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Community dynamics, synchronization, and controllability as example of collective motions

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Econophysics Point of View of Trade Liberalization: Community dynamics, synchronization, and controllability as example of collective motions*

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Abstract

In physics, it is known that various collective motions exist. For instance, a large deformation of heavy nuclei at a highly excited state, which subsequently proceeds to fission, is a typical example. This phenomenon is a quantum mechanical collective motion due to strong nuclear force between nucleons in a microscopic system consisting of a few hundred nucleons. Most national economies are linked by international trade and consequently economic globalization forms a giant economic complex network with strong links, i.e., interactions due to increasing trade. In Japan, many small and medium enterprises could achieve higher economic growth by free trade based on the establishment of an economic partnership agreement (EPA), such as the Trans-Pacific Partnership (TPP). Thus, it is expected that various interesting collective motions will emerge in the global economy under trade liberalization. In this paper, we present collective motions in trade liberalization observed in the analysis of the industry sector-specific international trade data from 1995 to 2011 and production index time series from 1998 to 2015 for G7 countries. We discuss the results and implications for three collective motions: (i) synchronization of international business cycle, (ii) immediate propagation of economic risk, and (iii) difficulty of structural controllability during economic crisis.

Keywords: International trade, International business cycle, Economic crisis, Community structure, Synchronization, Control theory

JEL classification: F40, F44

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I. Introduction

It has been known that various collective motions exist in Physics. For instance, large deformation of heavy nuclei at highly excited state, which is subsequently proceeds to fission, is a typical example. This phenomenon is quantum mechanical collective motion due to strong nuclear force between nucleons in a microscopic system consists of a few hundred nucleons. Most of national economies are linked by international trade and consequently economic globalization forms a giant economic complex network with strong links, i.e. interactions due to increasing trade. In Japan many small and medium enterprises would achieve higher economic growth by free trade based on the establishment of Economic Partnership Agreement (EPA), such as Trans-Pacific Partnership (TPP). Thus, it is expected that various interesting collective motions will emerge in global economy under trade liberalization.

The interdependent relationship of the global economy has become stronger due to the increase of international trade and investment [1, 2, 3, 4]. As a result, the international business cycle has synchronized and an economic crisis starting in one country now spreads across the world instantaneously. A theoretical study using a coupled limit-cycle oscillator model suggests that the interaction terms due to international trade can be viewed as the origin of this synchronization [5, 22]. We observed various kind of collective motions for economic dynamics, such as synchronization of business cycle [22, 23], on the giant economic complex network. The linkages between national economies play important role in economic crisis as well as in normal economic state. Once a economic crisis occurs in a certain country, the influence propagate instantaneously toward the rest of the world. For instance the global economic crisis initiated by the bankruptcy of Lehman Brothers in 2008 is still fresh in our minds. The global economic complex network might show characteristic collective motion even for economic crisis.

In this paper we analyzed the industry sector-specific international trade data from 1995 to 2011 to clarify the structure and dynamics of communities consist of industry sectors of various countries linked by international trade. Then we study G7 Global Production Network constructed using production index time series from January1998 to January 2015 for G7 countries. Collective motion of G7 Global Production Network was analyzed using complex Hilbert principal component analysis, community analysis for single layer network and multiplex networks, and structural controllability.

Section II explains methodologies used in analysis, and section III describes the international trade data and G7 production data. Section IV shows various results. Finally section V concludes the paper.

II. Methodology to detect Collective Motions

A. *Community Analysis*

a. Single Layer Network Community structure is detected using the maximizing modularity function [29, 30, 31] for a network constructed in the previous subsection,

$$Q_s = \frac{1}{2w} \sum_{ij} \left(w_{ij} - \frac{w_i^{\text{out}} w_j^{\text{in}}}{2w} \right) \delta(c_i, c_j), \tag{1}$$

$$w_i^{\text{out}} = \sum_j w_{ij}, \tag{2}$$

$$w_j^{\text{in}} = \sum_i w_{ij}, \tag{3}$$

$$2w = \sum_i w_i^{\text{out}} = \sum_j w_j^{\text{in}} = \sum_i \sum_j w_{ij}, \tag{4}$$

where $\delta(c_i, c_j) = 1$ if the community assignments c_i and c_j are the same, and 0 otherwise. w_{ij} are matrix elements representing the weighted adjacency matrix between node i and node j.

We identify community structure by maximizing modularity using the greedy algorithm [29, 30, 31] to each time slice of the global production network, and then identified the links between communities in adjoining years as described in the following subsection.

b. Jaccard Index Once the community structure is obtained for each year, the temporal evolution of the communities becomes an item of great interest. Therefore, we need to measure the similarity between communities c_i and c_j in adjoining years to obtain the linked structure of the communities. The measured similarity is the Jaccard index [32] defined as follows,

$$J(c_i, c_j) = \frac{|c_i \cap c_j|}{|c_i \cup c_j|}. \tag{5}$$

The range of the Jaccard index is defined as

$$0 \leq J(c_i, c_j) \leq 1. \tag{6}$$

c. Multiplex Network For a static network, a random network is often used as the null model. However, there are no known null models for a time-dependent network such as an global production network. Recently multiplex network analysis has been developed as a methodology for detecting the community structure in a time-dependent network [33]. The most of current algorithm is limited to application only to undirected network. Community structure for an undirected multiplex network is identified by maximizing the multiple modularity,

$$Q_m = \frac{1}{2w} \sum_{ijsr} \left(\left(w_{ijs} - \frac{w_{is} w_{js}}{2w_s} \right) \delta_{sr} + \delta_{ij} C_{jsr} \right) \delta(c_{is}, c_{jr}), \tag{7}$$

where the term $\delta_{ij} C_{jsr}$ is the inter-layer coupling term.

d. Robustness of Community Structure Robustness of identified community structure is confirmed by calculating the variation of information between unperturbed and perturbed networks[13]. First we make a perturbed network by changing links of an original unperturbed network with probability α without changing weighted out-degree distribution. Then we compare the community division $c = \{c_1, c_2, \cdots, c_K\}$ of the unperturbed network with the community division $c' = \{c'_1, c'_2, \cdots, c'_{K'}\}$ of the perturbed network. The comparison is made using the variation of information defined by

$$V(c, c') = H(c|c') + H(c'|c) = -\sum_{i}^{K} \sum_{i'}^{K'} \frac{n_{ii'}}{N} \log \frac{n_{ii'}}{n_{i'}} = -\sum_{i}^{K} \sum_{i'}^{K'} \frac{n_{ii'}}{N} \log \frac{n_{ii'}}{n_i}, \tag{8}$$

where $n_{ii'}$, n_i, $n_{i'}$, and N are the number of nodes belonging to community c_i in the community division c and community $c'_{i'}$ in the community division c', the number of nodes belonging to community c_i in the community division c, the number of nodes belonging to community $c'_{i'}$ in the community division c', and the total number of nodes in network, respectively.

We have the larger the variation of information $V(c, c')$ for the larger difference of community structure. Therefore the variation of information $V(c, c')$ is a good measure of the robustness of identified community structure.

B. *Synchronization*

a. Hilbert Transform The linear trend in value added time series $V(t)$ is removed by calculating the growth rate as follows,

$$v(t) = \log V(t) - \log V(t-1), \tag{9}$$

$$x(t) = \frac{v(t) - \mathrm{E}\left[v(t)\right]}{\sqrt{\mathrm{Var}\left[v(t)\right]}}. \tag{10}$$

Similarly, the linear trend in production index time series $p(t)$ is removed by calculating the growth rate as follows,

$$x(t) = \log p(t) - \log p(t-1). \tag{11}$$

This is a standard procedure of stationalization.

The Hilbert transform [24, 25, 26] of growth rate of value added time series $x(t)$ is then calculated as follows,

$$y(t) = H[x(t)] = \frac{1}{\pi} PV \int_{-\infty}^{\infty} \frac{x(s)}{t-s} ds, \tag{12}$$

where PV represents the Cauchy principal value. A complex time series is obtained by the use of the time series $y(t)$ as an imaginary part. Consequently, the phase time series $\theta(t)$ is obtained by the use of the following,

$$z(t) = x(t) + iy(t) = A(t)e^{i\theta(t)}. \tag{13}$$

Here, i is the unit imaginary number defined by $i^2 = -1$. The following example may help readers understand the idea of the Hilbert transform. Suppose the time series $x(t)$ is a cosine function $x(t) = \cos(\omega t)$, then the Hilbert transform of $x(t)$ will be $y(t) = H[\cos(\omega t)] = \sin(\omega t)$. Similarly, for a sine function $x(t) = \sin(\omega t)$, the Hilbert transform will be $y(t) = H[\sin(\omega t)] = -\cos(\omega t)$. Using Euler's formula $z(t) = \cos(\omega t) + i\sin(\omega t) = A(t)\exp[i\theta(t)]$, we can calculate the phase time series $\theta(t)$. Actual calculation in our analysis uses the discrete formulation [26].

b. Order Parameter as a Measure of Synchronization Here we will briefly explain the concept of synchronization. Suppose we have two oscillators, one with a phases of $\theta_1(t)$ and one with a phase of $\theta_2(t)$. While the amplitudes can be different, synchronization is defined as the phases locking $\theta_1(t) - \theta_2(t) =$ const..

