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Analyses of Aggregate Fluctuations of Firm Networks Based on the Self-Organized Criticality Model and Control Theory¹

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Abstract

This study examines the difference in the size of avalanches among industries triggered by demand shocks, which can be rephrased by control of the economy or fiscal policy, and by using the production-inventory model and observed data. In addition, we investigate how differently demand shocks affect each firm by using control theory that utilizes network topology. We obtain the following results. (1) The size of avalanches follows power law. (2) The mean sizes of avalanches for industries are diverse, but their standard deviations highly overlap. (3) We compare the simulation with an input-output table and with the actual policies and find them to be compatible. (4) The expectations about becoming involved in avalanches are diverse and depend on the industries. (5) It is difficult for service industries and small firms to be affected by the control. On the other hand, mining, manufacturing, and wholesale industries are strongly affected by the control. (6) If we clip a network in descending order of capital size, we do not lose the control effect.

Keywords: Aggregate fluctuation, Demand, Network, Firm, Production, Inventory, Control theory

JEL classification: D22; H32; E32

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1 Introduction

Giving stimulus to firms and prompting the spillover effect is a way for a government to affect its economy, which includes purchasing goods and services, giving grants to firms, and fine-tuning taxes. Governments consider fiscal policy as an important determinant of growth [1, 2]. Currently, analysis of input-output table is considered a strong tool to predict the spillover effect [3]. It enables us to obtain a single predicted value of the spillover effect caused by the stimulus. However, it is obvious that we cannot obtain the same result as that of the prediction, even if we use exactly the same volume as that used in the calculation. Nevertheless, we normally expect that the result should be substantially around the prediction. This concern, that, whether the expectation is correct is the main topic of this study.

If the size of the spillover effect is around the average of the prediction, it should be true that the propagation is never amplified or reduced through firm networks. However, Gabaix showed that if the firm size distribution is fat-tailed, the hypothesis breaks down [4]. In addition, Acemoglu et al. pointed out that microeconomic shocks may lead to aggregate fluctuations in the presence of intersectoral input-output linkages [5]. These studies suggest that the stimulus and spillover effect do not result in the proximity of the prediction. In other words, normal distribution is usually assumed, but this assumption is doubtful.

In addition, since the spillover effect must be the stochastic process, it seems improbable that each firm has the same probability of receiving the spillover effect. We can expect that the difference comes from the structure of the network. Hence, we focus on the network topology for each firm, which is calculated by the controllability of the network. The controllability can be rephrased by the possibility of being involved in the shock propagation.

This study reveals how demand shocks from outside cause the spillover effect. We use a micro model invented by Bak et al. [6] and the control theory [7]. In addition, we employ observed data. We clear up the following points: (1) the diversity of the spillover effect. (it must depend on industries in which shocks are given.); (2) the extent of getting involved in the spillover effect. (it must also depend on industries.); (3) the topological possibility of firms getting involved in the spillover effect; and (4) how we can clip a network without losing the effect of the spillover effect.

The remainder of this paper is organized as follows. In Section 2, we introduce the dataset. Section 3 describes the methodologies that we utilize in analyses. Section 4 presents the results. Finally, Section 5 concludes.

2 Data

We use datasets, TSR Company Information Database and TSR Company Linkage Database, collected by Tokyo Shoko Research (TSR), one of the major corporate research companies in Japan. The datasets are provided by the Research Institute of Economy, Trade and Industry (RIETI). The TSR data contain a wide range of firm information. As necessary information for our study, we use identification, capital, industry type, suppliers and clients. We construct an entire network of firms based on suppliers and clients. Note that there are up to 24 suppliers and up to 24 clients for each firm in the data. It may be considered that the constraint limits the number of links for each node. However, a node can be suppliers of other nodes without limitation, as long as those clients designate the node as a supplier, and vice versa. Therefore, the numbers of suppliers or clients are not limited to 24. In particular, we use the dataset collected in 2012. The number of firms, that is, nodes, is 1,109,549. The number of supplier-client ties, that is, links, is 5,106,081. This network has direction and the direction is important in our study.

We split firms based on industries. The industries are classified by the Japan Standard Industrial Classification [8]. We mainly use the division levels that have 20 classifications. However, we make alterations to the classification. Since the classifications “S: Government, except elsewhere classified” and “T: Industries unable to classify” are less important in our study, we omit them. In addition, we separate “I: Whole sale and retail trade” into wholesale and retail. The difference of the divisions is not negligible in our study because shocks from outside, such as fiscal policies, often occur in retail. Therefore, the division level in our study after alterations shows 19 industries.

Moreover, we use three industries at the group level to compare the effects of some Japanese fiscal policies. The groups are “5911: New motor vehicle stores,” “5931: Electrical appliance stores, except secondhand goods,” and “6821: Real estate agents and brokers.” The difference between the division and group levels is clear in the later industries and there is no concern about confusing the two levels.

We use an input-output table to compare the prediction of the spillover-effect size between a micro model and the table. As the closest table in time, we use the 2011 updated input-output table [9].

Figure 1 shows the degree distribution of the observed network. The red plots are the distribution of the observed network. An important point is that the distribution is fat-tailed, which means the distribution does not decay super-linearly. It seems that we can fit plots to a line $P \propto k^{-\lambda}$, where P

is the cumulative probability, k is the degree, and λ is a positive constant. If the degree distribution is the normal distribution, the plot is shaped as the blue plots in Figure 1. Since normal distribution exponentially decays, we can observe the blue plots decrease super-linearly on the log-log plot. How to create the random network is explained in Section 4. The reason we should compare the observed network with the random network is that the random network creates aggregate fluctuation that decays exponentially, as we show in Section 4. In other words, it tells us that if the random network is the network in the real economy, there is no fat-tailed aggregate fluctuation. However, the observed network is not the random network as we see here.

[Figure 1 here]

If a probability distribution or a cumulative probability distribution can be fitted to a line, it is said that the distribution is a power-law distribution. A network with a power-law distribution is often called a scale-free network. It has been pointed out that the power-law or scale-free nature of networks is a determinant of fat-tailed aggregate fluctuations [4]. Since the observed network is the scale-free network, we expect that the aggregate fluctuations of the network are fat-tailed.

3 Methodology

We use two methods. The first is a modified model [10] based on a production model [6]. The modified model enables us to conduct micro-level simulations and investigate the characteristics of aggregate fluctuations. The second method is control theory [11, 12, 13, 7]. By using this theory, we can know which nodes are pivotal to invoke aggregate fluctuations in a network by given demand and which nodes are likely to be involved in aggregate fluctuations.

3.1 Model of production and inventory

The model of production and inventory was originally invented by Bak et al. [6]. The model assumes that firms connect with each other on supply chains. Each firm has some amount of inventory. When a firm receives orders from clients, it supplies intermediate goods or services to clients. If the firm does not have enough inventory, it sends orders to suppliers. Therefore, cascades of orders and production sometimes occur. The size of the cascades can be defined by the total extent of production due to activated firms. Bak et al. showed that the distribution of the cascade size follows the power law. This result underlies recent network-based studies related to aggregate

fluctuations [4, 5]. That is, the cascade reaction can be understood as aggregate fluctuations. Here, for brevity, we call the cascade reaction an avalanche.

The result obtained by Bak et al. strongly depends on the regularity of supply chain network. A node has two suppliers and two clients in the regular network, except the nodes in the top and bottom layers. As we have already shown that the real supply-chain network is not a regular network but a scale-free network, the assumption is too strong to apply the model to the real supply-chain network. To mitigate the limitation of the regular network, Iino and Iyetomi generalized the model so that a node has arbitrary numbers of in-degree or out-degree and analyzed the nature of the generalized production model [10]. We employ their generalized model with a minor modification.

