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Abstract

This paper introduces team production into a two-sector Ricardian comparative advantage model having two types of agents, high-skilled and low-skilled, each with comparative advantage in one of the two sectors under self-production. A team is an organization in which one high-skilled agent manages low-skilled workers, allowing the latter to use their manager's knowledge, and thus resulting in a more efficient outcome than self-production. This paper conducts a comparative statics analysis to understand how the allocation of the high-skilled agents in a sector in which they do not have comparative advantage under self-production is affected by team production in that sector. The analysis provides two implications: First, team production changes the nature of comparative advantage, possibly leading to reallocation of the high-skilled agents from the sector for which they initially have comparative advantage to the other sector where the environment of team production improves. Second, the likelihood of shift is limited, and, in the case of shift, non-monotonic dynamics are likely to occur; namely, if a shift occurs, then redispersion follows.

Keywords: Team production, Ricardian comparative advantage, Local advantage
JEL classification: F11

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1 Introduction

Under globalization, the importance of regional resources as a source of comparative advantage has been increasing for local policy makers in developed countries in promoting local development. Global sourcing (Antràs et al., 2014) has made it difficult for local economies to have comparative advantage in easily reproducible activities such as mass production, because production costs, including labor cost, are higher in those economies than in less developed countries. In addition, it is also difficult for developed countries to have comparative advantage in skill-intensive activities such as management and research and development, because larger cities already have comparative advantage in those activities in the modern specialization of cities (Duranton and Puga, 2001, 2005).

In an approach to overcome this challenge, local governments promote the matching of high-skilled or creative workers in larger cities with local resources or workers. Whether this approach would be effective depends on how the allocation of the high-skilled across sectors is affected by cooperation between the high-skilled and low-skilled workers. To examine this issue, I construct a two-sector Ricardian comparative advantage model with team production and two types of agents, high-skilled and low-skilled. The two sectors are the global and local sectors; the former (latter) is defined as the one in which the high-skilled (low-skilled) have a comparative advantage. In both sectors, team production is allowed, connecting one high-skilled agent as manager to some low-skilled agents as workers. The cost of team production is the time spent for communication between the high-skilled manager and low-skilled workers; the benefit of team production is that the manager can leverage her knowledge effectively in a team and low-skilled workers can use their manager's knowledge. From an interpretation that global and local sectors correspond to larger and smaller cities, respectively, I introduce another form of team production in the local sector, where managers learn about their local advantages through communications with workers. While the benefit of this learning can be considered as a productivity gain, the cost is measured in terms of time.

Given the model, I conduct a comparative statics analysis of the share of the high-skilled in the local sector with respect to three parameters, communication cost of team production in the local sector, learning cost in the local sector, and productivity gain from learning, for millions of parameters for which they engage in the global sector initially. The main result is that the likelihood of shift of the high-skilled from the global to the local sector is limited, and even if a shift does occur, it is most likely associated with the inverted-U-shaped dynamics of allocation of the high-skilled. For example, a decrease in communication cost is first associated with a shift of the high-skilled from the global to the local sector and then their redispersion from the local sector. This non-monotonicity is a result of two counteracting forces: an increase in relative supply in the local sector, *a supply-expanding effect*, due to an increase in productivity from team production in that sector; and an increase in relative demand in the local sector good, *a demand-expanding effect*, due to a decrease in relative price in that sector caused by potential entrants. When team production represents only a small fraction of producers in the local sector, a decrease in communication cost in that sector results in the supply-expanding effect becoming smaller than the demand-expanding effect, excess demand in the local sector, and reallocation of the high-skilled from the global to the local sector. When team production represents a sufficiently large fraction of producers in the local sector, a decrease in communication cost results in the supply-expanding effect becoming larger than the demand-expanding effect and in a reallocation of the high-skilled from the local to the global sector.

This paper is related to two areas of research: First, the model extends Garicano and Rossi-Hansberg (2006) to multiple sectors and an additional form of team production. Second, it is related to knowledge creation, such as Berliant and Fujita (2012) . However, the focus of this study is on the collaboration between high-skilled and low-skilled agents, not between creative people.

The rest of this paper is organized as follows. Section 2 describes the structure of the model. Section 3 presents the policy implications derived from focusing on equilibria resembling the functional specialization of cities. Finally, Section 4 concludes the paper.

2 The Model

2.1 Environment

I consider a Ricardian closed economy with two competitive sectors, global g and local ℓ, and two types of agents, high-skilled h and low-skilled l. ℓ agents have a comparative advantage in ℓ sector, the relative price of good, denoted by $p > 0$. Production is specified as problem solving: Given one unit of time endowment, each agent draws one problem per unit of time, each unit associated with some level in $(0,1)$ of knowledge required to solve the problem. An i agent's level of knowledge in k sector is denoted by $k_i \in (0,1)$, and the law of large numbers implies that the sector-g (-ℓ) income of i agents is given by g_i ($p\ell_i$) under self-employment. I assume the following absolute and comparative advantages: $g < l < g_h$; $\ell_l < \ell_h$; and $g_l/\ell_l < g_h/\ell_h$. The relative supply of l agents is fixed at $\rho > 1$.

2.2 Team Production

In both sectors, team production á la Garicano and Rossi-Hansberg (2006) is allowed in addition to self-employment. More precisely, the economy has one common and one ℓ-specific form of team production.

2.2.1 Common Form

In a common form team in sector k, one h-agent manager and n_k l-agent workers constitute a team. First, the workers draw and try to solve problems by themselves, implying that $(1-k_l)n_k$ problems are left unsolved. Second, the workers pass $(1-k_l)n_k$ unsolved problems to their manager with communication cost c_k per unit of problems. Finally, the manager guides the workers on how to fix those problems if she knows, and the workers solve the problems. Thus, the team as a whole can solve $k_h n_k$ problems. The manager's time constraint determines the team size, $n_k = 1/[c_k(1-k_l)]$.

In this study, I focus on a case in which communication cost c_k satisfies

$$\frac{1}{\rho(1-g_l)} < c_g < \frac{g_h - g_l}{g_h(1-g_l)}; \quad \frac{1}{\rho(1-\ell_l)} < c_\ell < \frac{\ell_h - \ell_l}{\ell_h(1-\ell_l)}, \tag{1}$$

implying that team production is more productive than self-employment, and that l agents have no bargaining power in wage determination. Given this assumption, h managers exploit the rents of their teams, given by the zero-profit condition.

2.2.2 ℓ-specific Form

With an ℓ-specific team production, an h agent can invest her time (in addition to communication cost) to raise her productivity from ℓ_h to $a\ell_h$, where $a \in (1, \ell_h^{-1})$. This can be interpreted as follows: h managers can apply their knowledge suitable to activities in g sector to the ℓ sector and still earn income more than l agents do. However, on learning about the local advantages such as scenery, culture, and history, the quality of output increases further. Rather than simply designing a conventional building in beautiful scenery, designing a building in harmony with such nature makes the place more valuable. The time cost of learning is specifically given by the iceberg-type cost $\tau > 1$; that is, with learning, passing $(1-\ell_l)n_\ell$ unsolved problems in a team would cost the manager $\tau c_\ell(1-\ell_l)n_\ell$ units of time.

2.3 l-agent Choice

Since l agents cannot become managers and have no bargaining power,[2] income levels are equalized across self-employment and workers in teams in both sectors. The resulting environment is exactly the same as in a simple Ricardian model; that is, letting $p_l^* \equiv g_l/\ell_l$, l agents choose g sector if $p < p_l^*$ and ℓ sector otherwise. This optimal choice implies that the wage rate w_l of l agents can be given as follows: $w_l = g_l$ if $p < p_l^*$, and $w_l = p\ell_l$ otherwise.

