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Endogenous Business Cycles Caused by Nonconvex Costs and Interactions*

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Abstract

This paper shows that endogenous business cycles (inventory cycles) arise from a combination of nonconvex costs and economic interactions among firms. At the micro level, firm behavior is characterized by lumpiness, and the standard production-smoothing theory is empirically rejected. To account for this, a nonconvex cost function is assumed in our model. It might be expected that even if the microeconomic behavior is lumpy, that effect disappears at the aggregate level because of the law of large numbers. However, we show that if there exist interactions among firms, a regular endogenous cycle emerges at the aggregate level given that the degree of the interaction effect exceeds a critical point. That is, the randomly behaving microeconomic agents generate deterministic *collective behavior* via interactions. It offers an explanation for the Kitchin cycle.

Keywords: Kitchin cycle, Nonconvex cost function, Propagation of chaos, Bifurcation
JEL classification: E32, E23, D21

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1 Introduction

In this paper, we show that endogenous business cycles (the inventory cycle) arise from a combination of nonconvex costs and economic interactions among firms. In particular, we show that the aggregate of randomly behaving microeconomic agents generates deterministic *collective behavior* via interactions.

Economic fluctuations are certainly an important issue in economics, but what causes such fluctuations? This natural and fundamental question has not yet been answered in economics. For example, Cochrane (1994) demonstrates that popular economy-wide shocks (e.g., monetary shocks or oil prices) fail to explain the bulk of economic fluctuations. He writes, "What shocks are responsible for economic fluctuations? Despite at least two hundred years in which economists have observed fluctuations in economic activity, we still are not sure" (p. 295). Thus, it is very difficult to identify the origin of economic fluctuations. We cannot resort to *mysterious* aggregate exogenous shocks to explain them. However, because an economy is composed of many firms, it might be expected that the aggregate fluctuation stems from firm-specific shocks and inherits some properties from them.

At the micro level, economic activities are characterized by lumpiness and discreteness. Managers temporary shut down the plants or change the number of shifts for inventory adjustment. This behavior clearly contradicts the well-known production smoothing theory in microeconomics textbooks. In fact, the production smoothing theory is empirically rejected (see Blinder and Maccini (1991)). It is found that, when some fixed costs exist (for example, ordering costs), the cost curve is kinked and nonconvexity emerges, which implies that the cost-minimizing strategy of firms is production bunching (or the bunching of orders). This theory can account for the stylized fact that production is more volatile than sales (e.g. Hall (2000)). Therefore, the aim of this paper is to investigate how these firm-level characteristics are related to the aggregate fluctuations.

There are two different views concerning the effect of microeconomic characteristics on the aggregate fluctuations; one is that microeconomic characteristics disappear at the macroscopic level. Indeed, less attention has been paid to the role of idiosyncratic shocks in the macroeconomic literature simply because these shocks are considered to average out in the aggregate by the law of large numbers (LLN). This view is widely accepted in the literature and Lucas (1977)'s argument is a typical one [1] According to this view, the observed aggregate fluctuations must be explained by the presence of shocks that have a common origin across firms in the economy. By definition, they are *aggregate shocks*.

On the other hand, the second view, which attracts much attention in recent years, emphasizes the effects of interactions between sectors (or firms), especially input–output linkages. Although the LLN argument discussed above depends crucially on the independence assumption, interactions among firms are the fundamental aspects of the macroeconomy. In fact, positive comovement across sectors is a salient feature of the business cycle. In contrast to the LLN argument, it is emphasized that the effects of interactions between sectors (or firms) through input–output linkages, which propagate idiosyncratic shocks throughout the economy, cause the aggregate fluctuations that are unexplained by the usual aggregate shocks (e.g., Long and Plosser (1983), Carvalho (2010), Foerster et al. (2011), Acemoglu et al. (2012), and Carvalho and Gabaix (2013); for a review, Carvalho (2014)). The key element of models used in these studies is the existence of sectors that have disproportional impacts on the entire economy. This is due to the heterogeneity of input–output linkages; that is, sectors are not equally intense material suppliers. Shocks to general purpose technologies such as oil, electricity, iron and steel propagate to all sectors through the input–output linkages because most sectors rely on them. In this sense, the microeconomic shocks accounting for aggregate fluctuations in these studies can be regarded as *"pseudo–macroeconomic"* shocks. There are other literature following the second view and closely related our analysis, e.g., Bak et al. (1993) and Durlauf (1993), where nonconvex technology and (local) interactions are explicitly considered. Bak et al. (1993) demonstrate that small shocks to final goods can cause an "avalanche" of production increases via the supply chain.

Even though such interactions explain how the aggregate fluctuation can be caused by microeconomic shocks, there exist broad distinctions between our model and previous studies. In contrast to Carvalho (2010) and Acemoglu et al. (2012), in our model, each firm can hardly influence the outcome of the economy on its own. We assume that each firm is small compared with the economy as a whole. Furthermore, in contrast to Bak et al. (1993), in which shocks to final goods are as-

[1] Lucas (1977) says, "These changes [(the changes in technology and taste)] are occurring all the time and, indeed, their importance to individual agents dominates by far the relatively minor movement with constitute the business cycle. Yet these movements should, in general, lead to relative, not general price movements. ... in a complex modern economy, there will be a large number of such shifts in any given period, each small in importance relative to total output. There will be much "averaging out" of such effects across markets." (p. 19)

sumed to be exogenous, we assume that demand for the products depends on the overall economic condition. We assume that on the one hand, the behavior of a firm is affected by the state of the economy as a whole, but on the other hand, the economy is composed of the firms themselves. In other words, the macroscopic state of the economy not only is an aggregation of the firms, it also prescribes macroeconomic environment in which the firms engage in business activities. This feedback loop generates rich interesting phenomena. This idea is closely related to the "micro-macro loop" emphasized by Hahn (2002), where a macro variable acts as an externality. We show that this mechanism can generate *collective behavior* that is different from the motion of an individual firm.

On this point, our approach is close in spirit to *heterogeneous interacting agent models* (see e.g., Delli Gatti et al. (2009) and Stiglitz and Gallegati (2011); for a survey, see Hommes (2006)), especially to Aoki's methods (Aoki (1996, 2004), Aoki and Shirai (2000) and Aoki and Yoshikawa (2007)). Aoki and coauthors develop so-called *jump Markov processes*, where the evolution of the probability distribution is described by *master equations*. Although there is no doubt that Aoki's methods expand the scope of macroeconomic analysis, there exist some difficulties and situations that cannot be dealt with in his framework (see Section 4.1). In particular, in our model, firms' inventories are distributed continuously and affect firms' choice of production. That is, the system is described by an infinite dimensional random variable, that is, the distribution of inventories (and production).

By using the *propagation of chaos* instead of Aoki's methods, we present an alternative method to investigate how the system (that is, the probability distribution) behaves and changes its properties when we change the parameters. On the basis of nonconvexity in the cost function and the feedback effect, we show that a regular cyclical movement emerges given that the effect exceeds a threshold. This cyclical movement is endogenous and is an explanation for the *Kitchin cycle*.

The rest of the paper is organized as follows. Section 2 discusses the firm behavior characterized by nonconvexities, which can explain the empirical puzzle that the volatility of production is larger than that of sales. Section 3 discusses the importance of inventory movement for understanding the business cycle. Section 4 contains our main results and shows that the simple LLN cannot be applied and that an endogenous movement emerges. Section 5 concludes.

2 Firm Behavior: Production and Inventory

The standard cost function has been assumed to be convex in output and in the change of output. This means that for cost minimization, the manager of a firm must smooth its production by using inventories as a buffer stock (production-smoothing models; see, e.g., Holt et al. (1960)). This implies that production is less variable than sales.

However, the prediction of the production-smoothing models is known to be inconsistent with the empirical data (see Blinder and Maccini (1991)). In particular, the correlation between sales and inventories is positive, not negative as predicted by production-smoothing models. Firms do not use their inventories as a buffer.