In a limited case of const. $= 0$, discussion using a correlation coefficient is suitable. However, in the case of const. $\neq 0$, where the phase difference signifies a delay, a direct evaluation of the phase instead of the correlation coefficient is more adequate. This is because the correlation coefficient ρ varies depending on the delay δ. For example, in trigonometric function with the period of oscillation equal to 2π, we have $\rho = 1$ for $\delta = 0$, $\rho = 0$ for $\delta = \pi/2$, and $\rho = -1$

for $\delta = \pi$. This simple example shows that the correlation coefficient is not suitable for phases locking cases in which there is phase difference or delay.

The collective rhythm produced by the whole population of oscillators is captured by a macroscopic quantity, such as the complex order parameter $u(t)$, defined as follows,

$$u(t) = r(t)e^{i\phi(t)} = \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j(t)}. \tag{14}$$

The radius $r(t)$ measures the phase coherence, and $\phi(t)$ represents the average phase [18].

c. Complex Hilbert Principal Component Analysis Then the complex time series $z_\alpha(t)$ for sector α ($\alpha = 1, \cdots, N$) is normalized by

$$w_\alpha(t) = \frac{z_\alpha(t) - \langle z_\alpha\rangle_t}{\sigma_\alpha}, \tag{15}$$

where $\langle\cdot\rangle_t$ is the average over time $t = 1, \cdots, T$. The variance of $z_\alpha(t)$ is defined by

$$\sigma_\alpha^2 = \frac{1}{T}\sum_{t=1}^{T} |z_\alpha(t) - \langle z_\alpha\rangle_t|^2 = \langle|z_\alpha|^2\rangle_t - |\langle z_\alpha\rangle_t|^2. \tag{16}$$

The complex correlation matrix C is an $N \times N$ Hermitian matrix defined as,

$$C_{\alpha\beta} = \langle w_\alpha w_\beta^*\rangle_t. \tag{17}$$

The eigenvalue $\lambda^{(n)}$ and the corresponding eigenvector $\boldsymbol{V}^{(n)}$ satisfy the following relations [27, 28],

$$C\boldsymbol{V}^{(n)} = \lambda^{(n)}\boldsymbol{V}^{(n)}, \tag{18}$$

$$\boldsymbol{V}^{(n)*} \cdot \boldsymbol{V}^{(m)} = \delta_{nm}, \tag{19}$$

$$\sum_{n=1}^{N} \lambda^{(n)} = N, \tag{20}$$

$$C = \sum_{n=1}^{N} \lambda^{(n)}\boldsymbol{V}^{(n)}\boldsymbol{V}^{(n)\dagger}. \tag{21}$$

The superscripts n ($n = 1, \cdots, N$) denote denotes the eigenvalues, which is ordered in descending order $\lambda^{(n)} \geq \lambda^{(n-1)}$.

The filtered complex correlation matrix is given by

$$C_{\alpha\beta}^{(N_s)} = \sum_{n=1}^{N_s} \lambda^{(n)} \boldsymbol{V}_\alpha^{(n)} \boldsymbol{V}_\beta^{(n)\dagger} = r_{\alpha\beta} e^{i\theta_{\alpha\beta}}, \tag{22}$$

with the number of dominant eigen modes N_s, which is estimated using the rotational random shuffling procedure. If we consider the correlation matrix $C_{\alpha\beta}^{(N_s)}$ as an adjacency matrix, we obtain a network of production nodes linked each other with the corresponding correlation coefficients as weights. Note that the weight of the link $r_{\alpha\beta}$ between node α and node β ranges from 0 to 1, and the link has direction depending on the lead-lag relation between two nodes: β (α) leads α (β) if $\theta_{\alpha\beta}$ takes a positive (negative) value. Although the network constructed is in principle a complete graph, we select links with small lead-lag by setting the threshold θ_{th} on phase $\theta_{\alpha\beta}$ in the following analysis.

C. *Structural Controllability*

The theory of structural controllability is applied to complex network [34]. Dynamics of the system is often approximately described by a linear equation,

$$\frac{d\boldsymbol{x}}{dt} = \tilde{A}\boldsymbol{x}(t) + \tilde{B}\boldsymbol{v}(t), \tag{23}$$

Here $\boldsymbol{x} = (x_1(t), \ldots, x_N(t))^T$ is the state vector of the system. The N $\times$ N matrix $\tilde{A}$ is identical to transversed adjacent matrix and the N $\times$ M matrix $\tilde{B}$ identifies the driver node which is controlled from outside of the system. If the N $\times$ NM matrix K,

$$K = (\tilde{B}, \tilde{A}\tilde{B}, \tilde{A}^2\tilde{B}, \ldots, \tilde{A}^{N-1}\tilde{B}) \tag{24}$$

has full rank, that is

$$\mathrm{rank(K) = N}, \tag{25}$$

the system is controllable.

The driver nodes are identified by the maximum matching in the bipartite representation of the network [35]. Maximum bipartite matching is written as an integer linear programming as follows,

$$\begin{aligned} &\underset{y}{\text{maximize}} \quad \mathbf{1}^T\boldsymbol{y} \\ &\text{subject to} \quad \boldsymbol{y} \geq 0,\ A^T\boldsymbol{y} \leq 1, \end{aligned} \tag{26}$$

where A is the incidence matrix whose component s_{ij} is

$$a_{ij} = \begin{cases} 1 \text{ (if node } j \text{ is an endpoint of link } i) \\ 0 \text{ (otherwise)} \end{cases} \tag{27}$$

and $\boldsymbol{y}$ is the variable

$$\boldsymbol{y} = \begin{cases} 1 \text{ (if e} \in \text{F)} \\ 0 \text{ (if e} \notin \text{F)} \end{cases} \tag{28}$$

and $\mathbf{1}$ is a vector where all elements are equal to unity. The set F is a matching if each node is incident to at most one link in F.

We define the matchedness m to identify driver nodes as follows. In-degree $k_j^{(in)}$ and out-degree $k_j^{(out)}$ is calculated for each node j in the bipartite network after eliminating links with $y_i = 0$. If in-degree of node j is equal to 0 matchedness of node j is equal to $m_j = 0$, or else in-degree of node j is equal to 1 matchedness of node j is equal to $m_j = 1/k_l^{(out)}$. Here node l is the origin node spanning a link toward node j in the bipartite network. If $m_j = 0$, node j is a pure driver node. On the other hand, if $m_j = 1$, node j is a pure controlled node. Thus a node with small matchedness m is interpreted as a partial driver node. Note that the number of driver nodes is calculated by $N - \sum_j m_j$.

III. Data

A. *World Input-Output Database*

The World Input-Output Database has been developed to analyze the effects of globalization on trade patterns across a wide set of countries [19]. This database includes annual industry sector-specific international trade data on 41 countries and 35 industry sectors for the years 1995 to 2011. Therefore the number of nodes in the international trade network is equal to 1435.

Figure 1 depicts the growth of international trade. The average amount of trade per node increased monotonically from 1995 to 2011, except for the year 2008 in which there was a financial crisis in 2008 caused by the crash of the housing bubble in the US. As a result, the interdependent relationship of the global economy has become strengthened.

The industry sector-specific international trade network is specified by nodes of industry sector α of country A and links of trade amount $w_{A\alpha,B\beta}$ between industry sector α of country A and sector β of country B. The size of industry sector α of country A is measured by its value added.

Based on this basic understanding, we conducted community analysis with link identification and synchronization analysis using the World Input-Output Database.

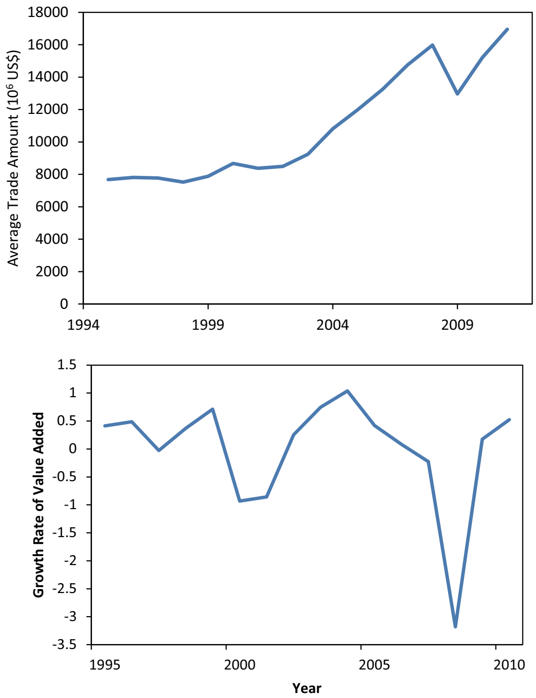

Figure 1: Trade and Business Cycles: (a) Amount of Import in USA and (b) Average growth rate of value added in USA

B. *Global Production Data*

G7 Global Production Data is complied by the use of monthly time series of production index, which is open from each government of G7 countries [36, 37, 38, 39]. Duration of the data is from January1998 to January 2015 and therefore includes the global economic crisis initiated by the bankruptcy of Lehman Brothers in 2008. The items of G7 Global Production Data are listed in Table 1.