2.4 h-agent Choice

In addition to self-employment, h agents can become manager of a team in either g or ℓ sector. In addition, if an h agent chooses to form a team in ℓ sector, she must also choose which type of team to form. For notational convenience, let g_s and ℓ_s denote self-employment in g sector and ℓ sector, respectively. Also, let g, $\ell_{w/}$, and $\ell_{w/o}$ denote a team in g sector, a team with learning in ℓ sector, and a team without learning in ℓ sector, respectively. Therefore, h agents choose any one of

[2] The former is not an exogenous assumption. That is, although l agents can form a team, there is no productivity gain, and so no agents would be willing to participate in such a team.

$\{g_s, \ell_s, g, \ell_{w/}, \ell_{w/o}\}$. Note that for a chosen form $f \in \{g, \ell_{w/}, \ell_{w/o}\}$ of a team, productivity z_f and team size n_f are determined implying that the wage rate $w_{h,f}$ of h manager is given by $w_{h,f} = (z_f - w_l)n_f$.

Since the wage rate $w_{h,f}$ of managers depends on the wage rate w_l of l workers, which in turn depends on the relative price p, h-agent's choice should be discussed conditional on p.

2.4.1 Indifference Curves: $p < p_l^*$

If $p < p_l^*$, the wage rate w_l of l workers can be given by $w_l = g_l$. On account of comparative advantage, h agents choose g_s if their choice is self-employment. However, note that g_s is never chosen by h agents when $p < p_l^*$. This is simply because $w_{h,g} > g_s$, which holds under (1).

Therefore, h agents are effectively faced with three options: g, $\ell_{w/}$, or $\ell_{w/o}$. The following equations are associated with the indifference curves:

$$I_{\ell_{w/}\sim\ell_{w/o}} \quad : \quad \frac{pa\ell_h - g_l}{\tau c_\ell(1-\ell_l)} = \frac{p\ell_h - g_l}{c_\ell(1-\ell_l)} \quad \Longrightarrow \quad p = \frac{g_l}{\ell_h}\frac{\tau - 1}{\tau - a}(\tau \neq a) \tag{2}$$

$$I_{g\sim\ell_{w/}} \quad : \quad \frac{g_h - g_l}{c_g(1-g_l)} = \frac{pa\ell_h - g_l}{\tau c_\ell(1-\ell_l)} \quad \Longrightarrow \quad p = \frac{g_l}{a\ell_h}\left[1 + \tau\frac{c_\ell(1-\ell_l)}{c_g(1-g_l)}\frac{g_h - g_l}{g_l}\right] \tag{3}$$

$$I_{g\sim\ell_{w/o}} \quad : \quad \frac{g_h - g_l}{c_g(1-g_l)} = \frac{p\ell_h - g_l}{c_\ell(1-\ell_l)} \quad \Longrightarrow \quad p = \frac{g_l}{\ell_h}\left[1 + \frac{c_\ell(1-\ell_l)}{c_g(1-g_l)}\frac{g_h - g_l}{g_l}\right]. \tag{4}$$

For notational convenience, let $p_{\ell_{w/}\sim\ell_{w/o}}$ denote the relative price corresponding to the indifference curve associated with $\ell_{w/} \sim \ell_{w/o}$. In a similar manner, I use similar notations for the other cases. When emphasizing that the relative price is a function of some parameter θ, I use an expression such as $p_{\ell_{w/}\sim\ell_{w/o}}(\theta)$.

2.4.2 Indifference Curves: $p > p_l^*$

If $p > p_l^*$, the wage rate w_l of l workers can be given by $w_l = p\ell_l$. On account of comparative advantage, h agents also choose g_s if their choice is self-employment. In this case, preferring g to g_s is not necessarily the case since their choice depends on the relative price p.

Therefore, h agents are effectively faced with four options: g_s, g, $\ell_{w/}$, or $\ell_{w/o}$. The following equations are associated with the indifference curves:

$$I_{\ell_{w/}\sim\ell_{w/o}} \quad : \quad \frac{pa\ell_h - p\ell_l}{\tau c_\ell(1-\ell_l)} = \frac{p\ell_h - p\ell_l}{c_\ell(1-\ell_l)} \quad \Longrightarrow \quad \tau = \frac{a\ell_h - \ell_l}{\ell_h - \ell_l}, \tag{5}$$

$$I_{g\sim\ell_{w/}} \quad : \quad \frac{g_h - p\ell_l}{c_g(1-g_l)} = \frac{pa\ell_h - p\ell_l}{\tau c_\ell(1-\ell_l)} \quad \Longrightarrow \quad p = \frac{g_h}{\ell_l + (a\ell_h - \ell_l)\frac{c_g(1-g_l)}{\tau c_\ell(1-\ell_l)}}, \tag{6}$$

$$I_{g\sim\ell_{w/o}} \quad : \quad \frac{g_h - p\ell_l}{c_g(1-g_l)} = \frac{p\ell_h - p\ell_l}{c_\ell(1-\ell_l)} \quad \Longrightarrow \quad p = \frac{g_h}{\ell_l + (\ell_h - \ell_l)\frac{c_g(1-g_l)}{c_\ell(1-\ell_l)}}, \tag{7}$$

$$I_{g_s\sim\ell_{w/}} \quad : \quad g_h = \frac{pa\ell_h - p\ell_l}{\tau c_\ell(1-\ell_l)} \quad \Longrightarrow \quad p = g_h\frac{\tau c_\ell(1-\ell_l)}{a\ell_h - \ell_l}, \tag{8}$$

$$I_{g_s\sim\ell_{w/o}} \quad : \quad g_h = \frac{p\ell_h - p\ell_l}{c_\ell(1-\ell_l)} \quad \Longrightarrow \quad p = g_h\frac{c_\ell(1-\ell_l)}{\ell_h - \ell_l}, \tag{9}$$

$$I_{g_s\sim g} \quad : \quad g_h = \frac{g_h - p\ell_l}{c_g(1-g_l)} \quad \Longrightarrow \quad p = \frac{g_h}{\ell_l}[1 - c_g(1-g_l)] \tag{10}$$

3 Results

3.1 h-agent Choice in (θ, p) Coordinates

Assuming that the initial parameters satisfy

$$1 < c_g(1-g_l) + c_\ell(1-\ell_l)\frac{\ell_l}{\ell_h - \ell_l}, \tag{11}$$

h-agent's choice can be summarized in (θ, p) coordinate,[3][4] where θ is any one of three parameters of interest, learning cost τ (Figure 1-3), productivity gain a (Figure 4), and ℓ-sector communication cost c_ℓ (Figure 5-6). In case of τ, three patterns exist: Pattern 1 (Figure 1), where $c_\ell/c_g < c_1^*$; Pattern 2 (Figure 2), where $c_\ell/c_g > c_1^*$ and $\hat{\tau}_2 > 1$; and Pattern 3 (Figure 3), where $c_\ell/c_g > c_1^*$ and $\hat{\tau}_2 < 1$. In case of c_ℓ, there are two patterns: Pattern 1 (Figure 5), where $\hat{\tau}_1 < \tau$, and Pattern 2 (Figure 6), where $\tau < \hat{\tau}_1$.