Here, we describe the model used in our analyses. For every time step t and every firm i , a new amount of inventory is decided based on the following equation.

$$z_i(t+1) = z_i(t) - s_i(t) + y_i(t),$$

where $z_i(t)$ is the amount of inventory of firm i at time t , $s_i(t)$ is the amount of orders received by firm i at time t , and $y_i(t)$ is the amount of production conducted by firm i at time t . The equation renews the inventory and is depicted in Figure 2(a). We assume that (1) the firm equally sends orders to its suppliers, (2) each firm produces one unit of production from one unit of material that it obtains, and (3) a firm produces the minimum goods necessary to meet requests from its consumers. Assumptions (1) and (2) simply result in a production feature in which $y_i(t)$ is a multiple of n_i , where n_i is the number of suppliers for the firm i . In addition, the assumption (3) results in $z_i(t) \leq n_i$. Based on the inventory renewal equation and the assumptions, the amount of production $y_i(t)$ is given by

$$y_i(t) = \begin{cases} 0 & (s_i(t) \leq z_i(t)) \\ n_i & (z_i(t) < s_i(t) \leq z_i(t) + n_i) \\ \vdots & \vdots \\ a_i(t) \cdot n_i & (z_i(t) + (a_i(t) - 1)n_i < s_i(t) \leq z_i(t) + a_i(t)n_i), \end{cases} \quad (1)$$

where $a_i(t)$ is the number of orders that firm i places with each supplier. $a_i(t)$ is calculated by a ceiling function

$$a_i(t) = \left\lceil \frac{s_i(t) - z_i(t)}{n_i} \right\rceil.$$

Since the quantity of the received orders $s_i(t)$ is the sum of orders to firm i ,

$$s_i(t) = \sum_j a_j(t),$$

where j is one of clients of firm i . If a firm does not have a client and the firm needs to produce, the firm regarded as belonging to primary industry and is assumed able to produce an arbitrary amount of production.

[Figure 2 here]

The first orders are placed from outside. Depending on the analyses, a firm is selected from all firms or firms in a specific industry to place an order. The selection is uniformly random.

Two firms may mutually supply or a long step of supply chains may form a cycle. It is possible that firms on a loop indefinitely produce goods or services, although this never occurs in the real economy. Iino and Iyetomi assume that firms have randomly assigned potential values, which is an analogy of electrostatics. A firm with more potential than another can supply, which is similar to water flow. Although this assumption helps avoid a loop and is useful for analyzing the nature of randomly created networks, it is not particularly clear how to assign a value to each firm. Here, we make a simple assumption. A firm that has already supplied products, that is, a firm that is already in a propagation process, is ignored as a supplier. Figure 2(b) shows an example of a loop with three firms. Firm 3 ignores firm 1 when it needs production. More precisely, the supply link from firm 1 to firm 3 is tentatively ignored.

Since the observed data include all industries, it may be considered that it is unnatural to contemplate inventory for service industries. This is because it is not understandable to consider inventory for intangible products, such as insurance or healthcare. However, service industries *do* have inventory. That is, if any service is ready to be used, it should be considered as inventory. For example, a vacant hotel room ready for use incurs cost. Therefore, we can discuss all industries that deal with tangible or intangible products in the same network.

3.2 Control theory

Control theory tells us whether a network is controllable with a given set of nodes to be controlled directly [11, 12, 13, 7]. Here, control means that through directly controlled nodes, other nodes are controlled indirectly. “Controllable” means that an arbitrary state of a network can lead to any desired state. Controllability fits with the motivation for fiscal policies, because they provide

stimulus to a certain set of firms and are intended to control firms directly. Moreover, through fiscal policies, governments to indirectly control other firms that are affected by directly controlled firms.

Control theory can be described formally as follows. A link indicates that there is a relationship in which one node affects another. In addition, we can assume that a network receives stimulus from the outside and propagate the stimulus through a given relationship. Based on this setup, we simply consider the following equation to depict the system.

$$\frac{d\mathbf{x}(t)}{dt} = A\mathbf{x}(t) + B\mathbf{u}(t),$$

where the vector $\mathbf{x}(t) = (x_1(t), \dots, x_N(t))^T$ is a state of N nodes at time t , the $N \times N$ matrix A is a diagram of links, the vector $\mathbf{u}(t) = (u_1(t), \dots, u_M(t))^T$ signifies the strength of outside controllers, and the matrix B is the $N \times M$ matrix ($M \leq N$) that indicates which drivers (nodes that take stimulus from outside) are connected to outside controllers. The system depicted by the equation is controllable if the following $N \times NM$ matrix

$$C = (B, AB, A^2B, \dots, A^{N-1}B)$$

has full rank. That means

$$\text{rank}(C) = N.$$

Figure 3 shows an example of a simple system. There is a network with four nodes and four links. In addition, there are two outside controllers. The network is controllable. The matrixes A and B correspond to the network. There are two outside controllers in the example but one outside controller is in theory enough to control for any network.

[Figure 3 here]

Once a set of driver nodes is given, we can calculate controllability. However, if we aim to test all sets of drivers, the calculation time is $O(2^N)$. The observed data are $N = 1,109,549$ and it is obvious that we cannot test all sets without thinking. Moreover, since a set of drivers that corresponds to all nodes can obviously control a network, finding the sets for a minimum number of drivers is also important. Liu et al. developed an algorithm to effectively obtain all sets of drivers that are controllable [7].

Sets of drivers have multiple configurations, even if they have a minimum number of drivers. We

return to Figure 3 to consider the example. The nodes x_1 and x_2 are drivers in panel (a) and the network is controllable. However, it is obvious that we can choose x_1 and x_3 for drivers and those are also a minimum set of drivers. The variable configuration gives us the following distinctions for nodes: (1) necessary driver: a node that is always chosen as a driver in any configuration of drivers; (2) necessary follower: a node that is never chosen as a driver in any configuration of drivers; and (3) ordinary: a node that is possibly chosen as a driver.

Fiscal policy and control theory are compatible. For example, it seems that it is more efficient not to choose necessary followers as targets of the fiscal policy than ordinaries or necessary drivers, because we can expect indirect effects delivered to necessary followers.

4 Results

In this section, we first show the fat-tailed nature of avalanches and their diversity over industries. Thereafter, we show that control theory is useful for revealing how each firm is subject to the spillover effect.

We start with the results of avalanches comparing a random network and the observed network. In a random network, every pair of nodes is connected according to constant probability p . The expected number of links of the random network with p is $\binom{N}{2}p$, where N is the number of nodes. The random network is created so that the network has the same number of nodes and the same number of links as the observed network. The observed network has 1,109,549 nodes and 5,106,081 links. Therefore, we set p as approximately 8.30×10^{-6} . As a result, we obtain a random network with 1,109,549 nodes and 5,366,223 links. This accordance is necessary to compare the two networks.

The experiments proceed as follows for both networks. (1) At time t , a firm is randomly chosen from all the firms. (2) A chosen firm sells a unit of product. (3) An avalanche is calculated. (4) Repeat (1)–(3) for 1 billion times (t proceeds from 1 to 1 billion.) For an avalanche size, that is, aggregated production, we obtain

$$Y(t) = \sum_i y_i(t),$$

for every time step t .

Figure 4 shows the avalanche sizes for the two networks. The red plots are for the observed network and the blue plots are for the random network. The random network obviously decays fast and it seems that the distribution cannot be fitted to a line. On the other hand, the observed

network has a part that can be fitted to a line. This result, that a scale-free network has a fat-tailed avalanche size, has already been shown analytically [10] and has been proved partly under some constraints [14].