Correlation matrices were estimated for time windows of 5 year. Each time window is slided by 1 years, thus we have 13 period where correlation matrices were estimated. For instance, period 1 corresponds to the duration from 1998 to 2003 and labeled by year 2001. Similarly period 2 corresponds to the duration from 1999 to 2004 and labeled by year 2002, and so on.

IV. Results

A. *Community Structure of Trade Network*

We identified the community structure for each time slice of the international trade network. Figure 8 (a) and (b) lists examples of community structures obtained for 1997 and 2009. There were 7 communities for 1997, and 8 for 2009. Because we have a few small community, the number of major community is equal to six. Temporal change of modularity Q is shown in Fig. 8 (c). The value of obtained modularity Q is about 2, which depends of threshold of the weight of links $w_{A\alpha,B\beta}$. In this community analysis we applied threshold of weight $w_{ij} > 10^7$ US$. This means that about half of links are included in the analysis. If we increase the threshold of weight, we have larger value of modularity Q.

Then the robustness of identified community structure was confirmed using the variation of information $V(c, c')$. The obtained results are shown in Fig. 3 for 1995 and 2011. From the value of modularity Q and the dependence of the variation of information on the probability of changing links α, we can say for a fact that the community structure is barely identified with the threshold of weight $w_{ij} > 10^7$ US$.

B. *Linked Communities of Trade Network*

The temporal evolution of communities is characterized by the link relations for communities in adjoining years. The similarity of communities c_i and c_j in adjoining years was measured by using the Jaccard index $J(c_i, c_j)$. Figure 9 shows the Jaccard indices between 1995 and 1996, and the indices between

Table 1: G7 Global Production Data

	Japan	
1	JP01	*Steel products*
2	JP02	*Nonferrous metal products*
3	JP03	*Fabricated metal products*
4	JP04	*Transportation equipments*
5	JP05	*Ceramic, stone and clay products*
6	JP06	*Chemical products*
7	JP07	*Petroleum and coal products*
8	JP08	*Plastic products*
9	JP09	*Pulp and paper products*
10	JP10	*Textile products*
11	JP11	*Food and tobacco*
12	JP12	*Miscellaneous*
13	JP13	*Mining*
14	JP14	*Electric appliances*
15	JP15	*General machinery*
16	JP16	*Precision machinery*
	USA	
17	US01	*Food*
18	US02	*Beverage and tobacco product*
19	US03	*Textile mills*
20	US04	*Textile product mills*
21	US05	*Apparel*
22	US06	*Leather and allied product*
23	US07	*Wood product*
24	US08	*Paper*
25	US09	*Printing and related support activities*
26	US10	*Petroleum and coal products*
27	US11	*Chemical*
28	US12	*Plastics and rubber products*
29	US13	*Nonmetallic mineral product*
30	US14	*Primary metal*
31	US15	*Fabricated metal product*
32	US16	*Machinery*
33	US17	*Computer and electronic product*
34	US18	*Electrical equipment, appliance, and component*
35	US19	*Transportation equipment*
36	US21	*Furniture and related product7*
37	US22	*Miscellaneous*
	Canada	
38	CA01	*Goods-producing industries*
39	CA02	*Service-producing industries*
40	CA03	*Industrial production*
41	CA04	*Non-durable manufacturing industries*
42	CA05	*Durable manufacturing industries*
43	CA06	*Energy sector*
	Germany	
44	DE01	*Capital goods*
45	DE02	*Durable consumer goods*
46	DE03	*Intermediate goods*
47	DE04	*Non-durable consumer goods*
	France	
48	FR01	*Capital goods*
49	FR02	*Durable consumer goods*
50	FR03	*Intermediate goods*
51	FR04	*Non-durable consumer goods*
	Italy	
52	IT01	*Capital goods*
53	IT02	*Durable consumer goods*
54	IT03	*Intermediate goods*
55	IT04	*Non-durable consumer goods*
	Great Britain	
56	GB01	*Capital goods*
57	GB02	*Durable consumer goods*
58	GB03	*Intermediate goods*
59	GB04	*Non-durable consumer goods*

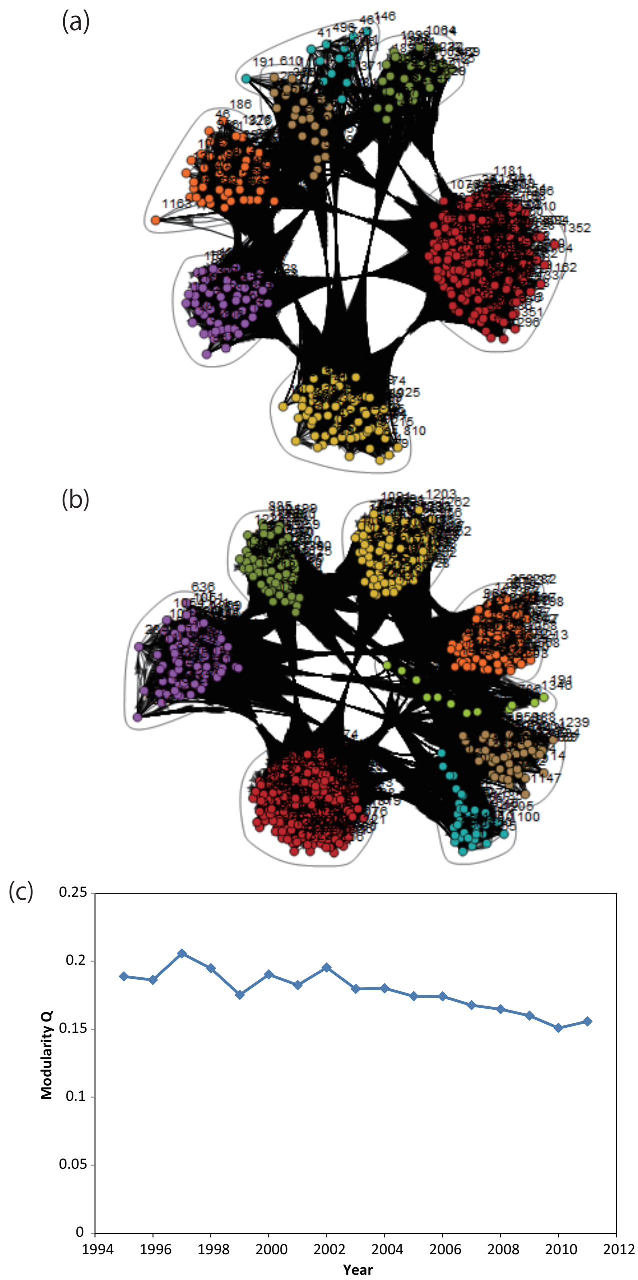

Figure 2: Community Analysis: (a) community structure in 1997, (b) community structure in 2009 and (b) temporal change of modularity

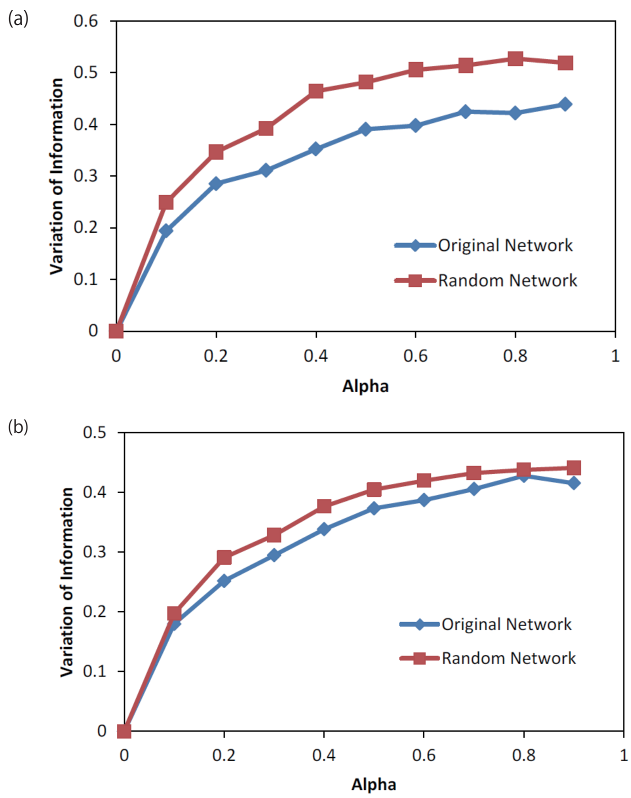

Figure 3: Variation of Information: (a) 1995 and (b) 2011

Table 2: Linked communities

Year	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
linked comm1	c_1	c_1	c_1	c_2	c_1	c_1	c_2	c_1	c_1	c_1	c_1	c_1	c_1	c_1	c_1	c_1	c_1
linked comm2	c_2	c_3	c_2	c_3	c_4	c_2	c_4	c_3	c_3	c_3	c_2	c_3	c_2	c_2	c_3	c_4	c_2
linked comm3	c_3	c_4	c_3	c_4	c_2	c_3	c_3	c_2	c_2	c_2	c_3	c_2	c_3	c_3	c_2	c_2	c_3
linked comm4	c_4	c_6	c_4	c_5	c_5	c_4	c_5	c_4	c_5	c_4	c_4	c_4	c_4	c_4	c_4	c_3	c_4
linked comm5	c_5	c_5	c_5	c_7	c_7	c_5	c_6	c_5	c_6	c_5	c_5	c_5	c_5	c_5	c_6	c_6	c_5
linked comm6	c_6	c_7	c_6	c_6	c_6	c_6	c_7	c_6	c_7	c_6	c_6	c_6	c_6	c_6	c_7	c_8	c_6

2010 and 2011. Communities were arranged in decreasing order of the number of industry sectors in each community. For instance 95c1 (community c_1 in 1995) is the largest, and 95c2 is the second largest. The indices for node 95c2 were displaced vertically relative to node 95c1 for better visibility. All other indices were displaced vertically in the same manner.