Under the above assumption, there exists a range of g_s of some positive measure in (θ, p) coordinate. The condition can be rewritten as $p_{g\sim g_s} < p_{g_s\sim\ell_{w/o}}$, where $p_{g\sim g_s}$ and $p_{g_s\sim\ell_{w/o}}$ are given by (10) and (9), respectively. By imposing the above assumption, I focus on a "severe" situation for ℓ sector, in that the shift in h-agent's choice from g sector to ℓ sector is not smooth.

[3] Under condition (11), there is a region of self-employment in sector g, that is, g_s, in (θ, p) coordinate. This implies that as the relative price p of sector-ℓ good increases from a sufficiently low level, although h agents choose team production in g sector eventually, they might choose self-employment in g sector for some intermediate level of p. Therefore, condition (11) introduces a disadvantage in shifting the high-skilled from g sector to ℓ sector. This allows for considering the effects of a decrease in τ and c_ℓ and an increase in a in shifting the high-skilled in a severe situation.

[4] The definitions of thresholds are given in Appendix A.1.1–A.1.3.

3.2 Relative Supply Curves

Given h-agent's choice in Subsection 3.1, the shifts of the relative supply curve of ℓ good for decreasing τ, increasing a, and decreasing c_ℓ are obtained as shown in Figure 7-10, Figure 11-14, and Figure 15-21, respectively.[5] For ease of exposition, the lower and upper bounds of (τ, a, c_ℓ) are omitted.

3.3 Numerical Experiment

3.3.1 Equilibrium of Interest

In this study, I focus on the simplest case of agents' preference specified by a Cobb–Douglas function with expenditure shares α_g and α_ℓ of g and ℓ goods, that is, $\alpha_g + \alpha_\ell = 1$, implying that the relative demand for ℓ good is given by αp^{-1}, where $\alpha \equiv \alpha_\ell/\alpha_g > 0$. Depending on the ratio α, various equilibria can be obtained, and hence there can be several "dynamics" of ℓ-sector share λ_h in h agents when τ or c_ℓ decreases or a increases.

Therefore, I focus on the equilibria with the following properties: First, α must satisfy

$$\frac{g_l}{g_h}[\rho c_g(1-g_l)-1] < \alpha < [\rho c_g(1-g_l)-1][1-c_g(1-g_l)]. \tag{12}$$

Second, the equilibria must hold such that $\max\{\hat{\tau}_2, 1\} < \tau$, which is equivalent to $1 < a < \hat{a}_2$, given that $\tau > 1$; or $\hat{c}_{\ell,1} < c_\ell$ if $\hat{\tau}_1 < \tau$ and $\tilde{c}_{\ell,1} < c_\ell$ otherwise.

In this type of equilibrium, where the equilibrium relative price of sector-ℓ good satisfies $p_l^* < p < p_{g\sim g_s}$, all h agents are initially g-team managers, whereas l agents are either employed by those managers or self-employed in ℓ sector. To some extent, this captures the functional specialization of cities (Duranton and Puga, 2005), in the sense that production in the 2nd-nature industries, corresponding to g sector, features organizations where the high-skilled specialize in skill-intensive activities and the low-skilled specialize in less-skill-intensive activities under supervision from their headquarters.

The possible scenarios of the dynamics of λ_h are illustrated in Figure 24-36. For decreasing τ, increasing a, and decreasing c_ℓ, Scenario 0-4, Scenarios 0-1 and A1-A2, and Scenarios 0-4 and C1-C6, respectively, apply.[6]

3.3.2 Results of Monte Carlo Simulation

Finally, for each of the three parameters of interest, I conduct Monte Carlo simulation, where one million samples satisfying (1), (11), (12), and $\max\{\hat{\tau}_2, 1\} < \tau$ are generated at random, and a comparative statics analysis is conducted for each sample. In this experiment, the upper bounds, $\bar{\rho}$ and $\bar{\tau}$, must be set for the relative supply ρ of ℓ agents and the learning cost τ, while the above conditions give the other restrictions to the parameter ranges. [7] I then compute the share of each possible scenario in the

[5] The definitions of the relative quantities are given in Appendix A.1.4.

[6] These possible scenarios of comparative statics of λ_h are identified analytically.

[7] Appendix A.2 describes the procedure of Monte Carlo simulations.

samples, referred to as “measure” below.

Compared with the other two parameters, a decrease in ℓ-sector communication cost c_ℓ is most effective in shifting h agents from the global to local sectors, in that the measure of Scenario 0 is lowest for most of the pair $(\bar{\rho}, \bar{\tau})$ of the upper bounds for the relative supply of l agents and learning cost (Figure 22).

As for $\ell_{w/}$-specific parameters (τ, a), no clear ranking exists, in that the measure of Scenario 0 tends to be higher in τ-λ_h dynamics when $\bar{\tau}$ is low whereas the contrary holds when $\bar{\tau}$ is high (Figure 22). This suggests that effective policy targets depend on cases. When learning cost τ is high ($\bar{\tau}$ is high), team production with learning becomes costly, limiting the effect of increasing productivity gain a.

I also report the measure of each scenario other than Scenario 0 for each dynamics in Figure 23. The left, center, and right panels give the measures of scenarios in τ-λ_h, a-λ_h, and c_ℓ-λ_h dynamics, and the lower, middle, and upper panels give the measures of scenarios for low, middle, and high $\bar{\tau}$, respectively.

At least two properties common across all dynamics exist. First, as the measure of Scenario 0 decreases from changes in $\bar{\tau}$, the measures of other scenarios shift upward.

Second, for a lower $\bar{\rho}$, there could be an equilibrium with $\lambda_h \in [0,1)$, while an equilibrium with $\lambda_h = 1$ does occur for a higher $\bar{\rho}$. More specifically, for a low $\bar{\rho}$, Scenario 2 (or Scenario A1 in the case of a-λ_h dynamics) and Scenario 1 are likely to occur, whereas the scenarios with $\lambda_h = 1$ occur with zero probability for a sufficiently low $\bar{\rho}$. However, for a high $\bar{\rho}$, scenarios with $\lambda_h = 1$ are more likely to occur, and among those scenarios, Scenario 4 in τ-λ_h dynamics, Scenario A2 in a-λ_h dynamics, and Scenario 3 in c_ℓ-λ_h dynamics are of measures comparable with those of the scenarios with $\lambda_h \in [0,1)$.

The second property arises from the general equilibrium effects of the relative supply ρ of l agents. When ρ is relatively large, the relative demand for ℓ good is more likely to be higher than the relative supply, resulting in all h agents engaging in team production in ℓ sector, because the supply for g good including that from self-employment of l agents, those not employed by h agents, increases as the relative supply ρ of l agents increases.

An important policy implication observed in Figure 23 is that when encouraging team production in ℓ sector, that is, with improvement in cost and benefit (τ, a, c_ℓ), the effects on λ_h are likely to be non-monotonic.[8] Specifically, Scenario 2 (or Scenario A1) has the highest measure except for Scenario 0.[9] The key to understanding non-monotonicity is that improvement, for example, a decrease in communication cost c_ℓ, has two counteracting forces: an increase in relative supply of ℓ sector good,

[8] According to the *Labor Force Survey 2014* compiled by the Statistics Bureau, Ministry of Internal Affairs and Communications, the sample value of ρ is 7.6, calculated as ratio of the total number of employed excluding “Administrative and managerial workers,” “Engineers,” and “Other professional and engineering workers” to the number of the three occupations. Under this value of ρ, the most likely scenario except for Scenario 0 is characterized with monotonicity for all the three parameters of interest.

