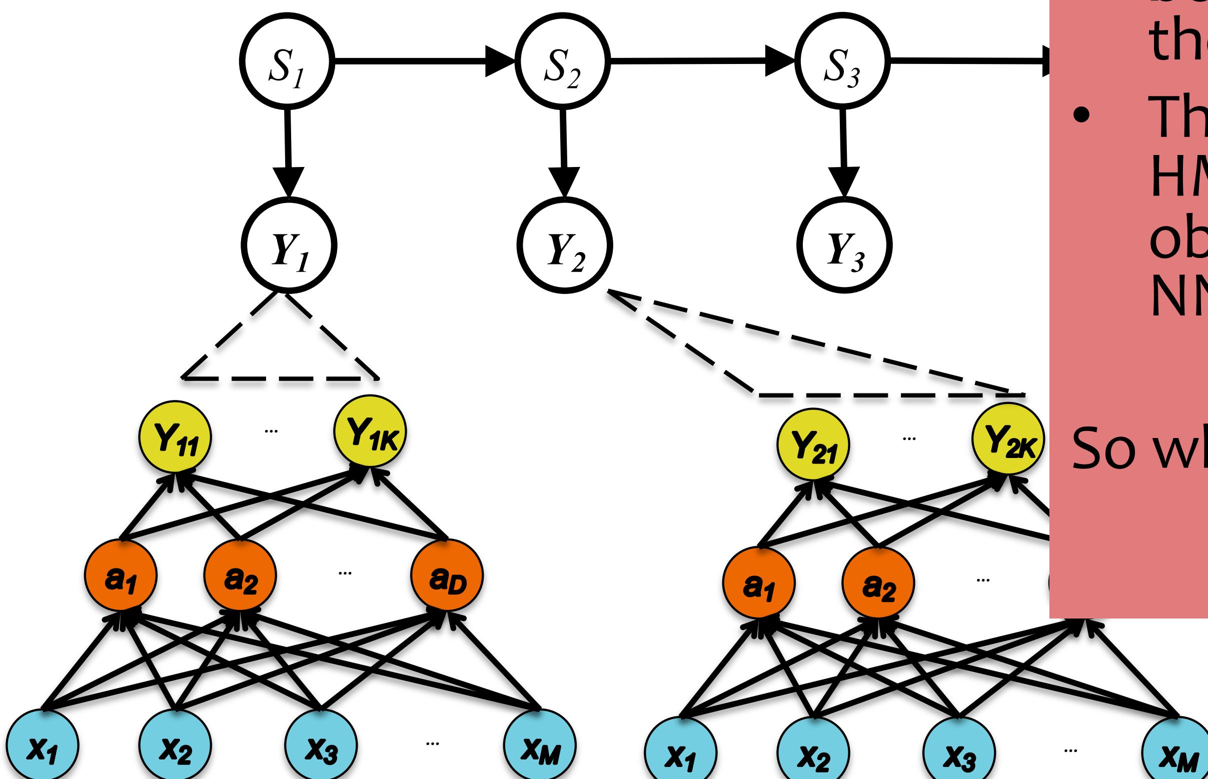
Discrete HMM state: $S_t \in \{/p/, /t/, /k/, /b/, /d/, \ldots, /g/\}$

Continuous HMM emission: $Y_t \in \mathcal{R}^K$

HMM: $p(\mathbf{Y}, \mathbf{S}) = \prod_{t=1}^{T} p(Y_t|S_t) p(S_t|S_{t-1})$

$$p(Y_t|S_t = i) = b_{i,t} = \sum_k \frac{Z_k}{((2\pi)^n \mid \Sigma_k \mid)^{1/2}} e$$

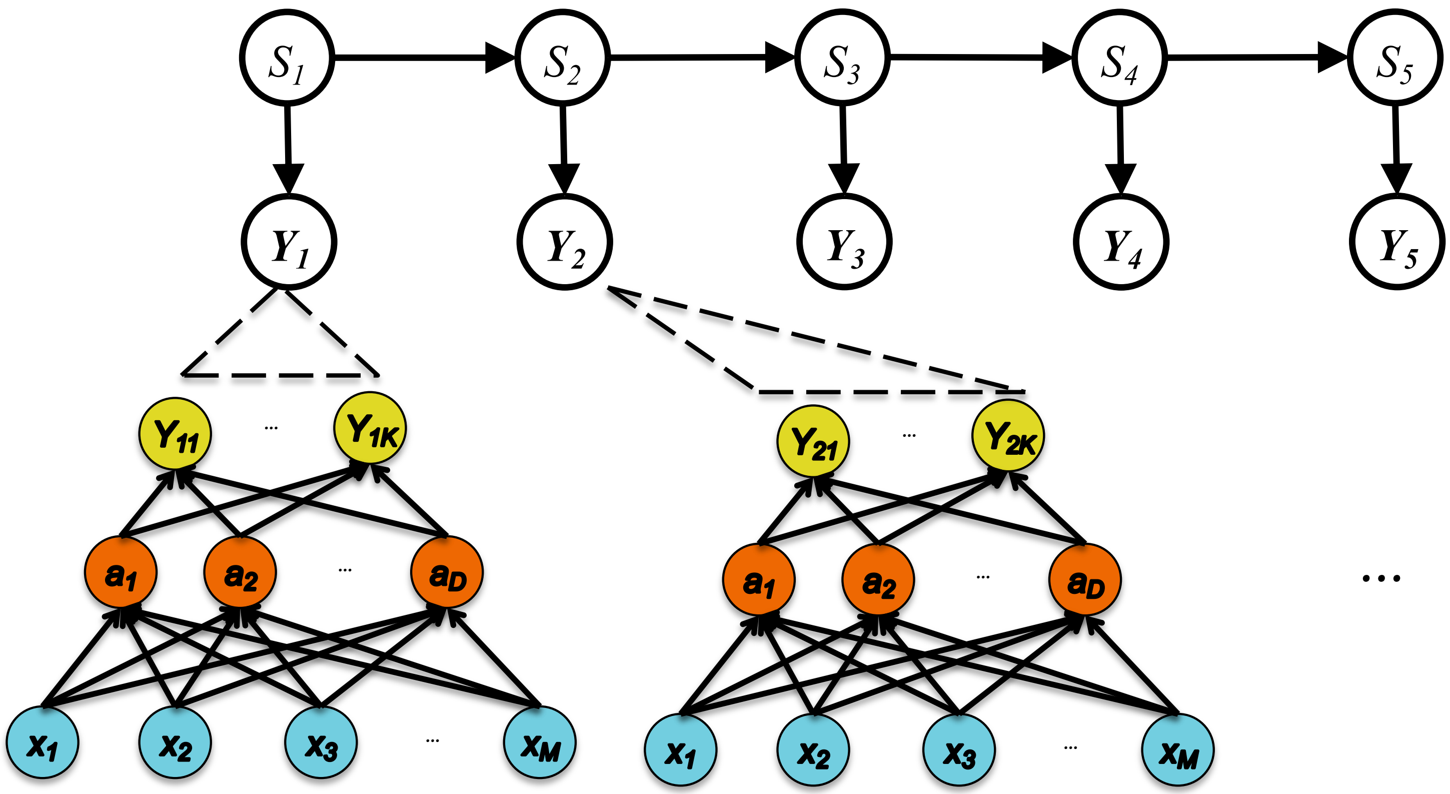



Lots of oddities to this picture:

- **Clashing visual notations** (graphical model vs. neural net)
- HMM generates data **top-down,** NN generates **bottom-up** and they meet in the middle.
- The "observations" of the HMM are not actually observed (i.e. x's appear in NN only)

So what are we missing?

# Hybrid: NN + HMM



$a_{i,j} = p(S_t = i | S_{t-1} = j)$
$b_{i,t} = p(Y_t | S_t = i)$

# Hybrid: NN + HMM

**Forward-backward algorithm:** a "feed-forward" algorithm for computing alpha-beta probabilities.

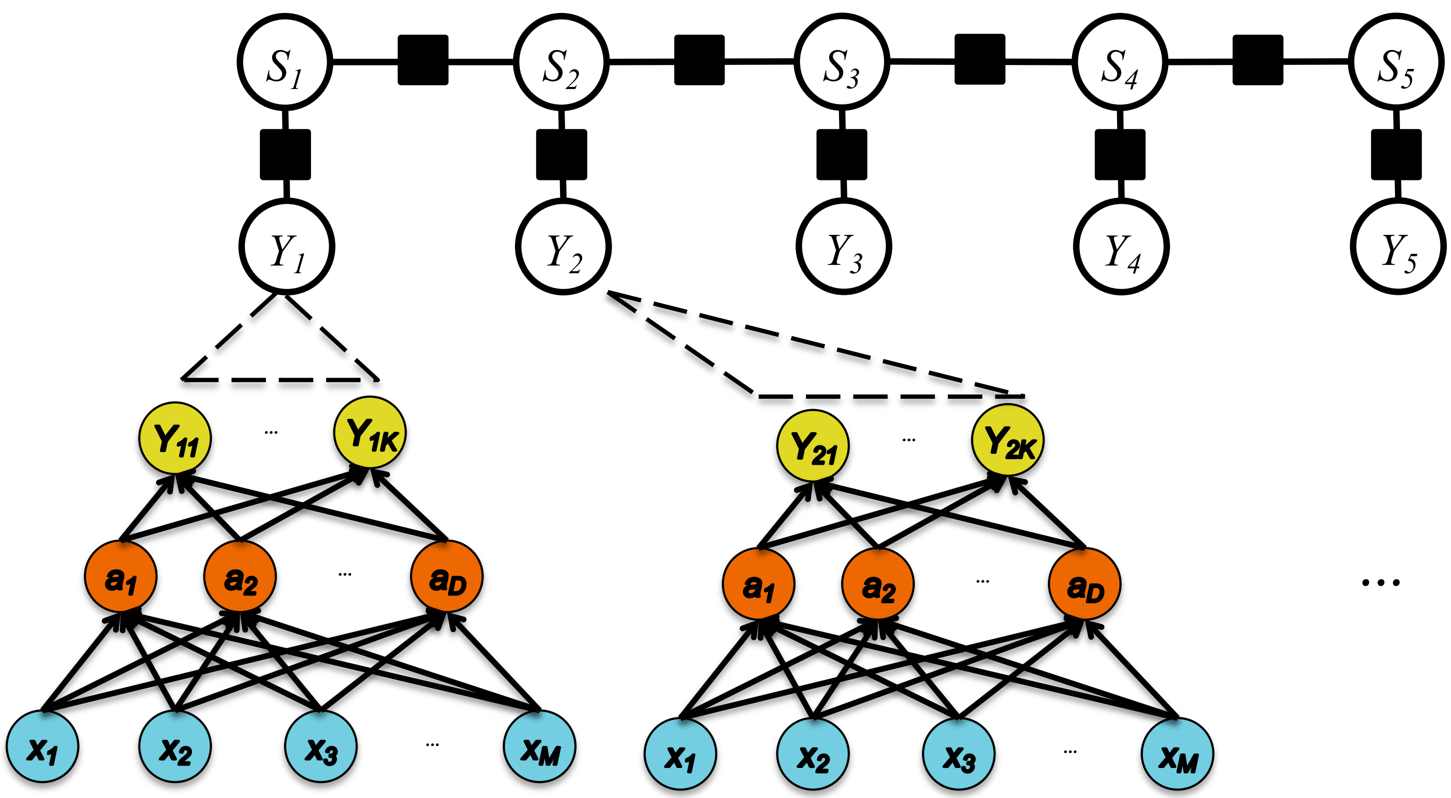
$$\alpha_{i,t} = P(Y_1^t \text{ and } S_t = i \mid model) = b_{i,t} \sum_j a_{ji} \alpha_{j,t-1}$$