Blinder and Maccini (1991) present a well-known (S, s) model in which a firm places an order of size $S - s$ whenever its inventories reach the lower bound, s. They show that it is optimal for the firm to place infrequent large orders when fixed costs of ordering exists, which leads to bunched orders. The inventory series is characterized by a sawtooth pattern. The (S, s) model is strongly supported by empirical data (e.g., Hall and Rust (2000)). Although they emphasize retail and wholesale inventories, that is, the lumpiness of the delivery process, the bunching of orders by the retail sector can induce production bunching in the manufacturing sector even though the latter has the usual increasing marginal costs. Cooper and Haltiwanger (1992) point out this possibility, saying, "Downstream bunching of orders by retailers may be the source of upstream production bunching by manufactures" (p.116).

In relation to these studies, a close examination of data at the micro level (especially for the automobile industry) reveals that changes in production are quite lumpy. Managers may shut their plants down for a week or change the number of shifts, varying production. Ramey (1991), Cooper and Haltiwanger (1992), Bresnahan and Ramey (1994), and Hall (2000) focus on the nonconvexity of the cost function to explain these behaviors. They show that when there are fixed costs associated with opening the plant and adding an additional shift, production bunching is an optimal strategy. For example, Cooper and Haltiwanger (1992) present a simple model and show that a start-up cost for a production run and a constant marginal cost of production lead to production bunching.

To illustrate how the cost function associated with such fixed costs might look, a simple non-convex cost function is depicted in Figure 1. If a manager has to produce, on average, output

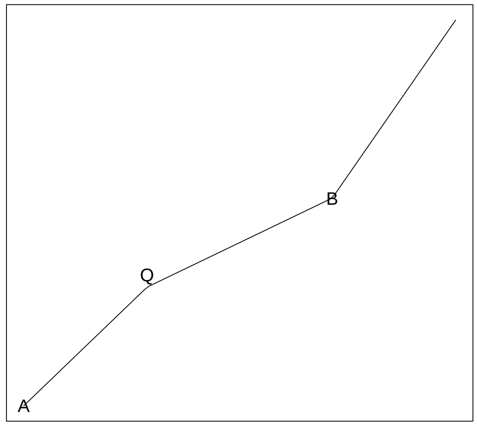

Figure 1: A nonconvex cost function. The horizontal (vertical) axis is quantity (costs).

$Q \equiv (A + B)/2$, the average cost can be reduced by alternating between production at A and B rather than production at Q, that is, by production bunching. It is clear that the nonconvexity leads to excess volatility of production. Furthermore, this nonconvexity is quantitatively important to explain the variation of output. Bresnahan and Ramey (1994) write, "[M]ost of the variance of output comes from varying hours over the nonconvex portions of the cost function, rather than from varying hours over the convex portions of the cost function" (p. 610).

The question arises of whether the automobile industry is representative of all manufacturing or is a just special one. On this point, Mattey and Strongin (1997) consider two extremes of technology types. "Pure assemblers" adjust their output through varying plants' work period, that is, through temporary plant shutdowns, adding or dropping shifts, and adding overtime hours (Saturday work). The automobile and transportation industries are typical examples. The other type is "pure continuous processing" operations, where output adjustment is carried out through varying the instantaneous flow rates of production rather than the work period margin. Mattey and Strongin (1997) conclude that "pure assembly" is a better characterization for manufacturing given that that the plant work period margin is commonly used.

Moreover, among these output adjustment margins, changes in the number of shifts are quantitatively important. Bresnahan and Ramey (1994) show that at a quarterly frequency, changes in the number of shifts account for 40% of plant-level output volatility in the automobile industry and is the most important contributors to the variation of output. Shapiro (1996) shows that close to half of the changes in employment in the U.S. manufacturing take place on late shifts. Thus, we focus on changes in the number of shifts in the following analysis.

The discussion above suggests that the behavior of firms is as follows. For the sake of simplicity, we assume that the firms choose one of two production states, high and low (the same simplification can be found in the literature; see, for example, Bak et al. (1993) and Durlauf (1993)). Suppose that a manufacturing firm has sufficient inventories (or the wholesale and retail inventories that the firm supplies) and that demand is low. The firm chooses a low production state (e.g., one-shift production) to reduce its inventories. After eliminating the excess inventories, the firm waits for demand to improve. If this happens, the firm adds a new shift to the existing line and increases its output. Even if the sales forecast is overestimated, it is optimal for a manager to maintain the high production for a while because of the fixed costs. After it replenishes its inventories, the firm lays off the workers on the second shift and returns to the initial state.

Note that the above pattern of behavior is not a deterministic path, but is exposed to various idiosyncratic shocks. Suppose first that the demand (or sales) of a firm indexed by i, s_t^i, fluctuates around $\overline{S}$,

$$s_t^i = \overline{S} + \xi_t^i \tag{1}$$

ξ_t^i represents a temporary demand shock with mean 0, that causes unintended inventory investment. We write $\xi_t^i \equiv -\frac{\sigma_2 dW_t^i}{dt}$, where W_t^i is a standard Brownian motion, σ_2 is a constant, and $\frac{dW_t^i}{dt}$ is the formal derivative with respect to t. We normalize $\overline{S} = 0$. By definition, inventory investment can be written as the difference between production and sales,

$$dy_t^i = (x_t^i - s_t^i)dt \tag{2}$$

where x_t^i denotes the production of firm i. We assume that the production is described by the motion in the so-called double-well potential.

$$dx_t^i = (-V'(x_t^i) - ey_t^i)dt + \sigma_1 dW_{1,t}^i, \quad V(x) = \frac{1}{4}x^4 - \frac{1}{2}x^2 \tag{3}$$

where $W_{1,t}^i$ is a standard Brownian motion and $\sigma_1, e > 0$ are constants. The stochastic term represents various idiosyncratic shocks that affects the target level of production—for example, changes in the price of materials. The potential function $V(x)$ is shown in Figure 2. The region

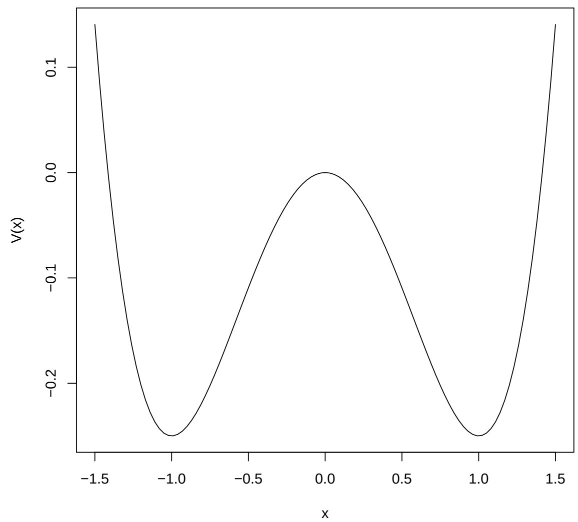

Figure 2: The potential function $V(x)$.

around $-1(+1)$ corresponds to low(high) production state. This model is a generalization of a two-state Markov chain. Suppose, for example, that $e = 0$. Because -1 and 1 are the local minima, x_t^i stays around there until a large shock occurs, at which point x_t^i goes toward the other local minimum. Thus, the path of x_t^i alternates between low and high production. The second term on the right-hand side, ey_t^i, represents the effect of the inventories on the manager's decision. That is, if y_t^i is large, the manager is likely to choose low production around $x_t^i = -1$.

Combining these equations, the behavior of firm i is described by the following two dimensional stochastic differential equations:

$$\begin{aligned} dx_t^i &= (-V'(x_t^i) - ey_t^i)dt + \sigma_1 dW_{1,t}^i, \quad V(x) = \frac{1}{4}x^4 - \frac{1}{2}x^2 \\ dy_t^i &= x_t^i dt + \sigma_2 dW_{2,t}^i \end{aligned} \tag{4}$$

where $W_{k,t}^i, k = 1, 2$ are independent Brownian motions and $\sigma_1, \sigma_2 > 0$. σ_1 and σ_2 represent the intensities of idiosyncratic shocks.