[9] Note that Scenario A1 has no decreasing phase in a-λ_h because the relative demand and supply have the same elasticity with respect to a. However, this phase can be interpreted as ineffectiveness of the policies for increasing productivity gain a.

a supply-expanding effect, due to an increase in productivity of team production in ℓ sector; and an increase in relative demand for ℓ sector good, *a demand-expanding effect*, due to a decrease in relative price p of ℓ sector good, caused by potential entrants.[10] When λ_h is increasing, the equilibrium relative price p is higher than p_l^*, implying that l agents who are not employed by h managers in g sector engage in ℓ-sector self-employment. Given that those l agents do not benefit from the improvement, the supply-expanding effect becomes small relative the demand-expanding effect, resulting in excess demand for ℓ good and thus reallocation of h agents from g sector to ℓ sector. When λ_h is decreasing, the equilibrium relative price p is less than p_l^*, implying that all producers in ℓ sector are teams managed by h agents. In this case, the decrease in c_ℓ makes the supply-expanding effect larger than the demand-expanding effect, leading to reallocation of h agents from ℓ sector to g sector.

4 Conclusion

This paper introduces team production á la Garicano and Rossi-Hansberg (2006) into a two-sector Ricardian comparative advantage model to examine how the allocation of creativity across sectors is affected by team production in the local sector. Although team production can be a tool for shifting creativity from the global to local sector, this study found that the likelihood of the shift is limited, and even in a case of shift, improving the environment of team production in the local sector is likely associated with the non-monotonic dynamics of creativity reallocation.

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[10] A decrease in relative price p of ℓ sector good indicates a downward shift of horizontal lines in the supply curve.

A Appendix

A.1 Definitions

A.1.1 Thresholds in (τ, p) Coordinate

The thresholds $\hat{\tau}_1$, $\hat{\tau}_2$, and $\hat{\tau}_3$ are obtained from (5), substituting $p_{g\sim g_s}$ into (6), and substituting p_l^* into (6), respectively:

$$\begin{aligned}\hat{\tau}_1 &\equiv \frac{a\ell_h - \ell_l}{\ell_h - \ell_l}, \\ \hat{\tau}_2 &\equiv \frac{a\ell_h - \ell_l}{\ell_l}\frac{1 - c_g(1 - g_l)}{c_\ell(1 - \ell_l)}, \\ \hat{\tau}_3 &\equiv \frac{g_l}{\ell_l}\frac{a\ell_h - \ell_l}{g_h - g_l}\frac{1 - g_l}{1 - \ell_l}\frac{c_g}{c_\ell}.\end{aligned}$$

Given (11), $\hat{\tau}_3 < \hat{\tau}_2 < \hat{\tau}_1$.

The threshold c_1^* is given by

$$c_1^* \equiv \frac{g_l}{\ell_l}\frac{a\ell_h - \ell_l}{g_h - g_l}\frac{1 - g_l}{1 - \ell_l}.$$

A.1.2 Thresholds in (a, p) Coordinate

The thresholds $\hat{a}_1$, $\hat{a}_2$, and $\hat{a}_3$ are obtained from solving (5) for a, substituting $p_{g\sim g_s}$ into (6), and substituting p_l^* into (6), respectively:

$$\begin{aligned}\hat{a}_1 &\equiv \left(1 - \frac{\ell_l}{\ell_h}\right)\tau + \frac{\ell_l}{\ell_h}, \\ \hat{a}_2 &\equiv \frac{\ell_l}{\ell_h}\left[1 + \tau\frac{c_\ell(1 - \ell_l)}{1 - c_g(1 - g_l)}\right], \\ \hat{a}_3 &\equiv \frac{\ell_l}{\ell_h}\left[1 + \tau\frac{c_\ell(1 - \ell_l)}{c_g(1 - g_l)}\frac{g_h - g_l}{g_l}\right],\end{aligned}$$

where the ranking $1 < \hat{a}_1 < \hat{a}_2 < \hat{a}_3$ holds.

A.1.3 Thresholds in (c_ℓ, p) Coordinate

The thresholds $\hat{c}_{\ell,1}$, $\hat{c}_{\ell,2}$, and $\hat{c}_{\ell,3}$ in Pattern 1 are obtained from substituting $p_{g\sim g_s}$ into (7), substituting p_l^* into (7), and substituting $p_{\ell_{w/}\sim\ell_{w/o}}$ given by (5) into (7), respectively:

$$\begin{aligned}
\hat{c}_{\ell,1} &\equiv \frac{\ell_h-\ell_l}{\ell_l(1-\ell_l)}[1-c_g(1-g_l)], \\
\hat{c}_{\ell,2} &\equiv \frac{g_l}{\ell_l}\frac{\ell_h-\ell_l}{g_h-g_l}\frac{1-g_l}{1-\ell_l}c_g, \\
\hat{c}_{\ell,3} &\equiv \frac{a-1}{\tau-a}\frac{g_l}{g_h-g_l}\frac{1-g_l}{1-\ell_l}c_g,
\end{aligned}$$

where the ranking $0<\hat{c}_{\ell,3}<\hat{c}_{\ell,2}<\hat{c}_{\ell,1}$ holds. The thresholds $\tilde{c}_{\ell,1}$ and $\tilde{c}_{\ell,2}$ in Pattern 2 are obtained from substituting $p_{g\sim g_s}$ into (6) and substituting p_l^* into (6), respectively:

$$\begin{aligned}
\tilde{c}_{\ell,1} &\equiv \frac{a\ell_h-\ell_l}{\ell_l}\frac{1-c_g(1-g_l)}{\tau(1-\ell_l)}, \\
\tilde{c}_{\ell,2} &\equiv \frac{g_l}{\ell_l}\frac{a\ell_h-\ell_l}{g_h-g_l}\frac{1-g_l}{1-\ell_l}\frac{c_g}{\tau},
\end{aligned}$$

where the ranking $0<\tilde{c}_{\ell,2}<\tilde{c}_{\ell,1}$ holds.

A.1.4 Relative Quantities

The relative quantities in Figure 7-21 are defined as follows:

$$s_g \equiv \frac{\ell_l}{g_h}[\rho c_g(1-g_l)-1], \quad s_{g_s} \equiv \frac{\ell_l}{g_h}\rho, \quad s_{\ell_{w/}} \equiv \frac{a\ell_h}{g_l}\frac{1}{\rho\tau c_\ell(1-\ell_l)-1}, \quad s_{\ell_{w/o}} \equiv \frac{\ell_h}{g_l}\frac{1}{\rho\tau c_\ell(1-\ell_l)-1} > s_{\ell_{w/}}.$$

A.2 Numerical Experiment

A.2.1 Algorithm

For each fixed set of upper bounds, $\bar{\rho}$ and $\bar{\tau}$, the algorithm below is used to generate random samples and conduct comparative statics.

Step 1: Generate samples of parameters $(\ell_l,\ell_h,g_l,g_h,\rho,c_g,c_\ell,a,\tau)$ of size of one million from the uniform distribution over the subset of parameters satisfying the stated conditions:

(a) Generate ℓ_l at random such that $0<\ell_l<(1-\bar{\rho}^{-1})$.

(b) Generate ℓ_h at random such that $[\bar{\rho}/(\bar{\rho}-1)]\ell_l<\ell_h<1$.

(c) Generate g_h at random such that $0<g_h<1$.

(d) Generate g_l at random such that $0<g_l<(\ell_l/\ell_h)g_h$.

(e) Generate ρ at random such that $\ell_h/(\ell_h-\ell_l)<\rho\leq\bar{\rho}$.