$$\beta_{i,t} = P(Y_{t+1}^T \mid S_t = i \text{ and } model) = \sum_j a_{ij} b_{j,t+1} \beta_{j,t+1}$$

$$\gamma_{i,t} = P(S_t = i \mid Y_1^t \text{ and } model) = \alpha_{i,t} \, \beta_{i,t}$$

**Log-likelihood:** a "feed-forward" objective function.

$$\log p(\mathbf{S}, \mathbf{Y}) = \alpha_{\text{END},T}$$



# A Recipe for Graphical Models

**Decision / Loss Function for Hybrid NN + HMM**

1. Given training data:

$$\{\boldsymbol{x}_i, \boldsymbol{y}_i\}_{i=1}^{N}$$

**Forward-backward algorithm:** a "feed-forward" algorithm for computing alpha-beta probabilities.

$$\alpha_{i,t} = P(Y_1^t \text{ and } S_t = i \mid model) = b_{i,t} \sum_j a_{ji} \alpha_{j,t-1}$$

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$$\gamma_{i,t} = P(S_t = i \mid Y_1^t \text{ and } model) = \alpha_{i,t}\, \beta_{i,t}$$

2. Choose each of these:
   - Decision function

$$\hat{\boldsymbol{y}} = f_{\boldsymbol{\theta}}(\boldsymbol{x}_i)$$

**Log-likelihood:** a "feed-forward" objective function.

$$\log p(\mathbf{S}, \mathbf{Y}) = \alpha_{\text{END},T}$$

   - Loss function

$$\ell(\hat{\boldsymbol{y}}, \boldsymbol{y}_i) \in \mathbb{R}$$

nt)

$$\eta_t \nabla \ell(f_{\boldsymbol{\theta}}(\boldsymbol{x}_i), \boldsymbol{y}_i)$$

How do we compute the gradient?

Recall...

Training

# Backpropagation

Graphical Model and Log-likelihood

Neural Network

**Backpropagation** is just repeated application of the **chain rule** from Calculus 101.

$$\boldsymbol{y} = g(\boldsymbol{u}) \text{ and } \boldsymbol{u} = h(\boldsymbol{x}).$$

How to compute these partial derivatives?

**Chain Rule:**

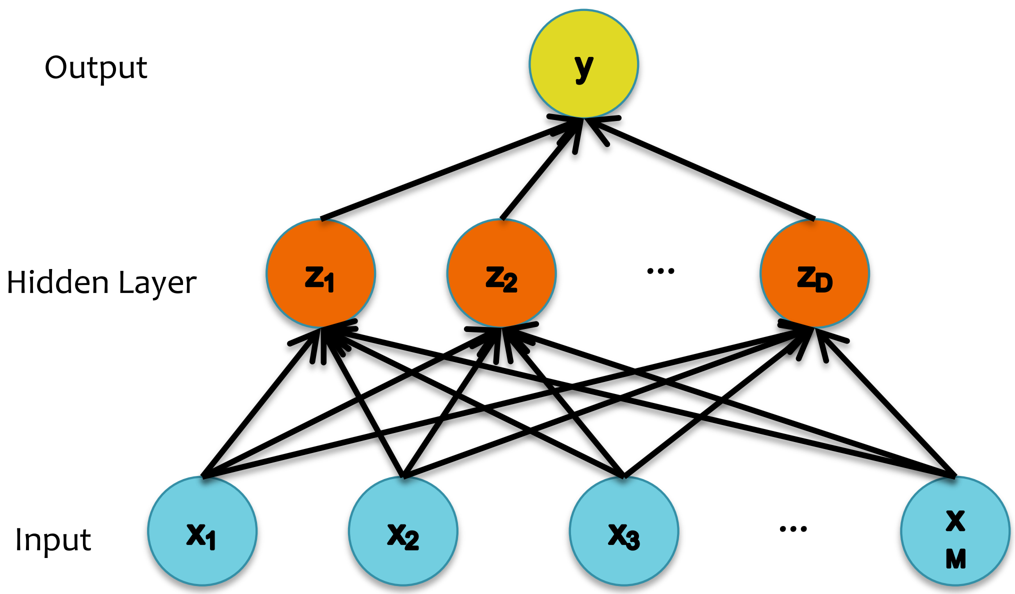
$$\frac{dy_i}{dx_k} = \sum_{j=1}^{J} \frac{dy_i}{du_j} \frac{du_j}{dx_k}, \quad \forall i, k$$

Recall...

# Training

# Backpropagation

What does this picture actually mean?




**(F) Loss**
$J = \frac{1}{2}(y - y^{(d)})^2$

**(E) Output (sigmoid)**
$y = \frac{1}{1+\exp(b)}$

**(D) Output (linear)**
$b = \sum_{j=0}^{D} \beta_j z_j$

**(C) Hidden (sigmoid)**
$z_j = \frac{1}{1+\exp(a_j)}, \ \forall j$

**(B) Hidden (linear)**
$a_j = \sum_{i=0}^{M} \alpha_{ji} x_i, \ \forall j$

**(A) Input**
Given $x_i, \ \forall i$

Recall...

# Training Backpropagation

Case 2: Neural Network

**Forward**

$$J = y^* \log q + (1 - y^*) \log(1 - q)$$

$$q = \frac{1}{1 + \exp(-b)}$$

$$b = \sum_{j=0}^{D} \beta_j z_j$$

$$z_j = \frac{1}{1 + \exp(-a_j)}$$

$$a_j = \sum_{i=0}^{M} \alpha_{ji} x_i$$

**Backward**

$$\frac{dJ}{dq} = \frac{y^*}{q} + \frac{(1 - y^*)}{q - 1}$$

$$\frac{dJ}{db} = \frac{dJ}{dy}\frac{dy}{db}, \quad \frac{dy}{db} = \frac{\exp(b)}{(\exp(b) + 1)^2}$$

$$\frac{dJ}{d\beta_j} = \frac{dJ}{db}\frac{db}{d\beta_j}, \quad \frac{db}{d\beta_j} = z_j$$

$$\frac{dJ}{dz_j} = \frac{dJ}{db}\frac{db}{dz_j}, \quad \frac{db}{dz_j} = \beta_j$$

$$\frac{dJ}{da_j} = \frac{dJ}{dz_j}\frac{dz_j}{da_j}, \quad \frac{dz_j}{da_j} = \frac{\exp(a_j)}{(\exp(a_j) + 1)^2}$$

$$\frac{dJ}{d\alpha_{ji}} = \frac{dJ}{da_j}\frac{da_j}{d\alpha_{ji}}, \quad \frac{da_j}{d\alpha_{ji}} = x_i$$

$$\frac{dJ}{dx_i} = \frac{dJ}{da_j}\frac{da_j}{dx_i}, \quad \frac{da_j}{dx_i} = \sum_{j=0}^{D} \alpha_{ji}$$

# Hybrid: NN + HMM

**Computing the Gradient:** $\nabla \ell(f_{\boldsymbol{\theta}}(\boldsymbol{x}_i), \boldsymbol{y}_i)$

*Forward computation*

$$\log p(\mathbf{S}, \mathbf{Y}) = \alpha_{\text{END},T}$$

$\alpha_{i,t} = \ldots$ (forward prob)

$\beta_{i,t} = \ldots$ (backward prop)

$\gamma_{i,t} = \ldots$ (marginals)

$a_{i,j} = \ldots$ (transitions)

