RIETI Discussion Paper Series 15-E-118

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The Research Institute of Economy, Trade and Industry
http://www.rieti.go.jp/en/

RIETI Discussion Paper Series 15-E-118
October 2015

Elastic Labor Supply and Agglomeration[1]

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Abstract

This study analyzes the interplay between the agglomeration of economic activities and interregional differences in working hours, which are typically longer in large cities, as normally they are more developed than small cities. For this purpose, we develop a two-region model with endogenous labor supply. Although we assume a symmetric distribution of immobile workers, the symmetric equilibrium breaks in the sense that firms may agglomerate when trade costs are intermediate and labor supply is elastic. We also show that the price index is always lower, while labor supply, per capita income, real wages, and welfare are always higher in the more agglomerated region.

Keywords: New trade theory, Endogenous labor supply, Symmetry break
JEL classification: J22, R13

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[1]This study is conducted as a part of the Project “Spatial Economic Analysis on Regional Growth” undertaken at Research Institute of Economy, Trade and Industry (RIETI). We thank M. Morikawa and J. Oshiro as well as the seminar audiences at RIETI and Kagawa University for their helpful comments and discussions.
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1 Introduction

While working hours are shorter in more developed countries (Ago et al., 2014), within a country, they are relatively longer in large cities, which are normally more developed than small cities. Rosenthal and Strange (2008) show that among professionals, working hours are longer in larger cities, which are comparable with more developed countries. Gicheva (2013) also shows that young highly educated workers work longer hours to pursue career advancement and to earn higher wages based on the 1979 cohort of the National Longitudinal Survey of Youth. Indeed, people working more than 48 hours per week roses from 16.6% to 24.3% between 1980 and 2005 in the United States (Kuhn and Lozano, 2008) as globalization has changed the market structure and increased incentives to produce the industry's best product in "winner-take-all"–type markets. According to 2006 Survey on Time Use and Leisure Activities in Japan, the working time (average time spent on work) is longer in denser prefectures which consist of (more developed) large cities.

According to Combes et al. (2008, p. 166), "Moving beyond the Krugman model in search of alternative explanations appears to be warranted in order to understand the emergence of large industrial regions in economies characterized by a low spatial mobility of labor." In this study, we consider that labor supply changes based on workers' choice of working hours rather than because of the relocation of firms. Workers prefer to adjust working hours than changing firms through interregional migration in the short run when shocks occur in the labor market. According to Nakajima and Tabuchi (2011), the annual gross migration between prefectures was 2.9% of the Japanese population for 1954-2005 and that between states was 1.1% of the U.S. population for 1989–2004. Braunerhjelm et al. (2000) also report the existence of low spatial labor mobility in EU countries.

Besides the low mobility of labor, economic activities are unevenly distributed. Puga (2002) reports that per capita income in the 10 richest regions of the EU was 3.5 times larger than that in the 10 poorest regions in 1992. Similarly, per capita GDP in the richest prefecture of Japan was about twice that of the poorest prefecture in 2011 (Cabinet Office, 2015). In the United States, the per capita GDP of the richest state was also about twice that of the poorest state in 2013 (U.S. Department of Commerce, Bureau of Economic Analysis, 2015).

Based on the foregoing, this study analyzes the interplay between the agglomeration of economic activities and interregional differences in working hours by using the framework of new economic geography pioneered by Krugman (1991). For this purpose, we develop a model of new economic geography by introducing endogenous labor supply without the interregional migration of labor. More specifically, we construct a two-region model with one differentiated good sector. Each agent is spatially immobile and chooses the optimal amount of labor supply as well as the consumption of the good. An increase in labor supply brings about disutility due to the labor burden, whereas it raises wage income. Therefore, each agent determines labor supply at which marginal disutility by labor equals the real wage, which is defined by the nominal income over the price index in the region.

Our main finding is that even if two regions are identical, the symmetric configuration of firms breaks if the elasticity of labor supply with respect to real wage is sufficiently high. That is, the emergence of an endogenous agglomeration is possible without assuming the spatial mobility of labor. This finding is in sharp contrast to studies of new trade theory such as Krugman (1980), where this symmetry never breaks.

The mechanism that brings about endogenous agglomeration occurs as follows. The real wage is higher in region that have more manufacturing firms. If firms agglomerate more, the price index decreases further, and thus, the real wage rises further. That is, the relative value of nominal income to labor disutility goes up. Since our model assumes an elastic labor supply unlike the familiar models that incorporate a fixed labor supply, the amount of labor supply rises in the agglomerated region. This leads to higher per capita income and a larger market size, which attracts manufacturing firms to the region. In summary, labor supply and the agglomeration of manufacturing firms have a positive correlation, whereas the migration of workers and the agglomeration of firms have a positive correlation in the new economic geography framework.

When the symmetry breaks, we have an asymmetric distribution of firms as a stable equilibrium, where the amount of labor supply is shown to be larger, while the nominal wage earning and per capita total income are higher in the agglomerated region. We also show that individual welfare is higher in the agglomerated region, implying that the higher nominal income and lower price index dominate the higher labor disutility in the agglomerated region.

Some studies have examined the endogenous agglomeration of firms without labor migration. Krugman and Venables (1995) introduce the input-output linkages that yield the agglomeration of firms in the absence of migration. Amiti and Pissarides (2005) assume training costs for skill formation, which serves as a proxy for labor migration, resulting in the emergence of firm agglomeration. Picard and Toulemonde (2006) consider labor unions that introduce wage rigidities so that unionized and high-wage firms agglomerate in a region in the absence of labor mobility. Our study presents a different mechanism of agglomeration under immobile labor, which is consistent with the above-mentioned facts on low labor mobility.

The remainder of the paper is organized as follows. In section 2, we present the basic model. Sections 3 and 4 assume the same population size while section 5 assumes different population sizes. In section 3, we characterize and examine the symmetric equilibrium of firm distribution. We show that when the symmetric equilibrium is unstable, asymmetric equilibria exist. In section 4, we analyze such asymmetric equilibria. Section 5 considers regional asymmetry in the sense that population size differs between regions. Section 6 concludes the paper.

2 The model

The economy consists of two regions, denoted $r = 1, 2$ and a manufacturing sector producing a differentiated good. Let L_r be the mass of immobile workers in region r, and n be the mass of mobile capital in the economy. We assume that one unit of capital is needed as a fixed requirement to produce each variety meaning that the total number of varieties of a differentiated good is n, which is exogenously given.