[Figure 4 here]

The results here tell us that uniformly random stimuli cause scale-free avalanche on the real network. That is, the average avalanche size is not a representative value of it. The input-output table analysis results in a single value of prediction, which means some representative value is used to predict the spillover effect. However, the power-law distribution does not have a typical scale. It seems that careful analyses are required by the input-output table.

From the viewpoint of fiscal policy, it is important to know how stimulus received by different industries causes differences. We conduct the experiments with a few changes to the previous experiments. A firm is randomly chosen from an industry. The industry is fixed through an experiment. In every experiment, 1 billion instances of demand are given. The experiments are conducted for the 19 industry divisions. Figure 5 shows the distributions of avalanche sizes. While we had expected that the distributions of avalanche sizes would have different shapes, this turns out to be untrue. As can be observed, there is no apparent difference in shapes.

[Figure 5 here]

Figure 6 shows the mean sizes of avalanches. However, as mentioned in the last paragraph, the size distribution is fat-tailed and therefore, the average is not a representative of the avalanche size. The error bars in Figure 6 show standard deviations. Since we already observed that the avalanche size has the power law distribution in Figures 4 and 5, we know that standard deviations or variances are large. We do not conduct the statistical test for the difference of the average because the 1 billion samples cause small standard errors and always show significance of the difference. Therefore, the test is pointless. Instead, it is important that standard deviations are overlapping. It seems that we cannot strongly expect that the spillover effect started from a specific industry that is certainly superior or inferior to other industries.

[Figure 6 here]

In Figure 6, the order of the industries that are aligned in the horizontal axis roughly shows the advancement from the primary industries. Since an advanced industry, such as manufacturing or

services, is in the downstream of supply chains and has long chains from the primary industries, it may be considered that the distance to the primary industries has a positive correlation coefficient with the avalanche size of an industry. However, we do not observe such a relationship, which can be attributed to the network structure. Scale-free network is a small-world network [15]. A firm has a short path to a hub. Since the hub yields a large $y_i(t)$, the magnitude of avalanches seems to be dominated by this mechanism.

It may be argued that the production model of this study is only a model and it is doubtful how much it can explain the actual economy. Therefore, we simply compare the size of avalanches and the inverse matrix of the input-output table [9]. The Pearson's correlation coefficient is 0.28. If we consider that the production model does not include the data of trade volume at all, it can be said that the coefficient is surprisingly large. It should be noted that the production model can simulate the variances that cannot be obtained from the input-output table.

We conduct further experiments that start from the group level of industries so that we can compare the effect of actual policies. The results of Figure 7 are based on the division level industries. Those industries are "5911: New motor vehicle stores" (CarSale), "5931: Electrical appliance stores, except secondhand goods" (ElectronicsSale), and "6821: Real estate agents and brokers" (HouseSale), which are at the group level. They correspond to the target industries of past Japanese fiscal policies: eco-vehicle tax breaks, eco-point system for housing system, and eco-point system for home electronics. The setup of experiments is the same as that for the previous experiments.

[Figure 7 here]

Since the government publishes the actual size of budgets and some institutes publish the estimated economic results, we can validate the predictability of the model. The government lost tax revenue corresponding to 241.0 billion Japanese yen (1.98 billion US dollars at an assumed exchange rate of 122 Japanese yen to 1 US dollar) in 2009 for the eco-vehicle tax breaks [16]. The economic result was estimated about 2,937.53 billion Japanese yen (24.08 billion US dollars) [17, 18]. However, this estimation does not include indirect effects of other industries. The leverage of the return to the investment is 12.18. The government spent 494.8 billion Japanese yen (4.06 billion US dollars) for the eco-point system for housing in total. The economic result was estimated at about 414.0 billion Japanese yen (3.39 billion US dollars) [19]. The leverage is 0.84. The government spent 692.9 billion Japanese yen (5.68 billion US dollars) for the eco-point system for home electronics in total. The economic result was estimated at about 5 trillion Japanese yen (40.98 billion US dollars)

[20]. The leverage is 72.16. Although the economic result and leverage are just estimations in the abovementioned studies, the eco-point system for housing is apparently small, as we find in our experiments. Three samples are small but we find no apparent contradiction in the results.

We observe the distributions of avalanche sizes that are stated from specific industries in Figure 5 and the shapes of the tails are similar. This can be interpreted as a certain supply chain always being used in those large avalanches and the supply chain may lie on particular industries. To examine the hypothesis, we obtain a different measure from the first experiments. The measure is how often firms in an industry become involved in avalanches. Note that we examine the avalanche sizes that start from specific industries thus far and a firm is randomly selected from all firms.

Surprisingly, as is observed in Figure 8, the extent to which a firm becomes involved in avalanches is sharply different. The sharpness is totally different from that observed in Figure 6. Wholesale and manufacturing are distinctly large and construction can be included in the largest group. This result means that firms in those industries are apparently always involved in large avalanches that start from any industry.

[Figure 8 here]

Thus far, we have used the production-inventory model and shown that the size of the spillover effect follows the power law. Next, we discuss each firm's possibility of being involved in the spillover effect. Here, we employ control theory to know that the possibility is determined by the structure of the network.

The results of the calculations are based on control theory are given in Figure 9. The large share of necessary driver means that the industries require direct controls. Service industries, including retail, have a large share of necessary drivers. This is mainly because most of them are connected to only the final consumer. In addition, the necessary drivers that do not have suppliers are relatively small firms. If a firm has a client firm, demand can possibly be propagated from the client, that is, the firm possibly can be controlled indirectly. The necessary follower is a stronger condition of indirect control. A necessary follower is a firm that should be controlled indirectly if the network should have minimum drivers. Mining, manufacturing, and wholesale have the large shares of the necessary followers. This means they can be controlled indirectly more easily than other industries.

[Figure 9 here]

Note that the input-output table includes the final consumer and similar information about the

necessary driver may be obtained from it. However, the information of the necessary follower cannot be obtained from the input-output table.

The control theory we adopt in this study cannot incorporate link weight. However, we may realize it by copying a node that has the same links according to the weight. This should be studied in the future work.

Since it is always difficult to affect the entire economy directly and indirectly through fiscal policy, it is useful to know what happens if we consider a partial network. We apply control theory to clipped networks. We examine two different ways to clip the observed network: random clipping and capital-order clipping. In random clipping, a certain amount of nodes are chosen randomly from the observed network and links that connect the chosen nodes are also reserved. In capital-order clipping, a certain amount of nodes is chosen by the descending order of the firm's capital size from the entire network and links between them are reserved.

We clip the observed network with five different fractions: 2^{-1} , 2^{-2} , 2^{-2} , 2^{-4} , and 2^{-5} . If the fraction is 2^{-1} , the observed network is clipped into a half size in the sense of the number of nodes. Figure 10 shows the results. The horizontal axis indicates the fraction of clipping. n_d is the ratio of the number of the necessary drivers to all nodes. We obtain unique networks from capital-order clipping but random clipping leads to different samples. Therefore, we obtain 10 samples for each fraction. The bars in the figure show the standard deviations. We observe that n_d decreases or hardly moves for capital-order clipping through different fractions. On the other hand, random clipping causes an increase of n_d , which means that the randomly clipped networks are difficult to control. If we want to partially affect firms, capital-order clipping seems better than random clipping.

[Figure 10 here]

The results of Figure 10 corroborate the earlier discussion on controllability (Figure 9). Figure 9 indicates that small firms tend to be the necessary drivers. Therefore, capital-order clipping can avoid those small firms to be chosen.

