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DebtRank Analysis of Financial Distress Propagation on a Production Network in Japan*

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Abstract

Supplier-customer relationship among firms in a production network is the arena where financial distress propagates from distressed debtors of customers to their creditors of suppliers. While the events of bankruptcies can be observed easily, the underlying contagion effect of financial distress can have considerable consequences such as a chain of bankruptcies. DebtRank is a model to quantify the propagation of financial distress, which has been applied recently for analyzing and evaluating systemic risk for interbank contagion. Because the production network in Japan, which comprises more than one million firms as nodes and millions of supplier-customer relationship as links, is much larger than the interbank credit network, it has been a formidable task to study the model of DebtRank on such a large-scale production network.

This work studies the financial distress propagation on the real data of a production network by employing an implementation of DebtRank on a supercomputer. We found that the DebtRank of individual firms has a significant correlation with firm-size with non-linearity, indicating that the DebtRank for big firms becomes much larger than what is expected naively. The analysis for individual sectors shows that, depending on the sector's position in the upstream and downstream, its DebtRank deviates from a linear relationship between DebtRank and sector size. In addition, one can measure vulnerability by using the DebtRank analysis, which is potentially useful to identify the likelihood of failures of firms in more vulnerable sectors.

Keywords: Supplier-customer network, Systemic risk, Supercomputer

JEL classification: G01, C88

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I. Introduction

Supplier-customer relationship among firms in production network is the arena where financial distress propagates from distressed debtors of customers to its creditors of suppliers. While the events of bankruptcies can be observed easily, the underlying contagion effect of financial distress can have considerable consequences such as chain of bankruptcies.

DebtRank (Battiston et al., 2012) is a model to quantify the propagation of financial distress, which has been recently applied for the analysis and evaluation of systemic risk mainly for interbank contagion (di Iasio et al., 2013; Tabak et al., 2013; Poledna and Thurner, 2014; Fink et al., 2014; Puliga et al., 2014). and also for a credit network between banks and firms by Aoyama et al. (2013). Typical size of those systems studied so far is hundreds, while the production network in a nation comprises more than a million of firms as nodes and millions of supplier-customer relationship as links, much larger than interbank credit network. Due to the problem in availability of real data and also in computation, little has been studied on the DebtRank analysis for such a large-scale production network.

This paper studies the financial distress propagation on real data of production network by employing the DebtRank methodology in order to model how financial distress propagates along supplier-customer relationships. Specifically, trade-credit has a crucial role in the model. Suppliers usually provide credit to their customers in trade anticipating payment in due time. If a customer delays or fails in the payment, then its suppliers may lose expected sales and profits, depending on relative exposure, potentially causing chain of failures and bankruptcies. We model this propagation of distress from downstream to upstream in the production network by the DebtRank. By assuming different initial configurations of idiosyncratic shocks on industrial sectors, and the model of distress propagation, we perform the quantification of financial distress, visualize the propagation, and evaluate the resulting ripple effect. The results have an implication of the fact that the present observation of the events of failures or bankruptcies underestimates the amount of financial stress in different parts of the network.

We briefly summarize the model in Section II, and describe the network structure of supplier-customer relationships in Japan comprising a million of firms and millions of supplier-customer relationships in Section III. Then we apply the model to the real data by using a supercomputer, which allows us to compute in a practical time. In Section IV, we show the results on statistical properties of DebtRank in relation to firm-size at the levels of individual firms and sectors, and also on vulnerability measures based on the DebtRank. Section V discusses several points, and Section VI summarizes the paper and implications.

II. DebtRank Method

The methodology of DebtRank was invented in Battiston et al. (2012) to quantify systemic risk in credit network among financial institutions. The network comprises of banks as nodes and financial dependencies among them as links. The quantity is a measure of how financial distress at a single institution or in a set of institutions can potentially make influence on others along the links of financial dependencies, namely exposure. The DebtRank and related methods have been recently studied for application to better evaluation of systemic risk in such financial networks and also in credit networks between banks and firms (see, for example, di Iasio et al. (2013); Tabak et al. (2013); Aoyama et al. (2013); Poledna and Thurner (2014); Fink et al. (2014); Puliga et al. (2014)).

Let us recapitulate the method of DebtRank in what follows. Consider a network with nodes $i = 1, 2, \cdots, N$ and with directed and weighted links. A link $j \Rightarrow i$ has a weight $0 < w_{ji} \leq 1$ that represents a relative exposure of i to j. At each time-step t in the computation, two variables of each node are updated:

- $h_i(t) \in [0, 1]$, the amount of financial distress of node i at time t.
- $s_i(t) \in \{\mathrm{U}, \mathrm{D}, \mathrm{I}\}$, respectively, the state of "Undistressed", "Distressed", and "Inactive" at time t.