We define link relation for the pair of communities with the largest Jaccard index between adjoining year. We observed clearly linked relationships for most of communities. For instance, nodes 95c1 (community c_1 in 1995), 95c2, 95c3, 95c4, 95c5, and 95c6 are linked to nodes 96c1 (community c_1 in 1996), 96c3, 96c4, 96c6, 96c5, and 96c7, respectively. Similarly, nodes 10c1 (community c_1 in 2010), 10c4, 10c2, 10c3, 10c6, and 10c8 are linked to nodes 11c1 (community c_1 in 2011), 11c2, 11c3, 11c4, 11c5, and 11c6, respectively. We used this means to identify five linked communities between 1995 and 2011 as shown in Table 2. The identified link structure shows that a six-backbones structure exists in the international trade network.

The composition of the linked communities was analyzed in terms of the marginal rank of the trade volume in countries and industry sectors. The respective compositions of the first to the fifth linked communities are listed in Table 3. Certain features of the compositions of these linked communities are briefly described below. The first linked community consists the metal and chemical materials and the electrical and transport equipment industries in primarily European countries. The second linked community and the fifth linked community contain similar industry sectors. Next, the second linked community and the fourth linked community contain similar industry sectors but different country group. The second linked community includes China and South Korea. While the fourth linked community includes the US and Canada. In addition, the third linked community consists of various countries from all over the world. The business sectors in this linked community are different from other linked communities, because the major sectors are trade and finance.

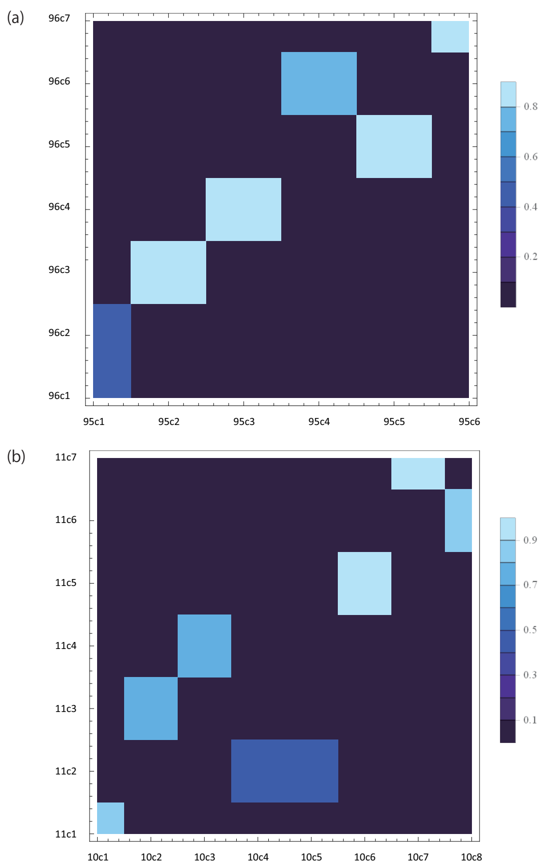

Figure 4: Jaccard index: (a) between 1995 and 1996, and (b) between 2010 and 2011

Table 3: Composition of Linked Communities

(a) Linked comm 1 (Total=24 trillions US$)

Rank	Share	Country	Industry sector
Largest	5.07%	United States	*Financial Intermediation*
2nd	4.92%	United States	*Renting of M&Eq and Other Business Activities*
3rd	4.91%	Rest of World	*Renting of M&Eq and Other Business Activities*
4th	3.41%	United Kingdom	*Renting of M&Eq and Other Business Activities*
5th	3.00%	United Kingdom	*Financial Intermediation*

(b) Linked comm 4 (Total=20 trillions US$)

Rank	Share	Country	Industry sector
Largest	37.67%	Rest of World	*Mining and Quarrying*
2nd	6.06%	Rest of World	*Coke, Refined Petroleum and Nuclear Fuel*
3rd	5.11%	Russia	*Mining and Quarrying*
4th	4.38%	Canada	*Mining and Quarrying*
5th	3.04%	Australia	*Mining and Quarrying*

(c) Linked comm 3 (Total=17 trillions US$)

Rank	Share	Country	Industry sector
Largest	9.03%	Rest of World	*Basic Metals and Fabricated Metal*
2nd	8.33%	Germany	*Basic Metals and Fabricated Metal*
3rd	4.74%	Germany	*Machinery, Nec*
4th	3.58%	Japan	*Basic Metals and Fabricated Metal*
5th	3.46%	United States	*Basic Metals and Fabricated Metal*

(d) Linked comm 2 (Total=16 trillions US$)

Rank	Share	Country	Industry sector
Largest	9.27%	Rest of World	*Chemicals and Chemical Products*
2nd	7.04%	Germany	*Chemicals and Chemical Products*
3rd	5.88%	United States	*Chemicals and Chemical Products*
4th	3.49%	France	*Chemicals and Chemical Products*
5th	3.37%	United States	*Wholesale Trade and Commission Trade, Except of Motor Vehicles and Motorcycles*

(e) Linked comm 5 (Total=15 trillions US$)

Rank	Share	Country	Industry sector
Largest	13.95%	China	*Electrical and Optical Equipment*
2nd	13.22%	Rest of World	*Electrical and Optical Equipment*
3rd	11.80%	United States	*Electrical and Optical Equipment*
4th	9.15%	Japan	*Electrical and Optical Equipment*
5th	8.08%	Germany	*Electrical and Optical Equipment*

(f) Linked comm 6 (Total=7 trillions US$)

Rank	Share	Country	Industry sector
Largest	16.76%	Germany	*Transport Equipment*
2nd	16.01%	United States	*Transport Equipment*
3rd	11.08%	Japan	*Transport Equipment*
4th	7.38%	France	*Transport Equipment*
5th	5.85%	United Kingdom	*Transport Equipment*

C. *Synchronization of International Business Cycles*

The phase time series $\theta_j(t)(j = 1, \cdots, 1435)$ for the growth rate of value added were evaluated for the years 1995 to 2011 using the methodology described in subsection a.

Figure 5 shows the polar plot of phase in 1997 for (a) all sectors, (b) community c_1, (c) community c_2, and (d) community c_3. In these polar plots, the complex order parameter $u(t)$ is indicated by a black asterisk. The respective amplitude for the order parameters are 0.483, 0.359, and 0.534 for community c_1, c_2, c_3, respectively. While the amplitude for all sectors is 0.253. The respective amplitude for the order parameter of each community was observed to be greater than the amplitude for all sectors. This means that active trade produces higher phase coherence within each community.

Figure 5 shows the polar plot of phase in 2009 are shown in for (a) all sectors, (b) community c_1, (c) community c_2, and (d) community c_3. In these polar plots, the complex order parameter $u(t)$ is indicated by a black asterisk. The respective amplitude of the order parameters are 0.758, 0.512, 0.801 for community c_1, c_2, and c_3, respectively. While the amplitude for all sectors is 0.662. Amplitude $r(t)$ and average phase $\phi(t)$ for each community is equivalent to those quantities for all sectors. The respective amplitude for the order parameter of each community was observed again to be greater than the amplitude for all sectors. It was noted that the amplitudes and average phases in 2009 are larger than the quantities in 1997. This relation clearly indicates that interdependent relationship of the global economy has become stronger.

Figure 6 shows the temporal change in amplitude for the order parameter $r(t)$ for the years 1996 to 2011. Phase coherence decreased gradually in the late 90's but increased sharply in 2001 and 2002. This temporal change might be related to the structural change in the international trade network discussed in subsection A. From 2002, the amplitudes for the order parameter $r(t)$ remain high except for the years 2005 and 2009. The decrease in 2009 was caused by financial crisis resulting from the housing bubble crash in the US but the cause of the decrease in 2005 is unclear.

Order parameters are estimated for the growth rate of the value added time series shuffled randomly with keeping auto-correlation [20, 21] and order parameter averaged over 1000 shuffled time series is plotted by black curve. This means that average order parameter is evidently larger than the systematic error of the analysis method. Therefore the synchronization observed for each linked communities is statistically significant. Thus it should be noted that existence of the synchronization in the international trade network is fully-clarified.