For the scale-free networks, it is analytically shown that the ratio of drivers depends only on the exponent of the degree distribution and the average of the degree [7]. (Note that the necessary driver is a part of the driver.) The equation is

$$n_d \approx \exp\left[-\frac{1}{2}\left(1 - \frac{1}{\gamma - 1}\right)\langle k \rangle\right].$$

Figure 11 shows the degree distributions of the networks. We observe that the capital-order clipping retains the shapes. That is, all networks are scale-free networks and the exponents seem to be constants. Since it seems that small firms have small degrees, they are cut first in capital-order clipping. As a result, the remaining firms are densely connected. Actually, the mean degree $\langle k \rangle$ for the fractions 2^0 , 2^{-1} , 2^{-2} , 2^{-2} , 2^{-4} , 2^{-5} , and 2^{-6} are 6.00, 6.98, 7.79, 8.42, 8.99, 9.20, and 9.46. Therefore, the mean degrees increase as the fraction becomes smaller. The equation shows that a large mean degree should result in a small ratio of drivers n_d . The ratio of the necessary driver decreases or hardly moves, as observed in Figure 11. Therefore, the ratio of the necessary drivers to total drivers increases for small networks. This discussion shows that capital-order clipping adds efficiency in the sense that drivers tend to be necessary drivers. This is because if we want to control the network, we should avoid unwanted cost and unpredictability.

[Figure 11 here]

Through analyses of the production model and control theory, we obtain how the spillover effect has diverse characteristics. In other words, the analyses of this study partly reveal the cause of the unpredictable change of the economy. Turning now to fiscal policy, the main concern would be the average size of the spillover effect, but it fluctuates, as shown in this study. Therefore, in addition to the representative size of the effect, we should observe how large the error is and the probability of the error. It should be added that disparity between firms can occur in an industry. Each firm has different sensitivity to the indirect effect, and therefore, the penetration of the fiscal policy may not be uniform. If the policy is based on a hypothesis, such as the trickle-down hypothesis, the implication posed by this study should be considered.

5 Conclusion

This paper analyzed the cause of the diversity of the spillover effect. We used observed data of transactions in Japan. For the data, we employed the production model and control theory. As a result, we confirmed the size of the spillover effect triggered by demand follows the power law. Therefore, the normal distribution, which is usually expected in analyses of input-output tables, cannot be a reliable assumption. Although we did not use the volume of trade, the results of the simulations show significant correlation coefficients. Moreover, the simulated avalanche sizes for the policies actually conducted correspond to the estimation given by the ex-post evaluations of the

policies. In addition, industries have diverse potential to become involved in avalanches. By using control theory, we were able to classify firms based on the need to control them. Firms have diverse possibilities of being controlled directly or indirectly. If we clip a network by capital order, we can effectively choose firms to be controlled.

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- [20] Board of Audit Japan. Report of the eco-point system for home electronics (In Japanese), 2012.

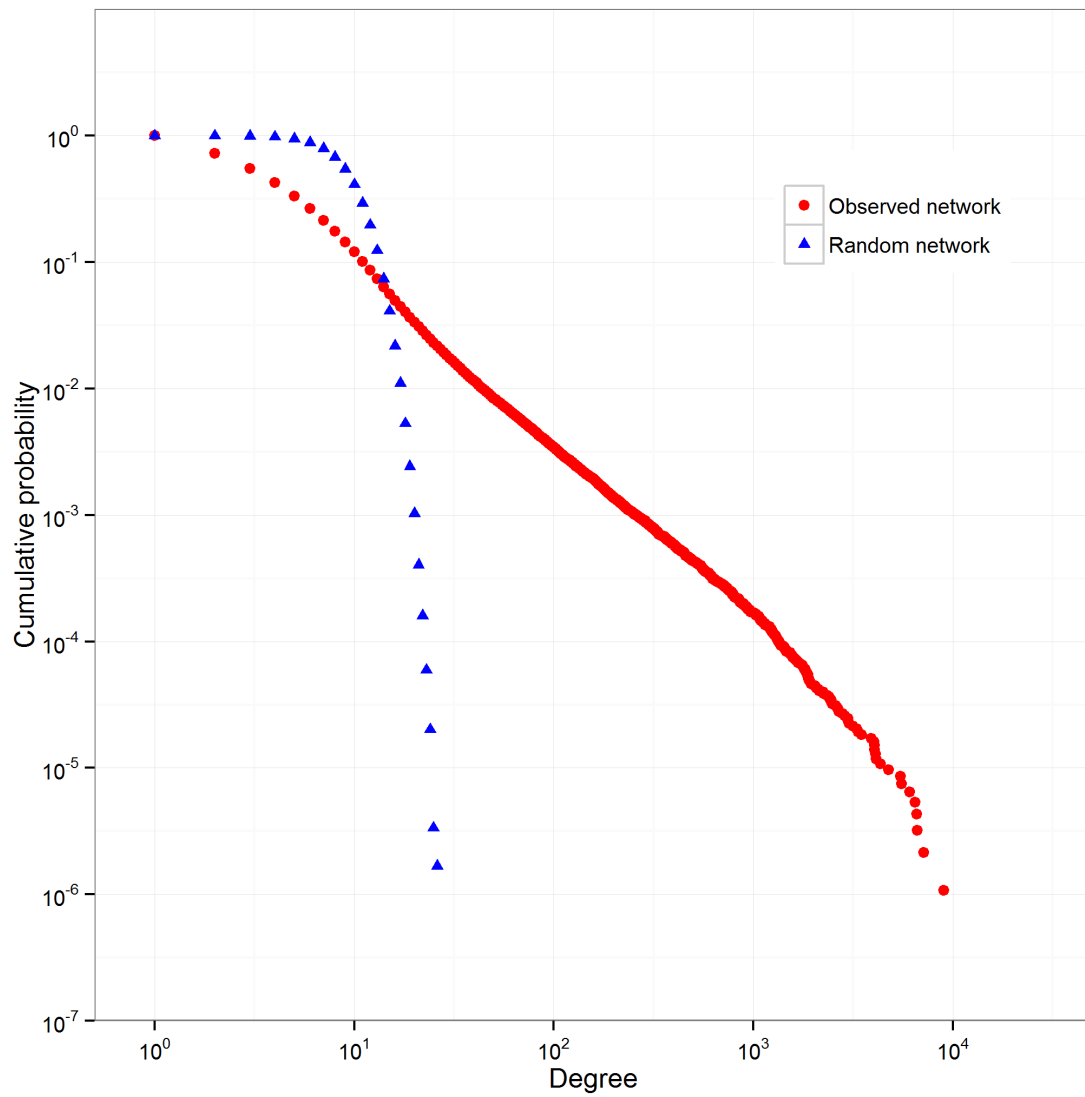
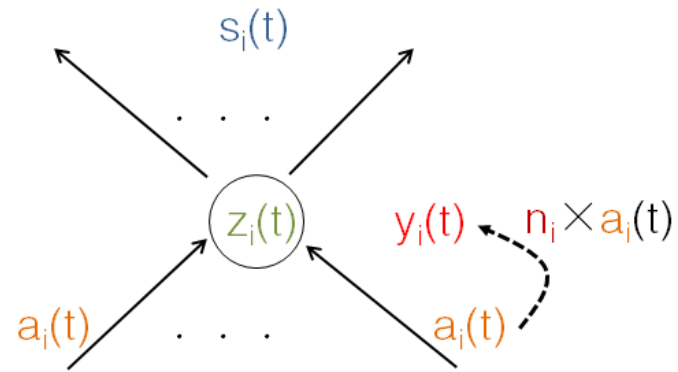


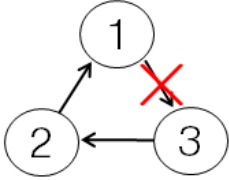
Figure 1: Comparison of degree distributions between the random network and observed network: The horizontal axis shows degree and the vertical axis shows cumulative probability. The blue plots indicate the random network. The red ones indicate the observed network.