The preferences of an agent located in region $r = 1, 2$ are given by:

$$U_r = \left[\int_0^n x_r(i)^\rho di\right]^{\frac{1}{\rho}} - \frac{1}{\alpha} l_r^\alpha, \qquad 0 < \rho < 1, \; \alpha > 1, \tag{1}$$

where $x_r(i)$ is the consumption of a variety indexed i in region r and l_r is the amount of labor supply, which reduces the utility since supplying labor reduces leisure time in region r. Each agent supplies labor and earns hourly wage w_r, which is used to purchase the good. She chooses the amount of labor supply, l_r, as well as the consumption of each variety, $x_r(i)$. Therefore, labor supply is elastic. In addition to the wage, she receives rewards from capital holding, a. Her income constraint is given by

$$a + w_r l_r = \int_0^n p_r(i) x_r(i) di, \tag{2}$$

where $p_r(i)$ is the price of variety i sold in region r.

From (1) and (2), we find the labor supply to be

$$l_r = \left(\frac{w_r}{P_r}\right)^{\theta} \tag{3}$$

where

$$P_r = \left[\int_0^n p_r(i)^{1-\sigma} di\right]^{\frac{1}{1-\sigma}}$$

is the price index, $\sigma \equiv 1/(1-\rho)$ is the elasticity of the substitution between differentiated varieties, and $\theta \equiv 1/(\alpha - 1)$ is the real wage elasticity of labor supply. We assume $\sigma > 1$ and $\theta > 0$ to satisfy the second-order conditions for utility maximization. Equation (3) shows that labor supply increases the real wage. On the one hand, when the nominal wage w_r increases, each agent raises labor supply in order to purchase the good. On the other hand, when price index P_r goes up, the value of real income goes down, which reduces labor supply.

We also find the individual demand for variety i produced in region r and consumed in region s as follows:

$$x_{rs}(i) = (a + w_s l_s) \frac{p_{rs}(i)^{-\sigma}}{P_s^{1-\sigma}} = \left(a + w_s^{1+\theta} P_s^{-\theta}\right) \frac{p_{rs}(i)^{-\sigma}}{P_s^{1-\sigma}}, \tag{4}$$

where the second equality is derived from the substitution of (3). Because of the symmetry of each variety, we drop i hereafter.

The interregional trade of the good incurs an iceberg type trade cost. If $\tau > 1$ units of the good are exported between two regions, only one unit reaches the destination. We define $\phi_{rs} = \phi \equiv \tau^{1-\sigma} < 1$ if $r \neq s$ and $\phi_{rs} = 1$ if $r = s$. The price index in region r can be expressed as

$$P_r = (n_r p_{rr}^{1-\sigma} + n_s p_{sr}^{1-\sigma})^{\frac{1}{1-\sigma}} \tag{5}$$

for $r, s = 1, 2$ $(r \neq s)$.

To produce x units of a differentiated good, mx units of labor are needed in addition to one unit of capital. The rewards from capital holding are the profits of firms. We assume that each agent has an equal share of capital, therefore, the total rewards from capital are equally shared by all agents. The profit of a manufacturing firm in region r is described as

$$\pi_r = (p_{rr} - m w_r) x_{rr} L_r + (p_{rs} - m\tau w_r) x_{rs} L_s, \tag{6}$$

where individual demand x_{rs} is given by (4) and the reward from capital holding per agent is given by

$$a = \frac{n_1 \pi_1 + n_2 \pi_2}{L_1 + L_2}. \tag{7}$$

Each manufacturing firm sets prices, p_{rr} and p_{rs}, to maximize the profits. The prices of the good are computed as

$$p_{rr} = \frac{\sigma m}{\sigma - 1} w_r, \qquad p_{rs} = \frac{\sigma m \tau}{\sigma - 1} w_r. \tag{8}$$

By substituting (8) into (6), we have

$$\begin{aligned} \pi_r &= \left(\frac{\sigma m}{\sigma - 1} w_r - m w_r \right) x_{rr} L_r + \left(\frac{\sigma m \tau}{\sigma - 1} w_r - m \tau w_r \right) x_{rs} L_s \\ &= \frac{m w_r}{\sigma - 1} \left(x_{rr} L_r + \tau x_{rs} L_s \right). \end{aligned} \tag{9}$$

Total labor supply and the total labor demand in region r are $l_r L_r$ and $n_r m \left(x_{rr} L_r + x_{rs} L_s \tau \right)$, respectively. Thus, the labor market clearing condition in region r is expressed as

$$l_r L_r = n_r m \left(x_{rr} L_r + x_{rs} L_s \tau \right). \tag{10}$$

One of the two labor market clearing conditions is redundant according to Walras' law.

By plugging (10) into (9), we obtain

$$\pi_r = \frac{w_r}{\sigma - 1} \frac{l_r L_r}{n_r}. \tag{11}$$

Hence, we have shown that *the profit of a firm is proportional to the sales per firm*, $w_r l_r / n_r$, which comprises the wage bill $w_r l_r$ and number n_r of firms. The profit is in proportion to the former, while inversely proportional to the latter. In the agglomerated region, the denominator n_r of (11) is larger, implying keen competition among firms there. To attain a spatial equilibrium, the numerator $w_r l_r$ of (11) should also be larger in the agglomerated region. This fact means that firms in the agglomerated region should offer a higher wage bill $w_r l_r$ to secure larger labor supply, which is due to a larger number of firms.

In the spatial equilibrium, the profit of each firm is the same between regions. That is, the spatial equilibrium conditions are given by

$$\Delta \pi \equiv \pi_1 - \pi_2 = 0, \tag{12}$$

and the labor market clearing condition (10). They lead to

$$\frac{n_1 / L_1}{n_2 / L_2} = \frac{w_1 l_1}{w_2 l_2}. \tag{13}$$

If the home market effect $n_1/L_1 > n_2/L_2$ is exhibited, then per capita income $a + w_1 l_1$ is higher in the larger region.

Lemma 1 *Per capita income is higher in an agglomerated region.*

If an agglomerated region is interpreted as a more developed region (i.e., a large city), then this agrees with the stylized facts in the urban economy: income per capita is higher in larger cities.