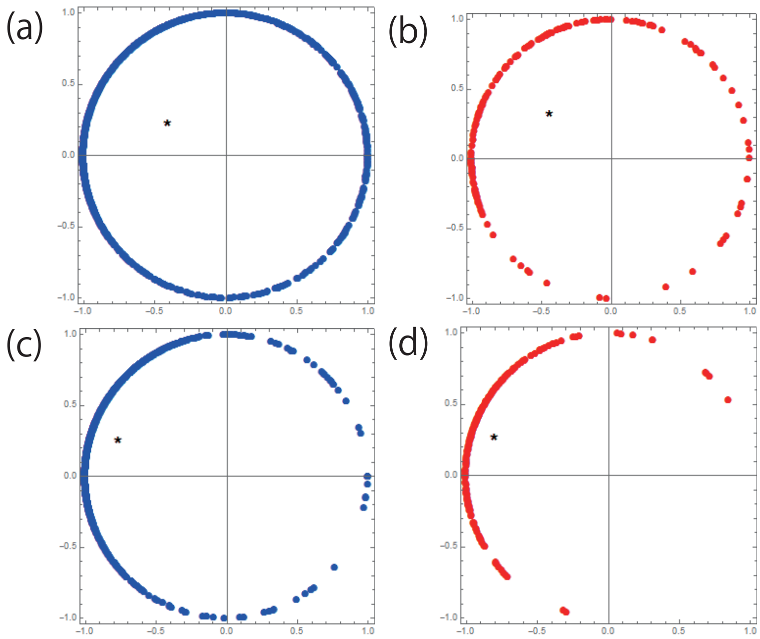

Figure 5: Polar plot of phase for growth rate of value added: (a) all communities in 1997, (b) community c_1 in 1997, (c) all communities in 2009, and (d) community c_1 in 2009

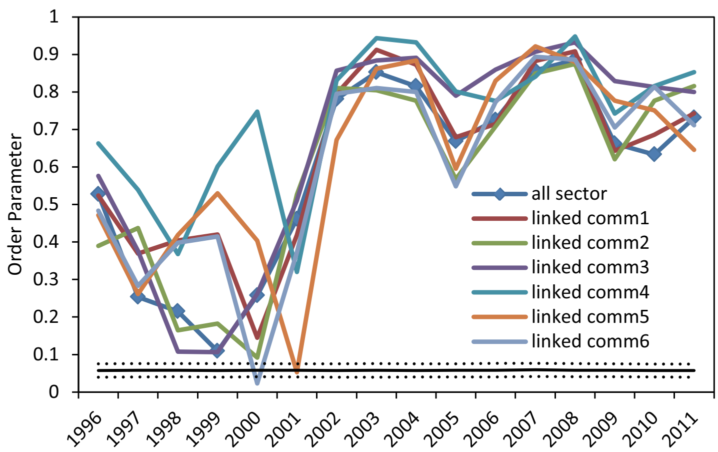

Figure 6: Temporal change of amplitude for the order parameter $r(t)$

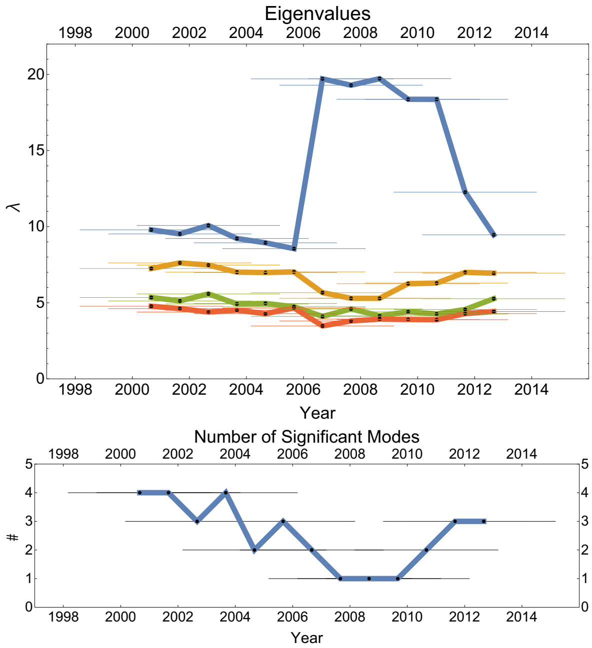

Figure 7: Eigenvalues of Significant Modes

D. *Significant Modes in Economic Crisis*

In the analysis of G7 Global Production Data, we obtained eigenvalues for the correlation matrix C from Eq.(18) and selected statistically significant modes using the rotational random shuffling. In the rotational random shuffling, the growth rate of production time series $x(t)$ was shuffled randomly with keeping autocorrelation [20, 21] and then eigenvalues for the correlation matrix were calculated for the randomly shuffled time series. If an eigenvalue for the original time series is larger than the largest eigenvalue calculated for the randomly shuffled time series, the eigenvalue or eigen mode is regarded as statistically significant. The largest four eigenvalues of each period are shown in the upper panel of Fig. 7. The lower panel depicts temporal change of the number of significant modes. Note that only a single eigen mode is significant for the periods that contain the sub prime mortgage crisis of 2008. This means that production for all industry sectors in G7 countries behaves similarly during economic crisis. The economic risk propagate instantaneously to All industry sectors in G7 countries and all industries decrease their production simultaneously.

E. *Community Structure of Production Network*

We identified the community structure for each time slice of the global production network constructed using the methodology described in section c. Figure 8 shows examples of community structures obtained for (a) 2004, (b) 2007, (c) 2010, and (d) 2013. Average value of modularity Q_s is 0.302 during 2001 and 2013. Maximum and minimum of modularity are 0.410 and 0.153, respectively. The means that the community structure is clearly identified for the global production network. The number of major communities varies between two and four. There were two major communities for the global economic crisis during 2007 and 2010. The observed community structure is consistent to the statistically significant modes obtained above. The number of major community in the period of normal economy (2004 and 2013) is larger than the period of economic crisis (2007 and 2010). The small number of statistically significant mode in economic crisis is observed as the small number of major communities in the global production network. This is interpreted again such that production for all industry sectors in G7 countries behaves similarly during economic crisis. The economic risk propagate instantaneously to All industry sectors in G7 countries and all industries look for new demand. Consequently new links (trade relations) are spanned beyond communities observed in the period of normal economy.

F. *Linked Communities of Production Network*

The temporal evolution of communities is characterized by the link relations for communities in adjoining years. The similarity of communities c_i and c_j in adjoining years was measured by the use of the Jaccard index $J(c_i, c_j)$. Figure 9 shows the Jaccard indices in adjoining years: (a) between 2003 and 2004, (b) between 2006 and 2007, (c) between 2009 and 2010, and (d) between 2012 and 2013. Communities were arranged in decreasing order of the number of industry sectors in each community. For instance community c_1 is the largest, and c_2 is the second largest.

We define link relation for the pair of communities with the largest Jaccard index between adjoining year. Most of communities show clearly linked relationships with communities of the adjoining year. For instance, Fig. 9 (a) shows that communities c_1, c_2, c_3, and c_4 in 2003 are linked to communities c_2, c_1, c_1, and c_3 in 2004, respectively. Similarly, Fig. 9 (c) shows that communities c_1 and c_2 in 2009 are linked to communities c_1 and c_2 in 2010, respectively. The obtained linked communities are summarized in Table 4, LinkedComm2, and LinkedComm3 for three periods: before the crisis, during the crisis, after

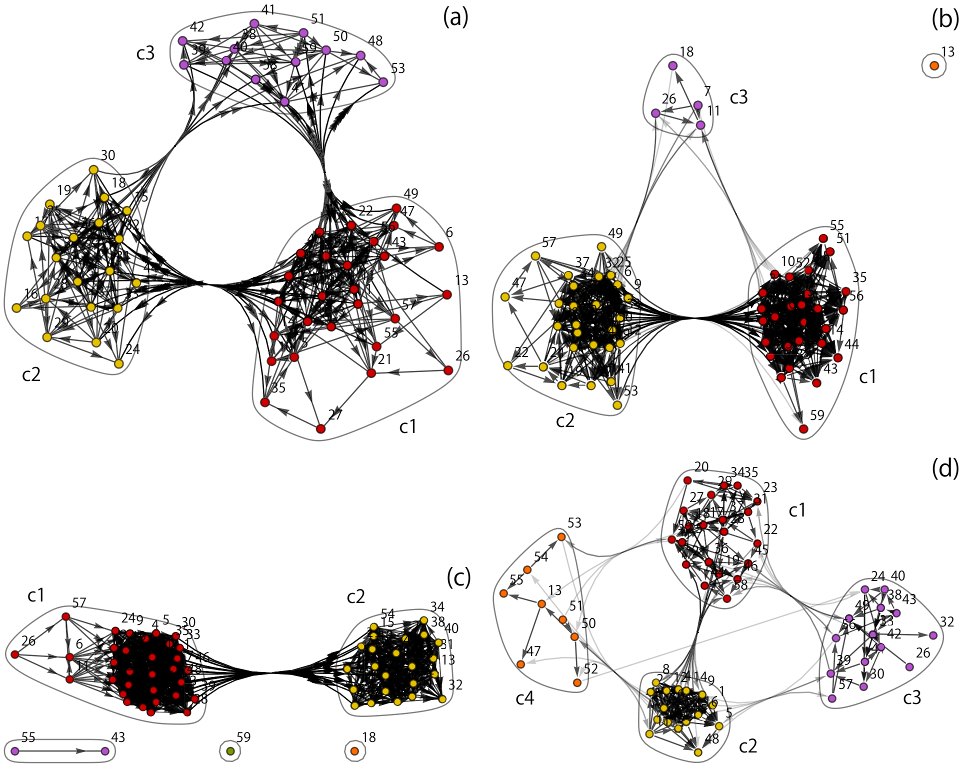

Figure 8: Community structure: (a) 2004, (b) 2007, (c) 2010, and (d) 2013

Table 4: Linked communities before the crisis

Year	2001	2002	2003	2004	2005	2006
Linked comm 1	c_1	c_2	$c_2 + c_3$	c_1	c_2	c_3
Linked comm 2	c_2	c_1	c_1	c_2	c_1	c_2
Linked comm 3	$c_3 + c_4$	c_3	c_4	c_3	c_3	c_1