← : flow of goods and services



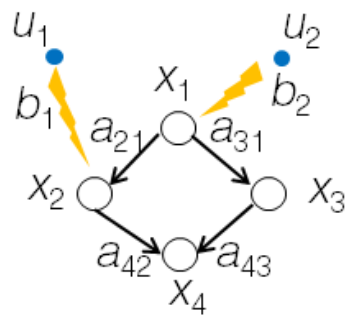
$z_i(t+1) = z_i(t) + y_i(t) - s_i(t)$
 $z_i(t)$: inventory
 $y_i(t)$: production
 $a_i(t)$: order to each supplier
 n_i : number of supplier
 $s_i(t)$: sold product

(a)



(b)

Figure 2: Generalized production and inventory model: (a) Scheme of the inventory renewal. Arrows show the flows of products. Therefore, orders of supplies are opposite. (b) Example of loop avoidance assumption. There are three firms 1, 2, and 3. Firm 1 asks firm 2 to supply products (given firm 1's inventory is not enough for demand.) Firm 2 asks firm 3 to supply products. However, firm 3 does not take supply from firm 1 and the link is ignored.



(a)

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 \\ a_{21} & 0 & 0 & 0 \\ a_{31} & 0 & 0 & 0 \\ 0 & a_{42} & a_{43} & 0 \end{pmatrix}$$

(b)

$$B = \begin{pmatrix} 0 & b_2 \\ b_1 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

(c)

Figure 3: Control theory: (a) A network and two outside controllers. This network is controllable. (b) Matrix of the network diagram. It corresponds to panel (a). (c) Matrix of the controller-driver diagram. It also corresponds to panel (b).

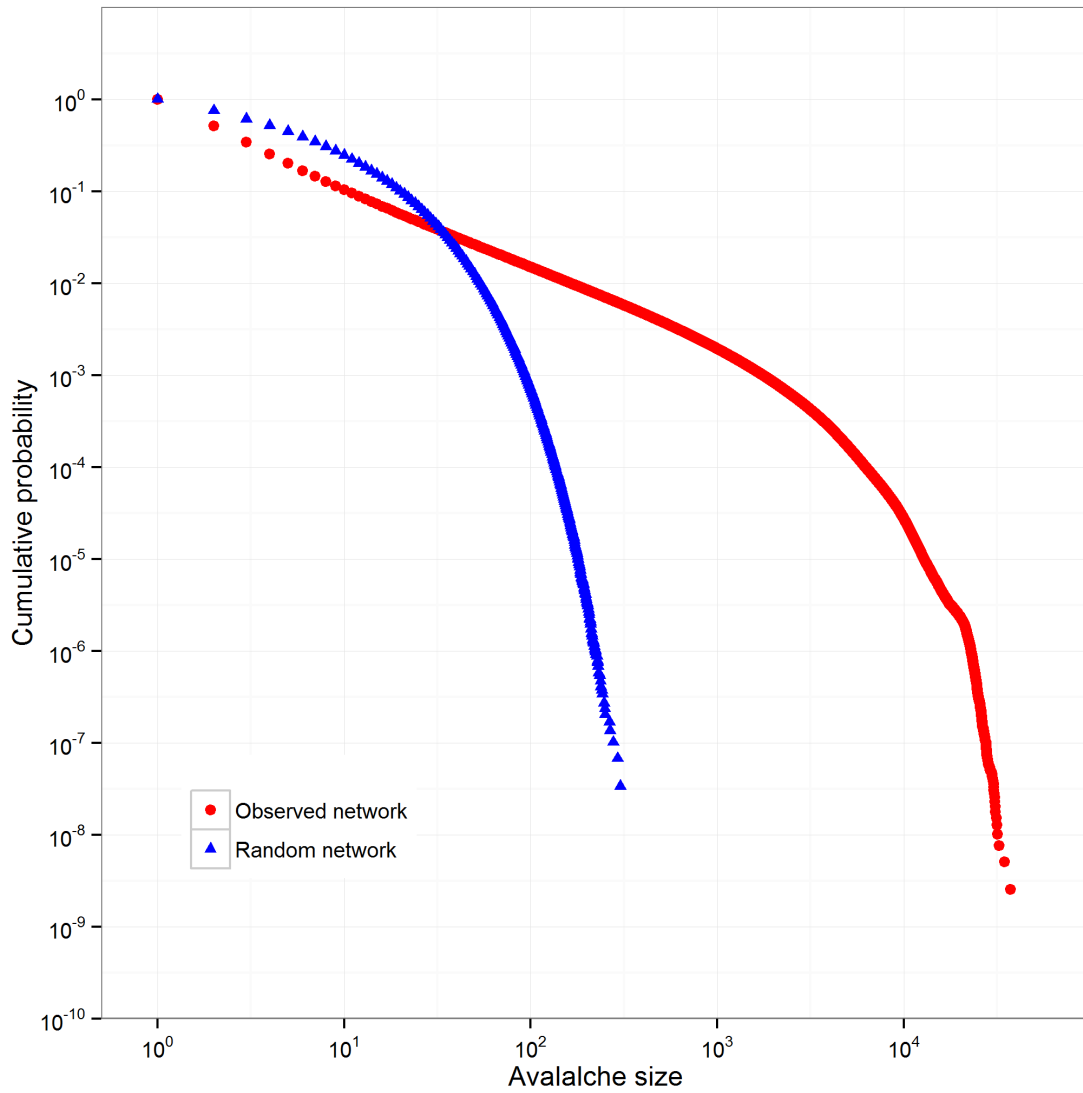


Figure 4: Comparison of avalanche size distribution between the random network and the observed network: The horizontal axis shows sizes of avalanches caused by demand. The vertical axis shows the cumulative probability. The blue plots indicate the random network. The red ones indicate the observed network. Although there are size zero avalanches, we ignore them because the main aim of the figure is to show the shapes of the tails and zero cannot be included into log plots.

