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Trade-offs in Compensating Transfers for a Multiple-skill Model of Occupational Choice

ICHIDA Toshihiro
Waseda University

Research Institute of Economy, Trade & Industry, IAA
The Research Institute of Economy, Trade and Industry
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Abstract

Using a multiple-skill model of occupational choice, we study the trade-offs faced by a benevolent government that aims at Pareto improvement from trade liberalization via incentive-compatible compensating transfers. When the transfers are designed after some liberalization has been realized, the trade-off is between Pareto improvement and overcompensation. When agents anticipate future transfer schemes, the trade-off is between the size of aggregate production gains and the amount of overcompensation.

Keywords: Multi-dimensional heterogeneity, Model of occupational choice, Pareto gains from international trade, Compensation schemes, Multi-dimensional human capital skills

JEL classification: F11, F16, H21, J24

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1 Introduction

There are many instances in economies where potentially Pareto-improving policy changes are blocked by opposition from groups that will be harmed by the change. For example, proposals for trade liberalization in a small open economy often encounter fierce political opposition within that economy, despite the fact that such liberalization typically results in aggregate gains to the economy as a whole. If the economic change is actually Pareto improving, then there should be nobody who opposes the change. In the real world, however, such changes create both winners and losers and "actual" Pareto improvement requires ex post income redistribution because such an improvement is merely "potential.". Why do some people clamor against such potentially Pareto-improving change? In most cases, it is because the actual execution of income transfers by the government is seldom done after the change (liberalization) has been implemented. In practice, compensating redistribution seldom takes place and losers are often left uncompensated. Even when some redistribution schemes are carried out, they are not done in full. The lack of satisfactory redistributing transfers is the main reason that potentially Pareto-improving changes have so many opponents. This raises a question: Why and how is compensating redistribution unsatisfactory? The aims of this paper are to elucidate the reasons why governmental compensation programs often fail and to identify the nature of trade-offs in such programs.

There are two strands of criticism about compensating redistribution schemes. The first is that the compensation coverage is insufficient. Current compensation schemes are said to be imperfect in that coverage is too limited and amount of compensation is too small relative to actual losses, and so some losses are left untreated. The second strand of criticism is that the existing compensating transfers are overcompensating (relative to intended consequences), and the money is thus wasted because many transfers reach those who were not originally targeted. This problem of *overcompensation* requires some explanation. There are some schemes to compensate those who lose from changes (such as trade liberalization). For example, in the United States, the Trade Adjustment Assistance (TAA) Program provides unemployment insurance that is more generous for those who have lost jobs as a consequence of trade liberalization than for those who have lost jobs for other reasons. Some say TAA is wasting money since it is overcompensating. Any scheme that aims to compensate losers will specify the targeted group of individuals and the targeted amount. Two problems with such schemes are of particular concern: (1) among many instances of actual transfers, the transfer amount may be considered to be larger than what was originally intended by the policymakers; and (2) some of the money might be misdirected.[1]

If we examine these two criticisms, they seem contradictory on the surface because the first type seems to imply that compensation is insufficient and the

[1] Here, I am not talking about illegal actions such as fraud. There are instances in which some people obtain the subsidies from the government *legally*, but the government did not intend them to be recipients.

second type seems to imply that compensation is excessive. The purpose of this paper is to formalize one possible explanation for this seeming contradiction.

Imagine that an economy undergoes a change (such as trade liberalization) that brings about aggregate gains to the economy. Any such changes will have distributional consequences and thereby create winners and losers. The government aims at Pareto improvement by subsidizing losers with taxes collected from winners. Nevertheless, the lack of information about individuals and the limits on instruments available in the tax system may prevent the government from properly identifying who gained and who lost. This inability to completely identify affected parties may cause the seeming contradiction. If the government aims to help every loser in the economy, then it may end up subsidizing some who gained and did not need to be helped. This may result in overcompensation and, at the same time, exhaust the governmental budget intended to help the losers. When the problem of overcompensation is severe, the compensation scheme may run a deficit. In such a case, the policymakers may stop compensating transfers, either totally or partially. This may cause insufficient coverage of the compensation for losers. This paper is the first paper to explain the trade-offs faced by a government trying to conduct Pareto-improving transfers after an economic change that yielded aggregate gains.

The explanation in this paper uses the idea of Roy (1951) to model self-selection and occupational choice in the context of labor economics. We combine the Roy model with the framework provided by Ruffin (2001), whose model assumes technology with both quasi-specific and regular factors of production.[2] In this paper, individual agents in the economy are endowed with a multi-dimensional bundle of skills, together with other regular types of productive factors. The compensation with regard to regular factors can be taken care of by the commodity taxation scheme proposed by Dixit and Norman (1980, 1986). However, the design of a compensation scheme that accounts for the multi-dimensional vector of skills is not straightforward. Multi-dimensional human capital skills are embodied in workers and cannot be sold separately from the workers themselves in the market. Employers acquire the associated bundles of skills when they hire a worker. The importance of this bundling restriction was noted in Murphy (1986)[3] and in Ohnsorge and Trefler (2007).[4]

[2] Quasi-specific factors in Ruffin (2001) are similar to the multi-dimensional vector of skills used in this paper. The difference between my proposed model and the model of Ruffin is in the richness of the support for joint distribution of talent vectors. The proposed model assumes a continuum of atomistic agents jointly distributed over a unit square; the Ruffin model assumes the existence of only a finite number of points in the unit square. The dense nature of my proposed framework allows job switching by individual workers in response to economic changes. In Ruffin's framework, in contrast, there may not be any job-switching agents. In fact, Ruffin's paper does not examine the problem of whether job-switchers exist.

[3] I thank Professor Elhanan Helpman for bringing my attention to this old Ph.D. dissertation by Kevin Murphy. The difference between my model Murphy's is the nature of individual heterogeneity. Because Murphy wanted to look at the human capital investment decisions in a multiple-skill model, all the agents in his model are born to be identical ex ante. My model does not have an investment component for analysis and agents are assumed to be innately heterogeneous in a multi-dimensional skill space.

[4] Both Ohnsorge and Trefler (2004) and Ohnsorge and Trefler (2007) use multi-dimensional

In the model with multi-dimensional skills, workers sort themselves into the jobs that pay the most according to their skill sets. Among the skills possessed by workers, all except one (corresponding to the chosen job) are assumed to be latent talents with no ex post market value. The unused talents serve workers as second- and third-best alternatives that can be called on when choosing in which sector to work. For each worker, the latent skill with the highest return determines the opportunity cost of keeping the worker's current job. It is the inability of government to capture the exact size of this opportunity cost for individual agents that prevents it from designing a Pareto-improving taxation scheme that does not involve trade-offs.