As an initial configuration of distress at time $t = 0$, we suppose that

$$h_i(t = 0) = \begin{cases} h_{i0} & \text{if } i \in A \ , \\ 0 & \text{otherwise} \ , \end{cases} \tag{1}$$

starting from a set of distressed nodes, denoted by A (this can be a single node), and that

$$s_i(t = 0) = \begin{cases} \mathrm{D} & \text{if } i \in A \ , \\ \mathrm{U} & \text{otherwise} \ . \end{cases} \tag{2}$$

Then we update the distress according to

$$h_i(t) = \min \left[1, \ h_i(t-1) + \sum_{j : s_j(t-1) = \mathrm{D}} w_{ji} \, h_j(t-1) \right] \ , \tag{3}$$

where the summation is taken over for all the i's neighboring nodes j that are in the state of D at time $t-1$. The weight w_{ji} determines the strength of propagation. We denote the direction of propagation by $j \Rightarrow i$. Simultaneously we update the state by

$$s_i(t) = \begin{cases} \mathrm{D} & \text{if } h_i(1) > 0 \text{ and } s_i(t-1) \neq \mathrm{I} \ , \\ \mathrm{I} & \text{if } s_i(t-1) = \mathrm{D} \ , \\ s_i(t-1) & \text{otherwise} \ . \end{cases} \tag{4}$$

One can see that a node with state D becomes I at the next time, and then does not affect any others afterward. This avoids infinite number of repercussions in the propagation of distress. A node with state U becomes D, when distress reaches to it, and then affect neighboring nodes at the next time and becomes I. Note that a node with state I can continue to receive distress, while it does not affect any others.

After a finite number of time-steps, denoted by T, the propagation terminates resulting in a final configuration $h_i(t = T)$ for all the nodes. Finally, to define the total amount of distress in the entire network due to the initial set A of distressed nodes, it is customary to take the average for all the $h_i(T)$ except for $i \in A$ as

$$D_A = \sum_{i \notin A} \hat{a}_i \, h_i(T) \tag{5}$$

where

$$\hat{a}_i = \frac{a_i}{\sum_{j \notin A} a_j} \tag{6}$$

and a_i is the size of the node i such as assets, sales and so forth. D_A is called the DebtRank for the system due to the propagation of distress starting from the nodes in A. If A is a single node $\{i\}$, then it represents how much an individual node can affect the entire network. Note that one is discarding the effect of initially given distress by excluding them in the summation above, because we do not want to include the trivial effect due to the given initial amount of distress.

Once the sets of nodes and links with weights w_{ji} are given as well as the attributes of nodes including the size a_i, it is straightforward to compute the DebtRank. In fact, the algorithm is quite similar to breadth-first search, an elementary graph algorithm (see Cormen et al. (2001)). It is, however, the fact that it takes impractical time, say more than a day, to compute each node's DebtRank values for our data comprising of a million nodes. To overcome the difficulty, we employ one of the world-fastest supercomputers, called K-supercomputer, and parallelize the computation on the CPU cores in its facility of the RIKEN AICS so that the speed of computation becomes much faster, within an hour or much less (see Terai et al. (2016) for the details). We believe that this point is important in the simulation under a number of scenarios and initial conditions.

III. Data

Our data for production network is based on a survey done by Tokyo Shoko Research (TSR), one of the leading credit research agencies in Tokyo, in the contract with the Research Institute of Economy, Trade and Industry (RIETI). We utilize the three datasets of TSR Kigyo Jouhou, Kigyo Soukan Jouhou and Kigyo Tousan Jouhou for basic information for more than a million firms, millions of supplier-customer and ownership links among firms, and a list of bankruptcies, respectively. The data were compiled at the timing of July 2011.

Let us denote a supplier-customer link as $i \to j$, where firm i is a supplier to another firm j, or equivalently, j is a customer of i. We extracted only the supplier-customer links for pairs of "active" firms to exclude inactive and failed firms by using a flag in the basic information. Eliminating self-loops and parallel edges (duplicate links recorded in the data), we have a network of firms as nodes and supplier-customer links as edges. When viewed as an undirected network, it has a largest connected component (99% in terms of the number of firms), which we shall study in the

Table 1. Classification of industrial sectors (Japan Standard Industrial Classification, Rev. 12, November 2007). Numbers of firms (third column) and fractional numbers (fourth column) are based on the divisions according to primary industry of each firm.

id	divisions	#firms	#firms(%)
A	agriculture, forestry	6,821	0.69
B	fishing	1,001	0.10
C	mining and quarrying	1,332	0.14
D	construction	337,206	34.19
E	manufacturing	156,847	15.90
F	electricity, gas, heat, water supply	648	0.07
G	information and communications	23,441	2.38
H	transportation and storage	33,246	3.37
I	wholesale and retail trade	249,610	25.31
J	finance and insurance activities	6,054	0.61
K	real estate activities	34,325	3.48
L	professional, scientific, technical	33,757	3.42
M	accommodation and food service	14,617	1.48
N	arts, entertainment and recreation	16,015	1.62
O	education/learning support	3,651	0.37
P	human health and social work	21,004	2.13
Q	compound services	5,586	0.57
R	other service activities	41,021	4.16
	Total	986,185	100.00

following. Denoting the number of nodes and links by N and M respectively, we have

$$N = 986,185 \ , \tag{7}$$

$$M = 4,402,270 \ . \tag{8}$$

as a result of our pre-processing. The largest connected component is often called a giant weakly connected component (GWCC).