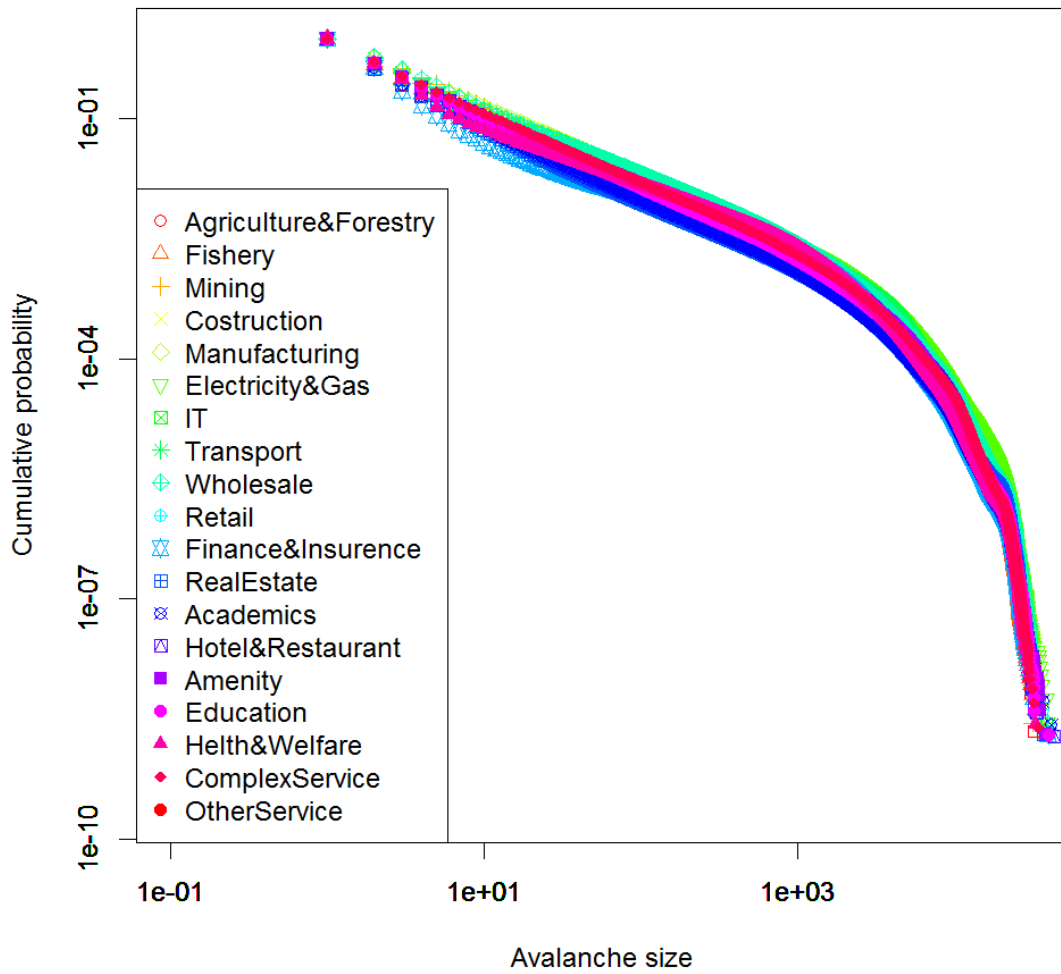


Figure 5: Comparison of avalanche size distribution between industries: The horizontal axis shows sizes of avalanches caused by demand. There is repeated demand for a firm that is chosen randomly from the industry. The vertical axis shows the cumulative probability. Although there are size zero avalanches, we omit them because the aim of the figure is to show the shapes of the tails and zero cannot be included in log plots.

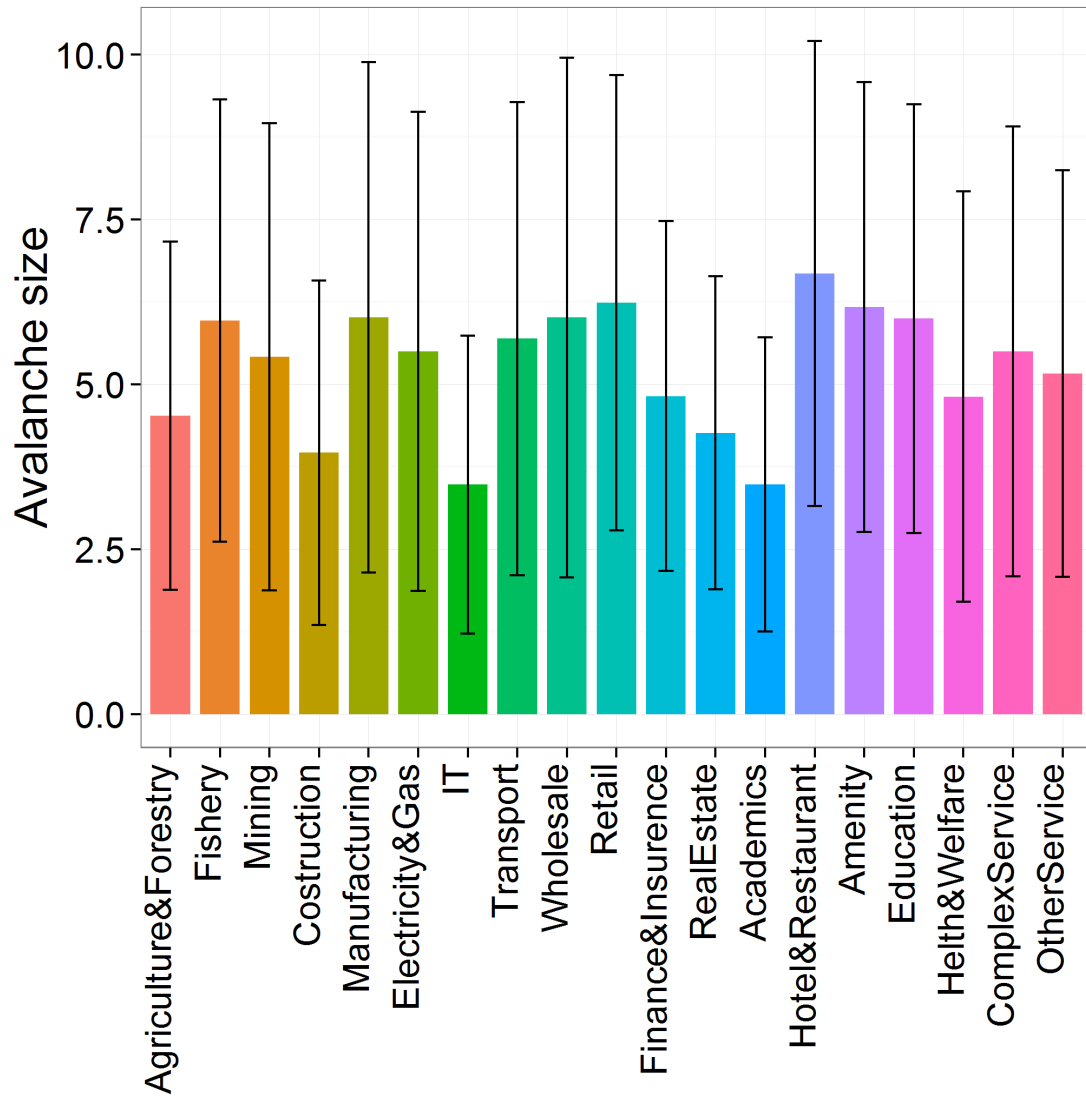


Figure 6: Comparison of average avalanche size starting from specific industry: The horizontal axis lists the industries. There is repeated demand for a firm that is chosen randomly from the industry. The vertical axis shows the average of avalanches. The error bars show standard deviations.

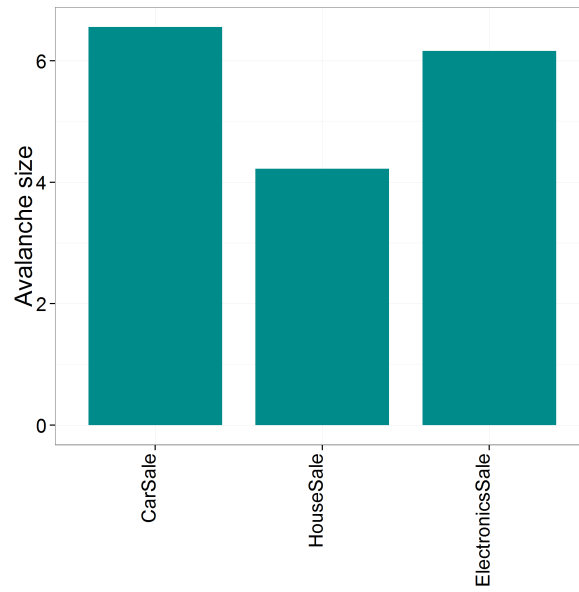


Figure 7: Simulation for industries to compare fiscal policies: The horizontal axis lists the industries. They correspond to the target industries of past Japanese fiscal policies: eco-vehicle tax breaks, eco-point system for housing, and eco-point system for home electronics. The vertical axis shows the average of avalanches. There is demand for a firm in each industry.