This paper focuses on the nature of trade-offs involved with compensating transfers in the context of a multi-dimensional skill model of occupational choice based on individual comparative advantages. The type of trade-off changes according to the timing of the announcement of the economic policy of interest. While the basic logic behind the model is general, we will use trade liberalization as an example to motivate our approach. Specifically, we consider an economy that is initially in a steady state with a tariff (or some other type of protection) that will be relaxed to promote free trade. The timing of compensating policies can be analyzed in two ways: sudden, unannounced liberalization; and delayed, pre-announced liberalization. We find that the trade-offs involved can be quite different between the two cases. Specifically, while the first trade-off (the unannounced case) is between the achievement of Pareto improvement and the existence of overcompensation, the second trade-off (the pre-announced liberalization case) is between the size of the aggregate production gains and the amount of wasteful overcompensation.

First, when the liberalization is sudden, agents in the economy do not anticipate the government's actions. Therefore, agents will change their occupations in response solely on the basis of their individual comparative advantages. Some agents will change jobs, and others will stay in the same sector. The aggregate production gain is maximized for the economy under the new relative price. The government's inability to tax according to the opportunity cost of unused talents will create overcompensation for some job-switching workers. If the government is determined to achieve Pareto improvement, then it cannot avoid overcompensation for some job-switchers. When the amount of overcompensa-

skill distribution, which is similar to the model of this paper. However, the model in this paper—which is based on my job market paper written in 2002—and the model of Ohnsorge and Trefler (2004, 2007) were independently discovered. There are two primary differences between my paper and those of Ohnsorge and Trefler: (1) Ohnsorge and Trefler (2007) looked at distributions of more primitive talents (such as quantitative skills and communication skills, which jointly determine the individual comparative advantage for production in different sectors, using Heckscher–Ohlin-type skill intensities) and (2) Ohnsorge and Trefler (2007) studied the source of comparative advantage by examining the higher moments of the joint distribution of talents, while this paper looks at the trade-offs faced by governments when administering compensating transfers. To explain the first difference more clearly, the notion introduced by the dissertation of Murphy (1986) is helpful. The modeling method used in Ohnsorge and Trefler is a "Becker–Lancaster" approach, and the modeling method used in this paper is a simple "Roy" approach in which skill categories are in one-to-one correspondence with sectors in the economy.

tion for job-switchers is large, the government may use up all the tax revenue from winners and may not be able to balance the budget for the scheme. If so, it might as well give up on achieving Pareto improvement as a policy goal and may resort to more moderate redistribution.

Second, when the liberalization is anticipated by agents, then those agents will adjust their behaviors accordingly. In the model in this paper, this happens for some (but not necessarily all) job-switching agents. When agents anticipate the future execution of compensating transfers, some counterfactual job-switchers (that is, those who would have changed job if the transfers were unanticipated) will stay in an industry anticipated to lose from the change. These job-switchers do so because they know they will receive compensation for staying in the losing sector. Another set of job-switchers will actually change their sectors because they can gain from switching, regardless of transfer scheme. Because there are some workers who, as a consequence of anticipation, do not change sectors, the degree of production efficiency is lower than in the case of unanticipated liberalization. However, the government can actually control the degree of overcompensation by determining the level of subsidy given to those who changed jobs from the losing sector to a new sector. That is, by subsidizing moves, the government can induce some counterfactual job-switchers to actually switch jobs, which enhances aggregate production efficiency. However, as more counterfactual job-switchers move to a new sector, overcompensation will rise, too. Therefore, the government faces a trade-off between the size of the increase in aggregate production efficiency and the size of overcompensation in the transfer program.

Davidson and Matusz (2006) investigated the similar problem of trade liberalization and compensation. In a sense, this paper provides a micro foundation for the model of their paper, which assumes one-dimensional worker heterogeneity.[5] Davidson and Matusz also discuss the contrast between job-stayers and job-switchers, and the problem of overcompensation. While they sought the best (most efficient) way to compensate losers, this paper focuses on the trade-offs faced by policymakers in constructing a compensation scheme.

The paper is organized as follows. Section 2 develops the basic model of occupational choice with heterogeneous agents having a multi-dimensional skill set. The examined model is a two-sector model produced by one general factor and a vector of occupational abilities. Section 3 conducts an analysis of the welfare of individual agents when there is no compensating distribution. Section 4 investigates various desirable and undesirable properties of possible compensating redistribution schemes. Section 5 discusses the trade-offs faced by a government when it tries to carry out the compensating redistribution schemes. The final section summarizes the results and suggests some possible extensions.

Although we use trade liberalization as an example to motivate our approach, the logical structure of the model is fairly general and can be applied

[5] In their model, workers have higher or lower ability. In this model, it is the strength of individual comparative advantages that induces agents to stay or switch jobs.

to any economic changes that will result in both aggregate economic gains and distributional consequences for each individual. The setup of the model is initially a small open economy with positive tariff, to which liberalization will be added. However, the basic workings of the model apply to economic changes occurring in a closed domestic economy as well. The main reason for considering a small open economy is the this reduces the length of the analysis[6] and focuses attention on the main topic of this paper: analysis of the trade-offs involved with compensating redistribution schemes.

2 Basic Model

Consider a small open economy that produces two outputs, X and Y, whose market prices are denoted by P_X and P_Y, respectively. These two output goods are produced by combining two types of input factors held by each individual: multi-dimensional occupation specific talents (abilities) and generic factors.

The economy comprises a continuum of heterogeneous atomless agents whose collective measure can be normalized to unity. An agent is characterized by a two-dimensional vector of occupational abilities (θ, τ) jointly distributed according to $F(\theta, \tau)$, whose density is written $f(\theta, \tau) > 0$ everywhere over a compact and convex space $\Theta \subset \mathbb{R}^2_+$. While the type of space can be general, we examine the case of a unit square $\Theta = [0, 1] \times [0, 1]$ here, and use this square in the diagrams and in the analysis of the compensation scheme. Each component of a vector (θ, τ) represents the size of an occupation-specific talent. The size of θ (respectively, τ) corresponds to the agent's ability to produce output X (respectively, Y). An agent $(\theta, \tau) \in \Theta$ is also endowed with $K(\theta, \tau) \geq 0$ units of generic factors of production. We do not specify a distribution for these, except that the total amount available in the economy is written is $\overline{K}$; that is, the full employment condition

$$\int_{(\theta,\tau)\in\Theta} K(\theta, \tau) dF(\theta, \tau) = \overline{K} \tag{1}$$

holds. Hence, the total factor endowments held by an agent (θ, τ) can be written as a tuple $(\theta, \tau; K(\theta, \tau))$, which we abbreviate to $(\theta, \tau; K)$.

There are several important assumptions about agents and factors of production that may be different from those in regular models. We summarize these assumptions here as A1–A3:

A1: Skill Specificity An occupation-specific talent is specific to its particular sector. The marginal product is positive in the specific sector but zero in the other sector. To produce output X (respectively, Y), an agent must

[6] A setup with a small open economy makes analysis straightforward because changes in the economy appear as changes in the relative prices of outputs (i.e., in terms of trade). Otherwise, changes in output prices may come from sources such as changes in technology, endowments, and preferences. In either case, it would be necessary to analyze the welfare change from both price and technology (endowment/preference) changes.

use both the generic factor K (or k)[7] and the occupation-specific talent θ (respectively, τ). Although the general factor is used in both sectors, the occupation-specific talent θ (respectively, τ) has no value for production in the other sector Y (respectively, X).