Table 5: Linked communities during the crisis

Year	2008	2009	2010
Linked comm 4	c_2	c_1	c_1
Linked comm 5	c_1	c_2	c_2

Table 6: Linked communities after the crisis

Year	2012	2013
Linked comm 6	c_1	c_3
Linked comm 7	c_2	c_1
Linked comm 8	c_3	c_2
Linked comm 9	c_4	c_4

the crisis, respectively. The temporal change of communities structurer, i.e. community dynamics is regarded as an example of collective motion.

We obtained three linked communities before the crisis. Linked communities 1 to 3 are characterized by countries and corresponds to Europe, US, Canada, respectively. Japan distributed to three communities.

Then two Linked communities (linked communities 4 and 5) were obtained for the period of the crisis. Linked communities 4 and 5 are characterized by sectors. For instance, linked community 4 is composed by sectors: *Steel products*, *Transportation equipments*, *Chemical products*, *Pulp and paper products*, *Computer and electronic product*, and others. Linked community 5 is composed by sectors: *Fabricated metal products*, *Precision machinery*, *Textile products*, and others.

We obtained four linked communities after the crisis. Linked communities 6 to 9 are characterized by countries as before. For instance, linked communities 6 to 9 corresponds to Canada, US, Japan, and Europe. Some European countries distributes to linked communities 6 and 7.

G. *Communities in Multiplex Production Network*

The dynamics of community structure is observed in the temporal change of community structure for the 13 layer multiplex network. We used parameter $r_s = 1$, inter slice coupling $C_{isr} = 0.8$. Color of each node corresponds to

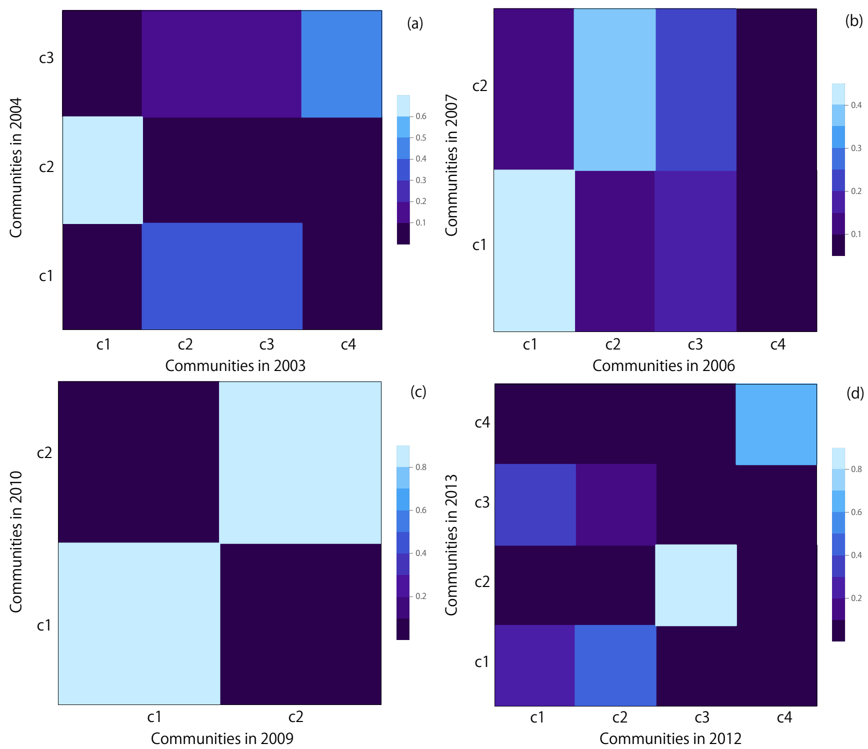

Figure 9: Jaccard index: (a) between 2003 and 2004, (b) between 2006 and 2007, (c) between 2009 and 2010, and (d) between 2012 and 2013

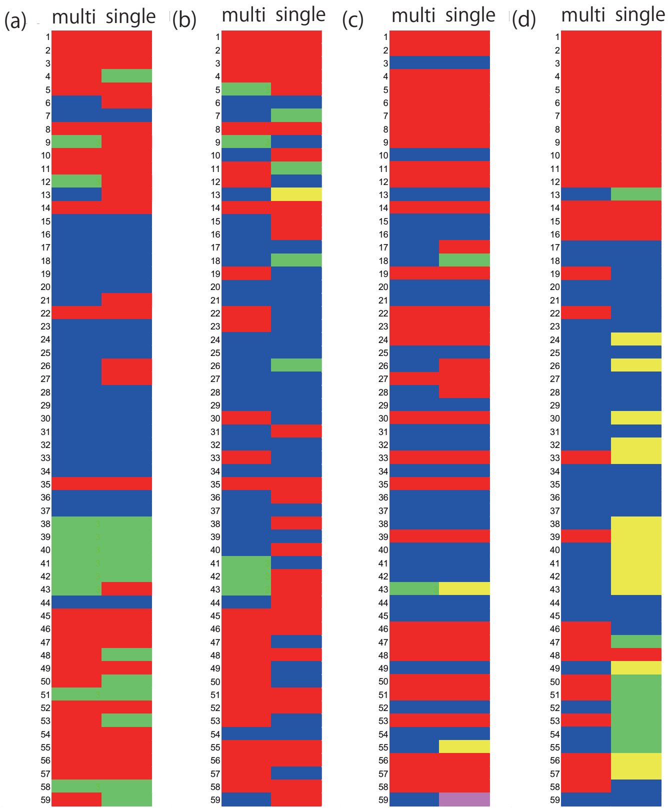

Figure 10: Comparison of communities between multiplex network and single layer network: (a) 2004, (b) 2007, (c) 2010, and (d) 2013

identified community and each number indicates layer from 2007 to 2013.

We compare community structures between multiplex network and single layer network for 2004, 2007, 2010, and 2013. Figure 10 depicts result of the comparison. The left and right column of each panel correspond to communities identified for multiplex network and single layer network, respectively. Although direction of links are ignored in community analysis for multiplex network, agreement of the two analysis is reasonably good. Note that the number of communities obtained for single layer network is larger than that obtained for multiplex network.

H. *Controllability of Production Network*

The number of driver nodes were identified by the use of Eqs. (26) to (28). Matchedness distribution $P_>(m)$ is shown in Fig. 11 for 2004, 2007, 2010, and

2013. It is clearly observed in panels (b) and (c) in Fig. 11 that many nodes have small value of matchedness m during economic crisis. Temporal change of the number of driver nodes is shown in Fig. 12. Note that the number of driver nodes increased during economic crisis from 2008 to 2010. During economic crisis the share of driver nodes n_D becomes about 80% of all the nodes, whereas n_D is about 60% during normal period. This means that we cannot expect to control global real economy by stimulating a relatively small number of nodes.

Partial driver nodes with matchedness less than or equal to 0.2 ($m_j \leq 0.2$) are collected for 2004, 2007, 2010, and 2013 and shown in Tables **??** to **??**, respectively. During normal period we have 16 nodes (27.1%) and 15 nodes (25.4%) for 2004 and 2013, respectively. On the other hand, during economic crisis we have 36 nodes (61.0%) and 47 nodes (79.7%) for 2007 and 2010, respectively.

If we look at the distribution of country, we notice the following: In 2004 during normal period, we have 0 node (0.0%), 1 node (25.0%), 2 nodes (50.0%), 3 nodes (75.0%), 2 nodes (50.0%), 5 nodes (18.8%), and 3 nodes (14.3%) for Canada, Germany, France, Great Britain, Italy, Japan, and USA, respectively. In 2013 during normal period, we have 0 node (0.0%), 1 node (25.0%), 1 node (25.0%), 1 node (25.0%), 0 node (0.0%), 4 nodes (6.3%), and 8 nodes (38.1%) for Canada, Germany, France, Great Britain, Italy, Japan, and USA, respectively. In 2007 during economic crisis, we have 3 nodes (50.0%), 3 nodes (75.0%), 2 nodes (50.0%), 2 nodes (50.0%), 1 node (25.0%), 12 nodes (12.5%), and 13 nodes (61.9%) for Canada, Germany, France, Great Britain, Italy, Japan, and USA, respectively. In 2010 during economic crisis, we have 5 nodes (83.3%), 4 nodes (100.0%), 4 nodes (100.0%), 2 nodes (50.0%), 1 node (25.0%), 14 nodes (12.5%), and 17 nodes (81.0%) for Canada, Germany, France, Great Britain, Italy, Japan, and USA, respectively. Therefore it is hard to find a country dominates driver nodes for both of normal period and economic crisis.