These equations duplicate the firm behavior discussed above. Suppose that x_t^i is near -1 and $y_t > 0$, that is, the firm has sufficient inventories and chooses low production. Because of the effect of y_t^i, x_t^i stays around -1 until y_t^i is sufficiently reduced. When $y_t^i < 0$, production, x_t^i, is pushed up by the shortage of inventories. Exceeding the top of the curve (around 0), x_t^i goes toward high production (+1), and the inventories are replenished. The stochastic terms $\sigma_1 dW_{1,t}^i$ and $\sigma_2 dW_{2,t}^i$ represent idiosyncratic shocks to firm i. For example, a good market condition $\sigma_2 dW_{2,t}^i < 0$ reduces the inventories beyond expectation and x_t^i might stay around +1 longer. Sample paths of equation (4) are depicted in Figures 3 and 4. Figure 3 shows that x_t^i oscillates between +1 and -1 with the stochastic noise. In Figure 5, the result of numerical simulations of $N = 20000$ independent copies of equation (4) is shown. It clearly shows the bimodality of production.

3 Inventory Investment and Business Cycles

3.1 The importance of Inventory Investment

As is well known, the inventory investment behavior is a key element in explaining aggregate fluctuations. For example, Blinder and Maccini (1991) demonstrate that the drop in inventory investment accounted for 87% of the drop in the GNP during the average postwar recession in the United States. In addition, a large part of short-run fluctuations (the business cycle frequencies) are explained by the behavior of inventory investment. Blinder (1981) says, "Inventory fluctuations are important in business cycles; indeed, to a great extent, business cycles are inventory fluctuations" (p. 500).

Furthermore, there is a consensus in the empirical literature that inventory movements are procyclical and that production is more volatile than sales at the sector and aggregate levels (see

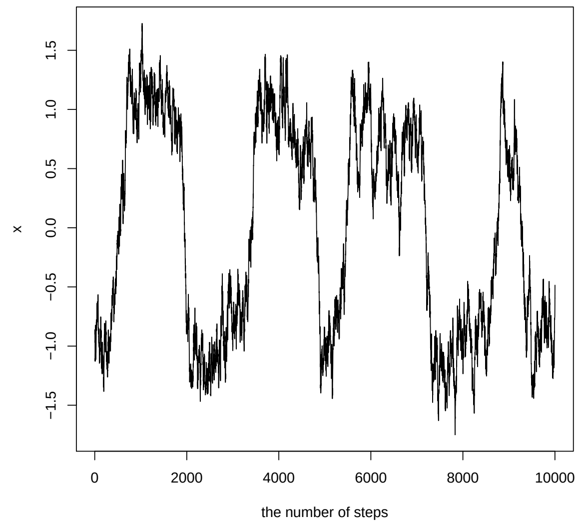

Figure 3: A sample path of production x in equation (4) with $\sigma_1^2 = \sigma_2^2 = 1/4$ and $e = 0.1$. The interval of a single time step, Δt, is 0.01. The horizontal axis is the number of steps.

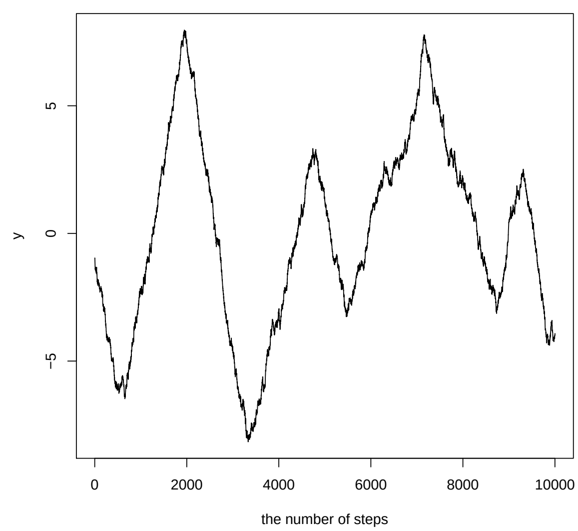

Figure 4: A sample path of inventories y in equation (4) with $\sigma_1^2 = \sigma_2^2 = 1/4$ and $e = 0.1$.

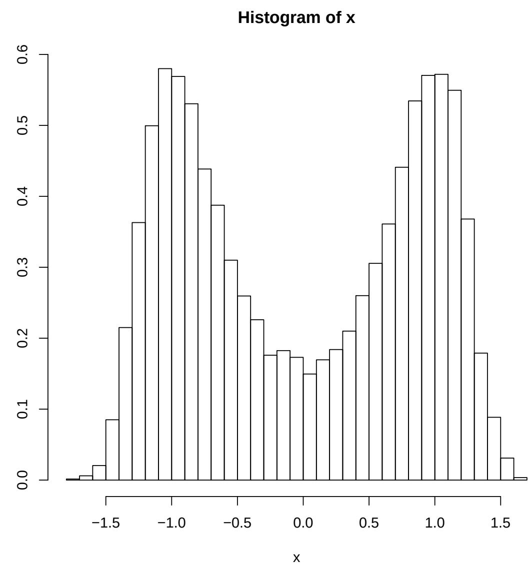

Figure 5: Histogram of $N = 20,000$ independent copies of x_t^i.

Ramey and West (1999) and the references cited in Section 2). As discussed in the previous section, these features contradict the production-smoothing theory, which predicts countercyclical inventory movements and smooth production. Thus, from a macroscopic point of view, inventories are considered *destabilizing* factors because they aggravate recessions by the reduction of inventories. In this regard, the inventory accelerator mechanism proposed by Metzler (1941) is a typical example that views inventory movements as destabilizing factors. Without the inventory accelerator, such cycles do not exist.

Interestingly, these "stylized facts" seem to depend on which frequencies we examine. Wen (2005) examines quarterly aggregate data from the U.S. and OECD countries and shows that production and inventories exhibit drastically different behaviors at low and high frequencies. According to his analysis, the procyclicality of inventory investment can be observed only at relatively low cyclical frequencies such as business-cycle frequencies (about 8–40 quarters per cycle). Excess volatility of production can be observed only at these frequencies. On the other hand, at a high frequency (2–3 quarters per cycle), production is less volatile than sales and inventory investment is strongly countercyclical. It can be considered that because of sluggish adjustments in production, managers cannot handle unexpected demand shocks at these high frequencies, and inventories act as buffer stock as production smoothing theory predicts.

As discussed above, which frequencies (or time scales) we examine is important. For example, Hall (2000) examines weekly data for automobile assembly plants and shows that two nonconvex margins (the change in the number of shifts and the temporary shutdown of the plant) play an important role in explaining production behavior. He particularly emphasizes intermittent production such as the weeklong temporary shutdown of plants (for a recent application of this model, see Copeland et al. (2011)). Although weeklong shutdowns used to vary output are relevant to high-frequency (weekly or monthly) production behavior, they are not suitable for explaining business cycles. Bresnahan and Ramey (1994) show that adding or dropping of an additional shift are substantially more important at the quarterly frequency than at the weekly frequency. In fact, they are the most important contributors to the quarter to quarter variation of output. Bresnahan and Ramey (1994) write, "While closing the plant temporarily might be important for the week-to-week variation in output, it might not be as important at the quarterly frequency" (p. 609). This is why we emphasize changes in the number of shifts to explain the business cycle in Section 2.

4 Aggregation: Interaction Effects

4.1 Interaction

In Section 2, we discussed the importance of lumpiness at the micro level. However, it is unclear whether this lumpiness has some impact on business cycles. It might be expected that because the real economy is composed of a large number of firms, lumpiness might be irrelevant at a macroscopic level. In particular, each firm is exposed to idiosyncratic shocks. Indeed, the conventional notion is that microeconomic behaviors would cancel each other out by the law of large numbers and that aggregate exogenous shocks are needed to explain business cycles (e.g., Lucas (1977)). The majority of theoretical models, including Kydland and Prescott (1982), use exogenous shocks to total factor productivity to derive aggregate fluctuations. According to these works, without aggregate shocks, microeconomic fluctuations have no aggregate implications.

