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Evolution of Standards and Innovation

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Abstract

We examine how a standard evolves when both a standard consortium or firm (incumbent) and an outside firm (potential entrant) innovate to improve the technology. The incumbent improves to deter entry, and the entrant can invest to counter the incumbent's attempt. We show that only when the technology is mature and inertia is sufficiently low will there be entry leading to the coexistence of both standards. When the technology is in its infancy, the incumbent deters entry by technology improvement (upgrade) for any level of inertia. The entrant is never able to drive the incumbent out of the market (replacement). Our results suggest that competition policy to control inertia is not a substitute for policies to promote technological innovation, and that coordination of the two policies is essential.

Keywords: Standards, Innovation, Technology, Upgrades, Entry deterrence, Standardization, Replacement effect

JEL classification: D43; K39; L15

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1 Introduction

We consider a situation in which there is an established standard and the owner (firm or consortium) of the standard technology may invest to improve the technology in an attempt to deter entry. The potential entrant is able to invest in technology for an alternative standard to counter the deterrence attempt or even drive the incumbent out of the market. Thus we have a framework where the standard can involve through upgrading (but no entry), entry and coexistence or entry and replacement, depending on investment choices.

Typically in markets with network effects such as standards, entry deterrence is achieved by increasing inertia (Farrel and Saloner, 1987) in the form of increasing switching cost (Klemperer, 1987a, 1987b). We focus on an alternative entry deterrence strategy, i.e., investing in technology improvement. Similarly, previous examinations of counter entry deterrence have been directly lowering switching cost, such as the entrant paying consumer's to switch (Chen, 1997). In our framework, counter deterrence is achieved by improving technology.

We develop a two-stage game, in the first stage of which the incumbent invests in upgrading the technology, and the entrant invests to improve its potential standard technology. These investments determine the qualities of the respective products. In the second stage, firms simultaneously choose prices, i.e., they engage in Bertrand competition. We model switching cost in a reduced form to capture the cost of forgoing the network effect and inertia such as installed bases, associated transaction costs (Chen, 1989) and includes both direct and indirect network effects of a standard (Matues and Regibeau, 1988, Clements, 2004). We do so in order to separate direct and indirect switching costs. Technology improvement by the incumbent *indirectly* increases switching cost since better technology and thus higher willingness to pay for the incumbent's standard increases the opportunity cost of switching. Similarly, improvement of entrant's technology indirectly lowers switching cost by reducing the difference of willingness to pay for incumbent and entrant's standards. We separate the traditional direct switching costs associate with standards from the indirect switching costs generated by technology improvement. In our framework, direct switching cost is an exogenous parameter.[1]

We adopt the approach used by Laffont, Rey, and Tirole (1998) to modeling differentiated products with elastic demand in the presence of heterogeneous consumers and applied to markets with switching cost by Aoki and Small (1999). Thus, our model is particularly applicable to a market such as the smartphone market, in which there are competing platforms, with each vendor being identified with a platform. Because consumers pay a fixed cost and per-unit fee, there is a cost of switching to a different provider. Incumbents and entrants also represent patent pools or a standard consortium, and consumers can be interpreted as manufactures that pay licensing royalties. Another applicable market is that for game consoles, considering the indirect payments that consumers make to the console manufacturer through games. Part of the price paid for a game goes to the console manufacturer in licensing fees.[2] The market analysis of stage two is a special case

[1] Case when switching cost is incumbent's strategy variable is analyzed in the Appendix.

[2] Both console and software are produced by a single firm, or at least production is coordinated.

of models of nonlinear price competition (Calem and Spulber, 1984; Oren, Smith, and Wilson, 1983) in the absence of switching costs. However, we provide a more complete characterization of price determination and welfare implications.

Bertrand competition in the second stage results in one of four outcomes according to the configuration of technology and the switching costs chosen in stage one: I only firm 0; II only firm 1; III coexistence (unique equilibrium); and IV coexistence (multiple equilibria). "Only firm 0" in regime I means that the incumbent deters entry through upgrading. "Only firm 1" in regime II means that the entrant's quality is so good that it drives the incumbent out of the market and the existing standard is replaced.

We characterize the subgame perfect Nash equilibrium (SPNE) of the whole game. Only regimes I and III are SPNE outcomes. Regime I occurs when technology improvement is not costly, for *any level* of inertia. In this case, the incumbent invests in technology improvement to successfully deter entry and the existing standard is upgraded. If technology improvement is costly *and* inertia low, incumbent and entrant quality are sufficiently similar for both firms to coexist in the market (Regime III). Regime II never occurs in equilibrium, i.e., entrant never replaces the incumbent. This is because by investing slightly more in stage one, firm 0 avoids being priced out of the market. In this case, the overall payoff is negative because profit is zero but investment is sunk.

In other words, given decreasing returns to technology investment, innovation costs are low when technology is in its infancy. In this case, incumbents can deter

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We do not model a two-sided market.

entry by upgrading even if inertia is low. When the technology is mature so that innovation costs are high, inertia must be also be sufficiently low for standards coexist. Consumers benefit in two ways: competition in the market and better technology. But in order to guarantee entry, both technological and market (low inertia) must be satisfied, implying need for competition policy at all stages of technology evolution.

Farrell and Saloner (1987) examined a situation in which firms can either adopt a technologically superior standard or rely on inertia. They showed that firms choose not to improve the standard when there is incomplete information. In their framework, technological superiority of the standard is exogenous to firms, and the choice of standard is a coordination problem. We endogenize the level of technology, with the possibility of two standards of different technology levels co-existing. Lack of coordination would be captured in our framework by the switching cost that consumers that switch to the new standard must incur (shown in the Appendix).

Cabral and Salant (2010) also consider firms that invest in improving the quality of a standard. They examine how moving from the coexistence of two standards to a unified standard affects the incentive to improve the standard, when the extent of improvement and thus increase in profits is predetermined. We allow each firm to choose their level of improvement which makes co-existence an option. They ignore the market interactions induced by quality improvement and assume that a single standard unambiguously increases the profits of both firms because of the network effect. In the context of our framework, one can interpret a move from coexistence (or incompatibility) to a single standard (or compatibility) as an infinite reduction in switching costs. In their framework, technology improvement is a predetermined single step, whereas in ours, the degree of technology improvement is chosen. Thus, according to Cabral and Salant (2010), firms choose to reduce switching costs either before or after investing in technology. The choice is not "which" but "which first". We focus on the "which" strategy by explicitly modeling consumer behavior.

In the next section, we briefly describe the product market and characterize the Bertrand equilibrium, given the technology. We characterize the choice of equilibrium technology in section 3, hence characterizing the SPNE. We examine the implications for the consumer and social surpluses in Section 3.1. We discuss policy implications in Section 5. All proofs are given in the Appendix. We also present outline of your analysis when switching cost in addition to technology investment is a strategic decision in the Appendix.

2 Framework

We develop a two-stage game played by two firms, firms 0 and 1. Firm 0 "owns" the current standard in the sense that it has a stake in, and controls, this standard. Firm 1 can enter the market if its technology and standard are sufficiently good. In stage one, both firms sequentially invest in the technology that determines the level of the standard. In stage two, firms engage in Bertrand price competition, given the technology investments made in stage one. Initially, firm 0 is the only firm in the market. Hence, firm 0 and firm 1 can be characterized as incumbent and entrant, respectively. We determine the SPNE strategies, technology investment choices, and prices.