Plugging (3), (4), (7), and (11) into utility (1) yields the indirect utility:

$$\begin{aligned} V_r &= \frac{a + w_r l_r}{P_r} - \frac{\theta}{\theta+1} l_r^{\theta+1} \\ &= \frac{(2\sigma-1)\, w_r^{1+\theta} P_r^{-\theta} + 2(\sigma-1)\, w_r^{1+\theta} P_r^{-\theta}}{2(\sigma-1) P_r} - \frac{\theta}{\theta+1}\left(\frac{w_r}{P_r}\right)^{\theta+1}. \end{aligned} \tag{14}$$

Define $\lambda \equiv n_1/n$ and $w \equiv w_1/w_2$, which are the endogenous variables to be determined by the two spatial equilibrium conditions, (10) and (12), with (3), (4), (7), and (11). Firms migrate to a region with a higher profit, meaning that ad hoc dynamics may be given by

$$\dot{\lambda} = \Delta\pi. \tag{15}$$

3 Symmetric Equilibrium

To focus on the symmetric equilibrium, we set an equal population size of regions $L_1 = L_2$, which is normalized to 1 in this section and the next section. It is apparent that there always exists a symmetric equilibrium for any values of the parameters. However, this equilibrium can be stable or unstable depending on the parameter values. We check its stability by totally differentiating the RHS of (15) with respect to λ and evaluating it at the symmetric equilibrium $(\lambda^*, w^*) = (1/2, 1)$ as follows:

$$\left.\frac{d\Delta\pi}{d\lambda}\right|_{(\lambda,w)=(1/2,1)} = \left.\frac{\partial\Delta\pi}{\partial\lambda} + \frac{\partial\Delta\pi}{\partial w}\frac{dw}{d\lambda}\right|_{(\lambda,w)=(1/2,1)} = Cg(\phi), \tag{16}$$

where $dw/d\lambda$ is computed by applying the implicit function theorem to (10), which is a function of λ and w. C is a positive constant and

$$g(\phi) \equiv -(2\sigma-1)(2\theta+1)\phi^2 + 2\left[(2\sigma-1)\theta + 3\sigma - 2\sigma^2\right]\phi - 1.$$

Therefore, the stability condition is reduced to $g(\phi) < 0$.

By examining this stability condition, we first find that the symmetric equilibrium is always stable if

$$\theta < \theta_B \equiv \sigma - 1 + \frac{2\sqrt{(\sigma-1)\sigma}}{2\sigma-1}. \tag{17}$$

Proposition 1 *Symmetry never breaks for a sufficiently inelastic labor supply such that $\theta < \theta_B$.*

This corresponds to the familiar result under an inelastic labor supply $\theta = 0$ in Krugman (1980), among others. When θ is small, labor supply is inelastic with respect to the real wage. Suppose that some manufacturing firms move to a region. The price index in the region that attracts firms decreases, which raises labor supply from (3). However, such an expansion of labor supply is small because labor supply is inelastic. On the contrary, labor demand increases according to the number of firms. Further, the tight labor market forces wage to rise, and thus the profits of firms reduce, which ensures the stability of the symmetric equilibrium.

Thus, *the symmetry break requires an elastic labor supply (large θ)*. Suppose θ is large enough and labor supply is elastic with respect to the real wage. Firms can expect large labor supply and agents can expect a higher real wage, which expands the market size in the destination region. More precisely, if $\theta > \theta_B$, the symmetric equilibrium is unstable when ϕ is in the interval of (ϕ_{B1}, ϕ_{B2}), where ϕ_{B1} and ϕ_{B2} are the solutions of $g(\phi) = 0$ and satisfy $0 < \phi_{B1} < \phi_{B2} < 1$. Otherwise, the symmetric configuration is a stable equilibrium.

Next, we check the possibility of a fully agglomerated equilibrium, $\lambda = 1$. If this is the case, the substitution of (4) into (9) yields the profit differential

$$\Delta \left.\pi\right|_{\lambda=1} = \left.(\pi_1 - \pi_2)\right|_{\lambda=1} = \frac{w_1 l_1}{(\sigma-1)n}\left[1 - \left(\frac{w_1}{w_2}\right)^{\sigma-1}\right]. \tag{18}$$

However, because labor supply in region 2 is $l_2 = 0$, the wage in region 2 is $w_2 = 0$ from (3). Hence, $\Delta \left.\pi\right|_{\lambda=1} = -\infty$, which violates the equilibrium condition. Therefore, full agglomeration is never an equilibrium.[1] Stated differently, manufacturing production is always carried out in both regions by immobile workers, whose labor supply is positive. Otherwise, they earn no income and consume no good.

We have seen that the symmetric equilibrium is unstable if $\theta > \theta_B$ and $\phi_{B1} < \phi < \phi_{B2}$ and that the fully agglomerated equilibrium never exists. Nevertheless, an equilibrium for any continuous utilities always exists, as shown by Ginsburgh *et al.* (1985), and a stable equilibrium always exists, as shown by Tabuchi and Zeng (2004).

[1] $w_2 = 0$ implies zero marginal cost under the CES setting. That is, the profit-maximizing price is zero, which leads to infinite demand and profits. Hence, each firm has an incentive to migrate to the empty region.

This finding suggests the existence of a partially agglomerated equilibrium that is stable if $\theta > \theta_B$ and $\phi_{B1} < \phi < \phi_{B2}$.

In sum, we establish the following proposition:

Proposition 2 *Assume $\theta > \theta_B$.*

(i) When $\phi \in [0, \phi_{B1}) \cup (\phi_{B2}, 1]$, the symmetric configuration $\lambda^* = 1/2$ is a stable equilibrium.

(ii) When $\phi \in (\phi_{B1}, \phi_{B2})$, the partially agglomerated configuration $\lambda^* \in (1/2, 1)$ is an stable equilibrium.

From Proposition 2, we can say that as trade costs steadily fall, the spatial distribution of economic activities is initially dispersed, then partially agglomerated, and then dispersed again given $\theta > \theta_B$. The equilibrium configuration is depicted in Figure 1, where the above mentioned transition is drawn as the red arrow. For a given $\theta > \theta_B$, falling trade costs move along the arrow, where the stable equilibrium distribution of firms runs from dispersion to partial agglomeration and then redispersion. It is worth noting that the agglomeration force is strong for intermediate trade costs compared with small and large ones.

3.1 Decomposition into the four effects

We can decompose the effects of the relocation of manufacturing firms to region 1 on the profit differential $\Delta\pi$ in the neighborhood of the symmetric equilibrium $(\lambda^*, w^*) = (1/2, 1)$ into the first term $\partial\Delta\pi/\partial\lambda$ and the second term $\partial\Delta\pi/\partial w \cdot dw/d\lambda$ in the stability condition (16). The first term is the *direct impact* of λ on $\Delta\pi$ with the fixed wage $w^* = 1$, whereas the second term is the *indirect impact* of λ on $\Delta\pi$ through the wage change in the labor market clearing condition (10). To examine these effects, we consider profit in region 1

$$\pi_1 = \frac{1}{\sigma - 1}\frac{w_1 l_1}{n_1}, \tag{19}$$

which consists of $w_1 = w$ and $n_1 = n\lambda$.