(f) Generate a at random such that

$$1 < a < \min\left\{\ell_h^{-1}, \frac{\ell_l}{\ell_h}\left(1+\bar{\tau}\frac{g_h}{g_l}\frac{\ell_h-\ell_l}{\ell_h}\right)\right\}.$$

(g) Generate c_g at random such that

$$\max\left\{\frac{1}{\rho(1-g_l)}, \frac{\ell_h-\ell_l}{\ell_h(1-g_l)}, \frac{1}{1-g_l} - \frac{\bar{\tau}}{1-g_l}\frac{\ell_l}{\ell_h}\frac{\ell_h-\ell_l}{a\ell_h-\ell_l}\right\} < c_g < \frac{g_h-g_l}{g_h(1-g_l)}.$$

(h) Generate c_ℓ at random such that

$$\max\left\{\frac{1}{\rho(1-\ell_l)}, \frac{\ell_h-\ell_l}{\ell_l(1-\ell_l)}[1-c_g(1-g_l)], \frac{a\ell_h-\ell_l}{\bar{\tau}\ell_l(1-\ell_l)}[1-c_g(1-g_l)]\right\} < c_\ell < \frac{\ell_h-\ell_l}{\ell_h(1-\ell_l)}.$$

(i) Generate τ at random such that $\max\{\hat{\tau}_2, 1\} < \tau \leq \bar{\tau}$.

Step 2: For each sample, construct equidistant grid points on the following closed interval of α under which an equilibrium relative price p satisfies $p_l^* \leq p \leq p_{g\sim g_s}$:

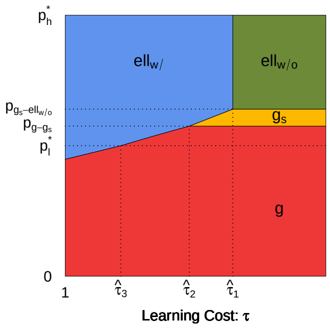
$$\frac{g_l}{g_h}[\rho c_g(1-g_l)-1] \leq \alpha \leq [\rho c_g(1-g_l)-1][1-c_g(1-g_l)],$$

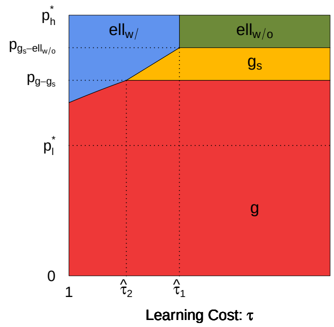
and compute the share of each scenario in the grid points.

Step 3: Compute the sample average of the share of each scenario.

Step 1 in the algorithm ensures the parameters satisfy the required conditions. The possible scenarios are illustrated in Figure 24-36, the conditions for which are omitted due to limitations of space. For $\bar{\tau}$, I consider three values: low (1.1), middle (1.5), and high (2.0).[11] $\bar{\rho}$ ranges from 2 to 20.

[11] Learning within ℓ team increases the time cost of passing unsolved problems by $100 \times (\tau - 1)\%$. $\bar{\tau}$ gives the upper bound for this increase, and "low," "middle," and "high" correspond to the maximal increase of 10%, 50%, and 100%, respectively.





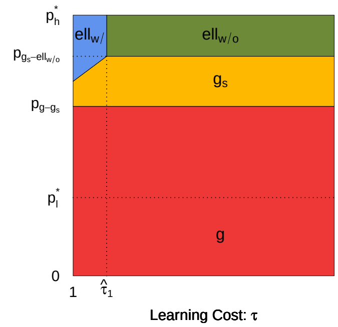


Figure 1: h-agent Choice in (τ, p) Coordinate, Pattern 1, $\frac{c_\ell}{c_g} < c_1^*$

Figure 2: h-agent Choice in (τ, p) Coordinate, Pattern 2, $\frac{c_\ell}{c_g} > c_1^*$, $\hat{\tau}_2 > 1$

Figure 3: h-agent Choice in (τ, p) Coordinate, Pattern 3, $\frac{c_\ell}{c_g} > c_1^*$, $\hat{\tau}_2 < 1$

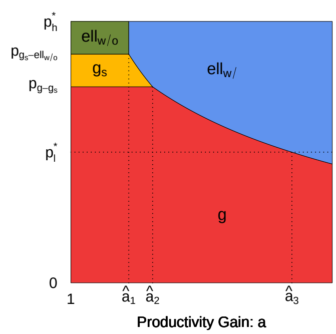
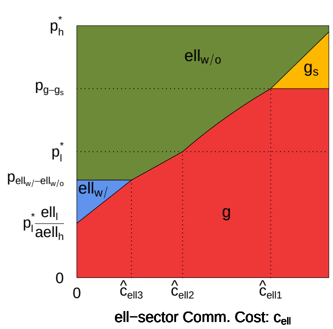


Figure 4: h-agent Choice in (a, p) Coordinate



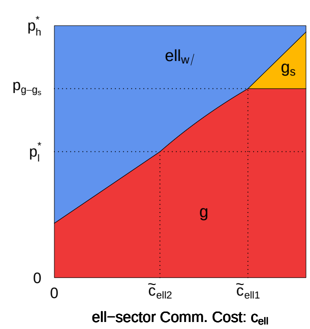


Figure 5: h-agent Choice in (c_ℓ, p) Coordinate, Pattern 1, $\hat{\tau}_1 < \tau$

Figure 6: h-agent Choice in (c_ℓ, p) Coordinate, Pattern 2, $\hat{\tau}_1 > \tau$