To represent the product market, we use a Hotelling model in which consumers are distributed uniformly over the interval [0,1]. Firm 0 is at point 0, and firm 1 is at 1. Each consumer purchases at most one unit of the good from one of the firms. When a consumer at $x \in [0,1]$ purchases from firm 0 at price p_0, his or her surplus is $v_0 - p_0 - tx$, where t is the per unit transportation cost. To purchase from firm 1, because the consumer must switch to a new standard, he or she incurs a switching cost S. The consumer's surplus is $v_1 - p_1 - S - t(1-x)$.

The intrinsic value of the products v_i are determined by the technology investments made in stage one. The established standard generates a technology level of $\overline{v}$; we assume that

$$v_i \geq \overline{v} \geq 2t. \tag{M}$$

Any positive investment in stage one by firm i implies that $v_i > \overline{v}$. The second inequality implies that a monopolist selling to all consumers charges a price of $v_i - t$. Because firm 0 is such a monopolist, all consumers who buy from firm 1 incur a switching cost of $S \geq 0$.

2.1 Bertrand Competition Equilibrium

The demand curve derived in the Appendix gives firm 0's profit as a function of (p_0, v_0) and (p_1, v_1). Standard analysis of the Hoteling model (outlined in the

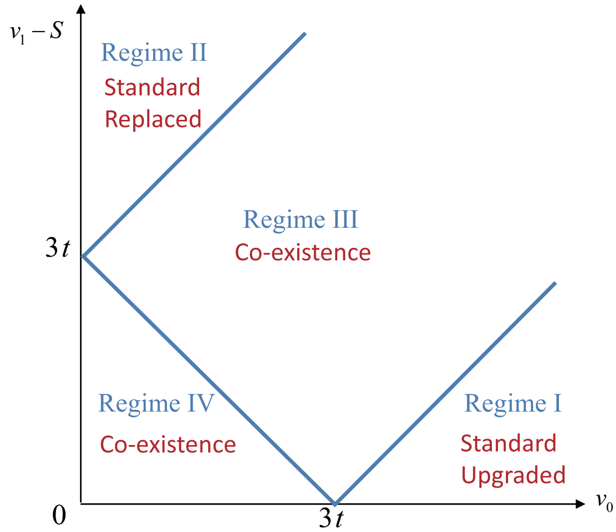

Figure 1: Stage Two (Bertrand Competition) Equilibrium

Appendix) yields the following Proposition characterizing Bertrand competition, which is illustrated in Figure 1.

Proposition 1. *Bertrand price competition results in one of four regimes.*

Regime I: If $v_1 - S \leq v_0 + 3t$, all consumers purchase from firm 0. The equilibrium prices are

$$p_0^* = v_0 - v_1 + S - t,\ p_1^* = 0.$$

Regime II: If $v_1 - S \geq v_0 - 3t$, all consumers purchase from firm 1. The equilibrium prices are

$$p_0^* = 0,\ p_1^* = v_1 - v_0 - S - t.$$

Regime III: If $v_0 + v_1 - S \geq 3t$ and $v_0 - 3t < v_1 - S < v_0 + 3t$, two firms coexist *in the market (unique equilibrium). The equilibrium prices are*

$$p_0^* = \frac{v_0 - v_1 + S + 3t}{3}, \; p_1^* = \frac{v_1 - v_0 - S + 3t}{3}. \tag{1}$$

Regime IV: If $v_0 + v_1 - S < 3t$, two firms coexist in the market (multiple equilibria). Then there is a continuum of equilibria. The equilibrium prices, indexed by $\alpha \in [0, 1]$, are

$$p_0^* = \frac{(3-\alpha)v_0 - (1-\alpha)(3t - v_1 + S)}{3} \tag{2}$$

$$p_1^* = \frac{(2+\alpha)(v_1 - S) - \alpha(3t - v_0)}{3}. \tag{3}$$

Regime I emerges when v_0 is large relative to $v_1 - S$. This occurs when the entrant is significantly less efficient than the incumbent or when the switching cost is large, or both. Entry does not result in any consumers switching to the new supplier in this regime. However, the presence of the entrant gives consumers a higher surplus. In particular, the surplus of the consumer at $x = 1$ increases from 0, under the incumbent monopolist, to p_1^* after entry. The marginal consumer is indifferent between switching and not switching.

Regime II occurs when v_0 is small relative to $v_1 - S$. In this case, the entrant is highly efficient and the switching cost is sufficiently low for all consumers to switch. The consumer at $x = 0$ has a positive surplus of p_0^*.

Under regimes III and IV, both firms make positive sales. Firms split the market equally when $v_0 = v_1 - S$, which is a subregime of regime III. However, because of the switching cost, the entrant must be more efficient in order to have the same market share. A more detailed explanation of the two co-existence are in the Appendix. If, in addition to assumption (M), we also assume that the entrant is sufficiently efficient, i.e., $v_1 - S \geq 2t$, then regime IV never occurs and the equilibrium is unique.

3 Equilibrium Investment

In this section, we examine the equilibrium investment choices in technology improvement. Both firms can invest in its own technology in order to increases v_i. We denote the technology improvement as $\Delta_i (i = 0, 1)$. Thus, given an existing quality level of $\overline{v}$, investment raises the quality level to

$$v_i = \overline{v} + \Delta_i$$

To simplify the analysis, we assume $\overline{v} \geq 3t$, which is stronger than assumption (M). Cost of investment is

$$C_i(\Delta_i) = \frac{\delta \Delta_i^2}{2}$$

where δ is the investment efficiency parameter. The expected payoffs are

$$\Pi_i(\Delta_i, \Delta_j) = \pi_i^*(\overline{v} + \Delta_i, \overline{v} + \Delta_j) - C_i(\Delta_i).$$

$\pi_i(\cdot)$ is equilibrium profits in the Bertrand price competition stage that is defined by Proof of Proposition 1. If there is no investment ($\Delta_0 = \Delta_1 = 0$), qualities

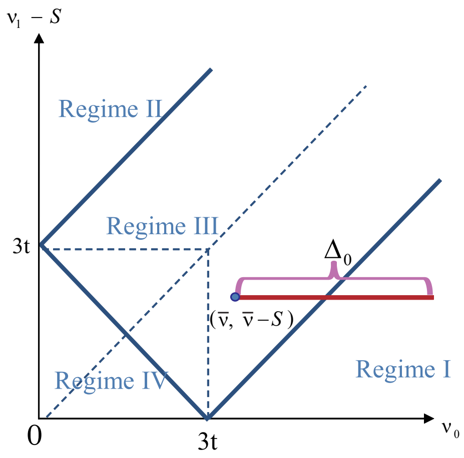

Figure 2: subgame ($\Delta_0 + S > 3t$)

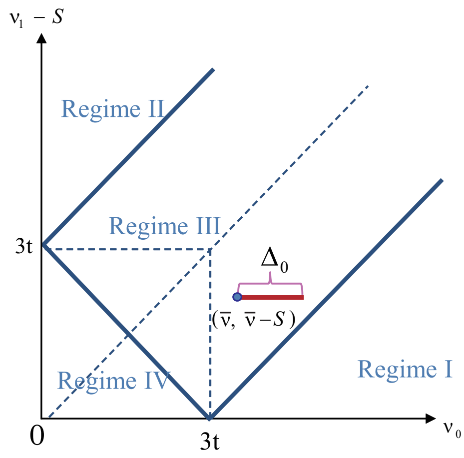

Figure 3: subgame ($\Delta_0 + S \leq 3t$)

$v_0 = v_1 = \overline{v}$ and regime III prevail. Firm 0 chooses Δ_0 to maximize profit. Once firm 0 has made its investment choice, two subgames are possible: $\Delta_0 + S > 3t$ (Figure 2) and $\Delta_0 + S \leq 3t$ (Figure 3).