On the one hand, the direct impact $\partial\Delta\pi/\partial\lambda$ in the first term of (16) is the changes in n_1 and l_1 because they are functions of λ. From (19), their impacts are in opposite directions: $\partial\pi_1/\partial n_1 < 0$ and $\partial\pi_1/\partial l_1 > 0$. An increase in the number of firms brings about the *competition effect*: the higher number of firms, the lower are profits, i.e., $\partial\pi_1/\partial\lambda = (\partial\pi_1/\partial n_1)\, n < 0$ given l_1 on the RHS of (19). An increase in the number of firms also generates the *price index effect*: an increase in the number of firms lowers the price index in region 1. When the price index is lowered, agents increase labor supply, which expands the market and raises profits, i.e., $\partial\pi_1/\partial\lambda = (\partial\pi_1/\partial l_1)\left[\partial\left(w_1/P_1\right)^\theta/\partial\lambda\right] > 0$ given n_1 on the RHS of (19).

On the other hand, the indirect impact $\partial\Delta\pi/\partial w \cdot dw/d\lambda$ in the second term of (16) is through the change in the wage. Since $\partial\Delta\pi/\partial w > 0$ always holds, the change $dw/d\lambda$ through the labor market clearing condition (10) matters. Figure 2 illustrates the labor market in region 1, where the upward sloping curve is the labor supply function given by (3):

$$l_1^S = \left(\frac{w}{P_1}\right)^\theta = k\left[\lambda + \phi\left(1-\lambda\right)w^{\sigma-1}\right]^{\frac{\theta}{\sigma-1}},$$

where $\partial l_1^S/\partial w > 0$ and $k \equiv \left(\frac{\sigma-1}{\sigma m}\right)^\theta n^{\frac{\theta}{\sigma-1}}$. The downward sloping curve is the labor demand function derived from (13) with respect to l_1 as follows:

$$l_1^D = \frac{n_1 w_2}{n_2 w_1} l_2 = \frac{k\lambda}{1-\lambda}\frac{\left(\phi\lambda w^{1-\sigma}+1-\lambda\right)^{\frac{\theta}{\sigma-1}}}{w},$$

where $\partial l_1^D/\partial w < 0$. Further, there is a unique intersection point of the two curves, which is the equilibrium (l_1^*, w_1^*). Figure 2(A) illustrates the shift in labor supply l_1^S due to the increase in λ, while Figure 2(B) presents the shift in labor demand l_1^D due to the increase in λ. The supply curve l_1^S shifts right because $\partial l_1^S/\partial\lambda \geq 0$ and this decreases the wage rate. We name this effect the *excess labor supply effect*. When λ increases, the number of firms in region 1 increases, which lowers the price index in region 1. When the price index in region 1 is lowered, agents in region 1 increase labor supply, since at the given nominal wage, the real wage in region 1 rises. Then, excess labor supply emerges with the increase in λ.

The demand curve l_1^D can shift right or left following the increase in the number of firms in region 1. We name this effect the *excess labor demand effect*. When λ increases, the number of firms is raised, which increases the labor demand. However, the increase in λ lowers the price index in region 1, which decreases labor demand there, since competition among firms in region 1 intensifies. If the former effect dominates the latter, the demand curve l_1^D shifts right and excess labor demand emerges as λ increases. On the contrary, if the latter effect outweighs the former, the demand curve shifts left. The increase in λ may increase or decrease the equilibrium wage depending on the shifts in the two curves.

It can be shown below that $dw/d\lambda < 0$ if the *excess labor supply effect* is strong, whereas $dw/d\lambda > 0$ if the *excess labor demand effect* is positive and strong. These two effects are new and do not exist in standard models with exogenous labor supply. Analysis of the indirect impact is somewhat complicated because we have to consider the labor market clearing condition.

The strength of these four effects depends on the freeness of trade ϕ as shown below. We examine the two extreme cases of near autarky and near free trade in the vicinity of the symmetric equilibrium $(\lambda^*, w^*) = (1/2, 1)$.

3.2 Near autarky $\phi \approx 0$

(i) The direct impact $\partial\Delta\pi/\partial\lambda$.

When trade is very costly, the price index is $\lim_{\phi\to 0} P_r = n_r^{\frac{1}{1-\sigma}} p_{rr}$ and profit is

$$\lim_{\phi\to 0} \pi_1 = \frac{kw\lambda^{\frac{\theta}{\sigma-1}-1}}{n}.$$

The sign of an increase in λ on π_1 depends on the exponent of λ, which is

$$\theta_B \equiv \sigma - 1 + \frac{2\sqrt{(\sigma-1)\,\sigma}}{2\sigma - 1}. \tag{20}$$

$$\frac{\theta}{\sigma-1} - 1 > \frac{\theta_B}{\sigma-1} - 1 = \frac{2\sqrt{(\sigma-1)\,\sigma}}{(\sigma-1)(2\sigma-1)} > 0$$

for all $\theta > \theta_B$ from (17). This fact implies that $\partial\pi_1/\partial\lambda > 0$, and hence, $\partial\Delta\pi/\partial\lambda > 0$ in autarky. That is, when trade is very costly, the price index effect dominates the competition effect, which encourages the symmetry break.

(ii) The indirect impact $\partial\Delta\pi/\partial w \cdot dw/d\lambda$.

Since we already know that $\partial\Delta\pi/\partial w > 0$, we investigate the sign of $dw/d\lambda$ on the labor market clearing condition (10). Near autarky, the labor supply curve l_1^S and labor demand curve l_1^D can be simplified as

$$\lim_{\phi\to 0} l_1^S = k\lambda^{\frac{\theta}{\sigma-1}}, \tag{21}$$

$$\lim_{\phi\to 0} l_1^D = \frac{k\lambda}{1-\lambda}\frac{(1-\lambda)^{\frac{\theta}{\sigma-1}}}{w}. \tag{22}$$

When ϕ is very small, the labor supply curve is almost vertical. From (21), an increase in λ raises labor supply l_1^S. Since in the equilibrium $l_1^S = l_1^D$ should be satisfied, labor demand l_1^D rises, which decreases w from (22). The labor demand curve can shift left or right with the increase in λ. When the labor demand curve shifts right with the increase in λ, the shift of the labor supply curve is larger than that of the labor demand curve because $\theta > \sigma - 1$, which results in $\partial w/\partial\lambda < 0$. When the labor demand curve shifts to left with the increase in λ, the reduction of wage is manifested by the shift of the labor demand curve. Excess labor supply because of the increase in λ reduces the wage near autarky.

(iii) The total impact $d\Delta\pi/d\lambda$.

Based on the foregoing, we have

$$\lim_{\phi\to 0} \left.\frac{d\Delta\pi}{d\lambda}\right|_{(\lambda,w)=(1/2,1)} = \left.\underset{+}{\frac{\partial\Delta\pi}{\partial\lambda}} + \underset{+}{\frac{\partial\Delta\pi}{\partial w}}\underset{-}{\frac{\partial w}{\partial\lambda}}\right|_{(\lambda,w)=(1/2,1)} < 0.$$

The positive first term implies that the price index effect dominates the competition effect, which tends to break the symmetry. However, the product of the second and third terms is negative, which implies that the excess labor supply effect dominates the excess labor demand effect, and thus discourages the symmetry break. The inequality means that *the excess labor supply effect dominates the other effects* for a prohibitive trade cost as a whole, meaning that the symmetry does not break near autarky $\phi \in [0, \phi_{B1})$.