$b_{i,t} = \ldots$ (emissions)

$$y_{tk} = \frac{1}{1+\exp(-b)}$$

$$b = \sum_{j=0}^{D} \beta_j z_j$$

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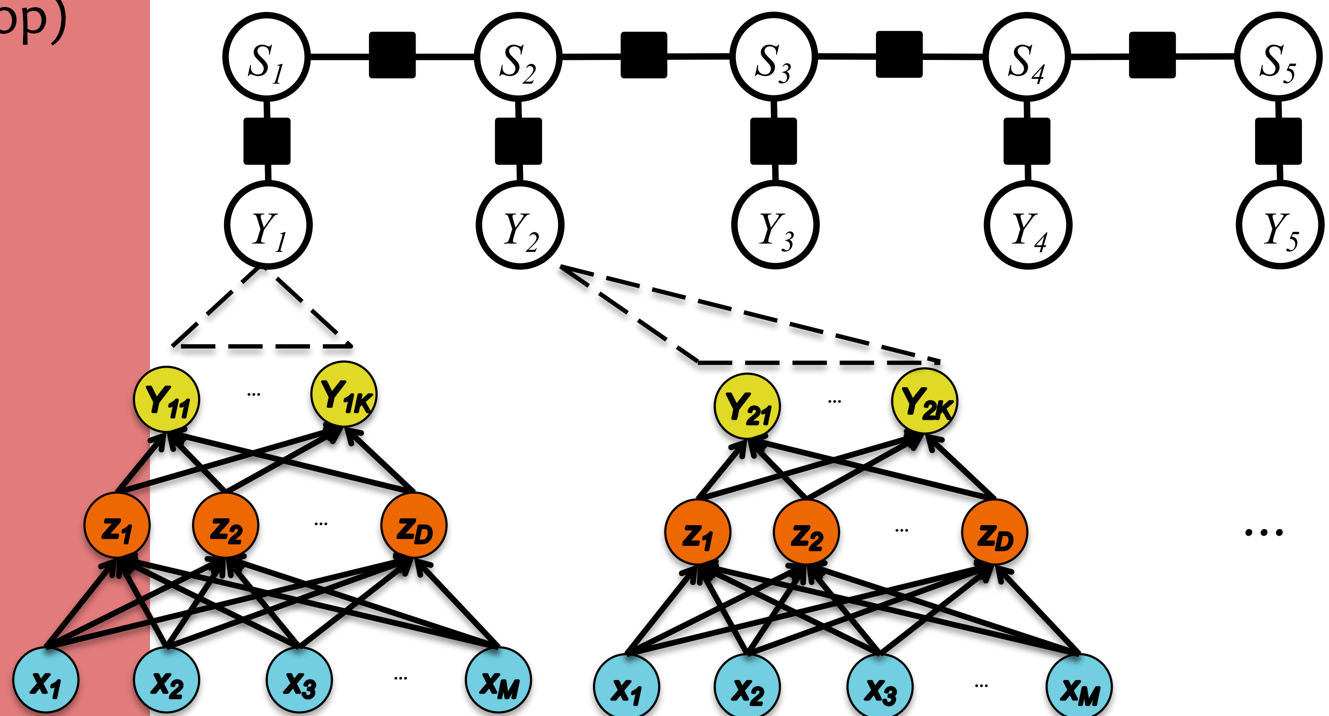

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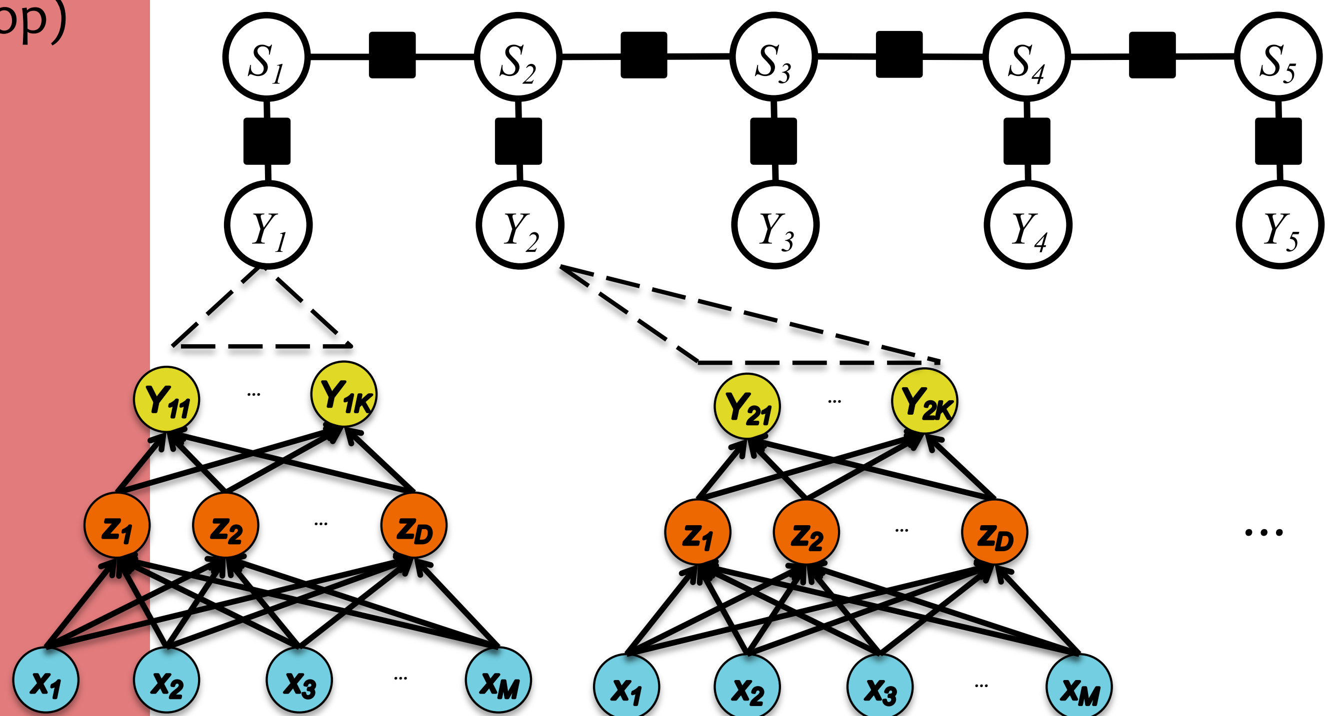

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$$z_j = \frac{1}{1 + \exp(-a_j)}$$

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*Backward computation*

$$\frac{dJ}{db_{i,t}} = \frac{\partial \alpha_{F_{model},T}}{\partial \alpha_{i,t}} \frac{\partial \alpha_{i,t}}{\partial b_{i,t}} = \left(\sum_j \frac{\partial \alpha_{j,t+1}}{\partial \alpha_{i,t}} \frac{\partial L_{model}}{\partial \alpha_{j,t+1}}\right)\left(\sum_j a_{ji} \alpha_{j,t-1}\right)$$

$$= \left(\sum_j b_{j,t+1} a_{ji} \frac{\partial \alpha_{F_{model},T}}{\partial \alpha_{j,t+1}}\right)\left(\sum_j a_{ji} \alpha_{j,t-1}\right) = \beta_{i,t} \frac{\alpha_{i,t}}{b_{i,t}} = \boxed{\frac{\gamma_{i,t}}{b_{i,t}}}$$

# Hybrid: NN + HMM

**Computing the Gradient:** $\nabla \ell(f_{\boldsymbol{\theta}}(\boldsymbol{x}_i), \boldsymbol{y}_i)$

*Forward computation*

$$J = \log p(\mathbf{S}, \mathbf{Y}) = \alpha_{\text{END},T}$$

$$\alpha_{i,t} = \ldots \text{(forward prob)}$$

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$$z_j = \frac{1}{1 + \exp(-a_j)}$$

$$a_j = \sum_{i=0}^{M} \alpha_{ji} x_i$$

*Backward computation*

$$\frac{dJ}{db_{i,t}} = \frac{\gamma_{i,t}}{b_{i,t}}$$

$$\frac{dJ}{dy_{t,k}} = \sum_{b_{i,t}} \frac{dJ}{db_{i,t}} \frac{db_{i,t}}{dy_{t,k}}$$

$$\frac{\partial b_{i,t}}{\partial Y_{jt}} = \sum_k \frac{Z_k}{((2\pi)^n \mid \Sigma_k \mid)^{1/2}} (\sum_l d_{k,lj}(\mu_{kl} - Y_{lt})) \exp(-\frac{1}{2}(Y_t - \mu_k)\Sigma_k^{-1}(Y_t - \mu_k)^T)$$

$$\frac{dJ}{db} = \frac{dJ}{dy}\frac{dy}{db}, \quad \frac{dy}{db} = \frac{\exp(b)}{(\exp(b) + 1)^2}$$

$$\frac{dJ}{d\beta_j} = \frac{dJ}{db}\frac{db}{d\beta_j}, \quad \frac{db}{d\beta_j} = z_j$$

$$\frac{dJ}{dz_j} = \frac{dJ}{db}\frac{db}{dz_j}, \quad \frac{db}{dz_j} = \beta_j$$

$$\frac{dJ}{da_j} = \frac{dJ}{dz_j}\frac{dz_j}{da_j}, \quad \frac{dz_j}{da_j} = \frac{\exp(a_j)}{(\exp(a_j) + 1)^2}$$

$$\frac{dJ}{d\alpha_{ji}} = \frac{dJ}{da_j}\frac{da_j}{d\alpha_{ji}}, \quad \frac{da_j}{d\alpha_{ji}} = x_i$$

# Hybrid: NN + HMM

**Computing the Gradient:** $\nabla \ell(f_{\boldsymbol{\theta}}(\boldsymbol{x}_i), \boldsymbol{y}_i)$

*Forward computation*

$$J = \log p(\mathbf{S}, \mathbf{Y}) = \alpha_{\text{END},T}$$

$$\alpha_{i,t} = \ldots \text{(forward prob)}$$

$$\beta_{i,t} = \ldots \text{(backward prop)}$$

$$\gamma_{i,t} = \ldots \text{(marginals)}$$

The derivative of the log-likelihood with respect to the neural network parameters!

$$a_j = \sum_{i=0}^{M} \alpha_{ji} x_i$$

*Backward computation*

$$\frac{dJ}{db_{i,t}} = \frac{\gamma_{i,t}}{b_{i,t}}$$

$$\frac{dJ}{dy_{t,k}} = \sum_{b_{i,t}} \frac{dJ}{db_{i,t}} \frac{db_{i,t}}{dy_{t,k}}$$

$$\frac{\partial b_{i,t}}{\partial Y_{jt}} = \sum_k \frac{Z_k}{((2\pi)^n \mid \Sigma_k \mid)^{1/2}} (\sum_l d_{k,lj}(\mu_{kl} - Y_{lt})) \exp(-\frac{1}{2}(Y_t - \mu_k)\Sigma_k^{-1}(Y_t - \mu_k)^T)$$

$$\frac{dJ}{db} = \frac{dJ}{dy}\frac{dy}{db}, \quad \frac{dy}{db} = \frac{\exp(b)}{(\exp(b)+1)^2}$$

$$\frac{dJ}{d\beta_j} = \frac{dJ}{db}\frac{db}{d\beta_j}, \quad \frac{db}{d\beta_j} = z_j$$