If we look at the distribution of sector, we notice the following: In 2004 during normal period, we have 4 nodes (21.1%) and 12driver nodes (30.0%)for durable/nondurable comsumer goods sectors and capital/intermediate goods, respectively. In 2013 during normal period, we have 4 driver nodes (21.1%) and 11 driver nodes (27.5%) for durable/nondurable comsumer goods and capital/intermediate goods, respectively. In 2007 during economic crisis, we have 6 driver nodes (31.6%) and 30 driver nodes (75.0%) for durable/nondurable comsumer goods and capital/intermediate goods, respectively. In 2010 during economic crisis, we have 12 driver nodes (63.2%) and 35 driver nodes (87.5%) for durable/nondurable comsumer goods and capital/intermediate goods, respectively. Here we have 19 sectors classified under the category as durable and non-durable consumer goods: JP10, JP11, JP14, US01, US02, US05, US18,

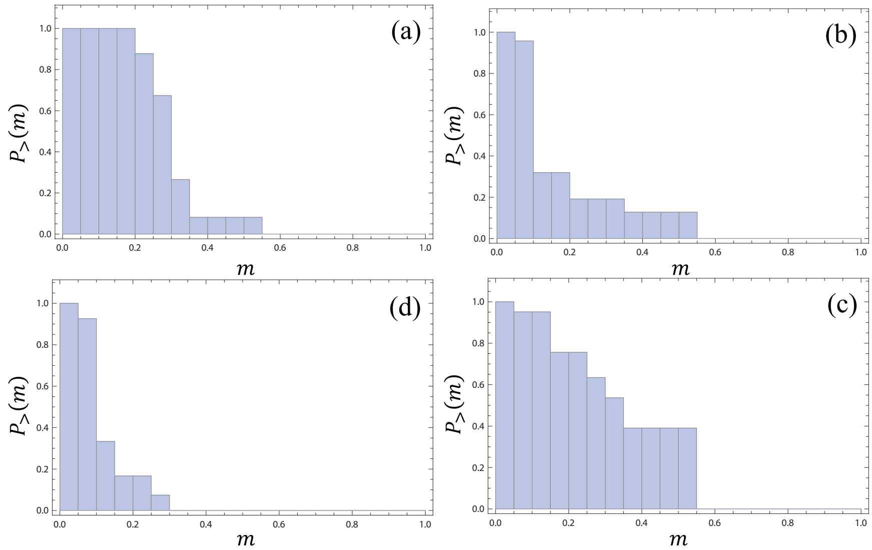

Figure 11: Matchedness Distribution: (a) 2004, (b) 2007, (c) 2010, and (d) 2013

Table 7: Driver Nodes with $m \leq 0.2$ in 2004

node	description	matchedness	comm
FR02	*Durable consumer goods*	0.166	c_1
US06	*Leather and allied product*	0.166	c_1
JP03	*Fabricated metal products*	0.166	c_1
GB02	*Durable consumer goods*	0.2	c_1
GB01	*Capital goods*	0.2	c_1
IT01	*Capital goods*	0.2	c_1
US10	*Petroleum and coal products*	0.2	c_1
JP12	*Miscellaneous*	0.2	c_1
JP10	*Textile products*	0.2	c_1
JP08	*Plastic products*	0.2	c_1
DE01	*Capital goods*	0.2	c_2
US16	*Machinery*	0.2	c_2
GB03	*Intermediate goods*	0.166	c_3
FR03	*Intermediate goods*	0.166	c_3
JP04	*Transportation equipments*	0.166	c_3
IT02	*Durable consumer goods*	0.2	c_3

US21, CA01, CA02, CA04, DE02, DE04, FR02, FR04, IT02, IT04, GB02, and GB04. The rest 40 sectors are classified under the category as capital and intermediate goods. Therefore we can say that capital/intermediate goods sectors are dominant over durable/nondurable consumer goods sectors for both of normal period and economic crisis.

Degree Distributions are shown in Fig. 13 for 2004, 2007, 2010, and 2013. The distributions have longer tail for the period of economic crisis as shown in Fig. 13 (b) and (c). Increase of the number of driver nodes during economic

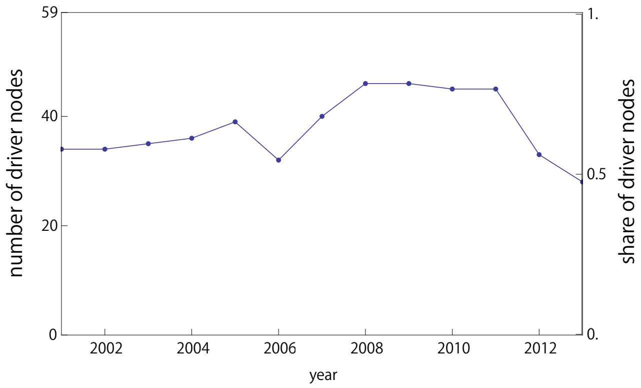

Figure 12: Temporal Change of the Number of Driver Nodes

Table 8: Driver Nodes with $m \leq 0.2$ in 2007

node	description	matchedness	comm
GB03	*Intermediate goods*	0.0625	c_1
DE03	*Intermediate goods*	0.0625	c_1
DE02	*Durable consumer goods*	0.0625	c_1
US15	*Fabricated metal product*	0.0625	c_1
JP16	*Precision machinery*	0.0625	c_1
JP15	*General machinery*	0.0625	c_1
GB01	*Capital goods*	0.0714	c_1
IT01	*Capital goods*	0.0714	c_1
FR01	*Capital goods*	0.0714	c_1
DE01	*Capital goods*	0.0714	c_1
CA03	*Industrial production*	0.0714	c_1
CA01	*Goods-producing industries*	0.0714	c_1
US19	*Transportation equipment*	0.0714	c_1
JP14	*Electric appliances*	0.0714	c_1
JP08	*Plastic products*	0.0714	c_1
JP05	*Ceramic,stone and clay products*	0.0714	c_1
JP04	*Transportation equipments*	0.0714	c_1
JP03	*Fabricated metal products*	0.0714	c_1
JP02	*Nonferrous metal products*	0.0714	c_1
JP01	*Steel products*	0.0714	c_1
FR03	*Intermediate goods*	0.0625	c_2
CA04	*Non-durable manufacturing industries*	0.0625	c_2
US22	*Miscellaneous*	0.0625	c_2
US17	*Computer and electronic product*	0.0625	c_2
US13	*Nonmetallic mineral product*	0.0625	c_2
US11	*Chemical*	0.0625	c_2
US04	*Textile product mills*	0.0625	c_2
US01	*Food*	0.0625	c_2
JP12	*Miscellaneous*	0.0625	c_2
JP09	*Pulp and paper products*	0.0625	c_2
US18	*Electrical equipment,appliance,and component*	0.166	c_2
US16	*Machinery*	0.166	c_2
US12	*Plastics and rubber products*	0.166	c_2
US09	*Printing and related support activities*	0.166	c_2
US07	*Wood product*	0.166	c_2
JP06	*Chemical products*	0.166	c_2

Table 9: Driver Nodes with $m \leq 0.2$ in 2010

node	description	matchedness	comm
US10	*Petroleum and coal products*	0	c_1
GB01	*Capital goods*	0.0588	c_1
FR04	*Non-durable consumer goods*	0.0588	c_1
FR03	*Intermediate goods*	0.0588	c_1
FR01	*Capital goods*	0.0588	c_1
DE04	*Non-durable consumer goods*	0.0588	c_1
DE03	*Intermediate goods*	0.0588	c_1
CA02	*Service-producing industries*	0.0588	c_1
US19	*Transportation equipment*	0.0588	c_1
US17	*Computer and electronic product*	0.0588	c_1
US14	*Primary metal*	0.0588	c_1
US12	*Plastics and rubber products*	0.0588	c_1
US07	*Wood product*	0.0588	c_1
JP12	*Miscellaneous*	0.0588	c_1
JP05	*Ceramic,stone and clay products*	0.0588	c_1
JP01	*Steel products*	0.0588	c_1
GB03	*Intermediate goods*	0.111	c_1
US11	*Chemical*	0.111	c_1
US03	*Textile mills*	0.111	c_1
JP14	*Electric appliances*	0.111	c_1
JP09	*Pulp and paper products*	0.111	c_1
JP08	*Plastic products*	0.111	c_1
JP07	*Petroleum and coal products*	0.111	c_1
JP04	*Transportation equipments*	0.111	c_1
JP02	*Nonferrous metal products*	0.111	c_1
US08	*Paper*	0.2	c_1
US06	*Leather and allied product*	0.2	c_1
US01	*Food*	0.2	c_1
JP11	*Food and tobacco*	0.2	c_1
JP06	*Chemical products*	0.2	c_1
DE02	*Durable consumer goods*	0.0588	c_2
JP16	*Precision machinery*	0.0588	c_2
IT01	*Capital goods*	0.0666	c_2
FR02	*Durable consumer goods*	0.0666	c_2
DE01	*Capital goods*	0.0666	c_2
CA05	*Durable manufacturing industries*	0.0666	c_2
CA04	*Non-durable manufacturing industries*	0.0666	c_2
CA03	*Industrial production*	0.0666	c_2
CA01	*Goods-producing industries*	0.0666	c_2
US22	*Miscellaneous*	0.0666	c_2
US21	*Furniture and related product*	0.0666	c_2
US18	*Electrical equipment,appliance,and component*	0.0666	c_2
US15	*Fabricated metal product*	0.0666	c_2
US09	*Printing and related support activities*	0.0666	c_2
US04	*Textile product mills*	0.0666	c_2
JP13	*Mining*	0.0666	c_2
JP10	*Textile products*	0.0666	c_2

crisis from 2008 to 2010 is discussed from the view point of heterogeneity in terms of degree distribution. The average number of degree $\langle k \rangle$ are 13.7, 25.1, 25.0, and 9.42 for 2004, 2007, 2010, and 2013, respectively.