3.3 Near free trade $\phi \approx 1$

(i) The direct impact $\partial\Delta\pi/\partial\lambda$.

When trade is almost costless, profit (19) is given by

$$\lim_{\phi\to 1} \pi_1 = \frac{kw\left[\lambda + (1-\lambda)\, w^{\sigma-1}\right]^{\frac{\theta}{\sigma-1}}}{n\lambda}.$$

Since wages are close to 1 near free trade, the bracketed term approaches 1. Therefore, $\partial\pi_1/\partial\lambda < 0$, which implies $\partial\Delta\pi/\partial\lambda < 0$. That is, when trade is costless, the competition effect is stronger than the price index effect, which interferes the symmetry break.

(ii) The indirect impact $\partial\Delta\pi/\partial w \cdot dw/d\lambda$.

We focus on the sign of $dw/d\lambda$ on the labor market clearing condition (10). Since wages are equalized between regions, the two curves in the vicinity of the symmetric equilibrium $(\lambda, w) = (1/2, 1)$ are simplified as

$$\lim_{\phi\to 1} l_1^S = k, \tag{23}$$

$$\lim_{\phi\to 1} l_1^D = \frac{k\lambda}{1-\lambda}. \tag{24}$$

When ϕ is close to 1, the labor supply curve is also almost vertical. Unlike autarky, from (24), an increase in λ raises labor demand l_1^D rather than labor supply l_1^S. Since $l_1^D = l_1^S$, labor demand l_1^S rises, which increases w from (23). That is, the shift in the labor demand curve is larger than that in the labor supply curve, which results in $\partial w/\partial\lambda > 0$. Excess demand for labor because of the increase in λ raises the wage near free trade. Hence, the indirect impact for autarky and free trade is opposite.

(iii) The total impact $d\Delta\pi/d\lambda$.

We get

$$\lim_{\phi\to 1} \left.\frac{d\Delta\pi}{d\lambda}\right|_{(\lambda,w)=(1/2,1)} = \left.\underset{-}{\frac{\partial\Delta\pi}{\partial\lambda}} + \underset{+}{\frac{\partial\Delta\pi}{\partial w}}\,\underset{+}{\frac{\partial w}{\partial\lambda}}\right|_{(\lambda,w)=(1/2,1)} < 0.$$

On the one hand, the negative first term implies that the price index effect is dominated by the competition effect, which stabilizes the symmetric equilibrium. On the other hand, the positive last term means that an increase in λ hardly affects the price index P_1 but raises the wage w_1 due to excess labor demand, which destabilizes the symmetric equilibrium. The inequality implies that *the competition effect outweighs the other effects* for costless trade, and hence, the symmetry is a stable equilibrium near free trade $\phi \in (\phi_{B2}, 1]$.

3.4 Intermediate trade costs $\phi \in (\phi_{B1}, \phi_{B2})$

When the trade costs are intermediate, the sign of $d\Delta\pi/d\lambda|_{(\lambda,w)=(1/2,1)}$ becomes positive, allowing the symmetry to break. This occurs under a sufficiently elastic labor supply (θ high), which acts as if the number of consumers changes. In such a case, in spite of the immobility of workers, firms agglomerate as in new economic geography because of *the price index effect and the excess labor demand effect.*

More precisely, we can say the following.

Lemma 2 *Assume that $\theta > \theta_B$. In the vicinity of the symmetric equilibrium, we have* *(i)* $\frac{\partial \Delta\pi}{\partial \lambda} \gtreqless 0$ *for* $\phi \lesseqgtr \phi_1 \equiv \frac{\theta-\sigma+1}{\theta+\sigma-1} \in (0,1)$,

(ii) $\frac{\partial \Delta\pi}{\partial w}\frac{\partial w}{\partial \lambda} \lesseqgtr 0$ *for* $\phi \lesseqgtr \phi_2 \equiv \frac{(\theta-2\sigma)(\sigma-1)+\sqrt{\sigma\left[\theta^2\sigma-4(\theta-\sigma)(\sigma-1)^2\right]}}{\theta(2\sigma-1)} \in (0,1)$.

In particular, if $\phi_2 < \phi_1$, then there exists $\phi \in (\phi_2, \phi_1)$. In this case, both the direct and the indirect impacts are positive, and thus, the total impact is also positive. Since the symmetric equilibrium is unstable, a stable asymmetric equilibrium must exist. Even if $\phi_2 \geq \phi_1$, a stable asymmetric equilibrium may occur for intermediate trade costs insofar as the *price index effect* and/or the *excess labor demand effect* are strong enough.

4 Asymmetric Equilibrium

In the case of the symmetric equilibrium, there is no room for international differential. However, in the case of the asymmetric equilibrium, which occurs if $\theta > \theta_B$ and $\phi_{B1} < \phi < \phi_{B2}$, wages and differ by region. To investigate such differentials, we explore the two equilibrium conditions in more detail.

By solving (5) for n_1 and n_2 and substituting them into the spatial equilibrium condition (12) and the labor market clearing condition (10) with $Q \equiv P_1/P_2$, they can be expressed by Q and w. By manipulating them, we can show the following statement on the wage differential.

Proposition 3 *If trade costs are high* $\phi < 1/(2\sigma-1)$*, the nominal wage,* w_r *is lower in the agglomerated region. Otherwise* $\phi > 1/(2\sigma-1)$*, the nominal wage is higher in the agglomerated region.*

The proof is presented in Appendix A. It is somewhat surprising that the wage is lower in the agglomerated region, which usually does not occur under new economic geography or new trade theory with immobile workers.

We explained in section 3.2 that excess labor supply because of the increase in λ reduces the wage near autarky. A similar intuition can be applied to high trade costs $\phi < 1/(2\sigma-1)$. We also explained in section 3.3 that excess labor demand because of the increase in λ raises the wage near free trade. A similar intuition is applied for low trade costs $\phi > 1/(2\sigma-1)$.

Finally, by examining the other differential indices, we are able to establish the following results.