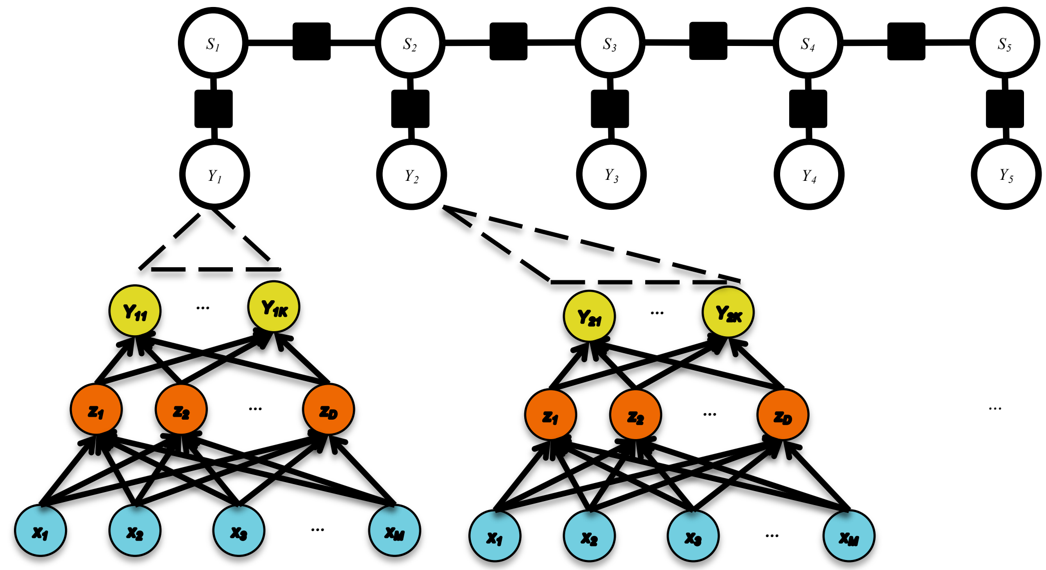
$$\frac{dJ}{dz_j} = \frac{dJ}{db}\frac{db}{dz_j}, \quad \frac{db}{dz_j} = \beta_j$$

$$\frac{dJ}{da_j} = \frac{dJ}{dz_j}\frac{dz_j}{da_j}, \quad \frac{dz_j}{da_j} = \frac{\exp(a_j)}{(\exp(a_j)+1)^2}$$

$$\frac{dJ}{d\alpha_{ji}} = \frac{dJ}{da_j}\frac{da_j}{d\alpha_{ji}}, \quad \frac{da_j}{d\alpha_{ji}} = x_i$$

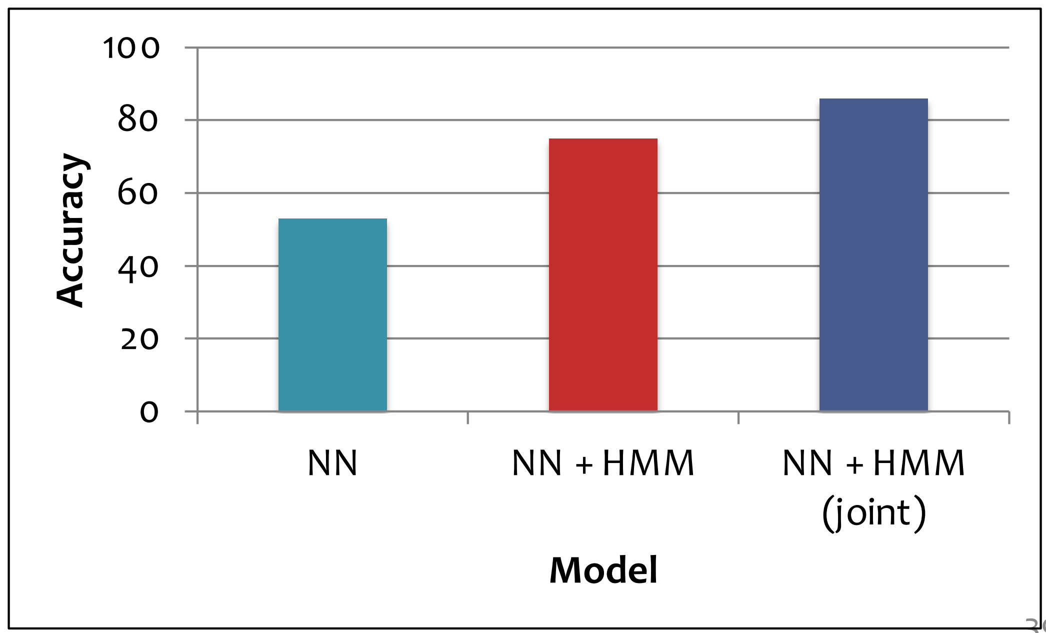
(Bengio et al., 1992)

# Hybrid: NN + HMM



**Experimental Setup:**

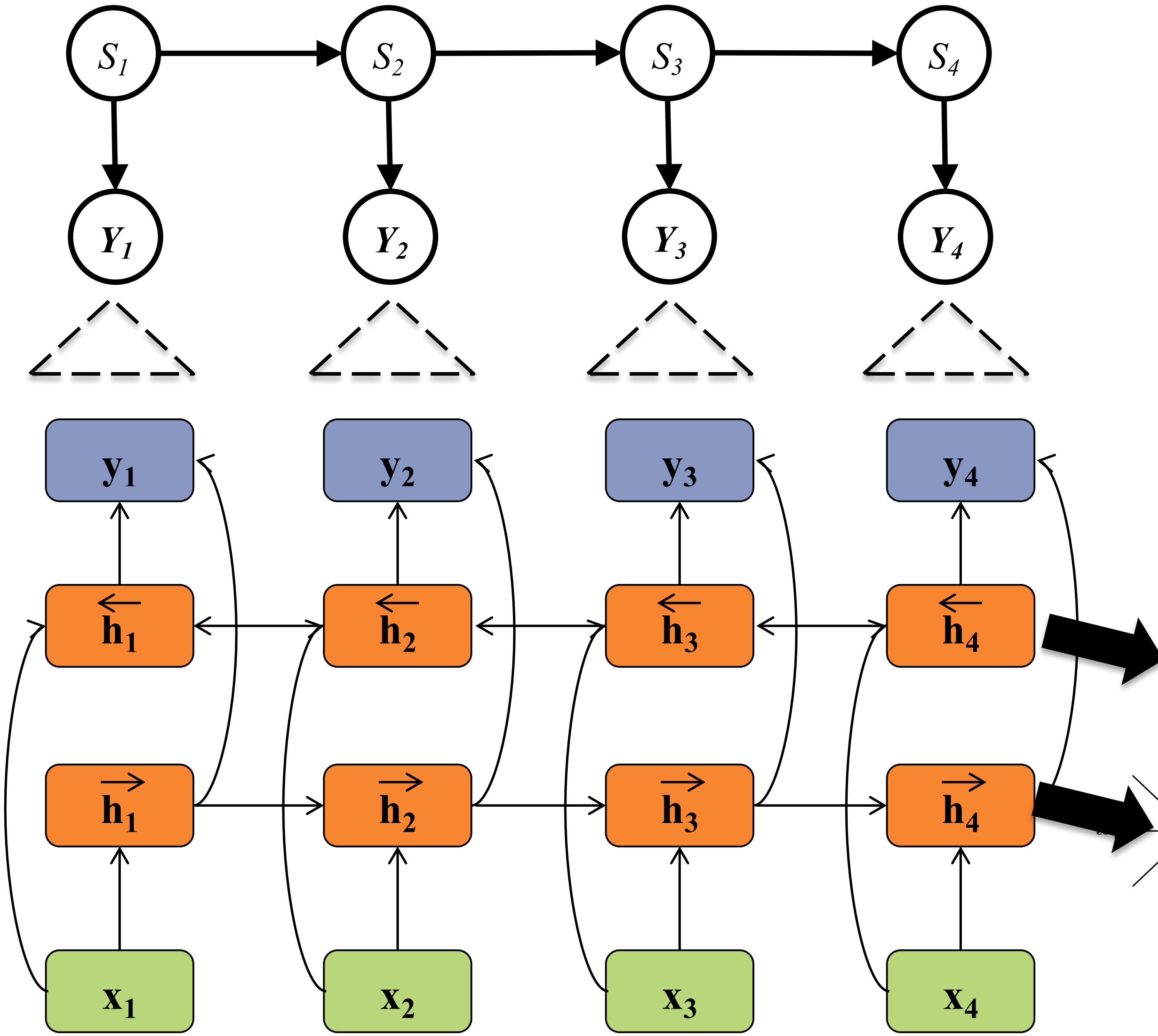
- **Task:** Phoneme Recognition (aka. speaker independent recognition of plosive sounds)
- **Eight output labels:**
  - /p/, /t/, /k/, /b/, /d/, /g/, /dx/, /all other phonemes/
  - These are the HMM hidden states
- **Metric:** Accuracy
- **3 Models:**
  1. NN only
  2. NN + HMM (trained independently)
  3. NN + HMM (jointly trained)