A2: Single Job (Skill Bundling) Each agent can choose only one job at a time. Because human capital skills are embodied in agents, the components of the skill set cannot be sold separately (see Murphy 1986.). In other words, a skill vector (θ, τ) must be sold as a package. Similarly to the setup in Ohnsorge and Trefler (2007), a bundle of skills cannot be decomposed. When an agent uses one component of the skill vector, the other component represents a latent skill, which is only potential.

A3: No Market for Skills There is not a complete market for occupation-specific talents. Although there is a market for generic factor K, so that agents can sell and buy a portion of their endowments, agents cannot sell a portion of their talents. Agents are assumed to be able to borrow money in order to buy generic factors because the generic factor itself can serve as collateral. However, we assume that agents cannot use talents as collateral for a monetary loan. This assumption is made to better reflect reality. For a justification, see a magazine article in *The Economist*[8] that supports the idea of imperfect capital markets for human capital investment. The magazine article says "For instance, borrowing to finance an investment in human capital may be difficult because would-be trainees lack collateral, or because the costs of administration and collection make such loans unattractive to private lenders." So, we assume that there is no market for skills per se.

Agents generate income according to the returns on ownership of endowment as represented in the tuple $(\theta, \tau; K)$. The income should consist of the returns from both the generic factor return and the two-dimensional skills vector. We assume that there is a market for generic factor K, and we let its price be denoted by r. An agent with a skill vector (θ, τ) is assumed to earn the residual profit for the *used* skill. In this model, each agent is a residual claimant for the *used* skill. Although the agent's skill vector has multiple components, the return from skill usage is from only one of the skills. The other skill component is a latent potential and has nothing to do with the income of the agent. This feature follows from the bundling and single-job assumptions together.

2.1 Technology and Supply Side

All economic agents share the same constant returns to scale technology. Both goods are produced with symmetric Cobb–Douglas production functions with a

[7] A capital letter indicates endowment and a lower-case letter indicates employment.
[8] "Investing in people," *The Economist*, March 26, 1994 p. 85.

parameter $a \in (0,1)$, which represents the share of the general factor income in total revenue:

$$\begin{cases} x(\theta,\tau) &= (k_X)^a \cdot (\theta)^{1-a}, \\ y(\theta,\tau) &= (k_Y)^a \cdot (\tau)^{1-a}, \end{cases} \tag{2}$$

where $x(\theta,\tau)$ and $y(\theta,\tau)$ are the potential amounts of production of each good by an agent with skill vector (θ,τ); and k_X and k_Y are the quantities of general factor K used for the production. Note that an agent (θ,τ) in (2) does not actually produce both $x(\theta,\tau)$ and $y(\theta,\tau)$ simultaneously; instead, the agent produces either in the equilibrium.

As a price taker in the markets for outputs and general factors, each agent takes the price vector (P_X, P_Y, r) as given and tries to maximize own residual claims. In a sense, we can treat every agent as self-employed. The agent's profit (residual) functions are the solutions to the following problems:

$$\begin{cases} \pi_X(\theta,\tau;P_X,P_Y,r) &= \max_{k_X} \{P_X \cdot x(\theta,\tau) - r \cdot k_X\}, \\ \pi_Y(\theta,\tau;P_X,P_Y,r) &= \max_{k_Y} \{P_Y \cdot y(\theta,\tau) - r \cdot k_Y\}. \end{cases} \tag{3}$$

Note that an agent (θ,τ) will calculate both values but ultimately choose only one sector in which to work. Thus, one of $\pi_X(\theta,\tau;P_X,P_Y,r)$ and $\pi_Y(\theta,\tau;P_X,P_Y,r)$ matters for an agent, and the other does not. Using the Cobb–Douglas production functions given by (2), the actual values of the profit functions can be calculated as

$$\begin{cases} \pi_X(\theta,\tau;P_X,P_Y,r) &= (P_X)^{\frac{1}{1-a}} \cdot (r)^{\frac{-a}{1-a}} \cdot \left(a^{\frac{a}{1-a}} - a^{\frac{1}{1-a}}\right) \cdot \theta \\ \pi_Y(\theta,\tau;P_X,P_Y,r) &= (P_Y)^{\frac{1}{1-a}} \cdot (r)^{\frac{-a}{1-a}} \cdot \left(a^{\frac{a}{1-a}} - a^{\frac{1}{1-a}}\right) \cdot \tau. \end{cases} \tag{4}$$

By comparing the two (potential) profits (residuals) in (4), the type space can be divided into two *job-assignment* partition groups: the group Θ_X of X producers, and the group Θ_Y of Y producers. That is,

$$\begin{cases} \Theta_X &= \left\{(\theta,\tau) \in \Theta : \tau < (P_X/P_Y)^{\frac{1}{1-a}} \cdot \theta\right\}, \\ \Theta_Y &= \left\{(\theta,\tau) \in \Theta : \tau > (P_X/P_Y)^{\frac{1}{1-a}} \cdot \theta\right\}, \end{cases} \tag{5}$$

where the division of labor line (i.e., the indifference curve for occupational choice) can be written as

$$\tau = (P_X/P_Y)^{\frac{1}{1-a}} \cdot \theta, \tag{6}$$

above which agents produce Y and below which agents produce X. The assignment of workers to particular sectors is done here according to the comparative advantage of individual agents, as in the models by Sattinger (1975), Rosen (1978), and Matsuyama (1992).

The output prices (P_X, P_Y) in the regular economic models will typically be determined by both supply and demand conditions in the equilibrium. However, this paper considers a small open economy where the change in (P_X, P_Y) will be given exogenously, which simplifies the model.

2.2 Demand Side

Consumers take market prices (P_X, P_Y, r) as given. For an agent with the skill vector (θ, τ), the utility function is given as $u(c_X, c_Y)$, and the profit maximization problem is

$$\max_{c_X, c_Y} u(c_X, c_Y) \quad \text{s.t.} \quad P_X \cdot c_X + P_Y \cdot c_Y \leq I(\theta, \tau). \tag{7}$$

To keep notation manageable and save space, we write (c_X, c_Y) instead of explicitly writing $(c_X(\theta, \tau), c_Y(\theta, \tau))$, to indicate the vector of consumption of two goods by a particular agent (θ, τ). The total income $I(\theta, \tau)$ of an individual (θ, τ) in equation (7) must be generated as the sum of the market values of all the possessed factors, including the residual profits from the *used* occupation-specific skill. This can be written as

$$I(\theta, \tau) = r \cdot K(\theta, \tau) + \max\{\pi_X(\theta, \tau; P_X, P_Y, r), \pi_Y(\theta, \tau; P_X, P_Y, r)\}, \tag{8}$$

where the actual size of residual profit is determined by the agent's self-selected occupation.[9]

In general, the utility functions of individual agents should be homothetic, monotonically increasing, strictly quasi-concave, and a twice continuously differentiable function. However, we assume a specific functional form here so as to make the following analysis tractable. Assume that all individuals have identical Cobb–Douglas preferences, spending half of their income on either good.[10] Now, we can use a convenient method for price normalization, taking the geometric mean of the two output prices to be unity. We can then choose a relative price parameter $p > 0$ so that $P_X = p$ and $P_Y = 1/p$ hold. This way, we can take the value of a consumer price index to be fixed with respect to any relative price change, and so we can compare the welfare of different states by simply looking at the income expressed as the parameter p.