However, recent theoretical investigations present another possibility: that aggregate fluctuations can result from microeconomic shocks. The distinct feature of this model is input–output linkages through which the shocks propagate to other sectors. For example, Long and Plosser (1983) construct a multisector model that generates comovement of sector outputs even though productivity shocks to each sector are independent. A positive productivity shock to sector i increases not only sector i's output, but also the output of other sectors that use i for materials. Carvalho (2010) shows that aggregate volatility depends on the structure of intersectoral linkages. In particular, when sectoral outdegrees follow a fat-tailed distribution, the aggregate volatility decays at a lower rate as the size of the economy tends to infinity. Shocks to general purpose technologies such as oil, electricity, iron, and steel propagate to all sectors because most sectors rely on them (see also Acemoglu et al. (2012) and Foerster et al. (2011)). Significant asymmetry—the presence of *hubs*—leads to aggregate fluctuations. In this sense, their model is closely related to the "granular hypothesis"(Gabaix (2011)) that there exist sectors (or firms) that have a disproportional impact on aggregate fluctuations[2].

Although the literature discussed above does not take into account nonconvex technology, nonconvexity and interaction are explicitly considered in Bak et al. (1993) and Durlauf (1993). Bak et al. (1993) assume production bunching: that a firm produces batches of two units of the product or nothing. They assume that positive shocks to final goods leads to orders for final goods. If these orders cannot be filled out of inventories, the final good producer starts production and sends orders to its suppliers for materials. If the inventories of the suppliers are less than the orders, the suppliers start production and send orders, and so on. They call this resulting cascade of production caused by a small shock to final goods an "avalanche." Note that in their model, the shocks to the final goods are exogenous.

However, it is plausible to assume that the shocks to the final goods also depend on economic conditions. That is, if a large fraction of firms expand their production, it would cause increases in the national income (or GDP) and in the sales of final goods. In general, a firm is affected by the condition of the entire economy, while the economy consists of the firms themselves. This feedback (or interaction) effect is an important aspect, and it will be shown to be the origin of business cycles.

In this sense, our model is closely related to Durlauf (1993), who explores the role of complementarities and the resulting stationary probability distribution. He assumes that each individual

[2] The asymmetry is crucial to derive aggregate fluctuations in their models. Dupor (1999) shows that when the network is more densely connected and the asymmetry disappears, the aggregate fluctuations caused by microeconomic shocks also disappear (the "irrelevance theorem"). Dupor (1999) says "the input–output structures in this class provide a poor amplification mechanism for sector shocks" (p.391).

industry chooses one of two types production: one (denoted as technology 1) is high production with a fixed cost and the other (technology 2) is low production without fixed costs (the nonconvexity assumption). In his model, by the complementarities, the relative productivity of technology 1 is enhanced when other industries in the reference group choose technology 1. This positive spillover effect implies that when the number of neighboring industries committing to technology 1 is high, the probability of a firm choosing technology 1 becomes large. He shows that when strong enough, these complementarities lead to multiple equilibria.

Although the assumption of the binary choice (technology 1 and 2) simplifies the analysis significantly, more heterogeneous situations can also be considered. In our model, firms' inventories are distributed continuously and affect firms' choice of production. Inventories act as a state variable of a firm and their behavior cannot be described by the binary choice model. We have to deal with the distribution itself—that is, with an infinite dimensional variable. In this sense, our model is more heterogeneous than that of Durlauf (1993). Of course, there is no *a priori* reason to assume that the distribution is stationary. It requires an alternative framework to investigate the time evolution of an economy at the macroscopic level.

A series of studies conducted by Aoki (Aoki (1996, 2004), Aoki and Shirai (2000) and Aoki and Yoshikawa (2007)) address this problem and present a more general framework called *jump Markov processes*. In this framework, the evolution of the probability distribution $P(i,t)$ is described by the following *master equation*

$$\frac{\partial P(i,t)}{\partial t} = \sum_{j \in S} q(j,i)P(j,t) - P(i,t)\sum_{j \in S} q(i,j) \tag{5}$$

Here, $q(i,j)$ is the transition rate from i to j ($i, j \in \{1, ..., n\} \equiv S$). The interpretation is clear. The first term on the right hand side of the master equation represents the probability flow into state i from all other states, whereas the second term represents the outflows of probability from state i. Thus, the master equation means that the rate of change of $P(i,t)$ is given by the difference between the inflows and outflows of probability.

Although there is no doubt that Aoki's methods expand the scope of macroeconomic analysis and can be applied to various problems (for an application to the Diamond search model, see Aoki and Shirai (2000)), there exist some difficulties, and this framework is not suited for our problem.

In particular, in our model, firms' inventories are heterogeneous and distributed continuously, and a diffusion process is considered. In such a situation, it is difficult to specify the transition rate and the master equation explicitly. In particular, it is difficult to investigate the nonstationary behavior of the probability distribution by solving master equations. We use the *propagation of chaos* instead of Aoki's methods to present an alternative method to investigate how the probability distribution behaves, without directly seeking the probability distribution itself.

4.2 McKean–Vlasov equation

We consider an economy consisting of a large number of firms and add an interaction term to equation (4). We assume that the sales of a firm, i, depend on the conditions of the overall economy,

$$s_t^i = h\langle x\rangle + \xi_t^i, \quad \langle x\rangle \equiv \frac{1}{N}\sum_{j=1}^{N} x_t^j \tag{6}$$

where $0 < h < 1$ and N is the number of firms in the economy. The assumption that h is less than 1 means that the sales do not increase as much as an increase in the national income. That is, h is the marginal propensity to consume. In addition, firms are assumed to adjust their production depending on the expectation of the sales. Incorporating these effects into our model, equation (4) is modified as

$$\begin{aligned} dx_t^i &= (-V'(x_t^i) - ey_t^i + D(E[s_t^i] - x_t^i))dt + \sigma_1 dW_{1,t}^i, \quad V(x) = \frac{1}{4}x^4 - \frac{1}{2}x^2 \\ dy_t^i &= (x_t^i - s_t^i)dt \end{aligned} \tag{7}$$

where $E[s_t^i]$ refers to the expectation of s_t^i and $D > 0$ represents the strength of the interaction effects. The stochastic term $\sigma_1 dW_{1,t}^i$ includes estimation errors of $E[s_t^i]$ by managers. Because $E[s^i] = \frac{h}{N}\sum_{j=1}^{N} x_t^j$, we finally obtain

$$\begin{aligned} dx_t^i &= (-V'(x_t^i) - ey_t^i + D(h\langle x\rangle - x_t^i))dt + \sigma_1 dW_{1,t}^i, \quad V(x) = \frac{1}{4}x^4 - \frac{1}{2}x^2, \quad \langle x\rangle \equiv \frac{1}{N}\sum_{j=1}^{N} x_t^j \\ dy_t^i &= (x_t^i - h\langle x\rangle)dt + \sigma_2 dW_{2,t}^i \end{aligned} \tag{8}$$

The interaction term $D(h\langle x\rangle - x_t^i)$ means that it pushes up the production of a firm, i, if the average of production is higher than the production of the firm. On the other hand, by definition, $\langle x\rangle$ consists of all firms in the economy. This is the interaction and feedback mechanism that we consider in our model.

More generally, using the empirical measure (δ_z denotes the Dirac measure at z) defined by

$$U_t^{(N)} = \frac{1}{N}\sum_{j=1}^{N} \delta_{z_t^j} \tag{9}$$

the system of equations in (8) can be written as

$$dz_t^i = b(z_t^i, U_t)dt + a(z_t^i, U_t)dW_t^i \tag{10}$$

where z_t^i is a d-dimensional vector. The coefficients depend on the empirical measure. Because the state of the system can be described in terms of the empirical measure, $U_t^{(N)}$ is much more important than the probability distribution of dN dimensional variables. Note that $U_t^{(N)}$ is also a random variable. In particular, we consider the limiting case, $N \to \infty$.