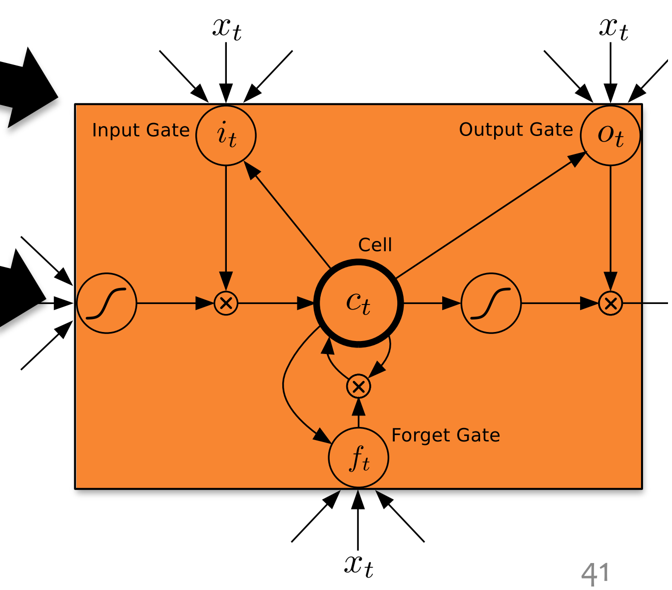
# HYBRID:
# RNN + HMM

# Hybrid: RNN + HMM

(Graves et al., 2013)




- Graves et al. (2013) uses a Deep Bidirectional LSTM
- Each hidden unit is an LSTM
- Deep → More than two layers

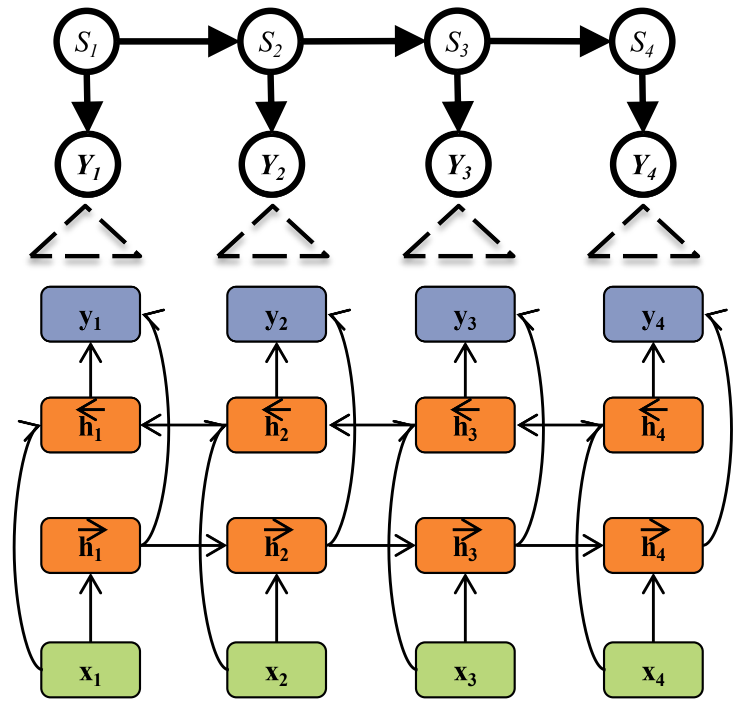


# Hybrid: RNN + HMM

(Graves et al., 2013)

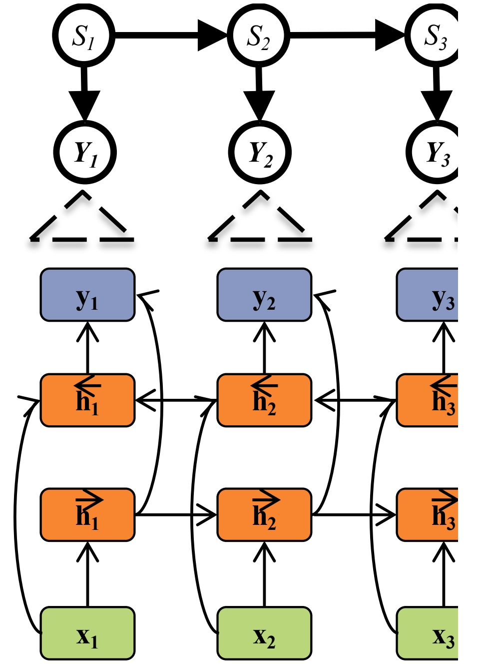
The model, inference, and learning can be **analogous** to our NN + HMM hybrid

- **Objective:** log-likelihood
- **Model:** HMM/Gaussian emissions
- **Inference:** forward-backward algorithm
- **Learning:** SGD with gradient by backpropagation



(Graves et al., 2013)

# Hybrid: RNN + HMM




**Experimental Setup:**

- **Task:** Phoneme Recognition
- **Dataset:** TIMIT
- **Metric:** Phoneme Error Rate
- **Two classes of models:**
  1. Neural Net only
  2. NN + HMM hybrids

| Training Method | Test PER |
|---|---|
| CTC | $21.57 \pm 0.25$ |
| CTC (noise) | $18.63 \pm 0.16$ |
| Transducer | $\mathbf{18.07 \pm 0.24}$ |

1. Neural Net only

| Network | Dev PER<br>Test PER |
|---|---|
| DBRNN | $19.91 \pm 0.22$<br>$21.92 \pm 0.35$ |
| DBLSTM | $17.44 \pm 0.156$<br>$19.34 \pm 0.15$ |
| DBLSTM (noise) | $16.11 \pm 0.15$<br>$\mathbf{17.99 \pm 0.13}$ |

2. NN + HMM hybrids

# HYBRID:
# CNN + CRF

# Markov Random Field (MRF)

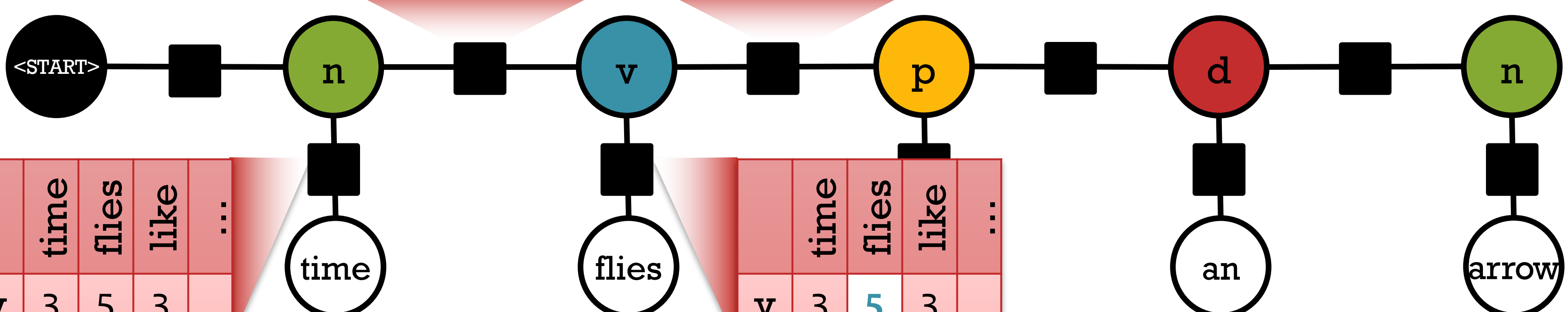
Recall...

Joint distribution over tags $Y_i$ <u>and</u> words $X_i$

$$p(\text{n, v, p, d, n, time, flies, like, an, arrow}) = \frac{1}{Z}(4 * 8 * 5 * 3 * \ldots)$$

| | v | n | p | d |
|---|---|---|---|---|
| v | 1 | 6 | 3 | 4 |
| n | 8 | 4 | 2 | 0.1 |
| p | 1 | 3 | 1 | 3 |
| d | 0.1 | 8 | 0 | 0 |

| | v | n | p | d |
|---|---|---|---|---|
| v | 1 | 6 | 3 | 4 |
| n | 8 | 4 | 2 | 0.1 |
| p | 1 | 3 | 1 | 3 |
| d | 0.1 | 8 | 0 | 0 |




| | time | flies | like | … |
|---|---|---|---|---|
| v | 3 | 5 | 3 | |
| n | 4 | 5 | 2 | |
| p | 0.1 | 0.1 | 3 | |
| d | 0.1 | 0.2 | 0.1 | |

| | time | flies | like | … |
|---|---|---|---|---|
| v | 3 | 5 | 3 | |
| n | 4 | 5 | 2 | |
| p | 0.1 | 0.1 | 3 | |
| d | 0.1 | 0.2 | 0.1 | |

# Conditional Random Field (CRF)

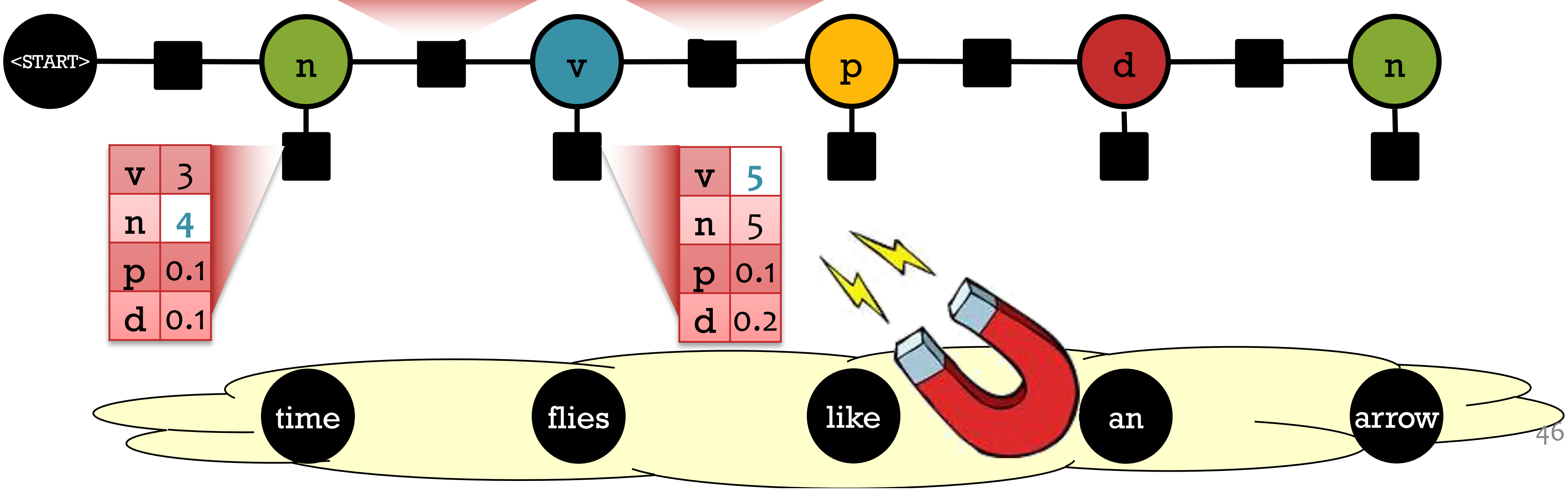
Recall...