Proposition 4 *In the asymmetric equilibrium, price index P_r^* is always lower, while labor supply l_r^*, wage earning $w_r^* l_r^*$, per capita nominal income $a^* + w_r^* l_r^*$, real wage w_r^*/P_r^*, and welfare V_r^* are always higher in the agglomerated region.*

The proof is presented in Appendix B. Price index P_r^* is lower in the agglomerated region because more firms supply varieties without trade costs. In this region, the relative value of the nominal wage to the price index is higher, which raises labor supply l_r^* from (3). Consequently, wage earning $w_r^* l_r^*$, per capita income $a^* + w_r^* l_r^*$ and real wage w_r^*/P_r^* are higher in the agglomerated region. These higher values outweighs the disutility from labor supply, which leads to higher welfare V_r^* in this region.

Proposition 4 states that labor supply is larger in the agglomerated region with higher nominal wage earning and per capita income. This is consistent with the facts presented in the introduction. Large labor supply brings about higher per capita income in this region, which expands its market size. This in turn attracts manufacturing firms. As a result, workers enjoy better access to a large market and are better off with a higher real wage and welfare in the agglomerated region.

Finally, we check if the home market effect is exhibited in the presence of an elastic labor supply. This effect is normally defined as "a more-than-proportional relationship between a country's share of the world production of a good and its share of world demand for the same good" (Crozet and Trionfetti, 2008). By using the equilibrium condition (13), we can easily show that if $\lambda > 1/2$, then

$$\frac{n_1}{n_2} > \frac{(a + w_1 l_1)\, L_1}{(a + w_2 l_2)\, L_2}$$

always holds. Furthermore, the effect is also defined that "countries tend to export those kinds of products for which they have relatively large domestic demand" (Krugman, 1980). This is true if the following ratio exceeds 1:

$$\frac{n_1 p_{12} x_{12}}{n_2 p_{21} x_{21}} = \frac{n_1 w_1^{1-\sigma}}{n_2 w_2^{1-\sigma}} \left(\frac{P_1}{P_2}\right)^{1-\sigma} \frac{a + w_1 l_1}{a + w_2 l_2} \tag{25}$$

We know from Proposition 4 that $P_1^{1-\sigma} > P_2^{1-\sigma}$, which implies $n_1 w_1^{1-\sigma} > n_2 w_2^{1-\sigma}$ because

$$P_1^{1-\sigma} - P_2^{1-\sigma} = (1 - \phi) \left(\frac{m\sigma}{\sigma - 1}\right)^{1-\sigma} \left(n_1 w_1^{1-\sigma} - n_2 w_2^{1-\sigma}\right).$$

We also know from Proposition 4 that $w_1 l_1 > w_2 l_2$. Thus, the three terms on the RHS of (25) are greater than 1 for all $\lambda > 1/2$. Hence, the home market effect is necessarily exhibited even under an elastic labor supply.

5 Different sized regions

So far, the mass of immobile workers was the same between regions. In this section, we consider the case of different population sizes between regions, $L_1 > L_2$, to explore the size effect on the spatial distribution of economic activities.

By using the parameter values of $(L_1, L_2, \sigma, \theta, n, m) = (2, 1, 3, 2, 1, 1)$, the interregional differential indices are plotted in Figures 3 and 4. In Figure 3, the blue curve is region 1's firm share $\lambda(= n_1/n)$, the red curve is the nominal wage differential w_1/w_2, and the yellow curve is the real wage differential $(w_1/P_1)/(w_2/P_2)$. In Figure 4, the blue curve is the utility differential V_1/V_2), the red curve is the differential in working hours l_1/l_2, and the yellow curve is the price index differential P_2/P_1. It is worth noting that all the curves are inverted U-shaped. The property remains the same for different parameter values.

Several observations can be made from these figures. First, as to the firm share, we observe $n_1/n_2 > L_1/L_2$ for all $0 < \phi < 1$, implying that the home market effect is always exhibited, as confirmed by most studies in new trade theory. Second, the nominal wage in the larger region is smaller for small ϕ, but larger for large ϕ, which is in accord with Proposition 3. Third, the price index is always lower, while labor supply, wage earning, per capita nominal income, real wage, and welfare are always higher in the larger region for all $0 < \phi < 1$. This finding is in accord with Proposition 4. Note that the second and third results are based on different population sizes, while Propositions 3 and 4 are based on the same population size. Finally, all the differential indices converge when the two regions are fully integrated $\phi = 1$.

Analytical results can be obtained in autarky and in full integration. In autarky $\phi = 0$, we have

$$\begin{aligned} \frac{n_1}{n_2} &= \frac{L_1}{L_2},\ \frac{w_1}{w_2} = \left(\frac{L_2}{L_1}\right)^{\frac{\theta}{\sigma-1}} < 1,\ \frac{l_1}{l_2} = \left(\frac{L_1}{L_2}\right)^{\frac{\theta}{\sigma-1}} > 1, \\ \frac{P_1}{P_2} &= \left(\frac{L_2}{L_1}\right)^{\frac{\theta+1}{\sigma-1}} < 1,\ \frac{V_1}{V_2} = \left(\frac{L_1}{L_2}\right)^{\frac{\theta+1}{\sigma-1}} > 1. \end{aligned}$$

From the comparative static analysis, we also have $\partial\lambda/\partial\phi > 0$ at $\phi = 0$, and $\partial\lambda/\partial\phi < 0$, $\partial w/\partial\phi < 0$ at $\phi = 1$. These results imply that the home market effect is shown to exhibit at least near autarky and near free trade, while the nominal wage in the larger region is shown to be smaller near autarky, but larger near free trade.

In full integration $\phi = 1$, we get convergence:

$$\frac{n_1}{n_2} = \frac{L_1}{L_2}, \ \frac{w_1}{w_2} = \frac{l_1}{l_2} = \frac{P_1}{P_2} = \frac{V_1}{V_2} = 1$$

as expected.

6 Conclusion

In this study, we introduced an elastic labor supply into the framework of new economic geography and examined the impacts of trade costs on the equilibrium outcomes of working hours and the spatial distribution of economic activities. Despite the symmetric distribution of immobile workers between two regions, we found that when trade costs are intermediate and labor supply is sufficiently elastic, the symmetry breaks. This finding is in sharp contrast to the body of literature on new economic geography. We also showed that the price index is always lower, whereas labor supply, wage earning, per capita income, real wage, and welfare are always higher in the agglomerated region.

The estimates of the wage elasticity of labor supply are less than 2 and those of the elasticity of substitution often exceed 3 in the literature. Since they satisfy the sufficient condition for stability given by (17), the symmetry does not break for all ϕ. This implies that an elastic labor supply is not a strong agglomeration force. For the emergence of agglomeration, labor migration may be needed as in the new economic geography framework. The introduction of labor migration into our model would be an important future extension. It might also be important for future studies to incorporate commuting costs into our model because they may be regarded as a part of working hours for workers, or to study how government policies on income tax or a commuting subsidy, for example, affect social welfare.