In the regime I subgame, depending on firm 1's investment choice, either regime I, regime II, or regime III prevails. The next lemma shows the final outcome under regime I .

Lemma 1. *When $\Delta_0 + S > 3t$, firm 1 invests nothing and its payoff is zero. Then, the final outcome is regime I.*

The next lemma shows that in the regime III subgame, either regime II or III prevails.

Lemma 2. *When $\Delta_0 + S \leq 3t$, firm 1's investment decision depends on investment efficiency and switching cost.*

(1) if $\delta > 1/3t$ and $9t(3t\delta - 1)/(9t\delta - 1) > S$, then firm 1's optimal investment results in regime III.

(2) Otherwise, firm 1 invests so that the final outcome is regime II.

If the final outcome is regime II, firm 0's payoff will be negative because it makes no profit. From the two lemmas, we obtain the next proposition.

Proposition 2. *SPNE outcomes of this game are as follows*

(1) If $\delta \leq 1/3t$ or $9t(3t\delta - 1)/(9t\delta - 1) < S$, firm 0 increases his quality of products enough to kick out firm 1. Firm 1 does not invest in this case. Then, the final outcome is regime I (upgrading and deterrence)

(2) If $\delta > 1/3t$ and $9t(3t\delta - 1)/(9t\delta - 1) > S$, both firms increases their quality and the final outcome is regime III (coexistence).

Both sufficiently high cost and low switching cost must be satisfied in order for the entrant to be improve its technology enough to so that the incumbent accommodates entry but not investing. Otherwise, incumbent is able to deter entry. That is, even when switching cost is low, incumbent is able to keep its technological advantage and upgrade to deter entry.

3.1 Welfare Analysis

It is useful to analyze welfare in the $(v_0, v_1 - S)$ space. The equilibrium consumer surplus and producer surplus for each of the four regimes (defined in Proposition 1) are summarized below. In regime IV, under which there are multiple equilibria, we choose the one that yields the highest payoff for the incumbent ($\alpha = 0$). The iso-consumer surplus lines are shown in Figure 4.

	Consumer Surplus	Producer Surplus
I	$v_1 - S + \frac{t}{2}$	$v_0 - v_1 + S - t$
II	$v_0 + \frac{t}{2}$	$v_1 - S - v_0 - t$
III	$\frac{(v_0 - v_1 + S)^2}{36t} + \frac{v_1 - S + v_0}{2} - \frac{5}{4}t$	$\frac{(v_0 - v_1 + S)^2}{9t} + t$
IV	$\frac{1}{2t}\left\{(t - \frac{v_1 - S}{3})^2 + (\frac{v_1 - S}{3})^2\right\}$	$v_0 - t - \frac{(v_0 - 2t)(v_1 - S)}{3t} + \frac{(v_1 - S)^2}{9t}$

Table 1: Consumer and Producer Surpluses by Regime

In both regimes I and II, consumers are served by only one of the firms. But because the prices reflect the alternative technology, surplus is function of non-producing firm's technology (and switching cost). In regime III, where both firms are in the market and two standards co-exist, consumers benefit technologies being closer. Regime IV consumer surplus is very sensitive to the equilibrium considered among the continuum. [3]

[3]We note that although the switching cost increases the consumer surplus, it is questionable whether this is procompetitive. In Regime IV, an increase in the switching costs increases the surplus for consumers who buy from the incumbent and reduces the surplus for those buying from the entrant. In addition, the proportion of those buying from the incumbent increases. This

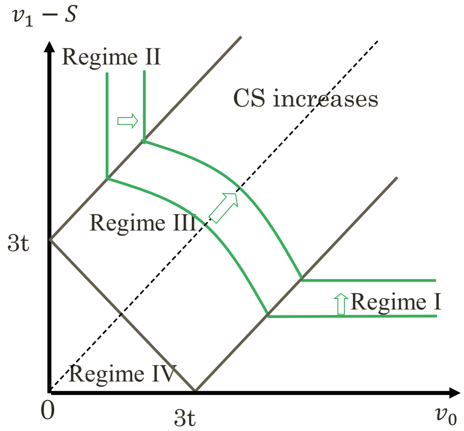

Figure 4: Iso-Consumer Surplus Curve

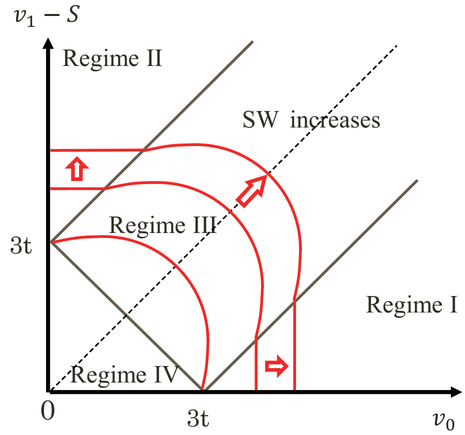

Figure 5: Iso-Social Surplus Curve

The iso-social surplus curves are presented in Figure 5, which shows the social benefits of equalizing $v_1 - S$ and v_0. Furthermore, if innovation costs are allocated carefully, there may be a distributional gain. For instance, allocations that increase c_0 more than c_1 may equate $v_1 - S$ and v_0.

Recall that in regime III,the consumer surplus decreases in S. In some regions of regime III, the social surplus may increase with S if gains in the producer surplus are sufficiently large. This occurs when $v_1 - S \leq -\frac{9}{5}t + v_0$. In these regions, firm 0 is significantly more efficient, which gives it substantial market power. In this case, whereas increasing the switching cost barely hurts consumers at the margin, producers gain significantly.

increases the total consumer surplus. An increase in switching costs increases consumer welfare by skewing the surplus distribution so that there are more people in the higher surplus consumer group (which benefits) and fewer in the lower surplus consumer group (which is disadvantaged). The producer surplus decreases for a similar reason.

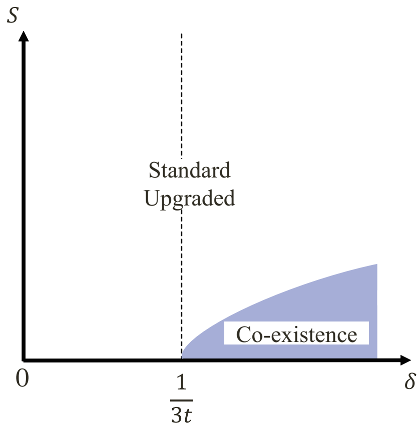

Figure 6: Equilibrium Outcome

4 Policy Implications

The preceding section considered welfare implication of this game. This section shows the policy implications from our results. Figure 6 shows the equilibrium outcome obtained by Proposition 2. We have already showed the social benefit of co-existence of standards. Ironically, innovation policy that subsidizes investment cost in mature technologies is not prudent if technology supports a standard. From national resource allocation point of view, it is more efficient to channel resources in frontier technologies where returns to public investment is higher. Our result suggests that supporting old technologies has other adverse effects, i.e., prolonging dominance of old standards. In order for market to benefit from competition, cost of investment in the underlying technology should be the true cost.