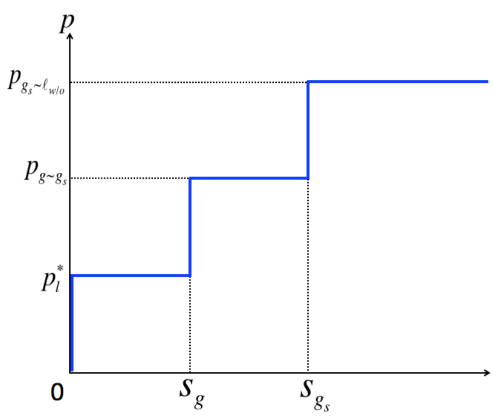


Figure 7: $\hat{\tau}_1 < \tau$

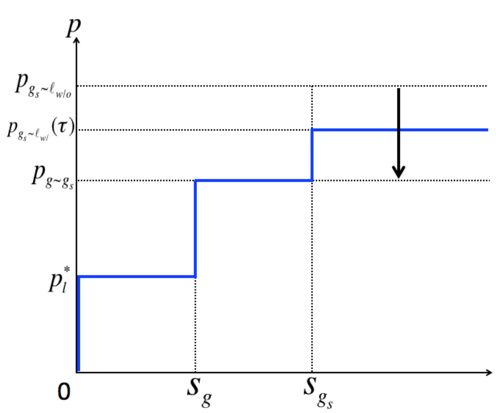


Figure 8: $\hat{\tau}_2 < \tau < \hat{\tau}_1$

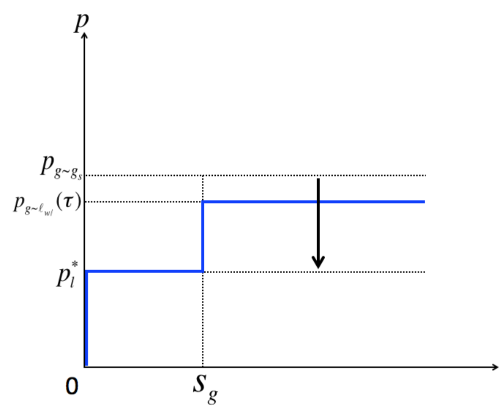


Figure 9: $\hat{\tau}_3 < \tau < \hat{\tau}_2$

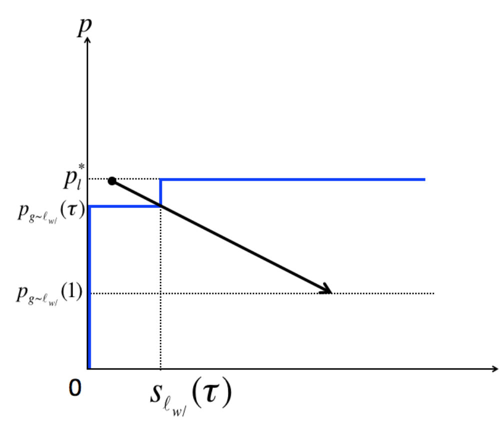


Figure 10: $1 < \tau < \hat{\tau}_3$

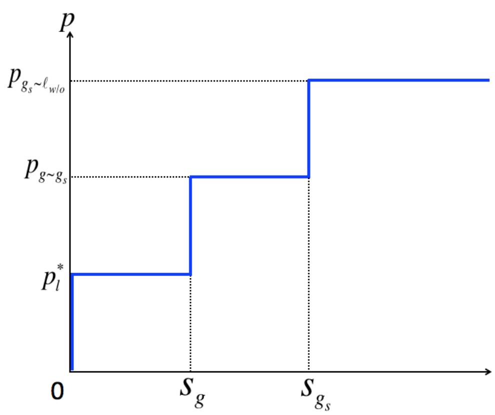


Figure 11: $1 < a < \hat{a}_1$

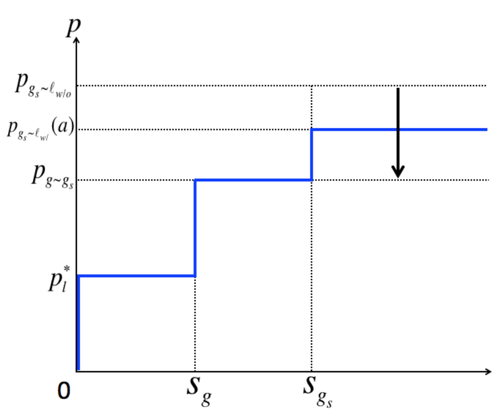


Figure 12: $\hat{a}_1 < a < \hat{a}_2$

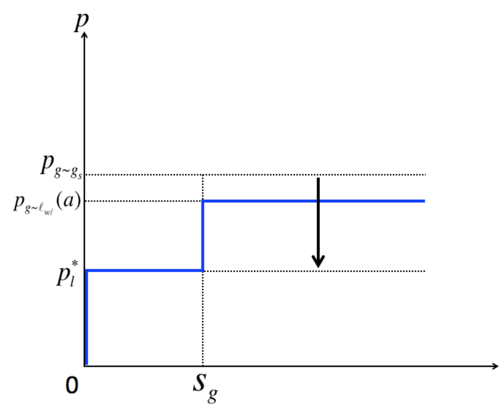


Figure 13: $\hat{a}_2 < a < \hat{a}_3$

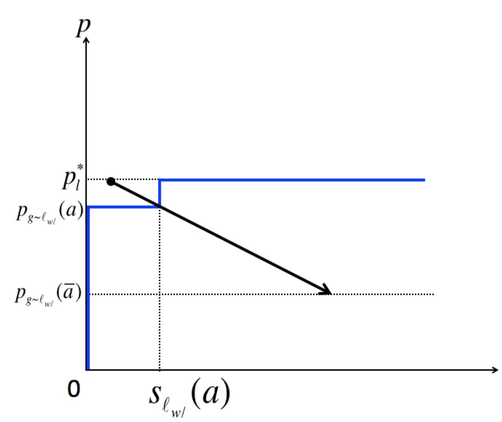


Figure 14: $\hat{a}_3 < a$

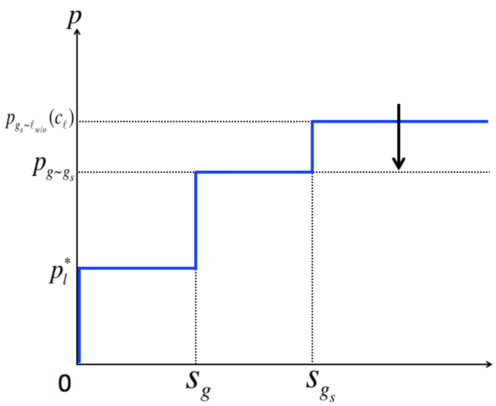


Figure 15: $\hat{c}_{\ell,1} < c_\ell$

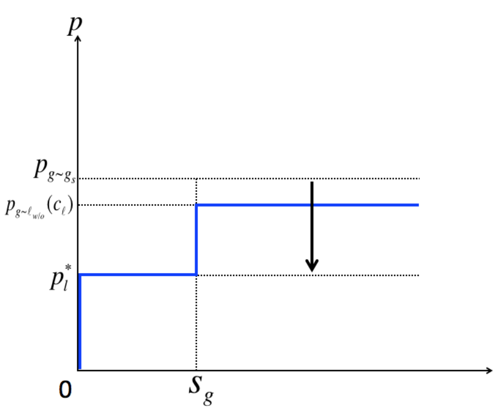


Figure 16: $\hat{c}_{\ell,2} < c_\ell < \hat{c}_{\ell,1}$