2.3 Factor-market Equilibrium

To derive the equilibrium condition for the factor market, consider the full employment condition for general factors with taking into account the occupational choices of the individual agents as given by the job-assignment partitions given in (5):

$$\iint_{\Theta_X} k_X(\theta, \tau) dF(\theta, \tau) + \iint_{\Theta_Y} k_Y(\theta, \tau) dF(\theta, \tau) = \overline{K}, \tag{9}$$

where $k_X(\theta, \tau)$ and $k_Y(\theta, \tau)$ are the quantities of general factor used (employed) in the actual production of each output. If we substitute the solutions $k_X(\theta, \tau)$

[9] If $(\theta, \tau) \in \Theta_X$, then $I(\theta, \tau) = r \cdot K(\theta, \tau) + \pi_X(\theta, \tau; P_X, P_Y, r)$; if $(\theta, \tau) \in \Theta_Y$, then $I(\theta, \tau) = r \cdot K(\theta, \tau) + \pi_Y(\theta, \tau; P_X, P_Y, r)$.

[10] The utility function can be written as $u(c_X, c_Y) = 2\sqrt{c_X \cdot c_Y}$ and its indirect utility function is $v(P_X, P_Y, I) = I/\sqrt{P_X P_Y}$.

and $k_Y(\theta,\tau)$ into the problem (3), using the production function (2) for each agent (θ,τ), then we can obtain the equilibrium equation for the factor price r:

$$r = a \cdot \overline{K}^{a-1} \left[P_X^{\frac{1}{1-a}} \cdot \iint_{\Theta_X} \theta dF(\theta,\tau) + P_Y^{\frac{1}{1-a}} \cdot \iint_{\Theta_Y} \tau dF(\theta,\tau) \right]^{1-a}. \tag{10}$$

Here, the size of the factor price depends on only exogenously given parameters, namely, output prices and the size of aggregated skills. Let the size of the aggregated skills be written as

$$\begin{cases} \iint_{\Theta_X} \theta dF(\theta,\tau) & \equiv & V_\theta(p) \\ \iint_{\Theta_Y} \tau dF(\theta,\tau) & \equiv & V_\tau(p) \end{cases}, \tag{11}$$

for which we can show that $V_\theta'(p) > 0$ and $V_\tau'(p) < 0$ hold. (A proof of this is given in Appendix A.1.)

Let us define the following notation.

$$s(p) \equiv p^{\frac{1}{1-a}} \cdot V_\theta(p) + p^{\frac{-1}{1-a}} \cdot V_\tau(p) \tag{12}$$

We can write $s(\cdot)$ as a function of the parameter p because both V_θ and V_τ in (11) depend on p.

Using (12), we rewrite the factor price equation (10) as a function of p:

$$r(p) = a \cdot \overline{K}^{a-1} \cdot s(p)^{1-a}, \tag{13}$$

where a and $\overline{K}$ are parameters.

2.4 Equivalence of National Income with Gross National Product

Equilibrium national income, $GNI(p)$, can also be expressed as a function of relative output prices, p.

$$\begin{aligned} GNI(p) &\equiv \int_{(\theta,\tau)\in\Theta} I(\theta,\tau) \\ &= r(p) \cdot \overline{K} + \iint_{\Theta_X} \pi_X(\cdot) dF(\theta,\tau) + \iint_{\Theta_Y} \pi_Y(\cdot) dF(\theta,\tau) \end{aligned} \tag{14}$$

We now present an intermediate result about the relation between national income and generic-factor income.

Lemma 1 *Generic-factor income is proportional to national income as expressed by the following equation.*

$$r(p) \cdot \overline{K} = a \cdot GNI(p) \tag{15}$$

This follows directly from equations (8), (13), and (14). The proportional relation in (15) arises because the production functions for the two sectors are Cobb–Douglas and symmetric. A detailed proof is presented in Appendix A.2.

Note also that national factor income is equal to the gross national product, GNP.

$$GNI(p) = GNP \equiv P_X \cdot \iint_{\Theta_X} x(\theta,\tau) dF(\theta,\tau) + P_Y \cdot \iint_{\Theta_Y} y(\theta,\tau) dF(\theta,\tau) \quad (16)$$

It can be easily shown that the relation in (15) is consistent with (16).

2.5 Goods-market Equilibrium

Now, we analyze the goods-market equilibrium. We are interested in two kinds of equilibria: one for trade (exogenously given output prices) and another for autarky. We investigate trade volumes for the trading equilibrium and derive the market-clearing conditions for the autarky equilibrium.

2.5.1 Trading Equilibrium

A trading equilibrium is represented by a net imports vector, $m(p)$, for a given relative price, p:

$$m(p) \equiv (ED_X(p), ED_Y(p)) = (C_X(p) - X(p), C_Y(p) - Y(p)),$$

where $ED_X(p)$ and $ED_Y(p)$ are the excess demand functions for sectors X and Y, respectively, and $C_X(p)$ and $C_Y(p)$ represent aggregate demand:

$$C_X(p) \equiv \iint_{\Theta} c_X dF(\theta,\tau) \text{ and } C_Y(p) \equiv \iint_{\Theta} c_Y dF(\theta,\tau),$$

and $X(p)$ and $Y(p)$ represent aggregate supply:

$$X(p) \equiv \iint_{\Theta_X} x(\theta,\tau) dF(\theta,\tau) \text{ and } Y(p) \equiv \iint_{\Theta_Y} y(\theta,\tau) dF(\theta,\tau).$$

At the trading equilibrium, the output markets are not required to clear while the factor markets must clear within the border.