Producers can use the standard strategically to increase their profit. For instance, the incumbent who owns standard may try to deter the entry by improving the technology of standard or increasing the switching cost. Upgrading a standard maintains its attractiveness to consumers, while investing in the installed base increases consumer costs of switching to the new standard. We can also develop a two-stage game, in the first stage of which the incumbent invests in upgrading and the installed base, and the entrant invests to improve its potential standard technology. If we assume that firm 0 can also control the switching cost strategically, we can obtain the following proposition.

Proposition 3. *When the switching cost is firm 0's strategic choice, SPNE outcomes of this game are as follows*

(1) If $\delta \leq 1/3t$, firm 0 increases his quality of products enough to kick out firm 1. Firm 1 does not invest in this case. Then, the final outcome is regime I (upgrading and deterrence)

(2) If $\delta > 1/3t$, both firms increases their quality and the final outcome is regime III (coexistence).

If investment costs are low, there is upgrading without entry but high investment costs lead to coexistence. Because of symmetry, investment costs are low for both incumbent and entrant. However, the incumbent can invest in the switching cost, which is a more efficient way of gaining a relative advantage and is thus able to deter entry. Even if we take into account the incumbent's strategic use of standard, our main results do not change.

5 Conclusion

We have shown how a potential entrant may overcome deterrence by the incumbent with non-passive investment in technology. The extent of incumbent's entry deterrence and entrant's counter attack depends on the age of the underlying technology. When the technology is in its infancy (investment cost efficiency parameter δ is large), incumbent can maintain its technological advantage even if switching cost is low. Incumbent can upgrades and deters entry.

As the technology matures and innovation costs increase, the incumbent's advantage shrinks. However, switching cost must be sufficiently low in order for prospect of entrant's investment to discourage the incumbent from improving. Only then can the entrant prevent entry deterrence and different standards coexist in the market.

There is always benefit to consumers from competition from entry. We have shown that there needs to be both low switching cost and technology maturity for entrant to successfully overcome entry deterrence. Competition policy and innovation policy are not substitutes. Both policies need to be coordinated. In particular, there needs to be caution over inertia, such as installed base, as the standard and its technology matures. Letting technology maturity run its course will not guarantee entry. And even then replacement will not never occur.

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Appendix

Derivation of Demand under Assumption (M)

We define the benchmarks, $\hat{x}_0(p_0)$, $\hat{x}_1(p_1)$, and $\hat{x}(p_0, p_1)$, by

$$v_0 - p_0 - t\hat{x}_0(p_0) = 0, \quad v_1 - p_1 - S - t(1 - \hat{x}_1(p_1)) = 0, \tag{4}$$

$$v_0 - p_0 - t\hat{x}(p_0, p_1) = v_1 - p_1 - S - t(1 - \hat{x}(p_0, p_1)). \tag{5}$$

All consumers to the left (right) of $\hat{x}_0(p_0)$ ($\hat{x}_1(p_1)$) derive positive utility from buying from firm 0 (firm 1). All consumers to the left (right) of $\hat{x}(p_0, p_1)$ derive greater utility from buying from firm 0 (firm 1). By definition, it must be that either (i) $\hat{x}_0(p_0) < \hat{x}(p_0, p_1) < \hat{x}_1(p_1)$, or (ii) $\hat{x}_0(p_0) \geq \hat{x}(p_0, p_1) \geq \hat{x}_1(p_1)$. In case (i), there is an interval of consumers in the middle that do not buy at all. In case (ii), all consumers buy, and there are three possibilities: all buy from firm 0 if $\hat{x}(p_0, p_1) \leq 0$; all buy from firm 1 if $\hat{x}(p_0, p_1) \geq 1$; and otherwise, both firms make positive sales. We have $\hat{x}_0(p_0) = (v_0 - p_0)/t$, $1 - \hat{x}_1(p_1) = (v_1 - S - p_1)/t$, and $\hat{x}(p_0, p_1) = (v_0 - p_0 - v_1 + S + p_1 + t)/2t$.

Proof of Proposition 1

The problem is to find the p_0 that maximizes

$$\pi_0 = \begin{cases} \pi_0^A = \frac{p_0(v_0-p_0)}{t} & \text{for } v_0 - p_0 \leq t - v_1 + S + p_1, \\ \pi_0^B = \frac{p_0(v_0-p_0-v_1+S+p_1+t)}{2t} & \text{for } t - v_1 + S + p_1 < v_0 - p_0 \leq t + v_1 - S - p_1, \\ \pi_0^C = p_0 & \text{for } t + v_1 - S - p_1 < v_0 - p_0. \end{cases}$$

Straightforward but tedious calculation yields the next lemma.

Lemma 3. *Firm 0's best-response correspondence $p_0 = R_0(p_1)$ is as follows.*

(1) If $t < v_0/3$, then

$$R_0(p_1) = \begin{cases} v_0 - v_1 + S + p_1 - t & \text{for } v_1 - S - p_1 \leq v_0 - 3t, \\ \frac{v_0-v_1+S+p_1+t}{2} & \text{for } v_1 - S - p_1 \geq v_0 - 3t. \end{cases}.$$

(2) If $t > v_0/3$, then

$$R_0(p_1) = \begin{cases} v_0 + v_1 - S - p_1 - t & \text{for } v_1 - S - p_1 \leq t - \frac{v_0}{3}, \\ \frac{v_0-v_1+S+p_1+t}{2} & \text{for } t - \frac{v_0}{3} \leq v_1 - S - p_1. \end{cases}.$$

(3) If $t = v_0/3$, then

$$R_0(p_1) = \frac{v_0 - v_1 + S + p_1 + t}{2} \textit{for all } v_1 - S - p_1 \geq 0.$$

Firm 1's best-response correspondence is obtained similarly and differs only because the switching cost must be taken into account in the profit function. By using the same argument applied to firm 0, the problem for firm 1 is to choose p_1 to maximize

$$\pi_1 = \begin{cases} \pi_1^A = \frac{p_1(v_1-S-p_1)}{t} & \text{for } v_1 - S - p_1 \leq t - v_0 + p_0, \\ \pi_1^B = \frac{p_1(t-v_0+p_0+v_1-S-p_1)}{2t} & \text{for } t - v_0 + p_0 < v_1 - S - p_1 \leq t + v_0 - p_0, \\ \pi_1^C = p_1 & \text{for } t + v_0 - p_0 < v_1 - S - p_1. \end{cases}$$

Lemma 4. *Firm 1's best-response correspondence $p_1 = R_1(p_0)$ is as follows.*

(1) If $t < (v_1 - S)/3$, then

$$R_1(p_0) = \begin{cases} \frac{v_1-S}{2} \text{ or } p_0 - t - v_0 + v_1 - S & \text{for } v_0 - p_0 \leq t - \frac{v_1-S}{2}, \\ p_0 - t - v_0 + v_1 - S & \text{for } t - \frac{v_1-S}{2} < v_0 - p_0 \leq v_1 - S - 3t, \\ \frac{t-v_0+p_0+v_1-S}{2} & \text{for } v_1 - S - 3t < v_0 - p_0. \end{cases} \cdot$$

(2) If $t > (v_1 - S)/3$, then

$$R_1(p_0) = \begin{cases} \frac{v_1-S}{2} & \text{for } p_0 - t - v_0 + v_1 - S \leq t - \frac{v_1-S}{2} \\ v_1 - S - t + v_0 - p_0 & \text{for } t - \frac{v_1-S}{2} < v_0 - p_0 \leq t - \frac{v_1-S}{3}, \\ \frac{t-v_0+p_0+v_1-S}{2} & \text{for } t - \frac{v_1-S}{3} \leq v_0 - p_0. \end{cases} \cdot$$