Let us examine how firms are located in the upstream and downstream of the entire network. To do so, we regard the network as a directed graph and find the so-called "bowtie" structure. A GWCC can be decomposed into the parts defined as follows (see Fig. 1):

NW The whole network.

GWCC Giant weakly connected component: the largest connected component when viewed as an undirected graph. An undirected path exists for an arbitrary pair of firms in the component.

DC Disconnected components: other connected components than GWCC.

GSCC Giant strongly connected component: the largest connected component

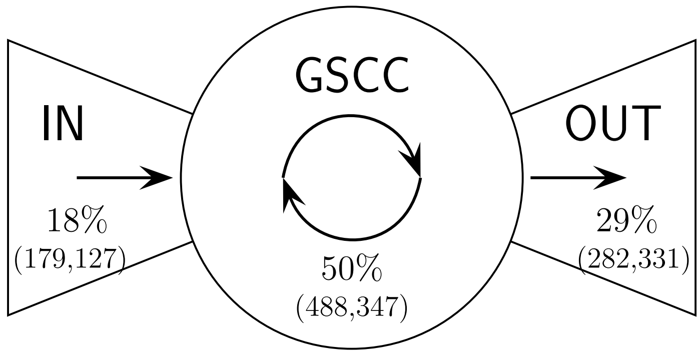

Figure 1. Bowtie structure for the production network including GSCC (giant strongly connected component) in which any pair of firms is mutually connected by a directed path; IN and OUT components comprised of firms in the GSCC's upstream and downstream sides respectively. See the main text for full explanation.

Component	#firms	Note
GWCC	986,185	100%
GSCC	488,347	50% × GWCC
IN	179,127	18% × GWCC
OUT	282,331	29% × GWCC
TE	36,380	4% × GWCC
Total	N =986,185	equal to GWCC

Table 2. Bowtie structure: sizes of different parts

when viewed as a directed graph. A directed path exists for an arbitrary pair of firms in the component.

IN The firms from which the GSCC is reached via a directed path.

OUT The firms that are reachable from the GSCC via a directed path.

TE "Tendrils"; the rest of GWCC (note that TEs may not look like tendrils).

It follows from the definitions that

$$\mathrm{NW} = \mathrm{GWCC} + \mathrm{DC} \tag{9}$$

$$\mathrm{GWCC} = \mathrm{GSCC} + \mathrm{IN} + \mathrm{OUT} + \mathrm{TE} \tag{10}$$

The shortest-path lengths (distances) from the GSCC and firms in the IN and OUT are given by:

Distance from GSCC to

	Distance	#firms
IN	1	172,526
	2	6,368
	3	221
	4	12
	Total	179,127

Distance from GSCC to

	Distance	#firms
OUT	1	269,555
	2	12,414
	3	350
	4	12
	Total	282,331

By comparing the over and under-presence of industrial divisions in each of these components, we can observe that in the portion of IN the numbers of firms in the sectors of real estate (K), agriculture and forestry (A), information and communications (G) are larger when compared with the corresponding sectors in the SCC. In the portion of OUT more abundant are human health and social work (P), accommodation and food service (M), education and learning support (O). This fact is reasonable, because these industries are mainly located either in the upstream or in the downstream. Nevertheless, all industries are basically embedded in the SCC with entanglement. We also analyze the community structure of network to obtain the result similar to the previous work (Fujiwara and Aoyama, 2010).

The diameter of a graph is the maximum length for all ordered pairs (i, j) of the shortest path from i to j. The average distance is the average length for all those pairs (i, j). We found that the average distance is 4.59 while the diameter is 22. This implies that the computation for the DebtRank will terminate at most within the time-steps corresponding to the diameter.

IV. Results for DebtRank

A. *Assumption*

Supplier-customer links are regarded as creditor-debtor relationships. A supplier depends on its customers for sales and profits. If one of the customers of them has financial distress, it may delay or even does not fulfill the payment, which results in the financial distress of the supplier. We assume that this is the most important channel for the propagation of distress. In fact, there are empirical and theoretical evidence for this assumption (see Battiston et al. (2007); Fujiwara and Aoyama (2010) and references therein). Thus if there is a supplier-customer relationship, $i \to j$, from firm i as a supplier and to firm j as a customer, it is assumed that the direction of the distress propagation in (3), $j \Rightarrow i$, is the *opposite* to the direction of supplier-customer link $i \to j$.