2.5.2 Autarky

Autarky is a special case in which the autarky price, p_A, makes $m(p_A) = (0,0)$. We now derive the conditions for the autarky equilibrium. Given the utility function $u(c_X, c_Y) = 2\sqrt{c_X \cdot c_Y}$, the aggregated Walrasian demand functions for goods X and Y can be written as

$$\left\{ \begin{array}{lcl} c_X(p, I(\theta,\tau)) & = & I(\theta,\tau)/2p \\ c_Y(p, I(\theta,\tau)) & = & p \cdot I(\theta,\tau)/2 \end{array} \right. \Longrightarrow \left\{ \begin{array}{lcl} C_X(p) & = & GNI(p)/2p \\ C_Y(p) & = & p \cdot GNI(p)/2 \end{array} \right. ,$$

where the left panel shows the individual demand functions and the right panel shows the market demand functions. By using the previous results, we can express the aggregate production in terms of p, as follows.

$$\left\{\begin{array}{lcl} x(\theta,\tau) & = & (ap/r(p))^{\frac{a}{1-a}} \cdot \theta \\ y(\theta,\tau) & = & (a/pr(p))^{\frac{a}{1-a}} \cdot \tau \end{array}\right. \Longrightarrow \left\{\begin{array}{lcl} X(p) & = & (ap/r(p))^{\frac{a}{1-a}} \cdot V_\theta(p) \\ Y(p) & = & (a/pr(p))^{\frac{a}{1-a}} \cdot V_\tau(p) \end{array}\right.$$

Thus, given the result in (15), when $p = p_A$, the following equations must hold.

$$\left\{\begin{array}{lcl} V_\theta(p) & = & \frac{\overline{K}}{2} \cdot \left(\frac{r(p)}{a \cdot p}\right)^{\frac{1}{1-a}} \\ V_\tau(p) & = & \frac{\overline{K}}{2} \cdot \left(\frac{p \cdot r(p)}{a}\right)^{\frac{1}{1-a}} \end{array}\right. \tag{17}$$

Substituting the equilibrium generic-factor return (13) into (17) yields the following autarky condition for aggregate employment of the specific occupational factors.

$$p^{\frac{1}{1-a}} \cdot V_\theta(p) = p^{-\frac{1}{1-a}} \cdot V_\tau(p) \mid_{p=p_A} \tag{18}$$

The equation (18) can also be written as

$$p_A^{\frac{2}{1-a}} = \frac{V_\tau(p_A)}{V_\theta(p_A)},$$

in which the right-hand side depends on the shape of the joint distribution of talent vectors.

3 Analysis of Welfare of Individual Agents before Transfers

This section analyzes the effects of trade liberalization on the welfare of individual agents. Throughout this section, we rule out possible compensation transfer schemes. Therefore, the current welfare analysis considers the situation before the introduction of any transfer schemes by the government.

We analyze the case in which trade liberalization will raise the relative price of good X. Thus, the price parameter was p_0 before liberalization and p_1, the price parameter after liberalization, is larger, so that $p_1 > p_0$. This small open economy is an exporter of X and an importer of Y.

Now, by using the price parameter p, the division of labor line (6) can be rewritten as

$$\tau = p^{\frac{2}{1-a}} \cdot \theta,$$

which will have a steeper slope after the liberalization because $p_1 > p_0$ and $\frac{2}{1-a} > 1$. Following the partitioning of the type space given by (5), let $\Theta_X(p_0)$ and $\Theta_Y(p_0)$ represent the **ex ante** partition of type space into a subspace for X producers and another for Y producers, and let $\Theta_X(p_1)$ and $\Theta_Y(p_1)$ represent each **ex post** partition. Then, the whole type space Θ can be divided into the following 3 partitions (See Figure 1.).

1. Job-stayers in sector X (workers who work in sector X both ex ante and ex post):
$$\Theta_{XX} \equiv \Theta_X(p_0) \cap \Theta_X(p_1)$$
2. Job-switchers who moved from sector Y to X (workers who work in sector Y ex ante and in sector X ex post):
$$\Theta_{YX} \equiv \Theta_Y(p_0) \cap \Theta_X(p_1)$$
3. Job-stayers in sector Y (workers who work in sector Y ex ante and ex post):
$$\Theta_{YY} \equiv \Theta_Y(p_0) \cap \Theta_Y(p_1)$$

Since $p_1 > p_0$, there are no job-switchers moving from X to Y, because $\Theta_X(p_0) \subset \Theta_X(p_1)$ and $\Theta_Y(p_0) \supset \Theta_Y(p_1)$ implies $\Theta_X(p_0) \cap \Theta_Y(p_1) = \emptyset$.

We can now summarize the results about welfare changes for individual agents.

Proposition 1 *The economic welfare of job-staying agents will improve (respectively, worsen) according to the increase (respectively, decrease) in the price of the goods that they produce.*

This proposition says that the welfare changes for the job-staying agents are the same as in the analysis for specific-factor owners in the specific-factors model of international trade. A proof of this is given in Appendix A.3.

Next, let us look at the welfare changes for job-switching agents.

Proposition 2 *Among those who are forced to switch jobs due to trade liberalization, there exist both those who gain and those who lose. Their sizes of gain or loss depend on the comparative advantages of the individual agents, which are determined by the relative sizes of the components of their skill vectors.*

The result of this proposition is intriguing because it may seem to be in contradiction with popular beliefs about the relation between liberalization and job losses. We tend to think that job losers (those who are forced to switch) should all be losers in their welfare, but the analysis shows that there are welfare gainers as well as losers among job losers. Furthermore, the analysis predicts that the amount of gain or loss depends on the comparative advantages of individuals.

Proof. Let the relative price parameter be given by p. The value of the profit function for agent (θ, τ) when the agent works for sector Y can be written as

$$\pi_Y(\theta, \tau; p) = \left[p^{\frac{-1}{1-a}} \cdot (r(p))^{\frac{-a}{1-a}} \cdot \left(a^{\frac{a}{1-a}} - a^{\frac{1}{1-a}} \right) \right] \cdot \tau. \tag{19}$$

By using $r(p)$ in (13), we can rewrite (19) as

$$\pi_Y(\theta, \tau; p) = \overline{K}^a (1-a) \cdot p^{\frac{-1}{1-a}} \, [s(p)]^{-a} \cdot \tau. \tag{20}$$

The profit of job-switchers $(\theta, \tau) \in \Theta_{YX}$ was $\pi_Y(\theta, \tau; p_0)$ ex ante and is $\pi_X(\theta, \tau; p_1)$ ex post. Note that

$$\pi_X(\theta, \tau; p) = \left[p^{\frac{1}{1-a}} \cdot (r(p))^{\frac{-a}{1-a}} \cdot \left(a^{\frac{a}{1-a}} - a^{\frac{1}{1-a}} \right) \right] \cdot \theta$$

for an agent who works for sector X. Similarly, we can write this using $\overline{K}$ and $s(p)$ as in (20).

Therefore, the percentage change of welfare for the job-switchers can be written as

$$\%\Delta\pi \equiv \frac{\pi_X(\theta, \tau; p_1) - \pi_Y(\theta, \tau; p_0)}{\pi_Y(\theta, \tau; p_0)} = \frac{(p_1)^{\frac{1}{1-a}} \left[s(p_1)\right]^{-a}}{(p_0)^{\frac{-1}{1-a}} \left[s(p_0)\right]^{-a}} \cdot \frac{\theta}{\tau} - 1. \tag{21}$$

The equation (21) can be thought of as an affine function of the comparative advantage parameter θ/τ because the output prices p_0 and p_1 are exogenously determined. To prove the proposition, we take three steps.