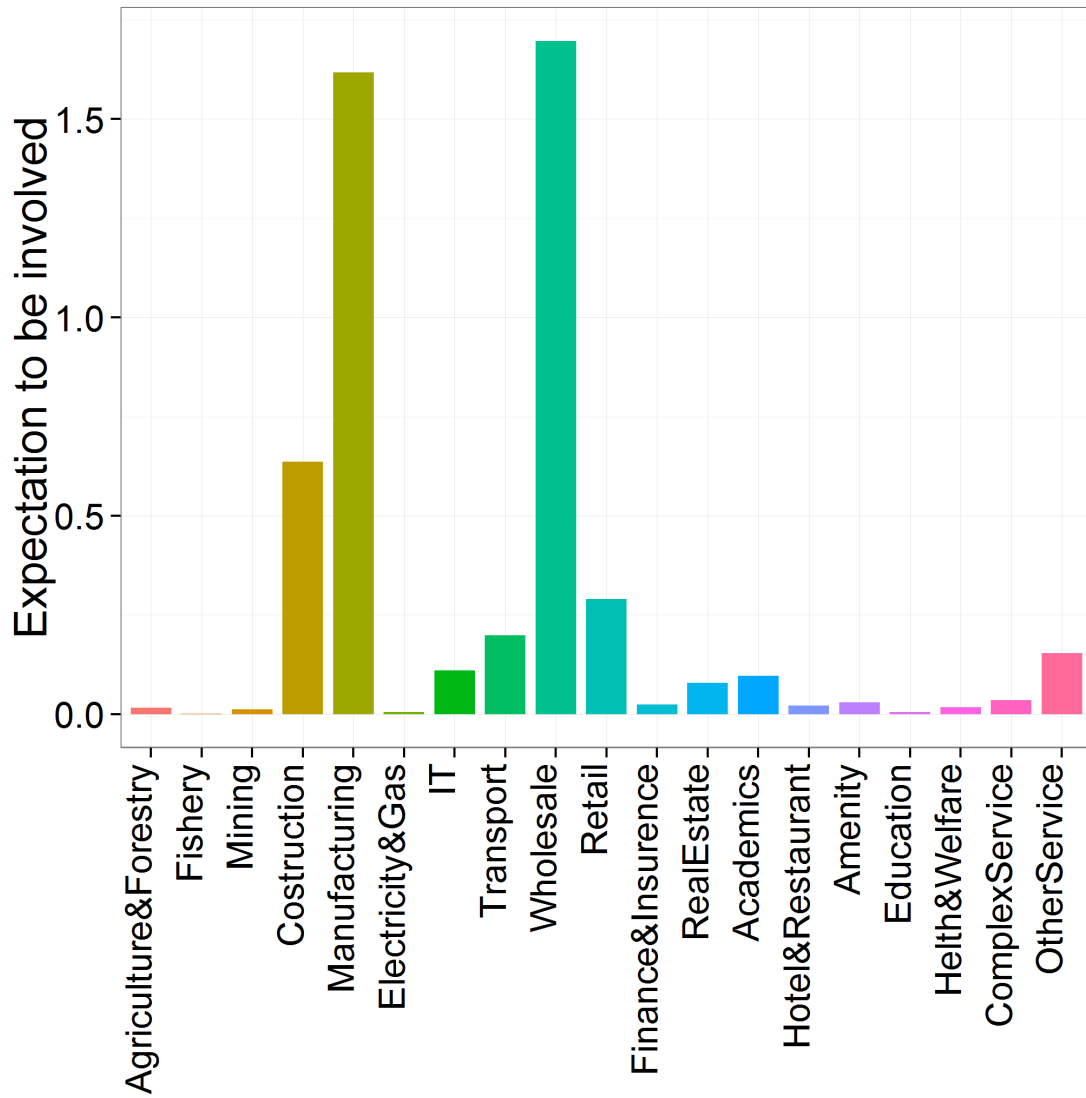


Figure 8: Difference of expectations for being involved in avalanches: The horizontal axis lists the industries. The vertical axis shows the expectation to be involved in avalanches per instance of demand. The firm with demand is chosen randomly from all the firms. The expectation in each industry is averaged over the firms in each industry.

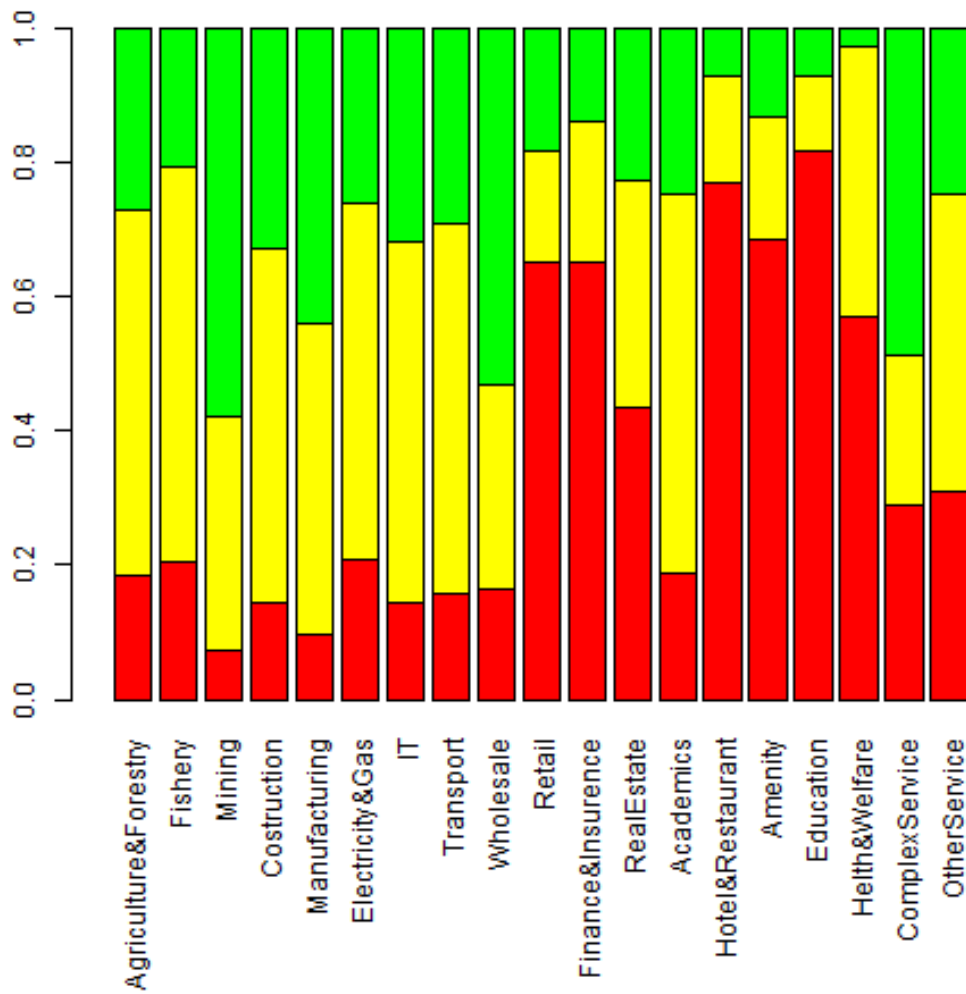


Figure 9: Share of node types obtained by the control theory: The horizontal axis lists the industries. The vertical axis shows the share of necessary followers (green), ordinary nodes (yellow), and necessary drivers (red).