It would be ideal to have information about the strength w_{ji} in (3) from the amount of trade, for example, which is not available in our data. We assume that the strength w_{ji} in (3), or relative exposure of the customer i to its suppliers j's is given by

$$w_{ji} = \frac{1}{\text{number of customers } i\text{'s of supplier } j} \ . \tag{11}$$

Because the data were collected from nomination of suppliers and customers that are most important to a particular firm under investigation. One could take into account the order of importance, but we simply assume that this is a reasonable approximation.

As for the attributes of firms, namely firm-size a_i in (5) and (6), we employed the amount of sales and the number of employees, which gave us qualitatively similar results, as far as statistical properties are concerned.

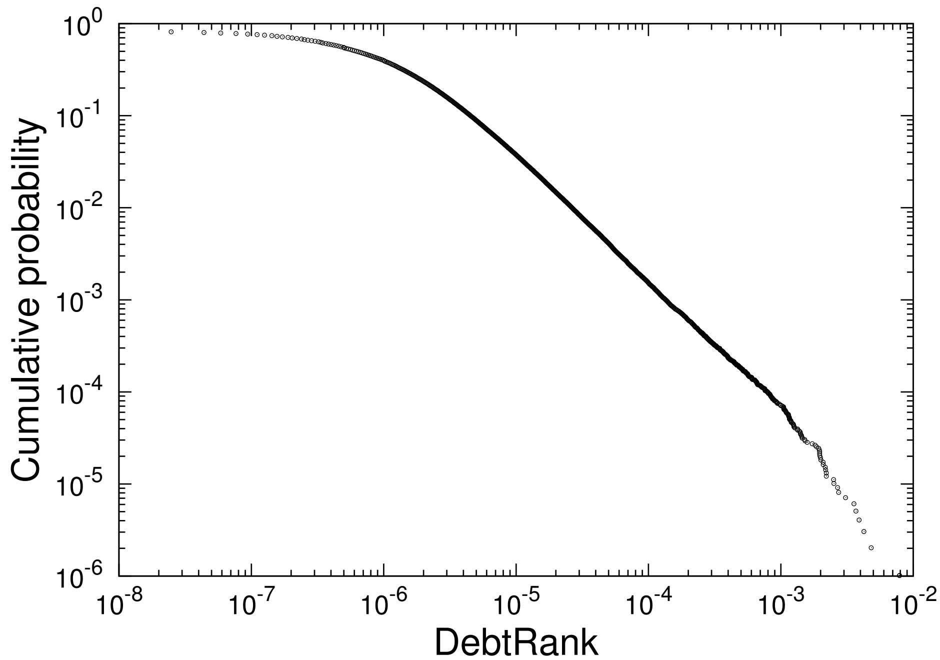

Figure 2. Cumulative distribution $P_>(x)$ for the DebtRank value x of individual firms (weighted by firm-size of sales). The distribution obeys a power-law, $P_>(x) \propto x^{-\alpha}$, in the region for large x, with exponent $\alpha = 1.28 \pm 0.01$.

B. *DebtRank for Individual Firms*

We computed the DebtRank values $x = D_{\{i\}}$ starting from each node i under distress. We assume that the initial value of the distress, $h_{i0} = 1$ in (1), the maximum value of distress in the model. The cumulative distribution $P_>(x)$ is shown in Fig. 2. One can observe that the distribution obeys a power law

$$P_>(x) \propto x^{-\alpha} \tag{12}$$

in the region for large x, where the value of α is given by

$$\alpha = 1.28 \pm 0.01 \tag{13}$$

estimated by maximum likelihood or Hill method (error at 99% significance level).

We remark that the distributions for the in-degree (number of suppliers) and out-degree (number of customers) of a firm also have power-laws with quantitatively similar exponents (see Fujiwara and Aoyama (2010)). In fact, there is a significant correlation between the value of DebtRank and degrees for each firm. This fact is not so trivial, because a big firm usually has a number of suppliers and customers, and the total amount of financial distress coming from those firms tend to be large, even if the weight tends to be small. On the other hand, a small firm typically depends on a limited number of customers, say one or two firm, being easily influenced by