1. First, consider job-switching agents near the ex ante division of labor line: $\tau = (p_0)^{\frac{2}{1-a}} \cdot \theta$. Agents on this line must have been indifferent, ex ante, to sector; thus, $\pi_X(\theta, \tau; p_0) = \pi_Y(\theta, \tau; p_0)$. Therefore, the agents must have gained exactly the same percentage as job-staying agents in sector X, from Proposition 1.

2. Next, consider job-switching agents near the ex post division of labor line: $\tau = (p_1)^{\frac{2}{1-a}} \cdot \theta$. Agents on this line must now be indifferent (ex post) to sector; thus, $\pi_X(\theta, \tau; p_1) = \pi_Y(\theta, \tau; p_1)$. Therefore, the agents must have lost exactly the same percentage as job-staying agents in sector Y, from Proposition 1.

3. From steps 1 and 2, we know that the welfare changes take positive value near the ex ante division of labor line and negative value near the ex post division of labor line. Because the affine function (21) is a continuous function of the parameter θ/τ, there exists (from the intermediate value theorem) a value of θ/τ such that the welfare change is $\%\Delta\pi = 0$. In fact, when the following relation holds, welfare gain becomes zero.

$$\theta/\tau = (p_0)^{\frac{1}{1-a}} \left[s(p_0)\right]^{-a} / (p_1)^{\frac{-1}{1-a}} \left[s(p_1)\right]^{-a} \tag{22}$$

   By rearranging the terms in (22), we can write the zero-gain line in the type space:

$$\tau = \frac{(p_1)^{\frac{-1}{1-a}} \left[s(p_1)\right]^{-a}}{(p_0)^{\frac{1}{1-a}} \left[s(p_0)\right]^{-a}} \cdot \theta. \tag{23}$$

   Agents above this are gainers and those below are losers.

This third step concludes the proof. ■

The percentage change in welfare given by equation (21) shows that it takes the same value along rays from the origin. The fortune and misfortune of job-switchers changes along with the slope of the ray from the origin. The steeper the slope on which agents are located is, the smaller the gains (the larger the losses) become. If we look at the agents near the ex ante division of labor line, $\tau = (p_0)^{\frac{2}{1-a}} \cdot \theta$, we know that the welfare of these agents should be the same as the job-staying agents for sector X. As we observe progressively steeper slopes of rays from the origin up to the zero-gain line, (23), we see the percentage change in the welfare gain decreasing. For those who are on the zero-gain line, the rate of welfare change is zero. If the slope to the agent is steeper than that of the zero-gain line and the agent is below the ex post division of labor line, $\tau = (p_1)^{\frac{2}{1-a}} \cdot \theta$, then the welfare of the agent is worsened. The degree of worsening becomes more severe as the slope (of rays from the origin) becomes steeper. For the agents near the ex post division of labor line, the welfare decreases as much as those who stayed in the losing sector Y. Therefore, if we were to draw the line of "iso-percentage change of welfare", then the lines should coincide with the rays from the origin. (See the left panel of Figure 2.)

4 Trade-offs in Compensation Schemes

The previous section examined the welfare changes of individual agents when there is no compensating transfer scheme. In this section, we analyze how the government constructs an optimal compensation scheme. The results of the preceding analysis have shown that there are both winners and losers among job switchers. Having analyzed the effect of a change in the terms of trade without compensation, we next consider a government redistribution policy that aims to achieve Pareto improvement (after the liberalization) and to avoid overcompensation.

In choosing the instruments of the compensation scheme, we follow the literature in avoiding the use of lump-sum compensation because of the associated informational requirements.[11] Therefore, we examine a compensation scheme that is based on factor taxes and commodity taxes.[12] (Atkinson and Stiglitz 1980, p. 20) Let us now formally define the compensation scheme.

Definition 1 *A* ***compensation scheme*** σ *is a combination of taxes and subsidies levied on the following variables: (1) output prices, (2) generic-factor prices, and (3) occupational rewards (residual profits). Tax/subsidy rates can be linear or nonlinear.*

[11] See, for example, Feenstra and Lewis (1994, p. 202).

[12] Negative taxes are considered to be the same as subsidies. This notion of factor taxes and commodity taxes has been adopted from the standard public economics textbook of Atkinson and Stiglitz (1980).

The taxes (or subsidies) on output prices are commodity taxes, and the taxes on both generic-factor prices and occupational rewards are factor taxes.[13] Following Dixit and Norman (1980, 1986) and Feenstra and Lewis (1994), we consider a two-stage compensation procedure.[14] Because both groups of authors aim to achieve ex post Pareto improvement, the first stage of their analysis focuses on making everyone in the economy as well off as they were under the ex ante price p_0. To arrive at this end, policymakers must use both commodity taxes and factor taxes—added to these, in the case of Feenstra–Lewis, are relocation subsidies. Both Dixit–Norman and Feenstra–Lewis proved that not only will government revenues from such first-stage schemes become non-negative, they will be redistributed back to individuals in the economy during the second stage.

Definition 2 *The **compensation scheme** σ can hypothetically be implemented in two stages: (1) In the **first stage**, the government tries to minimize the rents that accrue to individual agents; in other words, it seeks to capture all these rents in the form of positive revenue. Let us call this stage's result a σ_1 equilibrium; and (2) in the **second stage**, the government sends this positive revenue back to the individual agents by means of either a poll subsidy or a reduction of some commodity tax rate. Let us call the result of this second stage a σ_2 equilibrium. This σ_2 equilibrium can also be called a σ equilibrium, since the result of the second stage is also the final result of the whole compensation scheme.*

This separation between two stages is a hypothetical construct. In an actual implementation of a compensation scheme, the planners do not need to take two steps. In reality, the planners can implement directly the σ equilibrium. However, it is important to know theoretically whether we can construct a rent-neutral first stage. The purpose of this hypothetical construction of the first stage is to see whether the scheme can ensure Pareto gains (ex post) by moving as close as possible to an equilibrium in which all the individual agents in the economy are as well off as they were before. The hypothetical first stage may leave the government non-negative revenue, and the following second stage tries to distribute that surplus back to individual agents. This can be done either by poll subsidy or by lowering consumption taxes (raising factor subsidies). Since the technical requirements for the second-stage redistribution—notable among these being the *Weymark conditions* (Weymark 1979)—are closely examined in the work by Dixit and Norman (1986), we take these results as given.[15] Our

[13] In Dixit and Norman, "commodity taxes" include both commodity and factor taxes, simply because the authors use a general approach that does not distinguish outputs from inputs.

[14] Two-stage compensation schemes are common in the existing literature because of economists' preference for discussing efficiency without addressing equity issues. Indeed, rent-neutral economic policy is desirable because policy-induced arbitrary wealth redistribution should be avoided.

[15] The Weymark condition states that there is one good for which some consumers are net buyers and no consumer is a net seller. In traditional trade models, in which consumers are net sellers of factors of production and net buyers of consumer goods, this condition is automatically satisfied.

primary focus of analysis will be on the feasibility of constructing a first-stage equilibrium.