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Appendix A: Proof of Proposition 3

By manipulating the spatial equilibrium condition (13), we get

$$w^{\theta+1} = w^{\sigma-1} Q^{\theta} \frac{1-\phi Q^{\sigma-1}}{Q^{\sigma-1}-\phi}. \tag{26}$$

Likewise, the labor market clearing condition can be rewritten as

$$w^{\theta+1} = \frac{Q^{\theta}\left(1-\phi Q^{\sigma-1}\right)\left[Q^{\sigma-1}+\phi\left(2\sigma-1\right)\right]}{\left(Q^{\sigma-1}-\phi\right)\left[1+\phi\left(2\sigma-1\right)Q^{\sigma-1}\right]}. \tag{27}$$

By equating (26) with (27) and using the new variables $W \equiv w^{\sigma-1} \in (\phi^{\sigma-1}, \phi^{1-\sigma})$ and $R \equiv Q^{\sigma-1} \in (\phi, \phi^{-1})$, we have

$$\frac{W-1}{R-1} = \frac{2\sigma-1}{1+\phi\left(2\sigma-1\right)R}\left(\frac{1}{2\sigma-1}-\phi\right). \tag{28}$$

Because

$$\mathrm{sgn}\left(\frac{W-1}{R-1}\right) = \mathrm{sgn}\left(\frac{1}{2\sigma-1}-\phi\right)$$

from (28) and because

$$\mathrm{sgn}\left(1-R\right) = \mathrm{sgn}\left(\lambda-\frac{1}{2}\right), \tag{29}$$

from the definition of P_r and Q_r, we obtain that if $\lambda > 1/2$, then

$$w_1 \lesseqgtr w_2 \ \Leftrightarrow\ \phi \lesseqgtr \frac{1}{2\sigma-1}.$$

Appendix B: Proof of Proposition 4

Assume $\lambda^* > 1/2$.

(i) Proof of $P_1^* < P_2^*$. From (29), we immediately have

$$\mathrm{sgn}\left(\lambda^* - \frac{1}{2}\right) = \mathrm{sgn}\left(1 - Q^*\right) = \mathrm{sgn}\left(P_2^* - P_1^*\right).$$

(ii) Proof of $l_1^* > l_2^*$. From (3), we get

$$\frac{l_1^*}{l_2^*} = \left(\frac{w^*}{Q^*}\right)^{\theta} = \left(\frac{W^*}{R^*}\right)^{\frac{\theta}{\sigma-1}},$$

where W^* is a function of R^* given by (28). Because $\partial\left(W^*/R^*\right)/\partial R^* < 0$ and $W^*/R^*|_{R^*=1} = 1$ hold, we have

$$\mathrm{sgn}\left(\lambda^* - \frac{1}{2}\right) = \mathrm{sgn}\left(1 - R^*\right) = \mathrm{sgn}\left(l_1^* - l_2^*\right).$$

(iii) Proof of $w_1^* l_1^* > w_2^* l_2^*$. We showed in Lemma 1.

(iv) Proof of $w_1^*/P_1^* > w_2^*/P_2^*$. This is obvious from $l_1^* > l_2^*$ together with (3).

(v) Proof of $V_1^* > V_2^*$. From (14), we get

$$V_1^* - V_2^* = \frac{P_2^*}{2\left(\sigma-1\right)\left(\theta+1\right)Q^*}\left\{\left(\theta+1\right)\left[1 - \left(Q^*\right)^2\left(m^*\right)^{\frac{\theta+1}{\theta}}\right] - \left(2\sigma+\theta-1\right)Q^*\left[1 - \left(m^*\right)^{\frac{\theta+1}{\theta}}\right]\right\},$$

where $m^* \equiv l_1^*/l_2^*$. Since $\partial\left(V_1^* > V_2^*\right)/\partial m^* > 0$ holds for all $Q^* < 1$, we have $V_1^* - V_2^* > V_1^* - V_2^*|_{m^*=1} > 0$ for all $Q^* < 1$, i.e., for all $\lambda^* > 1/2$.