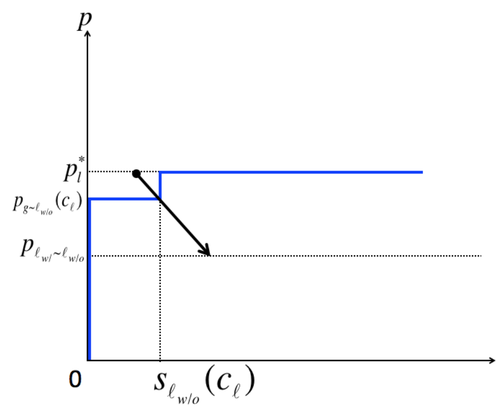


Figure 17: $\hat{c}_{\ell,3} < c_\ell < \hat{c}_{\ell,2}$

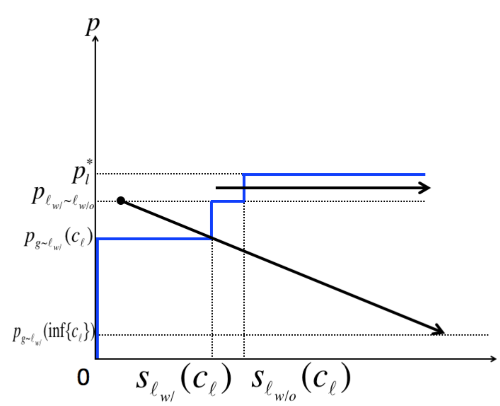


Figure 18: $(\inf\{c_\ell\} =)\underline{c}_\ell < c_\ell < \hat{c}_{\ell,3}$

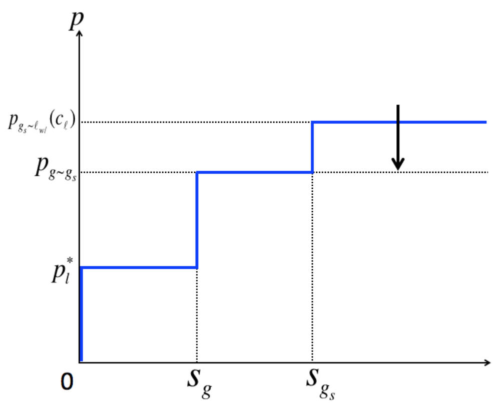


Figure 19: $\tilde{c}_{\ell,1} < c_\ell$

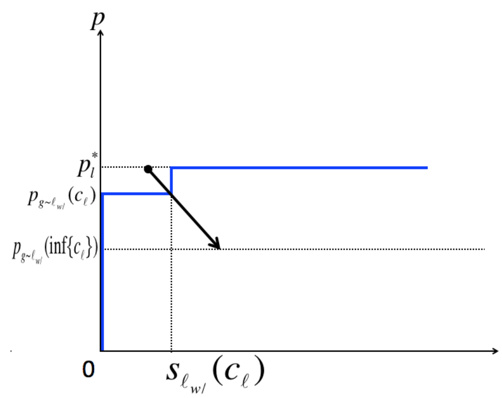


Figure 21: $(\inf\{c_\ell\} =)\underline{c}_\ell < c_\ell < \tilde{c}_{\ell,2}$

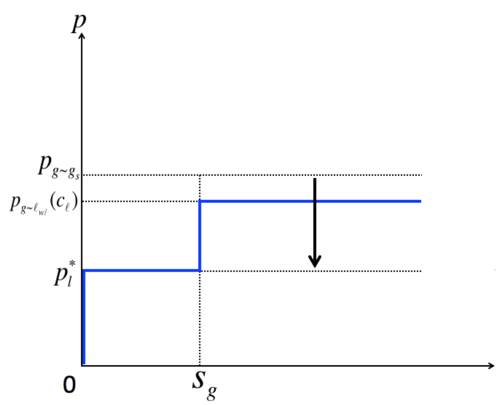


Figure 20: $\tilde{c}_{\ell,2} < c_\ell < \tilde{c}_{\ell,1}$

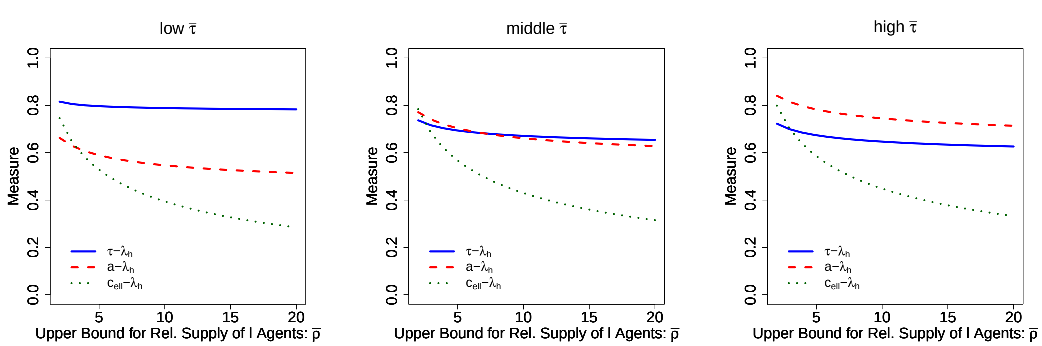


Figure 22: Measure of Scenario 0

Note: Low, middle and high $\bar{\tau}$ correspond to values of 1.1, 1.5, and 2.0, respectively.

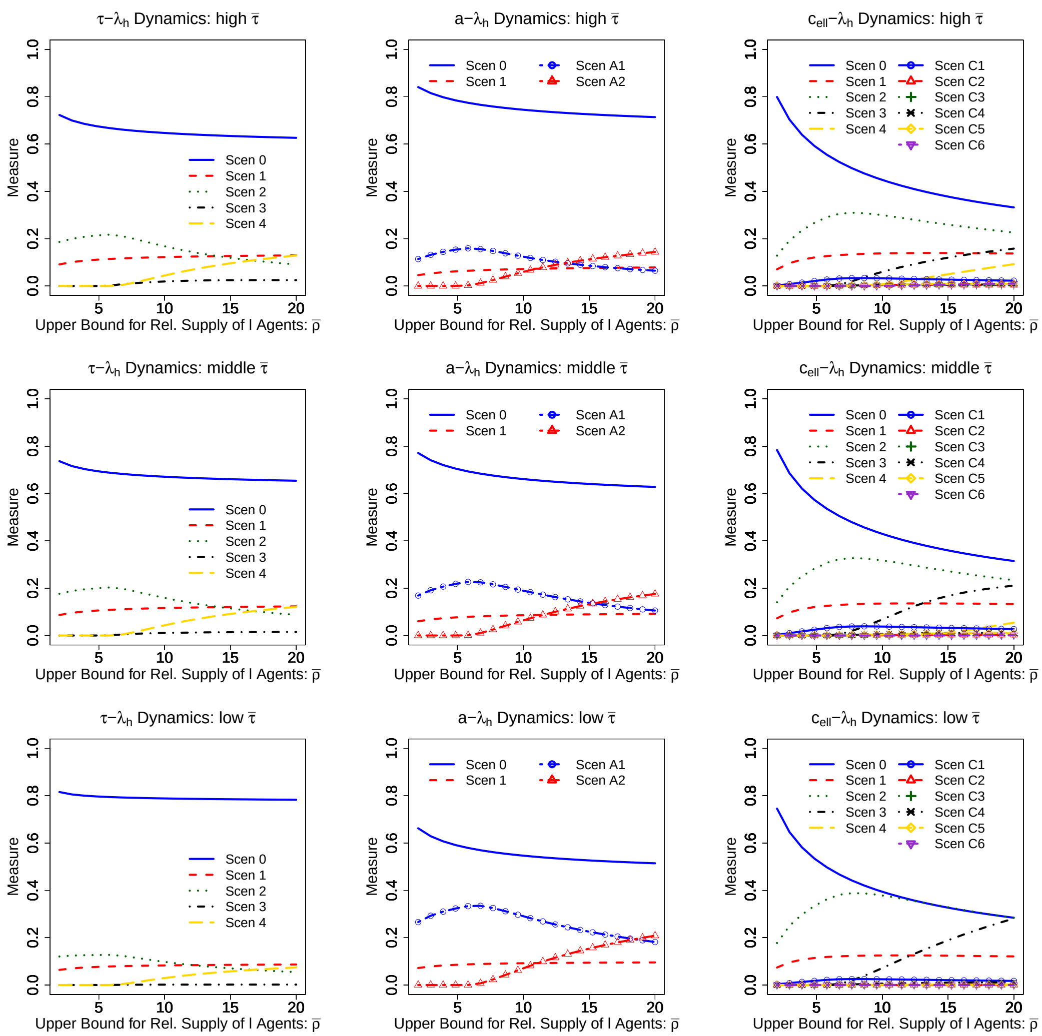


Figure 23: Measures of Scenarios in Dynamics

Note: Figures correspond to a middle value of $\bar{\tau}$, i.e., 1.5.