Let u_t denote the limit of the empirical measure (if it exists), that is, $u_t = \lim_{N\to\infty} U_t^{(N)}$ in the weak sense. In probability theory, since the seminal work by McKean (1967), a number of mathematicians have studied the limit of Equation (10), the mean-field equation (for reviews, see Sznitman (1991) and Gartner (1988)).

$$\begin{aligned} dz_t &= b(z_t, u_t)dt + a(z_t, u_t)dW_t && (11) \\ u_t(dz) &= \text{the law of } \; z_t && (12) \end{aligned}$$

The behavior of z_t is affected by the law of its solution. Furthermore, the law of the solution, u_t, satisfies the following equation.

$$\frac{\partial}{\partial t}u(z,t) = \sum_{k=1}^{d} \frac{\partial}{\partial x_k}(b_k(z,u)u(z,t)) + \sum_{k,l=1}^{d} \frac{\partial^2}{\partial x_k \partial x_l}(\sigma_{kl}(z,u)u(z,t)) \tag{13}$$

where $\sigma(z,u) = a(z,u)a^t(z,u)$. This is called the *McKean–Vlasov* equation. This equation is quite similar to the Fokker–Planck equation except that the coefficients depend on the measure u_t. It should be noted that this equation is nonlinear with respect to u_t.

In our case, Equation (8) can be written as

$$dz_t^{i,N} = f(z_t^{i,N})dt + g(z_t^{i,N})dW_t^i + \frac{1}{N}\sum_{j=1}^{N}\tilde{b}(z_t^{i,N}, z_t^{j,N})dt \tag{14}$$

$$z_t^{i,N} = \begin{pmatrix} x_t^{i,N} \\ y_t^{i,N} \end{pmatrix}, \quad f(z_t^{i,N}) = \begin{pmatrix} -(x_t^{i,N})^3 + x_t^{i,N} - ey_t^{i,N} \\ x_t^{i,N} \end{pmatrix}, \quad g(z_t^{i,N}) = \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{pmatrix},$$

$$dW_t^i = \begin{pmatrix} dW_{1,t}^i \\ dW_{2,t}^i \end{pmatrix}, \quad \tilde{b}(z_t^{i,N}, z_t^{j,N}) = \begin{pmatrix} \tilde{b}_1(z_t^{i,N}, z_t^{j,N}) \\ \tilde{b}_2(z_t^{j,N}) \end{pmatrix},$$

$$\tilde{b}_1(z_t^{i,N}, z_t^{j,N}) = \begin{cases} D(hx_t^{j,N} - x_t^{i,N}) & \text{if } \ |D(hx_t^{j,N} - x_t^{i,N})| \le K_1 \\ K_1 & \text{if } \ D(hx_t^{j,N} - x_t^{i,N}) > K_1 \\ -K_1 & otherwise \end{cases},$$

$$\tilde{b}_2(z_t^{j,N}) = \begin{cases} -hx_t^{j,N} & \text{if } \ |-hx_t^{j,N}| \le K_2 \\ K_2 & \text{if } \ -hx_t^{j,N} > K_2 \\ -K_2 & otherwise \end{cases}$$

where we have replaced the interaction b with $\tilde{b}$ and $K_1, K_2 > 0$. Although this technical assumption is needed for the following theorem, it is not expected to substantially affect the behavior of z_t^i and $U_t^{(N)}$ given that K_1 and K_2 are very large[3].

The corresponding mean-field equation is given by

$$\begin{aligned} dz_t^i &= f(z_t^i)dt + g(z_t^i)dW_t^i + \int \tilde{b}(z_t^i, v)u_t(dv)dt \\ u_t(dz) &= \text{the law of } \ z_t^i \end{aligned} \tag{15}$$

Assuming that the initial condition of $z^i, i = 1, ...N$ is drawn independently from the identical distribution, u_0, we obtain the following results.

[3]The stability analysis and numerical simulations in the following are performed with b.

Theorem 1 *1. The mean-field equation (15) is well-posed; that is, there exists a unique solution on* $[0, T]$ *for any* $T > 0$*.*

2. The process $z_t^{i,N}$ *converges in law to the solution of the mean-field equation (15),* z_t^i*, with speed* $1/\sqrt{N}$*; that is,*

$$\sup_N \sqrt{N} E[\sup_{t \leq T} \|z_t^{i,N} - z_t^i\|] < \infty \tag{16}$$

3. For any $k \in \mathbb{N}$ *and any k-tuple* $(i_1, ..., i_k)$*, the law of the process* $(z_t^{i_1,N}, ..., z_t^{i_k,N}, t \leq T)$ *converges to* $u_t \otimes ... \otimes u_t$*.*

Proof. Taking into account (15), the interaction, $\tilde{b}$, is bounded—that is, $\|\tilde{b}\|^2 \leq K$ for some $K > 0$. Hence, $\tilde{b}$ satisfies the linear growth condition of the interactions (H3) in Baladron et al. (2012). As is easily checked, other conditions about f, g, and $\tilde{b}$ (H1, H2, and H4 in Baladron et al. (2012)) are satisfied. Therefore, applying Theorems 2 and 4 in Baladron et al. (2012), our claim follows. ■

Property 3 in Theorem 1 is called the *propagation of chaos*[4]. This property means that the probability distribution of $\{(z_t^1, ..., z_t^m)\}$ evolves as if each element is independent when $N \to \infty$. The motions of k tagged particles approach independent copies of equation (15). Moreover, the following theorem is important for our analysis,

Theorem 2 *Property 3 in Theorem 1 is equivalent to* $U_t^{(N)}$ *(*$M(\mathbb{R}^2)$*-valued random variables, where* $M(\mathbb{R}^2)$ *denotes the set of probability measures on* $\mathbb{R}^2$*), which converges in law to the constant random variable* u_t (Proposition 2.2 in Sznitman (1991)).

This theorem states that the empirical measure tends to concentrate near u_t, the solution of the mean-field equation (15). That is, while each element behaves stochastically, the distribution is approximated by the deterministic process as N becomes large. In this sense, this can be considered as a form of the law of large numbers. It should be noted that there is no *a priori* reason to assume that u_t is stationary. In fact, as we will see later, the distribution u_t shows cyclical movement at some parameter values. In the following, we study the behavior of u_t when we change the parameters.

[4]This terminology comes from Kac.

4.3 Stability Analysis

In the previous subsection, the McKean–Vlasov equation, which governs the time evolution of the measure, $u(t) = \lim_{N\to\infty} U_t^{(N)}$, was introduced. Because the coefficients depend on $u(t)$, the equation is nonlinear with respect to $u(t)$. Thus, in practice, it is unrealistic to solve it explicitly. Therefore, to investigate the evolution of $u(t)$, an approximation method is needed.

It should be noted that in our model (equation (8)), each firm depends on $u(t)$ via mean fields, $\langle x \rangle$ and $\langle y \rangle$, and our primary concern is the behavior of these mean fields. Instead of investigating $u(t)$ directly, we consider the dynamics of lower moments of $u(t)$ (see, for example, Dawson (1983), Zaks et al. (2005) and Kawai et al. (2004).).