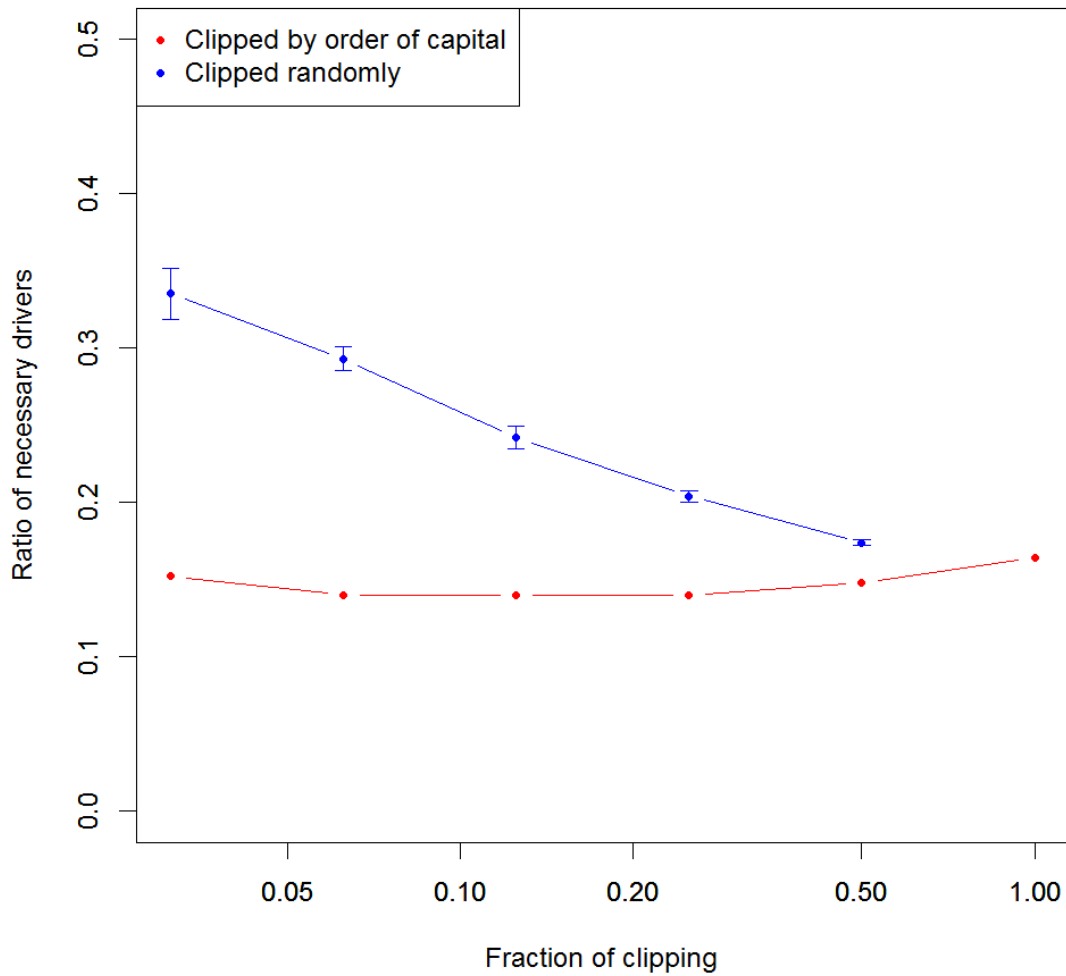


Figure 10: Comparison of necessary drivers between different clippings: The horizontal axis shows fractions of clipping. The vertical axis shows the ratio of necessary drivers. There are two different measures: the ratio of total capital of necessary drivers and the ratio of total number of necessary drivers. In addition, there are two different way of clipping: clipping in order of increasing capital and random clipping.

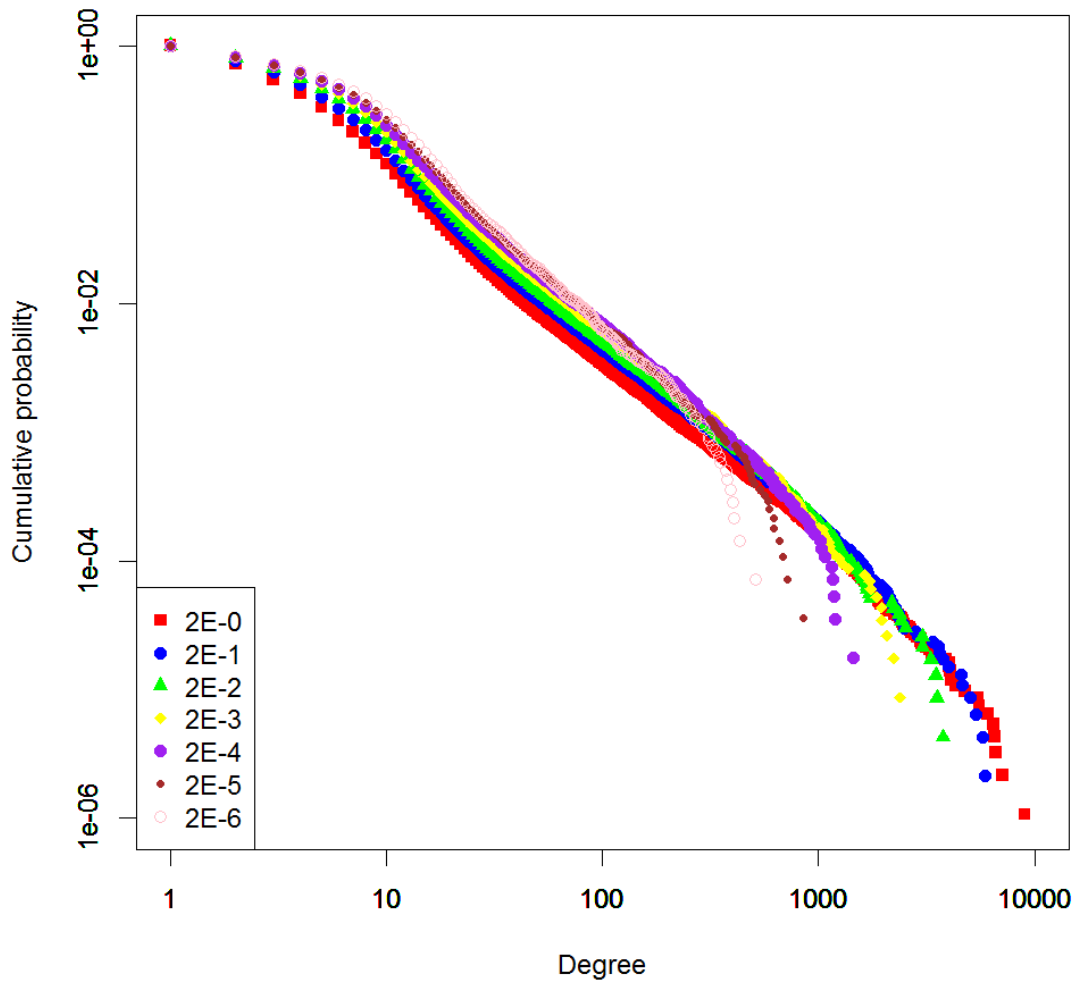


Figure 11: Comparison of degree distributions between clipped networks: The horizontal axis shows degree and the vertical axis shows cumulative probability. Each color and its number correspond to the ratio of clipping in order of increasing capital.