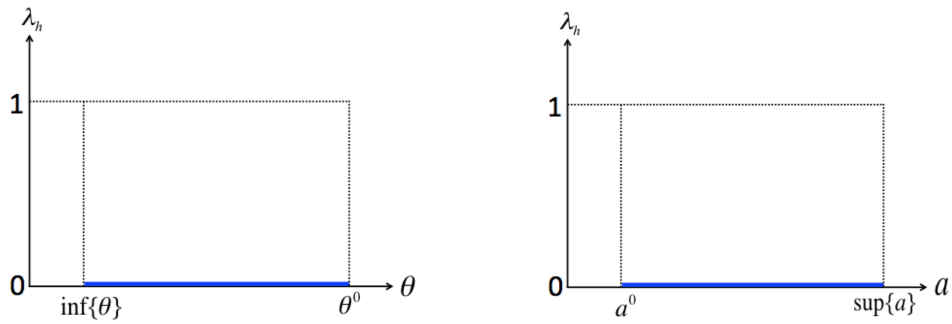


Figure 24: Scenario 0

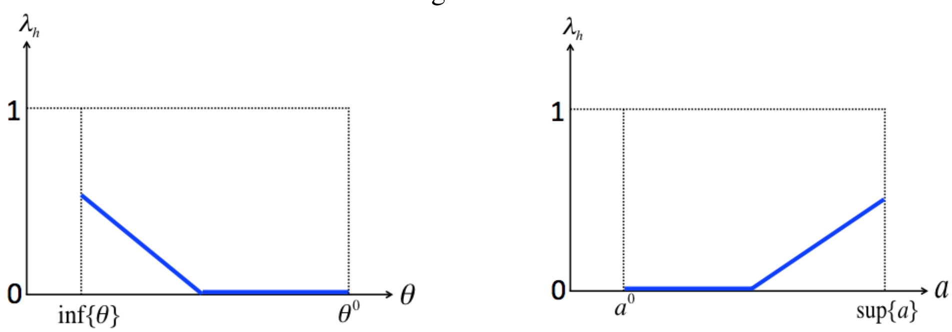


Figure 25: Scenario 1

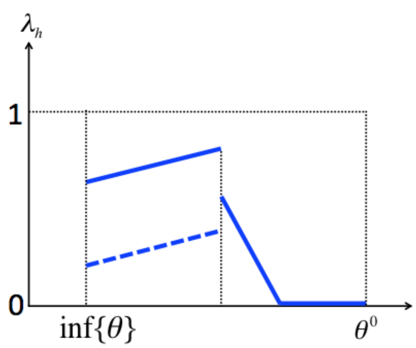


Figure 26: Scenario 2

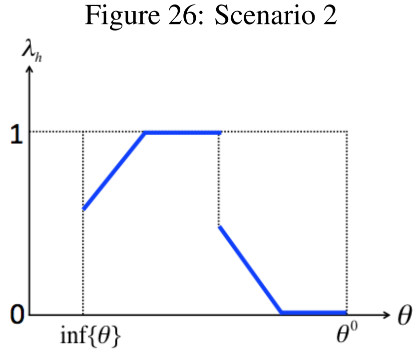


Figure 27: Scenario 3

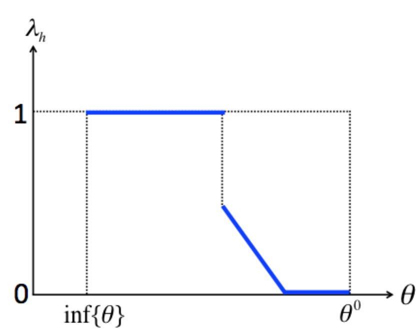


Figure 28: Scenario 4

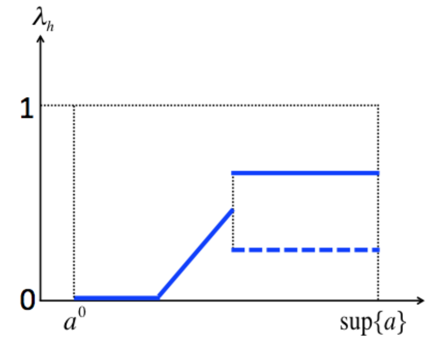


Figure 29: Scenario A1

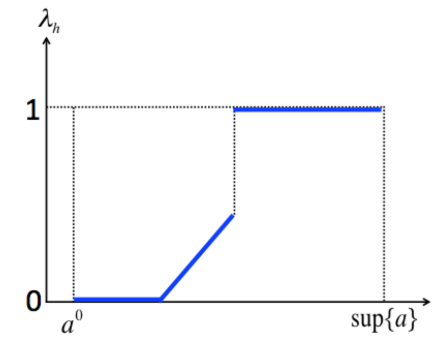


Figure 30: Scenario A2

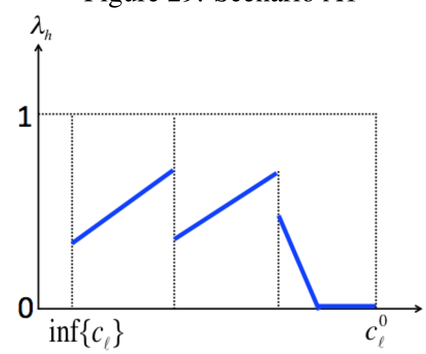


Figure 31: Scenario C1

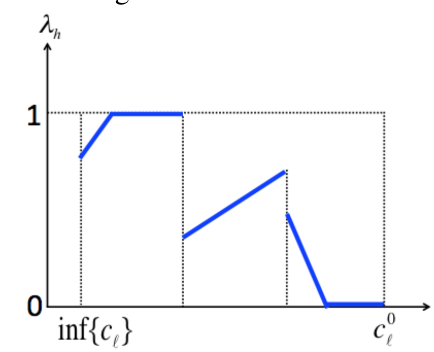


Figure 32: Scenario C2

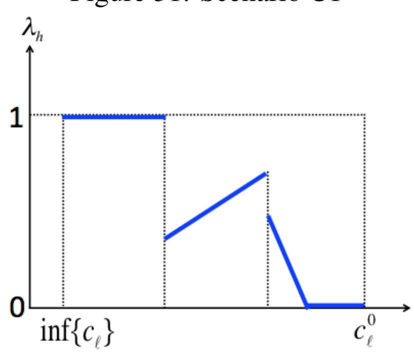


Figure 33: Scenario C3

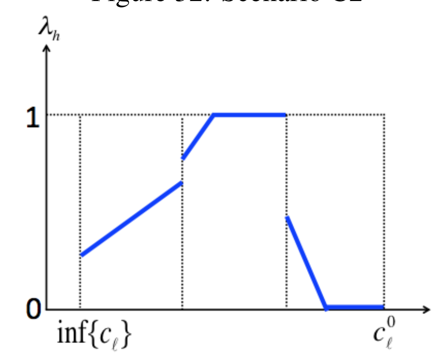
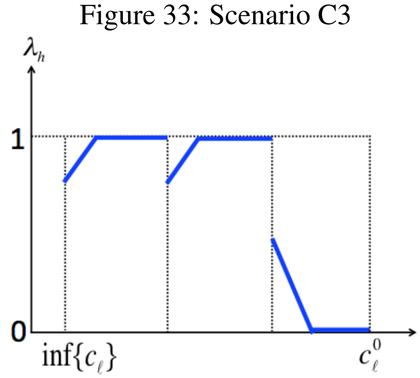


Figure 34: Scenario C4



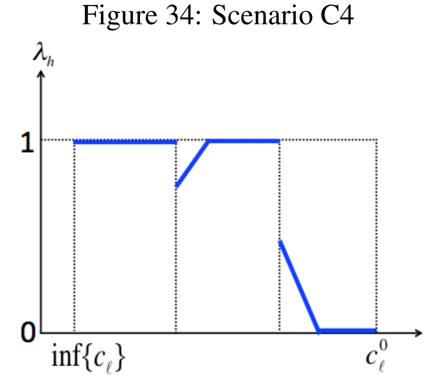


Figure 35: Scenario C5

Figure 36: Scenario C6