The maximum degree $k_{\max}$ are 26, 41, 37, and 19 for 2004, 2007, 2010, and 2013, respectively. If we assume the power-law degree distribution $P_>(k) = k^{-\gamma'}$for entire region from 1 to $k_{\max}$, power exponent γ' are estimated to be 1.25, 1.09, 1.129, and 1.38, for 2004, 2007, 2010, and 2013, respectively. Here $P_>$ is the cumulative probability. Thus we have power exponent γ for probability

Table 10: Driver Nodes with $m \leq 0.2$ in 2013

node	description	matchedness	comm
US19	*Transportation equipment*	0.125	c_1
US18	*Electrical equipment,appliance,and component*	0.125	c_1
US15	*Fabricated metal product*	0.125	c_1
US13	*Nonmetallic mineral product*	0.125	c_1
US12	*Plastics and rubber products*	0.125	c_1
US11	*Chemical*	0.125	c_1
US02	*Beverage and tobacco product*	0.125	c_1
GB04	*Non-durable consumer goods*	0.2	c_1
DE01	*Capital goods*	0.2	c_1
JP08	*Plastic products*	0.125	c_2
JP07	*Petroleum and coal products*	0.2	c_2
JP05	*Ceramic,stone and clay products*	0.2	c_2
JP03	*Fabricated metal products*	0.2	c_2
US10	*Petroleum and coal products*	0	c_3
FR04	*Non-durable consumer goods*	0	c_4

density $p(k) = k^{-\gamma}$ are 2.25, 2.09, 2.129, and 2.38, for 2004, 2007, 2010, and 2013, respectively. Therefore we have $\langle k \rangle \approx 12$ and $\gamma \approx 2.3$ during normal period. On the other hand, we have $\langle k \rangle \approx 20$ and $\gamma \approx 2.1$ during economic crisis. With these value of $\langle k \rangle$ and γ, an analytical formulae [34] gives $n_D \approx 0.25$ and $n_D \approx 0.40$ for normal period and economic crisis, respectively. This means that increase of the number of driver nodes during economic crisis is explained qualitatively by the heterogeneity in terms of degree distribution.

V. Conclusion

The interdependent relationship of the global economy has become stronger due to the increase of international trade and investment. Most of national economies are linked by international trade and consequently economic globalization forms a giant economic complex network with strong links, i.e. interactions due to increasing trade. In Japan many small and medium enterprises would achieve higher economic growth by free trade based on the establishment of EPA. Thus, it is expected that various interesting collective motions will emerge in global economy under trade liberalization.

We analyzed the industry sector specific international trade data to clarify the structure and dynamics of communities that consist of industry sectors in various countries linked by international trade. We applies conventional community analysis to each time slice of the international trade network data, the World Input-Output Database. This database contains the industry sector specific international trade data on 41 countries and 35 industry sectors from 1995 to 2011. Once the community structure was obtained for each year, the links between communities in adjoining years was identified by using the Jaccard index as a similarity measure between communities in adjoining years.

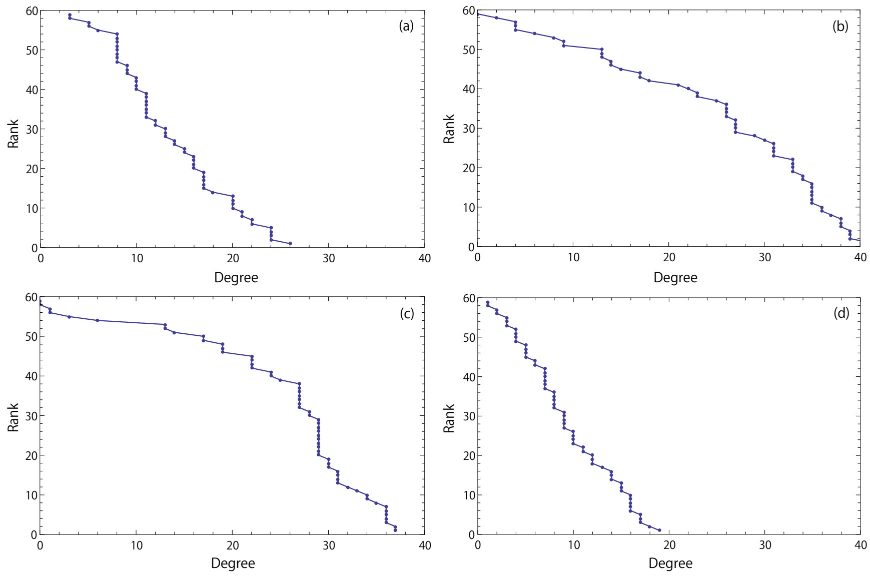

Figure 13: Degree Distribution: (a) 2004, (b) 2007, (c) 2010, and (d) 2013

The identified linked communities show that six backbone structures exist in the international trade network. The largest linked community is *Financial Intermediation* sector and *Renting of Machines and Equipments* sector in the US and the UK. The second is *Mining and Quarrying* sector in *Rest of World*, Russia, Canada, and Australia. The third is Basic Metals and Fabricated Metal in *Rest of World*, Germany, Japan, and the US. These community structure means that international trade is actively transacted among same or similar industry sectors. Furthermore, the robustness of the observed community structure was confirmed by quantifying the variation of information for perturbed network structure.

The theoretical study we conducted using a coupled limit-cycle oscillator model suggests that the interaction terms due to international trade can be viewed as the origin of the synchronization. We looked at international business cycle as the most important aspect of the collective motion in Economy. We used the Hilbert transform to evaluate the phase time series of the growth rate of value added for 1435 nodes and then estimated the complex order parameters for communities. The respective amplitude for the order parameter of each community was observed to be greater than the amplitude for all sectors. This means active trade produces higher phase coherence within each community. The temporal change in amplitude for the order parameter was studied for the years 1996 to 2011. Phase coherence decreases gradually in the late 90 ’s but increased sharply in 2001 and 2002. From 2002, the amplitudes for the order parameter remained high except for the years 2005 and 2009. The decrease in 2009 was caused by financial crisis resulting from the housing bubble crash in the US but the cause of the decrease in 2005 is unclear. Order parameters were estimated for the growth rate of the value added time series shuffled randomly with keeping rotationally and order parameter averaged over 1000 shuffled time series was plotted by black curve. Therefore the synchronization observed for each linked communities was statistically significant.

Then, we characterized features of collective motion during economic crisis in 2008 by studying G7 Global Production Network constructed using production index time series from January 1998 to January 2015 for G7 countries. G7 Global Production Network was analyzed by the use of complex Hilbert principal component analysis, community analysis for single layer network and multiplex networks, and structural controllability. First, complex Hilbert principal component analysis showed that only a single eigen mode was significant for the global economic crisis during 2007 and 2010. The community structure was clearly identified for the global production network with sufficiently large values of modularity. There were 2 major communities for the global economic crisis during 2007 and 2010, where as there were 4 major communities for normal economic state. Furthermore, community analysis for multiplex networks showed that agreement of the two analysis was reasonably good, although direction of links were ignored in community analysis for multiplex network. The number of communities obtained for single layer network was larger than that obtained for multiplex network.

Finally, study of structural controllability showed that the number of driver nodes increased during economic crisis from 2008 to 2010. During economic crisis the share of driver nodes n_D became about 80% of all the nodes, whereas n_D was about 60% during normal period. The observed increase in the number of driver nodes during economic crisis was explained qualitatively by the heterogeneity in terms of degree distribution. This means that we cannot expect to control global real economy by stimulating a relatively small number of nodes and furthermore it becomes more difficult to introduce some measure to control the state of global economy during the time of economic crisis than during the period of normal economy.

In conclusion, we observed various kinds of collective motions in global economy under trade liberalization, as we expected. Although many Japanese small and medium enterprises would achieve higher economic growth by free trade, we also need to pay attention to the fact that once negative economic shock occurred in a regional economy, it will propagate to the rest of the world instantaneously and we have no strong measure to control the economic crisis.

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