Setting $\varphi = (x - \langle x \rangle)^n (y - \langle y \rangle)^m$ and using Ito's formula, we have

$$\begin{aligned} d\varphi &= n(x-\langle x\rangle)^{n-1}(y-\langle y\rangle)^m dx_t^i + m(x-\langle x\rangle)^n(y-\langle y\rangle)^{m-1}dy_t^i \\ &+ \frac{1}{2}n(n-1)(x-\langle x\rangle)^{n-2}(y-\langle y\rangle)^m\sigma_1^2 dt + \frac{1}{2}m(m-1)(x-\langle x\rangle)^n(y-\langle y\rangle)^{m-2}\sigma_2^2 dt \end{aligned}$$

Then, taking into account equation (8) and using the Taylor expansion around $\langle x \rangle$ and $\langle y \rangle$, we obtain the following dynamical systems of moments:

$$\begin{aligned} \dot{\langle x\rangle} &= \langle x\rangle - \langle x\rangle^3 - 3\mu_{2,0}\langle x\rangle - \mu_{3,0} - e\langle y\rangle - D(1-h)\langle x\rangle \\ \dot{\langle y\rangle} &= (1-h)\langle x\rangle \\ \dot{\mu}_{2,0} &= -2D\mu_{2,0} - 2e\mu_{1,1} + 2(1-3\langle x\rangle^2)\mu_{2,0} - 6\langle x\rangle\mu_{3,0} - 2\mu_{4,0} + \sigma_1^2 \\ \dot{\mu}_{1,1} &= -D\mu_{1,1} - e\mu_{0,2} + (1-h)\mu_{2,0} + (1-3\langle x\rangle^2)\mu_{1,1} - 3\langle x\rangle\mu_{2,1} - \mu_{3,1} \\ \dot{\mu}_{0,2} &= 2(1-h)\mu_{1,1} + \sigma_2^2 \end{aligned} \tag{17}$$

where $\mu_{n,m} = \langle (x-\langle x\rangle)^n (y - \langle y\rangle)^m\rangle$ and $\langle\rangle$ denotes the expectation with respect to u_t, that is, $\int \varphi(v)u_t(dv)$. $\dot{}$ denotes the time derivative[5].

Now, we focus on a state $\langle x\rangle = \langle y \rangle = 0$ (called the *disordered state*). It corresponds to the situation where idiosyncratic shocks cancel each other out and no aggregate fluctuation appears. Note that there is always a stationary distribution with $\langle x\rangle = \langle y\rangle = 0$ satisfying equation (8)

[5] Equations for the higher moments can be deduced in a similar way, but become irrelevant under the Gaussian approximation. See below.

because of the symmetry of our model. At this stationary solution, other moments are determined by equation (17), that is,

$$0 = -2D\mu^*_{2,0} - 2e\mu^*_{1,1} + 2\mu^*_{2,0} - 2\mu^*_{4,0} + \sigma_1^2 \tag{18}$$

$$0 = -D\mu^*_{1,1} - e\mu^*_{0,2} + (1-h)\mu^*_{2,0} + \mu^*_{1,1} - \mu^*_{3,1} \tag{19}$$

$$0 = 2(1-h)\mu^*_{1,1} + \sigma_2^2 \tag{20}$$

We then apply the Gaussian approximation to investigate the linear stability of the state $\langle x \rangle = \langle y \rangle = 0$. The Gaussian approximation means that we approximate the system by the Gaussian distribution with time-varying parameters (see Zaks et al. (2005) and Kawai et al. (2004)). Because all the moments of Gaussian distributions are determined by the lower moments ($\langle x \rangle$, $\langle y \rangle$, $\mu_{2,0}$, $\mu_{1,1}$, $\mu_{0,2}$,), equation (17) becomes a closed-form expression. Specifically, $\mu_{3,0} = 0$, $\mu_{3,1} = 3\mu_{2,0}\mu_{1,1}$, and $\mu_{4,0} = 3\mu_{2,0}^2$ are used in our model.

Next, we carry out the standard linear stability analysis. As is easily checked, the Jacobian of the five dimensional system of ($\langle x \rangle$ $\langle y \rangle$ $\mu_{2,0}$ $\mu_{0,2}$ $\mu_{1,1}$) can be written in a block diagonal form.

$$\begin{pmatrix} A & \mathbf{0} \\ \mathbf{0} & B \end{pmatrix} \tag{21}$$

A is a 2×2 matrix and B is a 3×3 matrix. Therefore, the behavior of $\langle x \rangle$ and $\langle y \rangle$ around the disordered state can be determined solely by

$$A = \begin{pmatrix} 1 - 3\mu^*_{2,0} - D(1-h) & -e \\ (1-h) & 0 \end{pmatrix} \tag{22}$$

The eigenvalues are given by

$$\lambda_\pm = \frac{1}{2}\Big(1 - 3\mu^*_{2,0} - D(1-h) \pm \sqrt{(1 - 3\mu^*_{2,0} - D(1-h))^2 - 4(1-h)e}\Big) \tag{23}$$

We examine when the stability of the disordered state is lost—that is, when the condition that the real parts of the eigenvalues become 0. From (23), $\mu^*_{2,0} = \frac{1}{3}(1 - D(1-h))$ is implied. From (20),

$\mu^*_{1,1} = -\frac{\sigma_2^2}{2(1-h)}$. Substituting these values into equation (18), we obtain the following condition.

$$f_h(D) \equiv Dh(1 - D + Dh) = \frac{3}{2}\big(\sigma_1^2 + \frac{e\sigma_2^2}{1-h}\big) \equiv \frac{3}{2}\sigma^2 \tag{24}$$

$\sigma^2(\equiv \sigma_1^2 + \frac{e\sigma_2^2}{1-h})$ represents the intensity of idiosyncratic shocks. The left-hand side, $f_h(D)$, can be interpreted as the degree of interaction that generates order in the system. In particular,

$$\lim_{h \to 1} f_h(D) = D \tag{25}$$

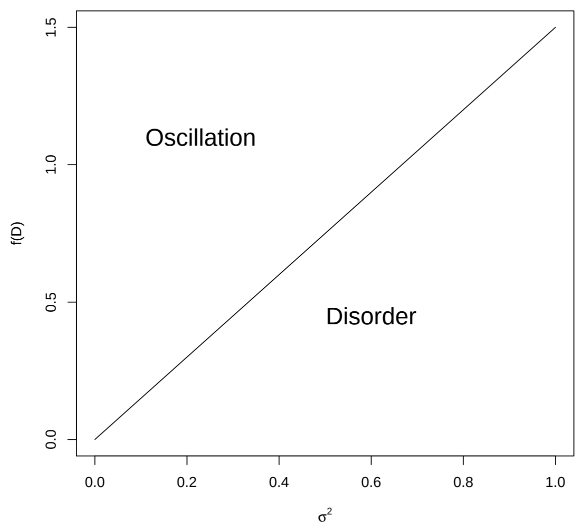

Figure 6: Equation (24).

When the two parameters, σ and D, satisfy this relation, bifurcation occurs. The interpretation is clear. When the interaction effect, D, is below the critical point, D^*, the idiosyncratic shocks dominate the system. Any order is destroyed by these shocks, and the system is close to the system with no interaction. Therefore, the simple LLN holds and any no order can be observed. The stationary distribution with $\langle x \rangle = \langle y \rangle = 0$ is stable. The microeconomic structure (e.g. lumpiness) is irrelevant to the explanation for the aggregate fluctuations.

However, when D exceeds the critical point, D^*, the situation changes completely. The linear stability analysis above shows that the stationary distribution with $\langle x \rangle = \langle y \rangle = 0$ is no longer stable. That is, the idiosyncratic shocks do not prevent the interaction from generating order in the system. This suggests the possibility of collective behavior in the system. In fact, as we will see in the following, regular cyclical behavior is observed at the aggregate level. In the next subsection, we carry out numerical simulations.

4.4 Simulation

Figures 7 to 13 show the results of simulations for $\langle x\rangle$ and $\langle y\rangle$ of equation (8) with different values of D (other parameters are fixed and $N = 20000$). In Figure 7 with a small value of D, there is no observable aggregate behavior. Only small variation around $\langle x\rangle = \langle y\rangle = 0$ exists. This is considered to be the finite number effect of N. It is consistent with our analysis in the previous section. The microeconomic shocks cancel each other out; therefore, lumpiness (or nonconvexity) at the firm level plays no role in the aggregate fluctuations.