First, we note some desirable and undesirable properties of the compensation scheme. The most important property is related to the concept of ex post Pareto efficiency.

Definition 3 *The compensation scheme σ is **weakly Pareto improving** if every individual is at least as well off as he or she was under p_0.*

Formally, the requirement for weak Pareto improvement is based on a comparison of individual welfare levels. In this model, these levels can be expressed as real income $I(\theta,\tau)$:

$$(I(\theta,\tau))^{\sigma} \geq (I(\theta,\tau))^{0}, \forall(\theta,\tau) \in \Theta, \tag{24}$$

where the superscript σ denotes individual welfare under the compensation scheme σ, and the superscript 0 denotes individual welfare under the ex ante p_0 situation. The real income of each individual $I(\theta,\tau)$ represents the welfare level in this model because real income is the same as individual indirect utility by our choice of price normalization.[16]

Another important property of the first-stage equilibrium is that of *rent neutrality.* A positive rent arises when a policy change or change in the environment raises individual welfare. The gain is a windfall profit in the sense of a Marshallian rent. For example, if the inequality

$$(I(\theta,\tau))^{\sigma} > (I(\theta,\tau))^{0} \tag{25}$$

is satisfied for an agent (θ,τ), then that agent derives a strictly positive rent, with a value of $(I(\theta,\tau))^{\sigma} - (I(\theta,\tau))^{0}$, from the policy shift under the compensation scheme σ.

We say little about the second-stage redistribution of positive government revenues. We simply reiterate that rent neutrality is a desirable feature of the first stage of a compensation scheme. Evidence for this is that the first-stage equilibria of both Dixit–Norman and Feenstra–Lewis are consistent with rent neutrality. Here is the formal definition of rent neutrality.

[16] Note that real income of an individual (θ,τ) can explicitly be written as a function of prices and rewards, that is, as $I(\theta,\tau;p,r(p),\pi_X(p),\pi_Y(p))$. If the output price parameter is p_0 ex ante, then

$$(I(\theta,\tau))^{0} = I(\theta,\tau;p_0,r(p_0),\pi_X(p_0),\pi_Y(p_0))$$

must hold. If the vector of the ad valorem tax (subsidy) rate for a tuple

$$(p_1,r(p_1),\pi_X(p_1),\pi_Y(p_1))$$

of prices and rewards can be written as $(t_p,t_r,t_{\pi X},t_{\pi Y})$, and

$$(T_p,T_r,T_{\pi X},T_{\pi Y}) \equiv (1-t_p,1-t_r,1-t_{\pi X},1-t_{\pi Y}),$$

then post-compensation real income should be written as

$$(I(\theta,\tau))^{\sigma} = I(\theta,\tau;T_p\cdot p_1,T_r\cdot r(p_1),T_{\pi X}\cdot\pi_X(p_1),T_{\pi Y}\cdot\pi_Y(p_1)).$$

Here, in order to keep the notation simple, we keep using both $(I(\theta,\tau))^{0}$ and $(I(\theta,\tau))^{\sigma}$.

Definition 4 *The first-stage compensation equilibrium σ_1 is **rent neutral** if all consumers have the same utility levels as in the ex ante situation (autarky). In other words, positive rents should all be accrued as government revenues.*

Dixit and Norman's original first-stage equilibrium is rent neutral. This is because all the consumers are in the same situation as they were in under ex ante (autarky is the ex ante situation in their model). Dixit and Norman generate this result by equating both output and input prices to their respective levels under autarky. Fixing input prices at the autarky level guarantees autarky-level incomes for consumers. If the policymaker were to fix output prices at the autarky level, consumers would be in the same utility-maximizing situation as they were under autarky, given that only income and output prices affect the consumer's problem. The same is true of the Feenstra and Lewis scheme. The only difference is that, in their paper, relocation subsidies are given to some consumers to compensate for the loss of income arising from positive adjustment costs associated with moving factors from one industry to another. Under the assumptions made by Feenstra and Lewis (1994), the government offers the smallest relocation subsidy consistent with some consumers being indifferent between moving and not moving to a new industry. Hence, the first-stage equilibrium in Feenstra and Lewis's scheme is also rent neutral.

As we show subsequently, in this paper, it is often difficult for the government to achieve a rent-neutral first-stage equilibrium. Sometimes, in order to achieve a Pareto improvement, the government cannot help providing positive rents to some groups of individual agents. We refer to this undesirable property as *overcompensation.*

Definition 5 *A scheme **overcompensates** a group of individuals if the government cannot help giving some positive rents to agents in the group in the first-stage compensation equilibrium σ_1 in order to make sure that nobody loses.*

Note that our definitions of overcompensation and rent neutrality represent two sides of the same coin. When the scheme is rent neutral, it does not overcompensate any group of consumers. By the same token, if the scheme is overcompensating some group, it cannot be rent neutral. However, we can identify who receives positive rents in the definition of overcompensation.

The other important property of the compensation scheme concerns the budget of the government.

Definition 6 *The compensation scheme, σ, is **self-financing** if it achieves non-negative government revenue in the first-stage equilibrium σ_1:*

$$B^{\sigma 1} \geq 0, \tag{26}$$

where $B^{\sigma 1}$ is the net government balance from the first-stage equilibrium of the scheme (i.e., the revenue from taxes minus the cost of subsidies).

This definition of a self-financing scheme is adapted from the definition of self-financing tariffs, as introduced by Ohyama (1972, p. 49). A compensation scheme based on taxes and subsidies applied to economic variables is self-financing if the government can balance its budget solely from the net revenue earned from the scheme.

Another important property of any compensation scheme is its informational feasibility. Despite the fact that much of the literature (on mechanism design) discusses the concept of "feasibility" in terms of non-negativity of governmental budgets (self-financing property), this paper separates the governmental budget issues (discussed above) from the information issues. In this paper, a scheme is feasible when the policy instruments of the government are based on observable (or, at least, taxable) variables.

Definition 7 *A scheme σ is **informationally feasible** if it is based solely on currently observable variables or on variables that are regularly considered as part of the **tax base**.*

This definition of informational feasibility is based on the observability of variables by the government. (Here, the phrase *observable* should not be interpreted literally. Observability relates to the concept of taxability. Therefore, we claim here that the variables are *observable* when the policymaker can use such variables as a tax base.) What are the observable variables? Which characteristics of individuals are observable to policymakers? We propose the following three reasonable assumptions about observability. (1) The government records information on aggregate variables. (2) Therefore, it has information on aggregate variables ex ante. (3) Only current data on individuals are observed at no cost.

These assumptions are somewhat realistic, because while most aggregate data are available in various forms, it is difficult to find past data that are specific to a particular individual. For example, the income tax rate is primarily determined by current income and does not usually depend on income from previous years.[17] Thus, individual data for the ex ante period are presumed to be costly to verify in the ex post period.[18]

[17] This relates to the absence of a cumulative-profit tax system. The late William Vickrey of Columbia University had been a proponent of such a system since the 1940s.