Conditional distribution over tags $Y_i$ given words $x_i$.
The factors and Z are now specific to the sentence $\boldsymbol{x}$.

$$p(\text{n, v, p, d, n} \mid \text{time, flies, like, an, arrow}) = \frac{1}{Z}(4 * 8 * 5 * 3 * \ldots)$$

| | v | n | p | d |
|---|---|---|---|---|
| v | 1 | 6 | 3 | 4 |
| n | 8 | 4 | 2 | 0.1 |
| p | 1 | 3 | 1 | 3 |
| d | 0.1 | 8 | 0 | 0 |

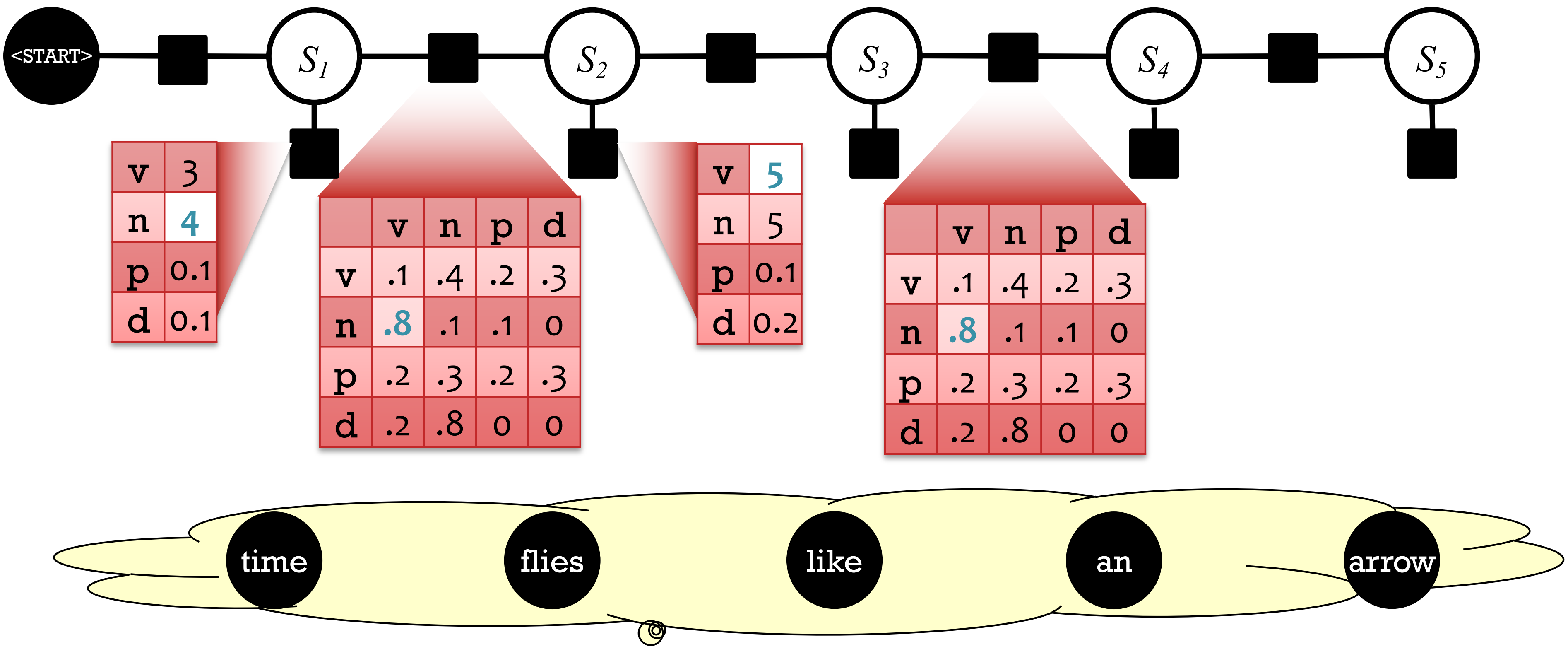
| | v | n | p | d |
|---|---|---|---|---|
| v | 1 | 6 | 3 | 4 |
| n | 8 | 4 | 2 | 0.1 |
| p | 1 | 3 | 1 | 3 |
| d | 0.1 | 8 | 0 | 0 |




| | |
|---|---|
| v | 3 |
| n | 4 |
| p | 0.1 |
| d | 0.1 |

| | |
|---|---|
| v | 5 |
| n | 5 |
| p | 0.1 |
| d | 0.2 |

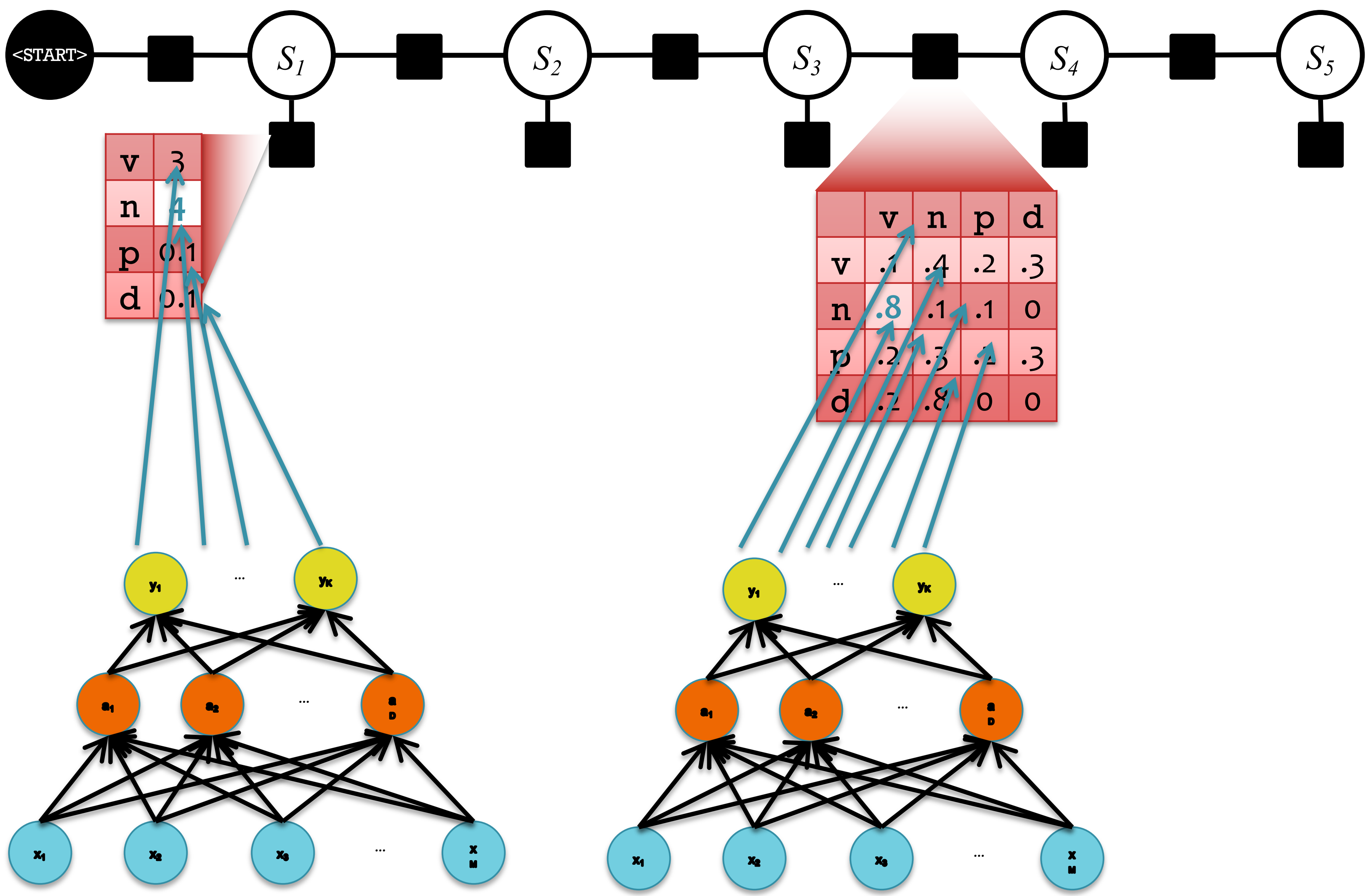
# Hybrid: Neural Net + CRF




- In a standard CRF, each of the factor cells is a parameter (e.g. transition or emission)
- In the hybrid model, these values are computed by a neural network with its own parameters

# Hybrid: Neural Net + CRF

*Forward computation*



(Collobert & Weston, 2011)

# Hybrid: CNN + CRF

- For **computer vision,** Convolutional Neural Networks are in **2-dimensions**
- For **natural language,** the CNN is **1-dimensional**