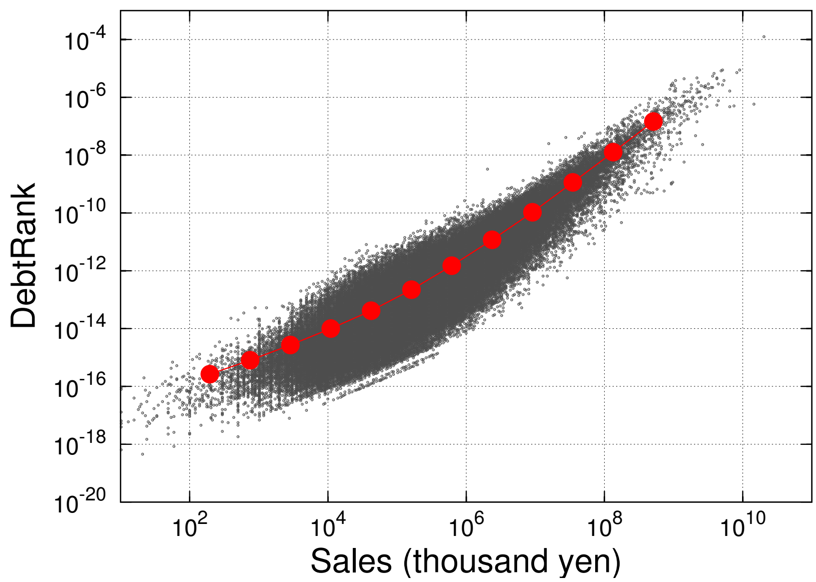

Figure 3. Scatter plot for the pair of the values of DebtRank and the amount of sales for individual firms. Also shown is the conditional average of DebtRank for firms present in each slice, corresponding to a particular size of firm (represented by each point in the red line).

others. Note that the resulting value of DebtRank tends to be small, because it is a weighted average by firm-size.

To examine the relation between the DebtRank and firm-size, we show the scatter plots in Fig. 3 for the amount of sales as firm-size, and also in Fig. 4 for the number of employees as an alternative measure of size.

We also show the conditional averages of DebtRank for firms with given ranges of firm-size, because the scatter plot is densely populated by points and can mislead the interpretation of the density. As obvious from the line for the conditional averages, it has an interesting non-linearity in the sense that the value of DebtRank becomes much larger than what is expected by a linear relation between the DebtRank and firm-size. This implies that big firms can have larger impact to the entire system than what one can naively expect from the numbers of suppliers and customers. Namely, "higher-order" structure of the network rather than degrees, certainly plays an important role.

C. *DebtRank for Sectors*

Let us turn our attention to the industrial sectors by computing the DebtRank D_A starting from the firms in the sector of A. We assume that the initial distress is given by $h_{i0} = 0.1$ in (1) for $i \in A$, and 0 otherwise. By this configuration we suppose that a relatively weak but simultaneous shock takes place overall in a particular sector. Let us assume that the level of sectoral classification is given as shown in Table 1, i.e. divisions from A to R which contains relatively a large number of firms.

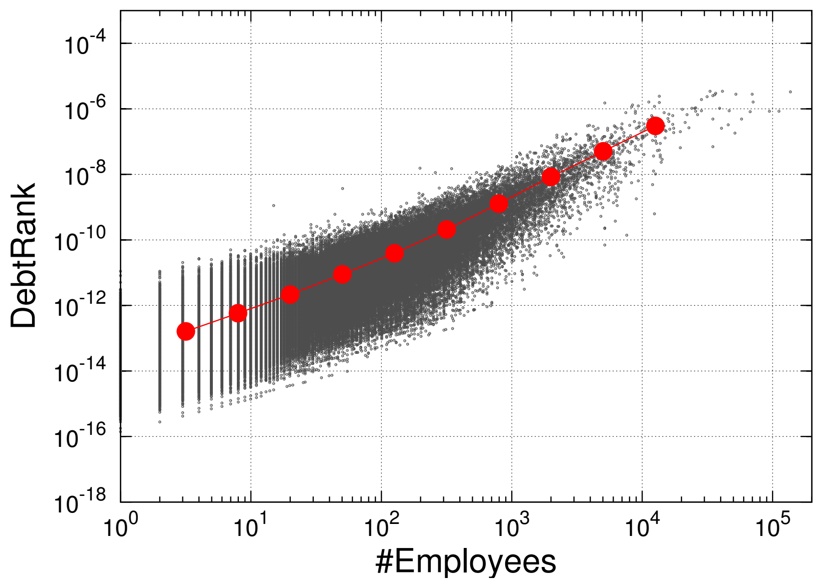

Figure 4. Scatter plot for the pair of the values of DebtRank and the number of employees for individual firms. Also shown is the conditional average of DebtRank for firms present in each slice, corresponding to a particular size of firm (represented by each point in the red line).

According to the definition in (5) and (6), we calculate D_A. Also we compute the total sum of firm-sizes of firms in the sector A so that one can estimate the size of the initial configuration of financial distress:

$$S_A = \sum_{i \in A} a_i \tag{14}$$

We show the scatter plot for the 18 sectors (A to R) in Fig. 5. Obviously one can see the power-law relation between S_A and D_A:

$$D_A \propto S_A^{-\beta} \tag{15}$$

where the exponent is estimated by $\beta = 0.92$ by minimum square estimate for the logarithmic values of the variables. Remember that we exclude the trivial effect due to the initial stress in the quantification of DebtRank.

The linear relationship between the logarithms of sector's size and DebtRank guides us to pay attention to deviation from it depending on individual sectors.