(3) If $t = (v_1 - S)/3$, then

$$R_1(p_0) = \frac{t - v_0 + p_0 + v_1 - S}{2} \textit{for all } v_0 - p_0 \geq 0.$$

In case (1), the value of $R_1(p_0)$ for $v_0 - p_0 \leq t - (v_1 - S)/2$ is $(v_1 - S)/2$ if $\pi_1^A(\frac{v_1 - S}{2}) \geq \pi_1^B(p_0 - t - v_0 + v_1 - S)$, and the value is $p_0 - t - v_0 + v_1 - S$ otherwise. It is unambiguously the case that $R_1(p_0) > v_0 - p_0$, which guarantees that this segment of the best-response function never contains the Nash equilibrium (in pure strategies). Because of the switching cost, firm 1 may not always want to sell to all consumers not buying from firm 0. However, because of assumption (M), firm 0 takes any opportunity to sell to a consumer who does not buy from firm 1. Using the best-response correspondences, we can characterize the Nash equilibrium prices and allocations.

For both firms, there is a case (case (2) for both) for which strategies can be strategic complements. Competition based on fixed fees is effectively competition based on prices that are strategic substitutes: when a rival firm lowers its fee, the firm's optimal response is to lower its fee. That is, when its rival increases demand, each firm finds it profitable to reduce its fee and to increase demand (to get back some of the lost demand caused by the rival lowering its fee). In doing so, each firm must forgo some of the surplus previously collected from its captive consumers. However, in case (2), if $v_1 - S - p_1 \leq t - \frac{v_0}{3}$, then in response to its rival's fee reduction, firm 0 finds it optimal to increase its own fee (and to lose further demand) to extract more surplus from its captive consumers. For

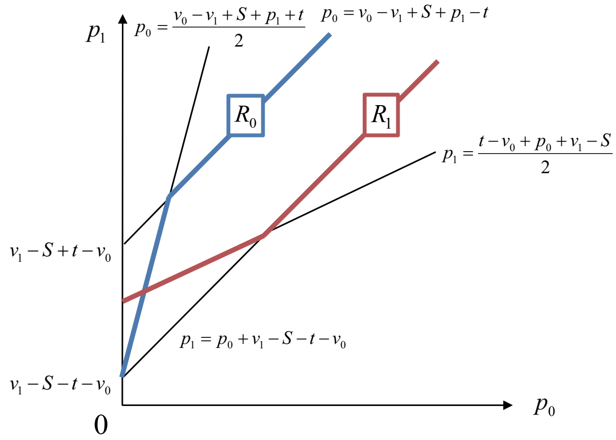

Figure 7: Best-Response Correspondences and Equilibrium in Regime III

this to be optimal, the reduction in demand induced by the fee increase must be small relative to the surplus; i.e., transportation cost (t) must be sufficiently large, which is the condition for case (2) to prevail. In addition, the marginal consumer's surplus must be small enough that it is not worth retaining that consumer ($v_1 - S - p_1 \leq t - \frac{v_0}{3}$). A similar argument holds for firm 1's strategic complementarity.

Regimes III and IV

Under regimes III and IV, both firms make positive sales. Firms split the market equally when $v_0 = v_1 - S$, which is a subregime of regime III. However, because of the switching cost, the entrant must be more efficient in order to have the same market share. The best-response correspondences and equilibrium under this regime are illustrated n Figure 7. The entrant does not reduce the final surplus by the whole amount of the switching cost because it takes into account the fact

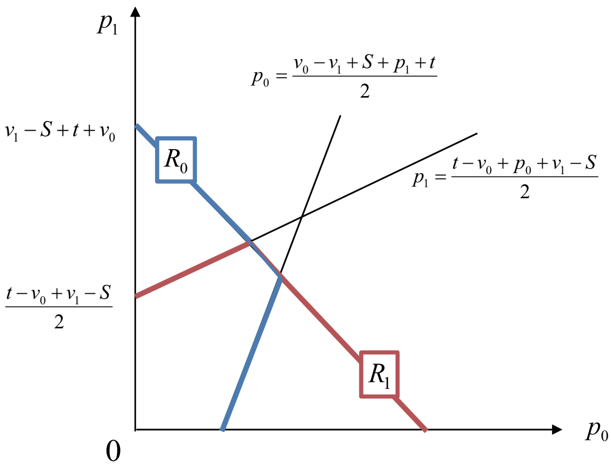

Figure 8: Best-Response Correspondences in Regime IV

that the incumbent will also reduce its surplus in response. This is a direct result of strategic complementarity. For both groups of consumers, the equilibrium surplus decreases with the switching cost. However, from (1), it is easy to show that the equilibrium fee only increases for the incumbent. An increase in the switching cost leads the incumbent to charge a higher fee and to increase its market share. Thus, its profit is increasing in the switching cost. Because the entrant has a lower market share and a lower fee, its profit decreases with the switching cost.

In regime IV, the intersection of the best-response correspondences is the closed line segment between points $(p_0, p_1) = \left(\frac{2v_0}{3}, \frac{v_0}{3} + v_1 - S - t\right)$ and $\Big(v_0 - t + \frac{v_1-S}{3}, \frac{2(v_1-S)}{3} + 2$ Among these equilibria, the most profitable for the incumbent is the one that generates the largest market share for the incumbent, $p_0^*(v_0, v_1, S) = t - \frac{v_1-S}{3}$. This corresponds to $\alpha = 0$ in the proposition and is at the lower right end of the relevant line segment in Figure 8. It is worth noting that this equilibrium coincides with the SPNE outcome were prices to be determined sequentially and were the incumbent to choose first. This is because the best-response correspondence of the entrant (the second mover) is kinked at this point, at which prices change from strategic substitutes to strategic complements. The equilibrium reflects the strategic substitute nature of the strategies. When the switching cost increases, the surplus of the incumbent's customers increases, whereas that of the entrant's customers decreases. Equations (2) and (3) clearly show that the equilibrium fees for both firms decrease with the switching cost. When switching costs increase, the entrant's equilibrium share decreases, and its fixed fee decreases. Hence, the entrant's profit unambiguously decreases with the switching cost. An increased switching costs reduces fees but raises the incumbent's market share. Thus, if the fee is relatively high, incumbent profits increase with the switching cost.

Proof of Lemma 1

First, we consider firm 1's response when firm 0's investment is sufficiently high ($\Delta_0 + S > 3t$). In this case, firm 1 must exit the market unless it can improve the quality of its product sufficiently. We consider the optimal investments in equilibrium. To determine firm 0's strategy, we must consider firm 1's response.