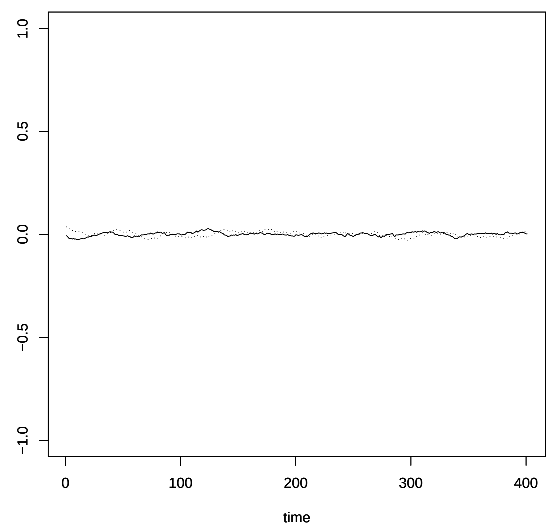

Figure 7: Simulation of equation (8) with $\sigma_1^2 = \sigma_2^2 = 1/4$, $e = 0.1$, $h = 0.9$ and $D = 0.1$. The solid (dashed) line is $\langle x\rangle(\langle y\rangle)$.

Figure 8 shows that when the interaction effect is large enough to compensate for the disturbance caused by the idiosyncratic shocks, different aggregate behavior appears and an endogenous cyclical movement is observed. This is consistent with the fact that the eigenvalues, (23), have an imaginary part different from 0 near the bifurcation point. Interestingly, the movement at the macroscopic level is more regular than at the firm level.

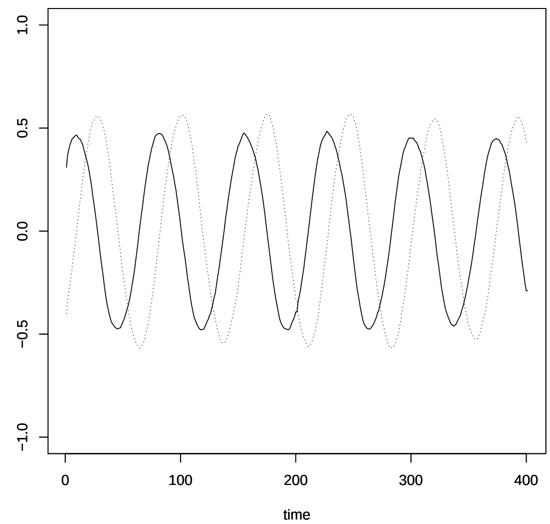

Figure 8: Equation (8) with $D = 0.32$. The other parameters are the same as in Figure 7.

Figure 10 shows the histogram of y when $\langle x \rangle = 0.00$ and $\langle y \rangle > 0$—that is, when the economy has excess inventories. It corresponds to a situation in which the economy goes through a phase of contraction to reduce the excess inventories. However, it should be noted that there is heterogeneity among firms and, as Figure 10 shows, some firms' inventories are running short. The same argument can be applied to Figure 12, where business is good, $\langle x \rangle > 0$. Under this favorable business condition, there exist firms that choose low production depending on their states. The motions of $\langle x \rangle$ and $\langle y \rangle$ are the averaging behaviors of firms in the economy.

However, the cyclical behavior of $\langle x \rangle$ and $\langle y \rangle$ can be observed to be significantly below the critical value $D^* = 0.92$ predicted by the stability analysis in the previous section. This is related to the fact that the resulting distribution is different from a Gaussian distribution. In particular, the marginal distribution of x_t^i shows clear bimodality. In Figure 14, we estimate the spectral density of the cycle of $\langle x \rangle$. This density peaks at 0.014—that is, the period of the cycle is 71. On the other hand, from (23), the frequency is given approximately by $2\pi/\sqrt{(1-h)e} = 0.016$ near the bifurcation point. The period predicted by the stability analysis is $1/0.016 = 63$, which is relatively close to the estimated value. Therefore, although the critical value of D is overestimated,

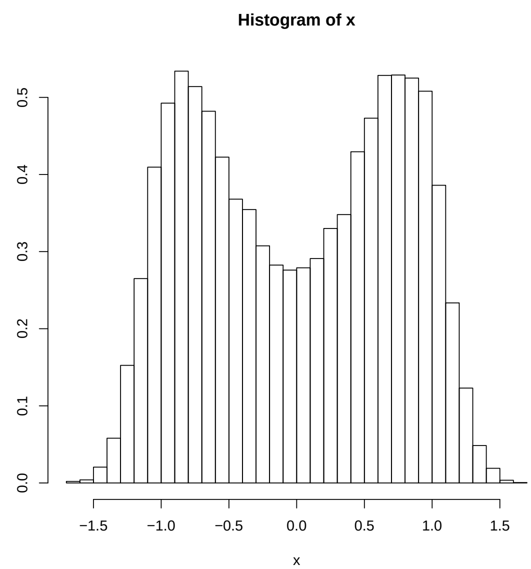

Figure 9: The histogram of x_t^i when $\langle x \rangle = 0.00$. The parameters are the same as in Figure 8.

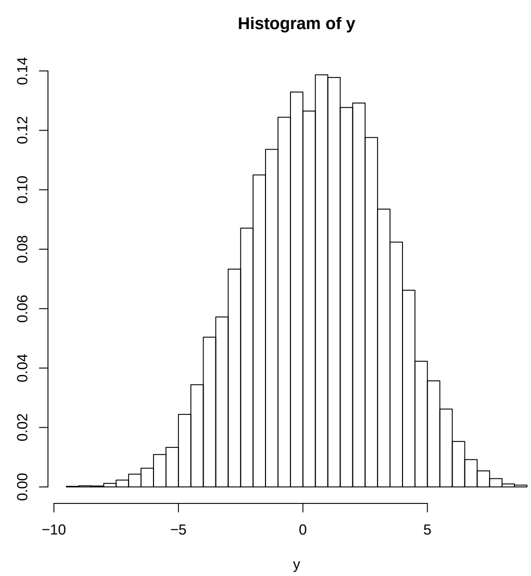

Figure 10: The histogram of y_t^i when $\langle x \rangle = 0.00$. It corresponds to an economy going through a phase of contraction due to excess inventories, $\langle y \rangle = 0.56$.

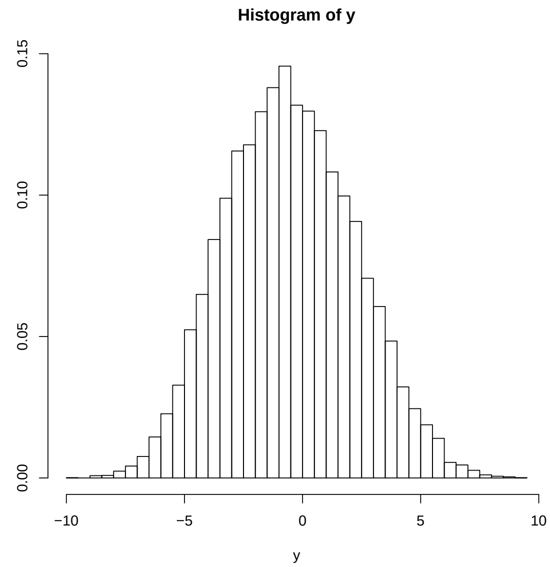

Figure 11: The histogram of y_t^i when $\langle x \rangle = 0.00$. It corresponds to an economy going through a phase of expansion, $\langle y \rangle = -0.52$.

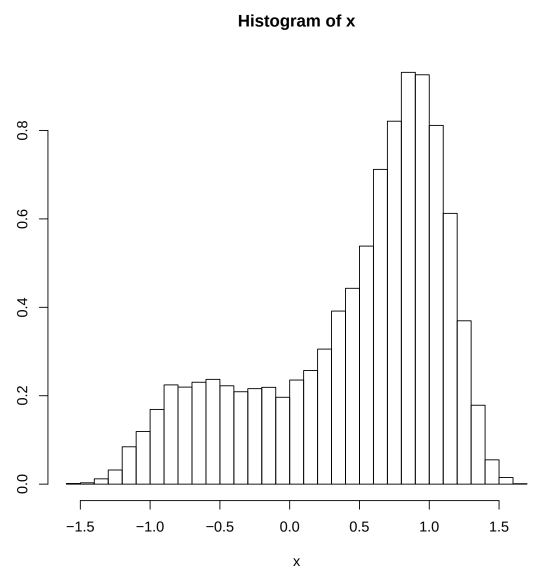

Figure 12: The histogram of x_t^i when $\langle x \rangle = 0.45$.