The sector of construction (D) has a smaller size than that for the sector of finance and insurance activities (J), but the former's impact is larger than the latter's. This implies that the one or more hops from the sector of construction in the upstream occupies more extended part of the network yielding larger impact. Similarly, the sector of accommodation and food service (M) has a size comparable to the sector of education/learning support (O), it has nearly a double of the impact to the whole system. By taking a close look at each sectors, one can see that the deviation

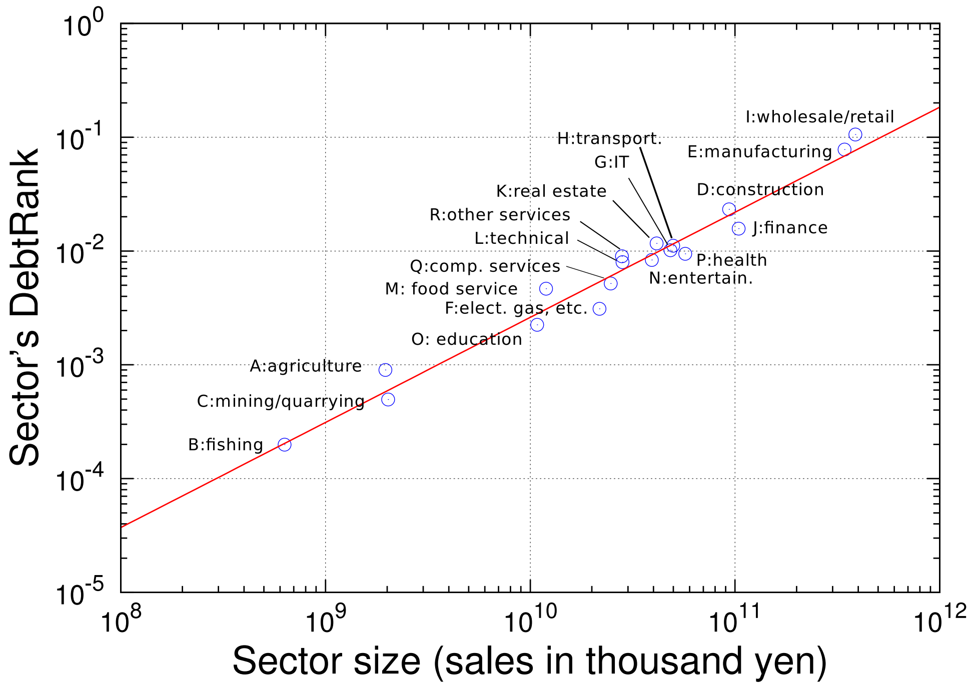

Figure 5. The size of each sector S_A (horizontal) and each sector's value of DebtRank D_A (vertical). The straight line is a power-law fit, $D_A \propto S_A^{-\beta}$, where $\beta = 0.92$ (MSE).

from (14) corresponds to the location of the sector in the upstream and downstream portions of the network, larger for upstream and smaller for downstream than what is expected by (14).

One is able to further divide the sectors into smaller ones, and to examine the relation between S_A and D_A in order to understand the relationship between the size of the stress and the location of the firms in the sector.

D. *Vulnerability of Sectors*

The financial distress brought to each sector can be used as a measure of vulnerability of firms in the sector under the condition that the initial distress starts from a single sector A.

To do so, one simply decomposes the DebtRank into different components, that is

$$D_A = \sum_g D_{A_g} \ , \tag{16}$$

where A_g is the sector from $g = 1, \ldots, G$. The quantity D_{A_g} is simply the decomposition of the sum given by (5) into other sectors except the initially distressed sector, and represents how much distress is propagated into the sector S_g. The larger the quantity is, the more vulnerable the corresponding sector is. It is a measure of vulnerability of each sector due to the initial configuration of distress.