Firm 1 does not invest $(\Delta_1 = 0)$

When firm 1 does not invest to improve product quality, the outcome is in regime I. Then, the producers' profits are given by

$$\pi_0 = \Delta_0 + S - t - \frac{\delta\Delta_0^2}{2},$$
$$\pi_1 = 0.$$

The optimal degree of quality improvement Δ_0^* solves the following:

$$\max_{\Delta_0} \pi_0 = \Delta_0 + S - t - \frac{\delta\Delta_0^2}{2}$$
$$s.t. \Delta_0 + S \geq 3t.$$

We define the Lagrangian

$$L_0 = \Delta_0 + S - t - \frac{\delta\Delta_0^2}{2} + \lambda(\Delta_0 + S - 3t).$$

Firm 0 chooses Δ_0 to maximize profit. Then, the Kuhn–Tucker conditions are

$$\frac{\partial L_0(\Delta_0)}{\partial \Delta_0} = 1 - \delta\Delta_0 + \lambda = 0,\ \ \Delta_0 \frac{\partial L_0(\Delta_0)}{\partial \Delta_0} = 0,$$

$$\frac{\partial L_0(\Delta_0 + S)}{\partial \lambda} = \Delta_0 + S - 3t \geq 0,\ \ \lambda \geq 0,\ \ \lambda \frac{\partial L_0(\Delta)}{\partial \lambda} = 0.$$

First, we consider the case in which $\Delta_0 + S = 3t$, $\lambda > 0$ when $3t - 1/\delta > S$, which gives

$$\Delta_0^* = 3t - S.$$

The optimal profits in this region are thus given by

$$\pi_0^* = 2t - \frac{\delta(3t-S)^2}{2}, \quad \pi_1^* = 0.$$

Second, we consider the case in which $\Delta_0 + S > 3t$, $\lambda = 0$ when $3t - 1/\delta < S$, which gives

$$\Delta_0^* = \frac{1}{\delta}.$$

The optimal profits in this region are thus given by

$$\pi_0^* = \frac{1}{2\delta} + S - t, \quad \pi_1^* = 0.$$

Firm 1 tries to move to regime III $(\Delta_0 + S - 3t < \Delta_1 < \Delta_0 + S + 3t)$

When firm 1 invests i n quality improvement and tries to move to regime III, producers' profits are given by

$$\pi_0 = \frac{(\Delta_0 + S - \Delta_1 + 3t)^2}{18t} - \frac{\delta\Delta_0^2}{2}, \pi_1 = \frac{(\Delta_1 - \Delta_0 - S + 3t)^2}{18t} - \frac{\delta\Delta_1^2}{2}.$$

We must consider firm 1's strategy. The optimal values of Δ_1^* are the solutions to

$$\max_{\Delta_1} \pi_1 = \frac{(\Delta_1 - \Delta_0 - S + 3t)^2}{18t} - \frac{\delta \Delta_1^2}{2}$$
$$s.t. \Delta_1 + 3t \geq \Delta_0 + S \geq \Delta_1 - 3t.$$

We define the Lagrangian

$$L_1 = \frac{(\Delta_1 - \Delta_0 + S + 3t)^2}{18t} - \frac{\delta \Delta_1^2}{2} + \lambda_1(\Delta_0 + S - \Delta_1 + 3t) + \lambda_2(\Delta_1 - \Delta_0 - S + 3t).$$

The Kuhn–Tucker conditions are

$$\frac{\partial L_1}{\partial \Delta_1} = \frac{(\Delta_1 - \Delta_0 - S + 3t)}{9t} - \delta\Delta_1 - \lambda_1 + \lambda_2 = 0,\ \Delta_1 \frac{\partial L_1}{\partial \Delta_1} = 0$$
$$\frac{\partial L_1}{\partial \lambda_1} = \Delta_0 + S - \Delta_1 + 3t \geq 0,\ \lambda_1 \frac{\partial L_1}{\partial \lambda_1} = 0,$$
$$\frac{\partial L_1}{\partial \lambda_2} = \Delta_1 - \Delta_0 + S + 3t \geq 0,\ \lambda_2 \frac{\partial L}{\partial \lambda_2} = 0.$$

We consider the case in which $\Delta_1 \geq 0$, $\lambda_1 = \lambda_2 = 0$ when $\delta < 1/9t$, which gives

$$\Delta_1^* = \frac{\Delta_0 + S - 3t}{1 - 9t\delta}.$$

The optimal profits in this region are thus given by

$$\pi_1^* = -\frac{(\Delta_0 + S - 3t)^2}{2(1 - 9t\delta)}.$$

When $\delta < 1/9t$, firm 1's equilibrium profit is negative. Thus, firm 1 does not choose this strategy.

Firm 1 tries to move to regime II $(\Delta_1 \geq \Delta_0 + S + 3t)$

When firm 1 invests sufficiently in quality improvement and tries to move to regime II , producers' profits are given by

$$\pi_0 = -\frac{\delta\Delta_0^2}{2}, \pi_1 = \Delta_1 - \Delta_0 - S - t - \frac{\delta\Delta_1^2}{2}.$$

We must consider firm 1's strategy. The optimal values of Δ_1^* are the solutions to

$$\max_{\Delta_1} \pi_1 = \Delta_1 - \Delta_0 - S - t - \frac{\delta\Delta_1^2}{2}$$
$$s.t. \Delta_1 \geq \Delta_0 + S + 3t.$$

We define the Lagrangian

$$L_1 = \Delta_1 - \Delta_0 - S - t - \frac{\delta\Delta_1^2}{2} + \lambda(\Delta_1 - \Delta_0 - S - 3t).$$

The Kuhn–Tucker conditions are

$$\frac{\partial L_1}{\partial \Delta_1} = 1 - \delta\Delta_1 + \lambda = 0,\ \Delta_1 \frac{\partial L_1}{\partial \Delta_1} = 0$$
$$\frac{\partial L_1}{\partial \lambda} = \Delta_1 - \Delta_0 - S - 3t \geq 0,\ \lambda\frac{\partial L_1}{\partial \lambda} = 0.$$

First, we consider the case in which $\Delta_1 = \Delta_0 + S + 3t$, $\lambda > 0$ when $\max\{3t, 1/\delta - 3t\} < \Delta$, which gives

$$\Delta_1^* = \Delta_0 + S + 3t.$$

The optimal profits in this region are thus given by

$$\pi_1^* = 2t - \frac{\delta(\Delta_0 + S + 3t)^2}{2}.$$

Second, we consider the case in which $\Delta_1 > \Delta_0 + S + 3t$, $\lambda = 0$ when $3t < \Delta_0 + S < 1 - 3t$, which gives

$$\Delta_1^* = \frac{1}{\delta}.$$

The optimal profits in this region are thus given by

$$\pi_1 = \frac{1}{2\delta} - \Delta_0 - S - t.$$

Optimal investment in this region

We can now consider firm 0's optimal investment in this region. Firm 1 has no incentive to move to region 3 because its profit is negative. Firm 0 prefers regime I to regime II . We can easily show that firm 1 has no incentive to move to regime II given firm 0's optimal investment in regime I . Therefore, in this region, firm 0 tries to maximize profit in regime I , and firm 1 does not invest in equilibrium.

Proof of Lemma 2

In this case, firm 0 invests little ($\Delta_0 + S \leq 3t$), and firm 1 can stay in the market unless firm 0 substantially improves product quality. To consider firm 0's strategy, we must take into account firm 1's response.