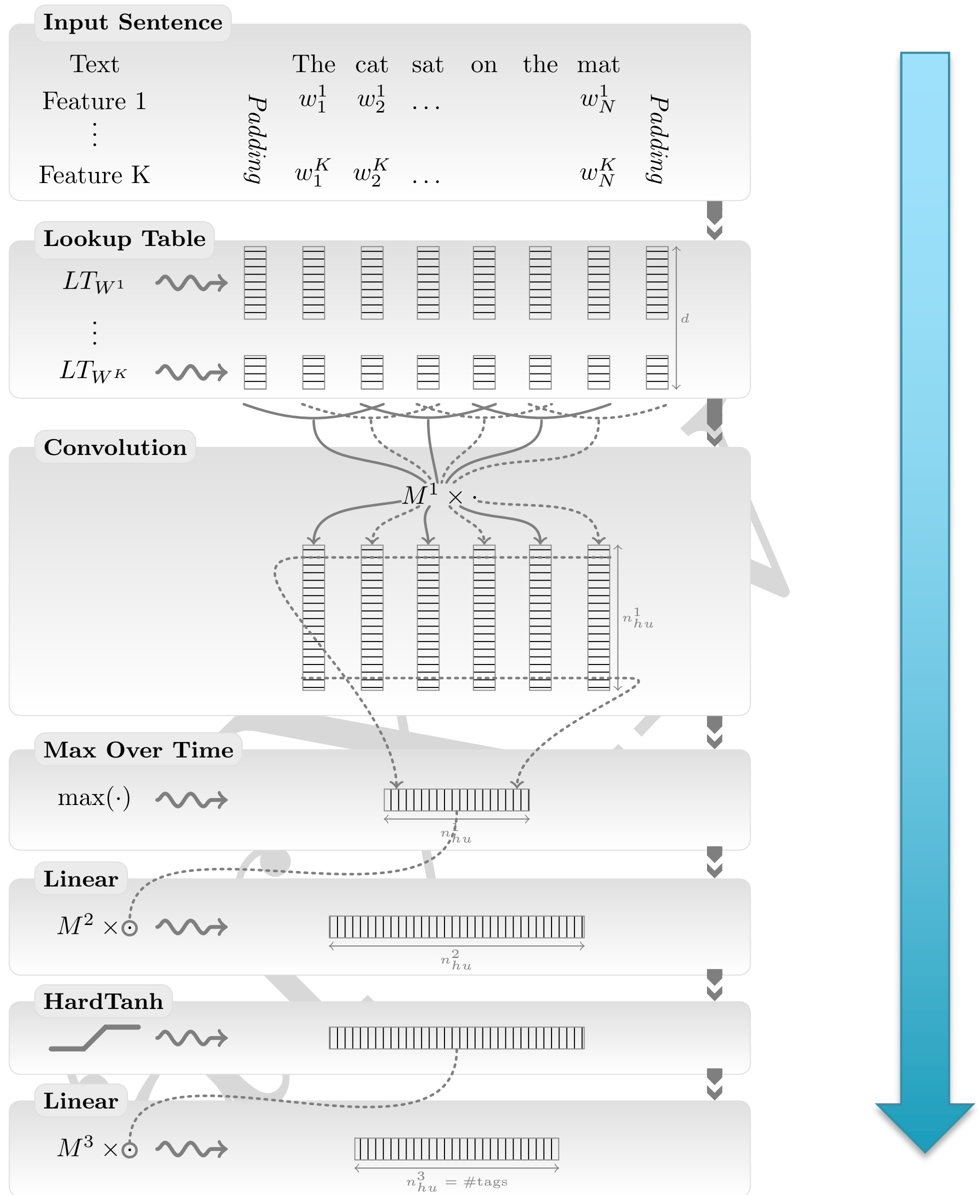



Figure from (Collobert & Weston, 2011)

# Hybrid: CNN + CRF

(Collobert & Weston, 2011)

"NN + SLL"

- Model: Convolutional Neural Network (CNN) with **linear-chain CRF**
- Training objective: maximize **sentence-level likelihood** (SLL)

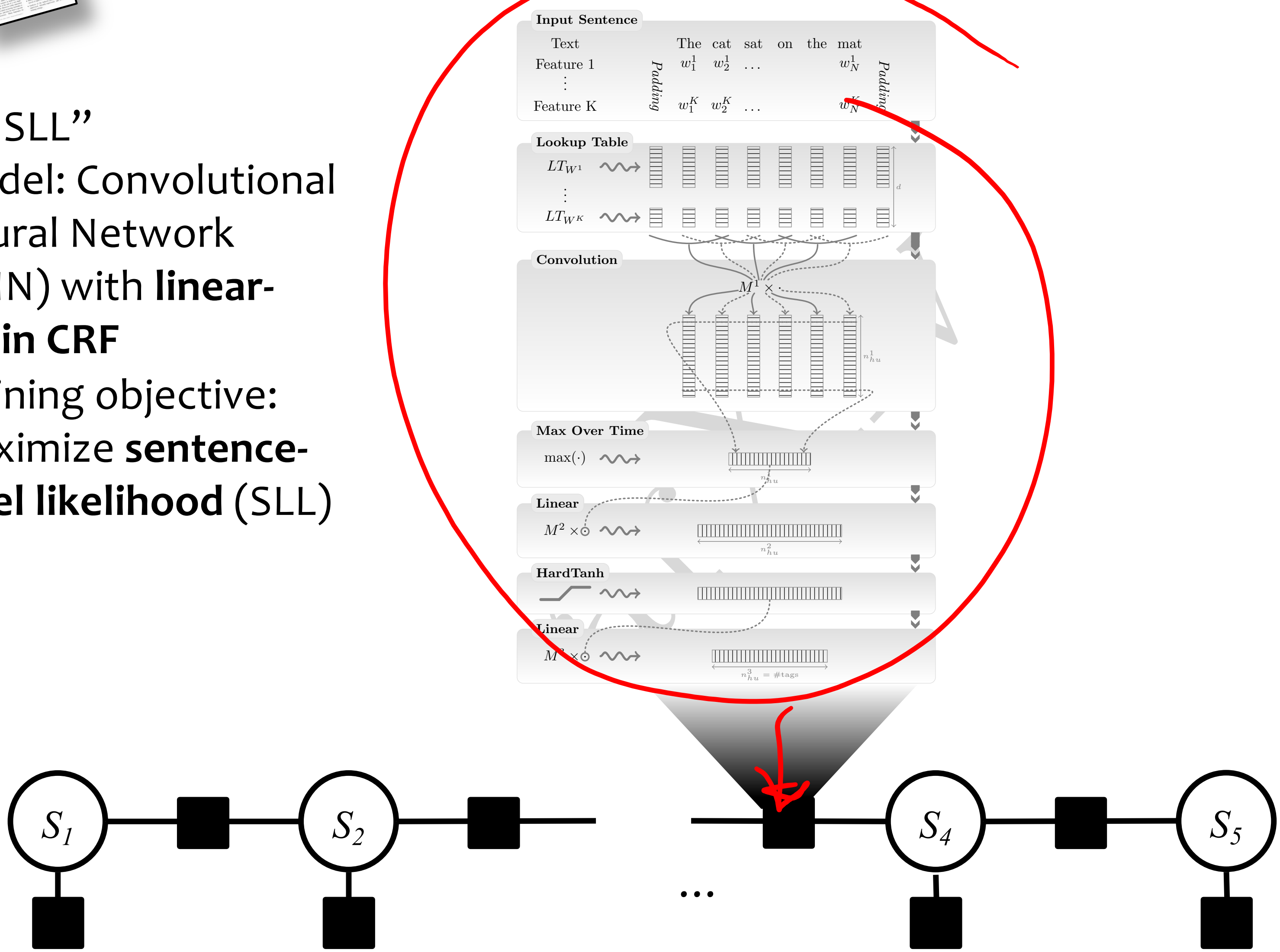



Figure from (Collobert & Weston, 2011)

(Collobert & Weston, 2011)

# Hybrid: CNN + CRF

"NN + WLL"

- Model: Convolutional Neural Network (CNN) with **logistic regression**
- Training objective: maximize **word-level likelihood** (WLL)