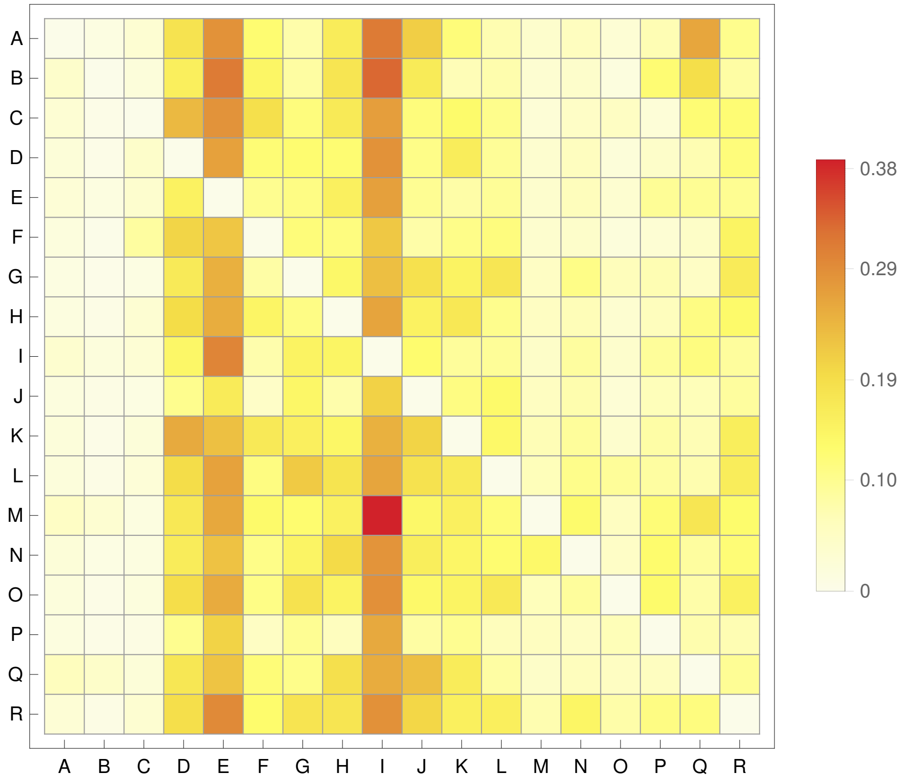

Figure 6. Vulnerability of each sector due to another sector's financial distress. Each row represents the initially distressed sector, from which financial distress propagates to other sectors in columns.

Fig. 6 shows the matrix of such vulnerabilities. Each row represents the initially distressed sector. Each column is the measure of vulnerability.

The result shows that depending on which sector is initially distressed, there is a heterogeneous propagation of distress into different sectors resulting into different levels of vulnerability. We see that this method of examining vulnerability can be employed to identify likelihood of failures of the firms in those more vulnerable sectors.

V. Discussion

We discuss about several points and policy implications.

Firstly, as one can see the methodology of DebtRank, the model is based on abstract quantification of financial stress. It is an important question how the model is related to financial states of individual firms in terms of stock and flow variables in balance sheets and profit-and-losses. In the application of DebtRank to financial institutions, there are recent works on this point in Battiston et al. (2015); Bardoscia et al. (2015). They attempt to clarify the link between financial distress defined and modeled by balance-sheet dynamics and the DebtRank, and found that the dynamics of DebtRank is naturally interpreted by the dynamics of balance-sheet and correspondingly defined variable of distress, such as debt and capital ratios.

In the application of production network, while there are related works such as Hazama and Uesugi (2012); Mizuno et al. (2014); Goto et al. (2015), which found various aspects of systemic risks on the production network, there is little work that simulates the entire system by using the dynamics based on the actual balance-sheets of individual firms and the model of DebtRank and its extension on the actual network. It would be valuable to relate those related works to the simulation.

Secondly, in the present model, we focus on the propagation of distress from downstream (customers) to upstream (suppliers), but not on the opposite direction from upstream to downstream. The latter is relevant in a typical case of the influence of price changes. When the price of raw materials goes up, the prices of commodities in the downstream eventually increase potentially affecting financial states in those firms. Another case is the external shock from supply-side propagating in a similar direction, due to a natural disaster, for example. Although these problems are not in the present scope focusing on demand-driven propagation of financial distress, they could be considered in the above mentioned dynamics of balance-sheet.

Thirdly, a distressed firm does not affect its neighboring firms once it becomes into the inactive state in the present model, as the dynamics is given by (3) and (4). Because it is usually the case that such a distressed firm may continue its business still affecting the neighbors, we may consider a variation of the original model so as to include amplification by such firms. Our estimation of DebtRank may be regarded, therefore, as a lower bound of the financial distress in the system. It would be an interesting problem to extend the model so that one can include the process of such amplification.

As a fourth point, we mention that one can employ the dataset in other snapshots available in the RIETI project corresponding to a recent year, so that one is able to compare the results for more than one network and to see how robust our results are, what are possible changes in the network structure, which are the results specific to the year 2011, right after the East Japan Disaster, and so forth, while we believe that the results stated in this paper do not depend on the particular year. In addition, one needs to examine *random networks*, preserving macroscopic variables such as degree (number of suppliers and customers) but otherwise random, as a null hypothesis for statistical validation. Comparison of results for other snapshots of production network and also randomized networks as a null hypothesis is an important future problem.

Concerning possible implications related to the Small and Medium Enterprise (SME) Agency's policies, we state the following point. The present major SME policies include safety-net guarantee program. This program supports SMEs whose business stability is threatened by external factors, such as a major customer's restricted operations or application for rehabilitation procedures, by making additional credit guarantees available. One of the strategies of this program is aimed at mitigating the possibility of chained bankruptcies of SMEs, each of which provides a credit of accounts receivable to its bankrupted customer of a large firm-size. The policy of making additional credit guarantees available should be based on a certain evaluation for the propagation of financial stress in the system, because of the budget constraint for the credit guarantees. The present model can serve as a benchmark for such evaluation.

VI. Summary

We apply the methodology of DebtRank (Battiston et al., 2012) to the propagation of financial distress along the supplier-customer links from the downstream of customers and to the upstream of suppliers. If a customer does not fulfill the payment to its suppliers due to a financial distress, then its suppliers are possibly under financial distress potentially causing propagation of distress.