Firm 1 tries to stay in regime III $(\Delta_0 + S - 3t < \Delta_1 < \Delta_0 + S + 3t)$

When firm 1 invests in quality improvement and tries to stay in regime III , producers' profits are given by

$$\pi_0 = \frac{(\Delta_0 + S - \Delta_1 + 3t)^2}{18t} - \frac{\delta\Delta_0^2}{2}, \pi_1 = \frac{(\Delta_1 - \Delta_0 - S + 3t)^2}{18t} - \frac{\delta\Delta_1^2}{2}.$$

We must consider firm 1's strategy. The optimal values of Δ_1^* are the solutions to

$$\max_{\Delta_1} \pi_1 = \frac{(\Delta_1 - \Delta_0 - S + 3t)^2}{18t} - \frac{\delta\Delta_1^2}{2}$$
$$s.t. \Delta_1 + 3t \geq \Delta_0 + S \geq \Delta_1 - 3t.$$

We define the Lagrangian

$$L_1 = \frac{(\Delta_1 - \Delta_0 - S + 3t)^2}{18t} - \frac{\delta\Delta_1^2}{2} + \lambda_1(\Delta_0 + S - \Delta_1 + 3t) + \lambda_2(\Delta_1 - \Delta_0 - S + 3t).$$

The Kuhn–Tucker conditions are

$$\frac{\partial L_1}{\partial \Delta_1} = \frac{(\Delta_1 - \Delta_0 - S + 3t)}{9t} - \delta\Delta_1 - \lambda_1 + \lambda_2 = 0,\ \Delta_1 \frac{\partial L_1}{\partial \Delta_1} = 0$$
$$\frac{\partial L_1}{\partial \lambda_1} = \Delta_0 + S - \Delta_1 + 3t > 0,\ \lambda_1 \frac{\partial L_1}{\partial \lambda_1} = 0,$$
$$\frac{\partial L_1}{\partial \lambda_2} = \Delta_1 - \Delta_0 - S + 3t > 0,\ \lambda_2 \frac{\partial L}{\partial \lambda_2} = 0.$$

We consider the case in which $\Delta_1 \geq 0$, $\lambda_1 = \lambda_2 = 0$ when $\delta > 1/9t$, which gives

$$\Delta_1^* = \frac{3t - \Delta_0 - S}{9t\delta - 1}.$$

Firm 0 takes into account firm 1's strategy to maximize profit. Then, the optimal values of Δ_0^* are the solutions to

$$\max_{\Delta_0} \pi_0 = \frac{\left(\Delta_0 + S - \frac{3t - \Delta_0 - S}{9t\delta - 1} + 3t\right)^2}{18t} - \frac{\Delta_0^2}{2}$$
$$s.t. \Delta_0 + S \geq \frac{3t - \Delta_0 - S}{9t\delta - 1} - 3t,$$
$$\frac{3t - \Delta_0 - S}{9t\delta - 1} \geq \Delta_0 + S - 3t.$$

In this section, we focus on the inner solution. Then, in equilibrium, the optimal investments are

$$\Delta_0^* = \frac{3t(3S\delta + 9t\delta - 2)}{81t^2\delta^2 - 27t\delta + 1},\ \ \Delta_1^* = \frac{27t^2\delta - 9t - 9St\delta + S}{81t^2\delta^2 - 27t\delta + 1}.$$

The optimal profits in this region are thus given by

$$\pi_0^* = \frac{t(3S\delta + 9t\delta - 2)^2}{2(81t^2\delta^2 - 27t\delta + 1)}, \quad \pi_1^* = \frac{\delta(27t^2\delta - 9t - 9St\delta + S)^2(9t\delta - 1)}{2(81t^2\delta^2 - 27t\delta + 1)^2}.$$

We must check that the following conditions are satisfied in equilibrium:

$$\begin{aligned}
\Delta_0^* + S < 3t &\iff -\frac{(9t\delta - 1)(27t^2\delta - 9t - 9St\delta + S)}{81t^2\delta^2 - 27t\delta + 1} < 0, \\
\Delta_0^* + S \geq \Delta_1^* - 3t &\iff \frac{3(9t\delta - 1)(3S\delta + 9t\delta - 2)}{81t^2\delta^2 - 27t\delta + 1} \geq 0, \\
\Delta_1^* \geq \Delta_0^* + S - 3t &\iff \frac{9t\delta(27t^2\delta - 9t - 9St\delta + S)}{81t^2\delta^2 - 27t\delta + 1} \geq 0.
\end{aligned}$$

The satisfaction of these conditions requires

$$\text{sign}\left(81t^2\delta^2 - 27t\delta + 1\right) = \text{sign}(3S\delta + 9t\delta - 2) = \text{sign}(27t^2\delta - 9t - 9St\delta + S).$$

We consider the case in which all signs are positive. (When all signs are negative, it is not possible to satisfy all conditions.) When $9t(3t\delta - 1)/(9t\delta - 1) > S$, sign$(27t^2\delta - 9t - 9St\delta + S)$ becomes positive. In this paper, we assume the switching cost S is positive. Thus, δ has to be larger than $1/3t$. It is clear that sign $(81t^2\delta^2 - 27t\delta + 1)$ and sign$(3S\delta + 9t\delta - 2)$ becomes positive when $\delta > 1/3t$. Therefore, all conditions are satisfied when δ exceeds $1/3t$ and $9t(3t\delta - 1)/(9t\delta - 1) > S$.

Firm 1 tries to move to regime II $(\Delta_1 > \Delta_0 + S + 3t)$

When firm 1 invests in quality improvement and tries to move to regime II , producers' profits are given by

$$\pi_0 = -\frac{\delta\Delta_0^2}{2}, \pi_1 = \Delta_1 - \Delta_0 - S - t - \frac{\delta\Delta_1^2}{2}.$$

We must consider firm 1's strategy. The optimal values of Δ_1^* are the solutions to

$$\max_{\Delta_1} \pi_1 = \Delta_1 - \Delta_0 - S - t - \frac{\delta\Delta_1^2}{2}$$
$$s.t. \Delta_1 \geq \Delta_0 + S + 3t.$$

We define the Lagrangian

$$L_1 = \Delta_1 - \Delta_0 - S - t - \frac{\delta\Delta_1^2}{2} + \lambda(\Delta_1 - \Delta_0 - S - 3t).$$

The Kuhn–Tucker conditions are

$$\frac{\partial L_1}{\partial \Delta_1} = 1 - \delta\Delta_1 - \lambda = 0,\ \Delta_1 \frac{\partial L_1}{\partial \Delta_1} = 0,$$
$$\frac{\partial L_1}{\partial \lambda} = \Delta_1 - \Delta_0 - S - 3t > 0,\ \lambda \frac{\partial L_1}{\partial \lambda} = 0.$$

First, we consider the case in which $\Delta_1 \geq 0, \lambda > 0$ when $1/\delta - 3t < \Delta_0 + S < 3t$, which gives

$$\Delta_1^* = \Delta_0^* + S + 3t.$$

The optimal profits in this region are thus given by

$$\pi_1^* = 2t - \frac{\delta(\Delta_0 + S + 3t)^2}{2}.$$

Second, we consider the case in which $\Delta_1 \geq 0, \lambda = 0$ when $\Delta_0 + S < \max\{3t, 1/\delta - 3t\}$, which gives

$$\Delta_1^* = \frac{1}{\delta}.$$

The optimal profits in this region are thus given by

$$\pi_1 = \frac{1}{2\delta} - \Delta_0 - S - t.$$

Optimal investment in this region

We can now consider firm 0's optimal investment in this region. Firm 0 prefers regime III to regime II . Therefore, both firms invest and stay in regime III when $\delta > 1/3t$ and $9t(3t\delta - 1)/(9t\delta - 1) > S$. Otherwise, regime II defines the equilibrium.

Proof of Proposition 2

We can now consider optimal investment.