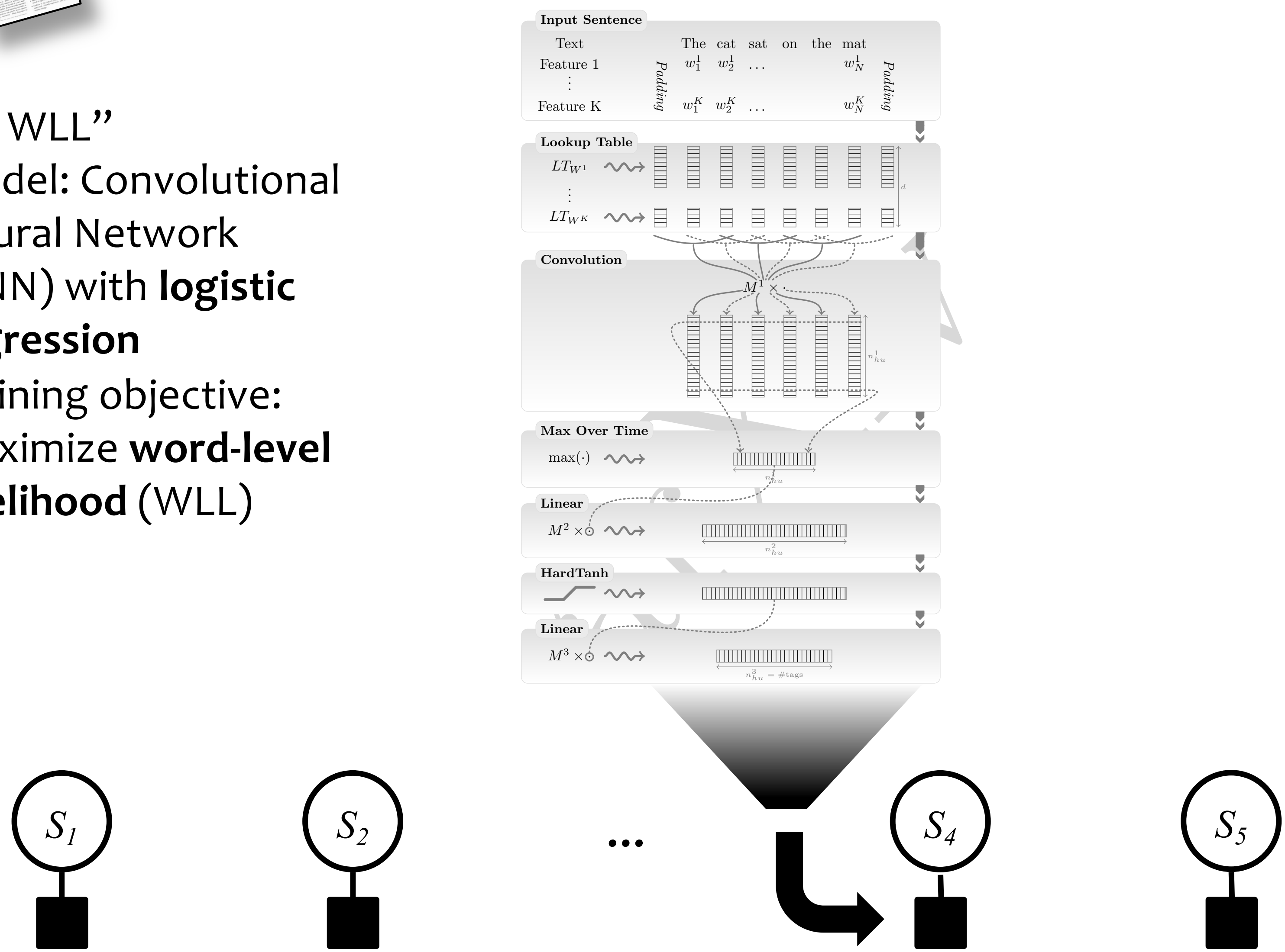



Figure from (Collobert & Weston, 2011)

(Collobert & Weston, 2011)

# Hybrid: CNN + CRF

**Experimental Setup:**

- **Tasks:**
  - Part-of-speech tagging (POS),
  - Noun-phrase and Verb-phrase Chunking,
  - Named-entity recognition (NER)
  - Semantic Role Labeling (SRL)
- **Datasets / Metrics:** Standard setups from NLP literature (higher PWA/F1 is better)
- **Models:**
  - Benchmark systems are typical – non-neural network systems
  - NN+WLL: hybrid CNN with logistic regression
  - NN+SLL: hybrid CNN with linear-chain CRF

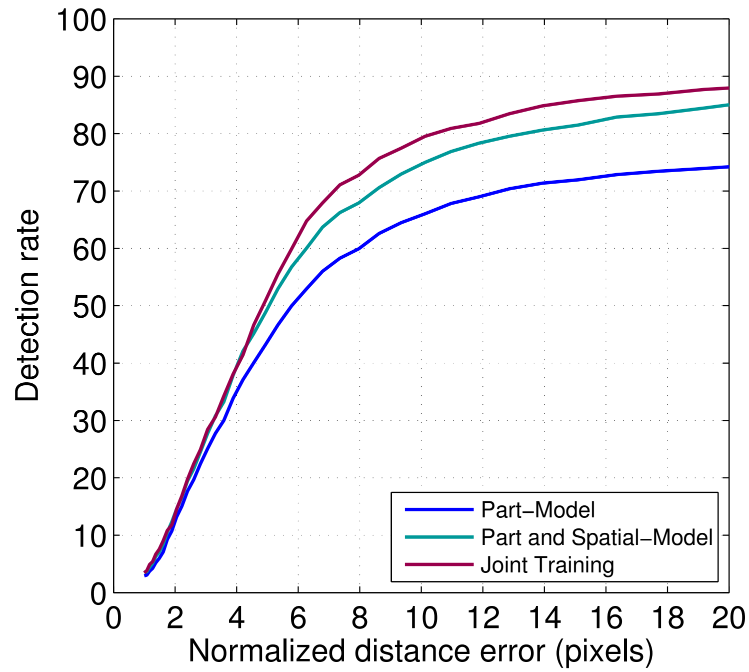
| Approach | POS (PWA) | Chunking (F1) | NER (F1) | SRL (F1) |
|---|---|---|---|---|
| **Benchmark Systems** | 97.24 | 94.29 | 89.31 | 77.92 |
| NN+WLL | 96.31 | 89.13 | 79.53 | 55.40 |
| NN+SLL | 96.37 | 90.33 | 81.47 | 70.99 |

(Thompson et al., 2014)

# Hybrid: CNN + MRF

## Experimental Setup:

- **Task:** pose estimation
- **Model:** Deep CNN + MRF



# Tricks of the Trade

- **Lots of them:**
  - Pre-training helps (but isn't always necessary)
  - Train with adaptive gradient variants of SGD (e.g. Adam)
  - Use max-margin loss function (i.e. hinge loss) – though only sub-differentiable it often gives better results
  - …
- A few years back, they were considered "**poorly documented**" and "requiring great expertise"
- Now there are lots of **good tutorials** that describe (very important) specific implementation details
- Many of them **also apply to training graphical models**!

# MBR DECODING

# Minimum Bayes Risk Decoding

- Suppose we given a loss function $l(\boldsymbol{y}', \boldsymbol{y})$ and are asked for a single tagging
- How should we choose just one from our probability distribution $p(\boldsymbol{y}|\boldsymbol{x})$?
- A minimum Bayes risk (MBR) decoder $h(\boldsymbol{x})$ returns the variable assignment with minimum **expected** loss under the model's distribution

$$h_{\boldsymbol{\theta}}(\boldsymbol{x}) = \operatorname*{argmin}_{\hat{\boldsymbol{y}}} \ \mathbb{E}_{\boldsymbol{y} \sim p_{\boldsymbol{\theta}}(\cdot|\boldsymbol{x})}[\ell(\hat{\boldsymbol{y}}, \boldsymbol{y})]$$

$$= \operatorname*{argmin}_{\hat{\boldsymbol{y}}} \ \sum_{\boldsymbol{y}} p_{\boldsymbol{\theta}}(\boldsymbol{y} \mid \boldsymbol{x}) \ell(\hat{\boldsymbol{y}}, \boldsymbol{y})$$

# Minimum Bayes Risk Decoding

$$h_{\boldsymbol{\theta}}(\boldsymbol{x}) = \underset{\hat{\boldsymbol{y}}}{\text{argmin}} \ \mathbb{E}_{\boldsymbol{y} \sim p_{\boldsymbol{\theta}}(\cdot \mid \boldsymbol{x})}[\ell(\hat{\boldsymbol{y}}, \boldsymbol{y})]$$

Consider some example loss functions:

The **Hamming loss** corresponds to accuracy and returns the number of incorrect variable assignments:

1 if $\hat{y}_i = y_i$
0 otherwise

$$\ell(\hat{\boldsymbol{y}}, \boldsymbol{y}) = \sum_{i=1}^{V} (1 - \mathbb{I}(\hat{y}_i, y_i))$$

The MBR decoder is:

from BP or ___

$$\hat{y}_i = h_{\boldsymbol{\theta}}(\boldsymbol{x})_i = \underset{\hat{y}_i}{\text{argmax}} \ p_{\boldsymbol{\theta}}(\hat{y}_i \mid \boldsymbol{x})$$

This decomposes across variables and requires the variable marginals.

# Minimum Bayes Risk Decoding

$$h_{\boldsymbol{\theta}}(\boldsymbol{x}) = \underset{\hat{\boldsymbol{y}}}{\operatorname{argmin}}\ \mathbb{E}_{\boldsymbol{y} \sim p_{\boldsymbol{\theta}}(\cdot \mid \boldsymbol{x})}[\ell(\hat{\boldsymbol{y}}, \boldsymbol{y})]$$

Consider some example loss functions:

The ***0-1* loss function** returns *1* only if the two assignments are identical and *0* otherwise:

$$\ell(\hat{\boldsymbol{y}}, \boldsymbol{y}) = 1 - \mathbb{I}(\hat{\boldsymbol{y}}, \boldsymbol{y})$$

$\begin{cases} 1 & \text{if } \hat{y} = y \\ 0 & \text{otherwise} \end{cases}$

The MBR decoder is:

$$h_{\boldsymbol{\theta}}(\boldsymbol{x}) = \underset{\hat{\boldsymbol{y}}}{\operatorname{argmin}}\ \sum_{\boldsymbol{y}} p_{\boldsymbol{\theta}}(\boldsymbol{y} \mid \boldsymbol{x})(1 - \mathbb{I}(\hat{\boldsymbol{y}}, \boldsymbol{y}))$$

$$= \underset{\hat{\boldsymbol{y}}}{\operatorname{argmax}}\ p_{\boldsymbol{\theta}}(\hat{\boldsymbol{y}} \mid \boldsymbol{x})$$

which is exactly the MAP inference problem!

# Minimum Bayes Risk Decoding

$$h_{\boldsymbol{\theta}}(\boldsymbol{x}) = \operatorname*{argmin}_{\hat{\boldsymbol{y}}} \; \mathbb{E}_{\boldsymbol{y} \sim p_{\boldsymbol{\theta}}(\cdot|\boldsymbol{x})}[\ell(\hat{\boldsymbol{y}}, \boldsymbol{y})]$$

Consider some example loss functions:

The ***0-1* loss function** returns *1* only if the two assignments are identical and *0* otherwise:

$$h_\theta(x) = \operatorname*{argmin}_{\hat{y}} \left[ \sum_y p(y|x) \left(1 - \mathbb{1}(\hat{y} = y)\right) \right]$$

$$= \operatorname*{argmin}_{\hat{y}} \underbrace{\left[ \sum_y p(y|x) \right]}_{\text{constant wrt } \hat{y}} - \underbrace{\left[ \sum_y p(y|x) \mathbb{1}(\hat{y} = y) \right]}_{= p(\hat{y}|x)}$$

$$= \operatorname*{argmin}_{\hat{y}} - p(\hat{y}|x)$$

$$= \operatorname*{argmax}_{\hat{y}} \; p(\hat{y}|x)$$

# MBR Decoders

**Q:** If loss(y, y*) additively decomposes in the same way as log p(y|x), can we efficiently compute the MBR decoder h(x) for that loss/model pair?

**A:** Yes.

How to do so is left as an exercise…