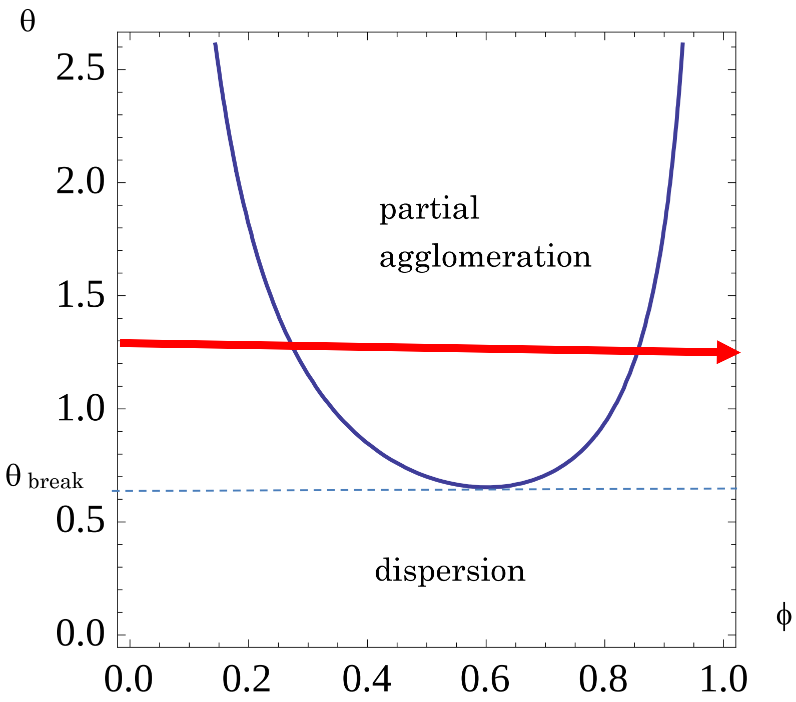

Figure 1:　Stable equilibrium distribution of firms

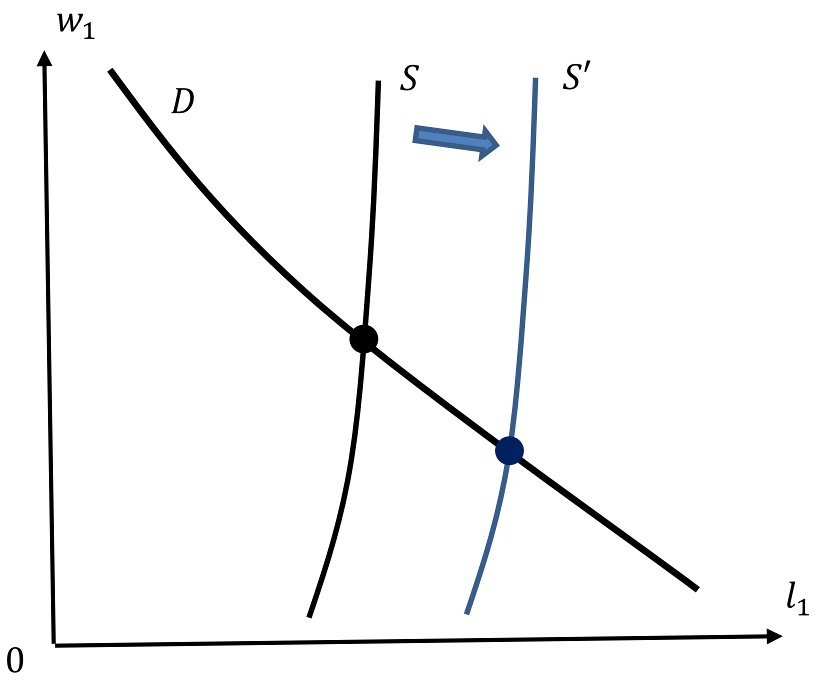

Figure 2(A):　Supply shift due to excess labor supply effect

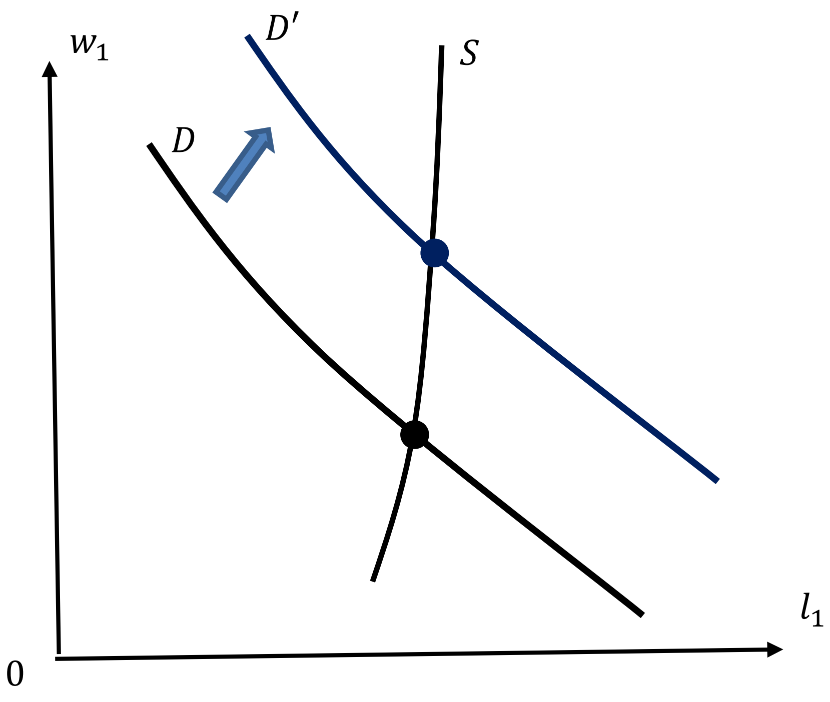

Figure 2(B):　Demand shift due to excess labor demand effect

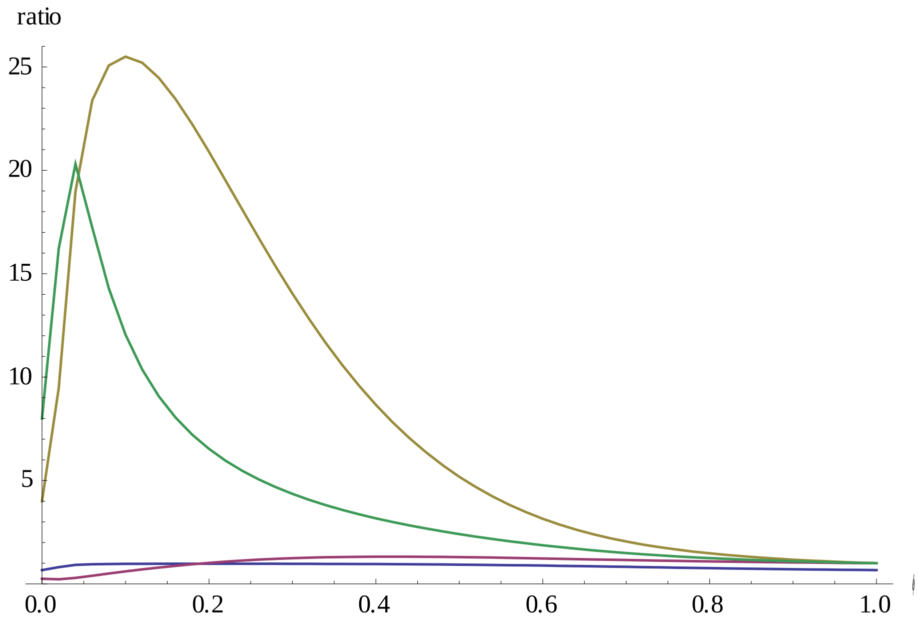

Figure 3(A):　Interregional differential indices

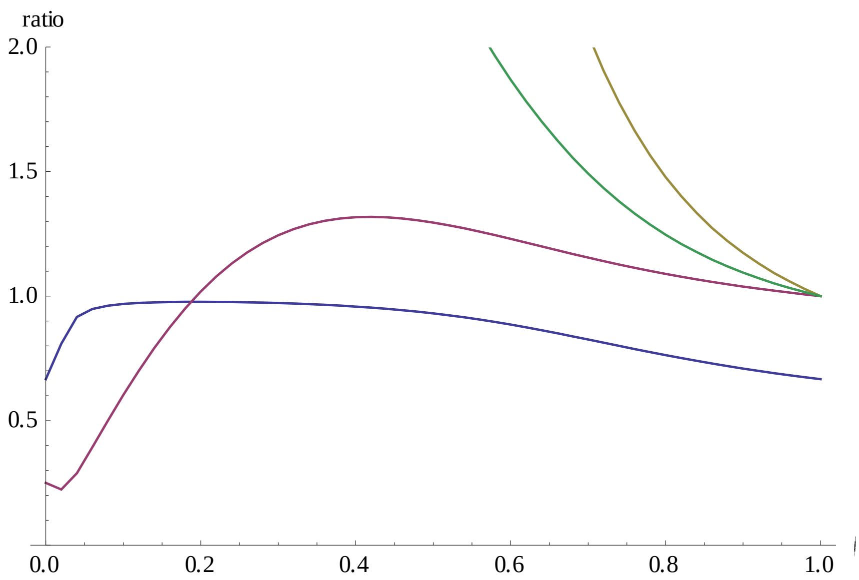

Figure 3(B):　Interregional differential indices (same as Figure 3(A) with a shorter vertical axis)