If quality improvement is costly ($\delta > 1/3t$) and switching cost is low ($9t(3t\delta - 1)/(9t\delta - 1) > S$)

When firm 0's investment is not sufficiently high ($(\Delta_0 + S \leq 3t)$), the region defines the equilibrium. When firm 0's investment is sufficient ($(\Delta_0 + S > 3t)$), the equilibrium is defined by regime I. We can easily show that, in this case, firm 0 makes more profit under regime III than under regime I . Thus, firm 0 tries to stay in regime III.

If quality improvement is not costly ($\delta \leq 1/3t$) or switching cost is high

When firm 0's investment is not sufficiently high ($(\Delta_0 + S \leq 3t)$), the equilibrium is located in region 2. When firm 0 does invest sufficiently ($(\Delta_0 + S > 3t)$), regime I defines the equilibrium. Thus, firm 0 tries to invest enough to prevent firm 1's entry.

Derivation of the Consumer Surplus

In a mature industry, the consumer surplus for the four regimes is given below.

$$
\begin{aligned}
\text{Regime I} : CS_I &= \int_0^1 (v_0 - p_0^* - tx)dx = v_1 - S + \frac{t}{2}, \\
\text{Regime II} : CS_{II} &= \int_0^1 (v_1 - S - p_1^* - t(1-x))dx = v_0 + \frac{t}{2}, \\
\text{Regime III} : CS_{III} &= \int_0^{\hat{x}(p_0^*, p_1^*)} (v_0 - p_0^* - tx)dx + \int_{\hat{x}(p_0^*, p_1^*)}^1 (v_1 - S - p_1^* - t(1-x))dx \\
&= \frac{(v_0 - v_1 + S)^2}{36t} + \frac{v_1 - S + v_0}{2} - \frac{5}{4}t, \\
\text{Regime IV} : CS_{IV} &= \int_0^{\hat{x}(p_0^*, p_1^*)} (v_0 - p_0^* - tx)dx + \int_{\hat{x}(p_0^*, p_1^*)}^1 (v_1 - S - p_1^* - t(1-x))dx \\
&= \frac{1}{2t}\left\{ \left(t - \frac{v_1 - S}{3} \right)^2 + \left(\frac{v_1 - S}{3} \right)^2 \right\}.
\end{aligned}
$$

Derivation of the Iso-Social Surplus Curves

These curves are obtained from the expressions below.

$$
\begin{aligned}
&\text{Regime I} : SS = CS + PS = -\frac{1}{2}t + v_0, \\
&\text{Regime II} : SS = -\frac{1}{2}t + v_1 - S.
\end{aligned}
$$

For the remaining regimes, by using the following partial derivatives, we obtain

$$\begin{aligned}
\text{Regime III}: \frac{\partial SS}{\partial v_0} &= \frac{1}{2}t + \frac{5v_0 - v_1 + S}{18t}, \\
\frac{\partial SS}{\partial (v_1 - S)} &= \frac{1}{2}t - \frac{5(v_0 - v_1) + S}{18t}, \\
\text{Regime IV}: \frac{\partial SS}{\partial v_0} &= \frac{3t - v_1 + S}{3t}, \\
\frac{\partial SS}{\partial (v_1 - S)} &= \frac{4(v_1 - S) - 3v_0 - 3t}{9t}.
\end{aligned}$$

Proof of Proposition 3

In addition to choosing Δ_0, incumbent can invest to increase S. We assume the following cost,

$$C_0(\Delta_0, S) = \frac{\delta(\Delta_0 + S)^2}{2}, C_1(\Delta_1) = \frac{\delta \Delta_1^2}{2}.$$

With this formulation, improving technology (increasing v_0 or equivalently Δ_0) and increasing switching cost S are symmetric. Firm 0's choice is to choose $\Delta_0 + S$ instead of Δ_0 to maximize profit. We may use the analysis we did for the maximization with respect to Δ instead of Δ_0 where $\Delta \equiv \Delta_0 + S$. The expected payoffs are

$$\begin{aligned}
\Pi_0(\Delta_0, \Delta_1, S) &= \pi_0^*(\overline{v} + \Delta_0, \overline{v} + \Delta_1, S) - C_0(\Delta_0, S), \\
\Pi_1(\Delta_0, \Delta_1, S) &= \pi_1^*(\overline{v} + \Delta_0, \overline{v} + \Delta_1, S) - C_1(\Delta_1).
\end{aligned}$$

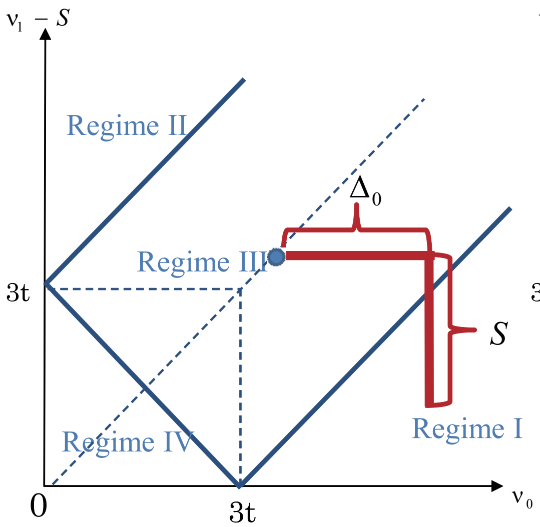

Figure 9: subgame ($\Delta > 3t$)

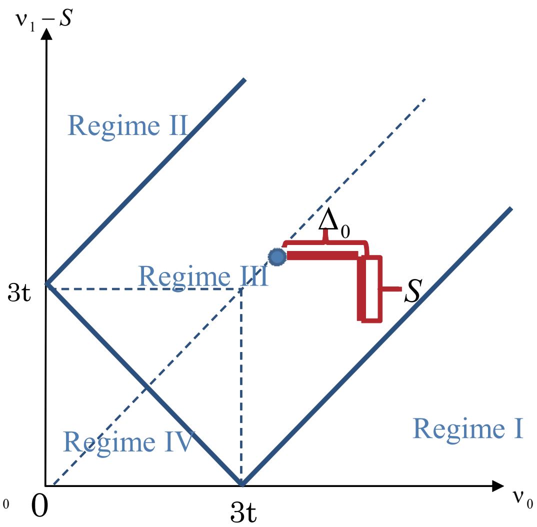

Figure 10: subgame ($\Delta < 3t$)

If there is no investment ($\Delta_0 = S = \Delta_1 = 0$), qualities $v_0 = v_1 = \overline{v}$ and regime III prevail. Once firm 0 has made its investment choice, two regimes are possible: $\Delta \equiv \Delta_0 + S > 3t$ (regime I, Figure 9) and $\Delta \equiv \Delta_0 + S \leq 3t$ (regime III, Figure 10).

In the regime I subgame, depending on firm 1's investment choice, either regime I, regime II, or regime III prevails. The next lemma shows the final outcome under regime I .

Lemma 5. *When $\Delta > 3t$, firm 1 invests nothing and its payoff is zero. Then, the final outcome is regime I.*

The next lemma shows that in the regime III subgame, either regime II or III prevails.

Lemma 6. *When $\Delta < 3t$, firm 1's investment decision depends on investment efficiency and switching cost.*

(1) if $\delta > 1/3t$, then firm 1's optimal investment results in regime III.

(2) Otherwise, firm 1 invests so that the final outcome is regime II.

If the final outcome is regime II, firm 0's payoff will be negative because it makes no profit. From the two lemmas, we obtain proposition 3.