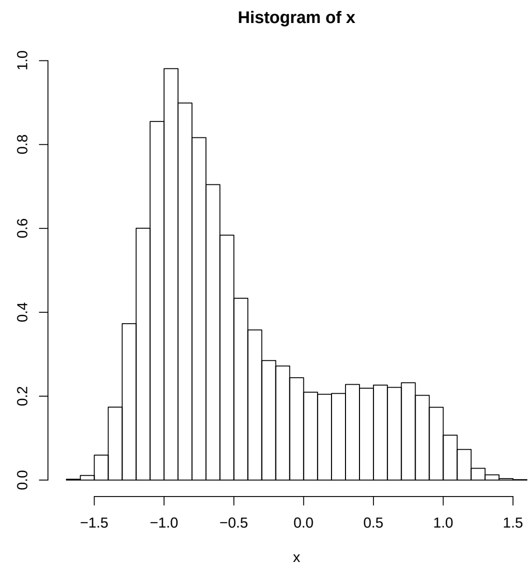

Figure 13: The histogram of x_t^i when $\langle x \rangle = -0.46$.

we conclude that the qualitative feature of our model is captured by the stability analysis[6].

This cyclical behavior of $\langle x \rangle$ and $\langle y \rangle$ is closely related to the well-known *Kitchin cycle*, which is usually explained as follows (see, e.g., Korotayev and Tsirel (2010)). Suppose that firms observe the improvement of their commercial situation. They manage the increase in demand by increasing production. The demand is filled with the supply, but the supply gradually becomes excessive because it takes some time for businesspeople to realizes that the supply exceeds the demand. This time lag generates an unexpected increase in inventories, which leads to the reduction of production to decrease the excessive inventories. After the inventories are sufficiently reduced, a new cycle of demand increase is initiated. The origin of these cycles is the time lags in the information.

At first glance, as shown in Figure 8, the behavior of $\langle x \rangle$ and $\langle y \rangle$ appears to be consistent with the above scenario. An increase in $\langle x \rangle$ is an increase in demand that leads to an increase in $\langle y \rangle$. The cycle of $\langle y \rangle$ lags behind that of $\langle x \rangle$. However, time lags at the firm level do not occur in our model. Because of nonconvex technology, businesspeople optimally choose the low or high production and increase or decrease their inventories. Furthermore, in contrast to Carvalho (2010), Acemoglu et al. (2012), and Gabaix (2011), each firm has a negligible impact on $\langle x \rangle$ and $\langle y \rangle$ as N is large. $\langle x \rangle$ and

[6] On this point, Zaks et al. (2005) reach the same conclusion. See Zaks et al. (2005) and the relevant discussion therein.

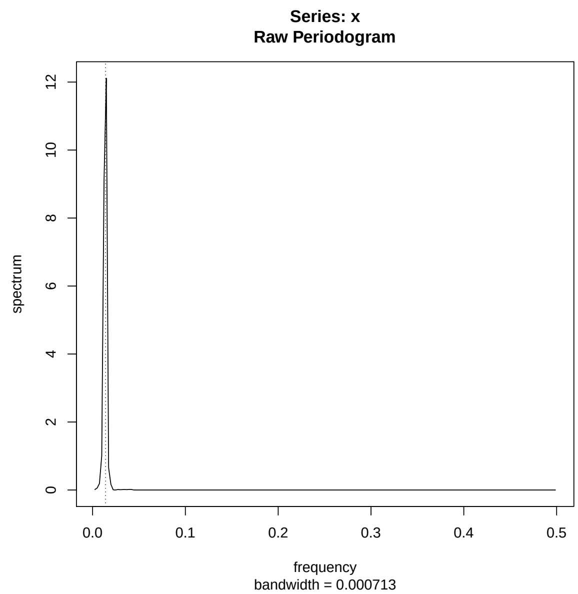

Figure 14: Spectral density of $\langle x \rangle$.

$\langle y \rangle$ are the average of firms in the economy; therefore, there is no representative firm corresponding to the motion of $\langle x \rangle$ and $\langle y \rangle$. Indeed, as shown in Figures 3, 4, and 8, the behavior of $\langle x \rangle$ and $\langle y \rangle$ is different from that of an individual firm, x^i and y^i. The behavior of $\langle x \rangle$ and $\langle y \rangle$ is a type of collective behavior that can only be observed at the macroscopic level.

Furthermore, the relation of the two cyclical behavior of $\langle x \rangle$ and $\langle y \rangle$ can be explicitly written. Summing both sides of equation (2) over i and dividing them by N, we obtain

$$\frac{1}{N}\sum_{i=1}^{N} dy_t^i = \frac{1}{N}\sum_{i=1}^{N}(x_t^i - s_t^i)dt = \frac{1}{N}\sum_{i=1}^{N}(x_t^i - h\langle x \rangle - \xi_t^i)dt = \Big((1-h)\langle x \rangle - \frac{1}{N}\sum_{i=1}^{N}\xi_t^i\Big)dt \tag{26}$$

Taking the limit, $N \to \infty$, we obtain the simple relation $\dot{\langle y \rangle} = (1-h)\langle x \rangle$ by the law of large numbers. This means that the aggregate inventory investment (the change in inventories) comoves with the aggregate production without a time lag. This prediction is consistent with empirical data (see, e.g., Table 2 in Stock and Watson (1999)).

5 Concluding Remarks

This paper investigated the relationship between microeconomic structures and business cycles. The standard production-smoothing theory has been empirically rejected in the literature; therefore, we focused on the nonconvex cost function. This hypothesis, which has empirical support, can explain the excess volatility of production. The issue is whether this microeconomic structure has a nontrivial effect at the aggregate level. If the interaction effect is taken into account, this problem becomes very complicated, and the LLN argument (e.g. Lucas (1977)) cannot be applied. In particular, we have to deal with the evolution of the distribution of production and inventories, that is, an infinite-dimensional random variable.

To investigate this problem, the propagation of chaos result is used in our model. Our model explicitly takes into account the feedback loop—that is, the macroscopic state of the economy not only is an aggregation of the firms but also prescribes the macroeconomic environment experienced by firms. We have shown that the empirical measure of production and inventories, $U_t^{(N)}$, converges to a $M(\mathbb{R}^2)$-valued constant variable, u_t, as N goes to infinity. This means that whereas each element behaves stochastically, the distribution is approximated by the deterministic process as N becomes large. In this sense, it can be considered a form of the law of large numbers. However, this does not imply that the distribution is stationary. In fact, this feedback loop together with nonconvex technology generates rich interesting phenomena and has been shown to be the origin of business cycles.

The standard linear stability analysis shows that the disorder state loses its stability, given that the interaction effect exceeds the critical point. This means that the interaction effect generates order in the system. With the help of numerical simulations, we have demonstrated that the resulting aggregate behavior shows regular cyclical movement without any aggregate exogenous shocks. This endogenous business cycle is an explanation for the Kitchin cycle. It should be noted that there is no representative firm corresponding to $\langle x \rangle$ and $\langle y \rangle$ and that the behavior of $\langle x \rangle$ and $\langle y \rangle$ is different from that of an individual firm, x^i and y^i. This is one example of the collective behaviors that can be only observed at the aggregate level and are crucial to macroeconomic analysis.

Finally, there exists other microeconomic behavior that is characterized by lumpiness (e.g., Cooper and Haltiwanger (2006)). Investigating how the microeconomic characteristics affect the aggregate fluctuations via interactions is a promising subject for future research.

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