Assuming that the propagation takes place in the opposite direction to the supplier-customer relation $i \to j$, namely from customers to suppliers, $j \Rightarrow i$, and that the strength of propagation W_{ji} is given by the inverse of the number of customers j's for firm i, we perform the simulation and computation of the DebtRank on a million of firms and millions of links by supercomputers including the world-fastest K-computer. Such computation under many different scenarios has been difficult in practical computation time.

We show that the distribution of DebtRank for individual firms obeys a power-law in a significant correlation between the DebtRank and size for each firm. This fact is not trivial in the sense that big firms are affected by many connected but less-depending firms, while smaller firms are strongly influenced by distress. There is an interesting non-linearity, namely that the DebtRank becomes much larger than what is expected by a linear relation between the DebtRank and firm-size. This implies that the role of big firms are usually underestimated.

By calculating the DebtRank of individual sector, we show that there is a linear relationship between the logarithms of sector's size and DebtRank, but also there is a deviation due to the location of the sector in the upstream and downstream portions of the network. This implies that the linear relationship between the logarithms of sector's size and DebtRank guides us to pay attention to deviation from it depending on individual sectors.

Finally, we show that one can measure vulnerability in the methodology of DebtRank which can be potentially useful to identify likelihood of failures of the firms in those more vulnerable sectors. One will be able to use simulations on supercomputers under many scenarios, models of financial distress propagation, and various initial configurations.

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References

Aoyama, H., S. Battiston, and Y. Fujiwara, "DebtRank Analysis of the Japanese Credit Network," Discussion Paper 13-E-087, Research Institute of Economy, Trade & Industry 2013.

Bardoscia, M., S. Battiston, F. Caccioli, and G. Caldarelli, "DebtRank: A microscopic foundation for shock propagation," 2015. mimeo.

Battiston, S., D. Delli Gatti, M. Gallegati, B. Greenwald, and J. E. Stiglitz, "Credit chains and bankruptcy propagation in production networks," *Journal of Economic Dynamics & Control*, 2007, *31*, 261–2084.

– , G. Caldarelli, M. D'Errico, and S. Gurciullo, "Leveraging the Network: A Stress-Test Framework based on DebtRank," 2015. mimeo.

Battiston, Stefano, Michelangelo Puliga, Rahul Kaushik, Paolo Tasca, and Guido Caldarelli, "DebtRank: too central to fail? Financial networks, the FED and systemic risk," *Scientific reports*, 2012, *2.* Article number, 541.

Cormen, T. H., C. E. Leiserson, R. L. Rivest, and C. Stein, *Introduction to Algorithms*, Cambridge, MA: MIT Press, 2001.

di Iasio, G., S. Battiston, L. Infante, and F. Pierobon, "Capital and Contagion in Finanial Networks," MPRA Paper 52141, Munich Personal RePEc Archive 2013.

Fink, K., U. Kruger, B. Meller, and L. H. Wong, "BSLoss – A Comprehensive Measure for Interconnectedness," Discussion Paper, Deutsche Bundesbank 2014.

Fujiwara, Y. and H. Aoyama, "Large-scale structure of a nation-wide production network," *European Physical Journal B*, 2010, *77* (4), 565–580.

Goto, H., H. Takayasu, and M. Takayasu, "Empirical Analysis of Firm-Dynamics on Japanese Interfirm Trade Network," in H. Takayasu, N. Ito, I. Noda, and M. Takayasu, eds., *Proceedings of the International Conference on Social Modeling and Simulation, plus Econophysics Colloquium 2014*, Springer Proceedings in Complexity Springer 2015, pp. 195–204.

Hazama, M. and I. Uesugi, "Measuring the Systemic Risk in Interfirm Transaction Networks," Working Papers Series 20, Center for Interfirm Network, Institute of Economic Research, Hitotsubashi University 2012.

Mizuno, T., W. Souma, and T. Watanabe, "The structure and evolution of buyer-supplier networks," *PloS one*, 2014, *9* (7), e100712.

Poledna, S. and S. Thurner, "Elimination of Systemic Risk in Financial Networks by Means of a Systemic Risk Transaction Tax," 2014. `http://arxiv.org/abs/1401.8026`.

Puliga, M., G. Caldarelli, and S. Battiston, "Credit Default Swaps Networks and Systemic Risk," *Scientific reports*, 2014, *4.* Article number, 6822.

Tabak, B. M., S. R. S. Souza, and S. M. Guerra, "Assessing the Systemic Risk in the Brazilian Interbank Market," Working Paper Series 318, Central Bank of Brazil 2013.

Terai, M., Y. Fujiwara, K. Minami, and F. Shoji, "Performance Analysis of the Graph Traversal Code Using DebtRank Algorithm for Economic Simulation," IPSJ SIG HPC Technical Report, Information Processing Society of Japan, Special Interest Groups, High Performance Computing 2016.