[18] In sum, the government can observe (and use as a tax base) the following variables:

- output prices, P_X, P_Y (both at the ex ante and ex post levels);
- generic-factor prices, r (both at the ex ante and ex post levels); and
- residual returns (profits) from the individual's current (ex post) occupation.
- In addition, we suppose that the government is able to observe the following two characteristics of individuals:
- which industry the individual is currently working in; and
- whether the individual has changed occupation.

We further suppose that the government cannot observe the following variables:

- individual consumption vectors;

Most of the above assumptions about observability are standard in the literature. (See, e.g., page 2 of Guesnerie (1995).) Given the assumption about the observability of profits, the following result can be used in subsequent analysis.

Remark 1 *Given the production set-up of the model, and given that the government can observe the residual profits of individuals, a profits tax does not distort individual behavior. In other words, individuals maximize their profits truthfully so long as the elasticity of the after-tax (subsidy) share, with respect to profit, is larger than -1. Formally, they do so whenever*

$$\varepsilon = \frac{\partial T/T}{\partial \pi/\pi} > -1, \tag{27}$$

where $T(\pi) = 1 - t(\pi)$, with π the residual profit and $t(\pi)$ an ad valorem tax rate (when $t(\pi)$ is negative, a subsidy rate).

See Appendix A.4 for a proof. Note also that the linear tax has an elasticity of $\varepsilon = 0$ and thus satisfies condition (27) automatically. In addition, given that individual agents are assumed to be acting truthfully, we conclude that the policymaker can observe each agent's currently used talents.

Remark 2 *Given the previous observation in Remark 1 about the truthfully maximized current levels of individuals' residual returns, the government can recalculate θ for X-producers and τ for Y-producers. The planner is able, from this, to infer the amount of talent being used, rather than the agent's endowment of latent talent.*

This is straightforward. If policymakers can condition their policy on current profits, then either

$$\begin{cases} \pi_X(\theta, \tau; p) &= \left[p^{\frac{1}{1-a}} \cdot (r(p))^{\frac{-a}{1-a}} \cdot \left(a^{\frac{a}{1-a}} - a^{\frac{1}{1-a}} \right) \right] \cdot \theta, \text{ or} \\ \pi_Y(\theta, \tau; p) &= \left[p^{\frac{-1}{1-a}} \cdot (r(p))^{\frac{-a}{1-a}} \cdot \left(a^{\frac{a}{1-a}} - a^{\frac{1}{1-a}} \right) \right] \cdot \tau. \end{cases}$$

Given the observability of aggregate variables, such as output prices, p and $1/p$, and the generic-factor return r, the inversion of profit to type is achieved by a simple calculation. One might also say that the profit is a strictly increasing function of the size of the type; in that case, any tax-subsidy rate that is proportional to the observed profit could be used. Hence, it is almost as if the government directly observes the type.

- individual generic-factor endowments;
- individual occupational-ability vectors; and
- residual returns (profits) from the individual's previous (ex ante) occupation.

5 Two Types of Compensation Schemes

Now that we have defined all the necessary properties of the compensation scheme and examined the relevant results, we examine the results of possible compensation schemes. We investigate two distinctive cases, characterized by the timing of implementation. In the first case, *an unanticipated compensation scheme*, the ex post situation (such as trade liberalization) occurs before the government announces that it will compensate those who lost from the change. In the second case, *an anticipated compensation scheme*, individual agents expect the compensation scheme to be implemented by the government once there is a change. In the following sections, we investigate these separately.

5.1 Unanticipated Schemes

First, we look at the case of surprise schemes. Despite the tradition of lump-sum compensation being introduced before trade (Mas-Colell, Whinston, and Green 1995, p.328), a more plausible and realistic policy is a "post-trade compensation scheme" (Kemp and Wan 1986, p. 99), in which the government first opens up to trade and then creates the compensation scheme to help those who lost from the change. Arguably, this unanticipated compensation scheme was applied in the 1960s. For example, in response to the Kennedy round of GATT multilateral tariff reductions, the United States government introduced the first TAA (trade adjustment assistance) program to accommodate workers displaced by the tariff reduction.

For designing an optimal compensating redistribution scheme, it is important to consider the property of Pareto improvement. To design such a scheme, the policymakers must be aware of the informational feasibility constraint because of the limited observability of the unused talents of individual agents. When the scheme comprises two stages, the policymakers try to accrue all rents in the form of governmental revenues in the first stage. Thus, the ideal first-stage equilibrium is rent neutral. Because of the informational feasibility constraint, however, this paper's model does not posit rent neutrality of the first-stage equilibrium. Nevertheless, we explore the process of creating a compensating scheme.

For analytic convenience, we focus on the case in which the price change occurs in one direction (the other case being completely symmetric). More specifically, this is the case in which the post-liberalization price is $p_1 > p_0$, and so there are job-switchers from sector Y to sector X.

Given the setup of the model as described in Section 2, we consider five cases (Cases I–V) relating to the gains and losses of different groups of individuals, as follows.

Case I. Generic factor owners all gain, since $r(p_1) > r(p_0)$. Specifically, the gain for those who own $K(\theta, \tau)$ is given by

$$(r(p_1) - r(p_0)) \cdot K(\theta, \tau) = a \cdot \overline{K}^{-(1-a)} \cdot \Big\{ [s(p_1)]^{1-a} - [s(p_0)]^{1-a} \Big\} \cdot K(\theta, \tau) > 0, \tag{28}$$

where $s(p)$ is from (12). Note that this group's gain from trade is proportional to the agent's endowment of the generic factor, $K(\theta,\tau)$. The multiplier component,

$$a \cdot \overline{K}^{-(1-a)} \cdot \left\{ [s(p_1)]^{1-a} - [s(p_0)]^{1-a} \right\},$$

is the same for all agents. Both a and $\overline{K}$ are parameters of the model. Given the relative price change, $p_0 \Longrightarrow p_1$, the values for both $s(p_0)$ and $s(p_1)$ are determined in the aggregate equilibrium. Because the policymaker knows the joint distribution of the talent vector, (θ,τ), he or she also knows the values of $V_\theta(p)$ and $V_\tau(p)$ and, hence, of $s(p_1)$ and $s(p_0)$. Thus, by imposing an ad valorem tax rate of

$$t_{r(p)} = \frac{[s(p_1)]^{1-a} - [s(p_0)]^{1-a}}{[s(p_1)]^{1-a}}, \tag{29}$$

on the market for generic factors, the policymaker has no difficulty in making the status of all owners of generic factors the same as that under autarky in the first-stage equilibrium.

Case II. Job-stayers in sector X—those who are in the area $\tau < (p_0)^{\frac{2}{1-a}}\theta$—all gain, since $\pi_{X1}(p_1) > \pi_{X0}(p_0)$ when $p_1 > p_0$. Specifically, the gain for those who have talent θ is given by

$$\pi_{X1}(p_1) - \pi_{X0}(p_0) = \overline{K}^a (1-a) \cdot \left(p^{\frac{1}{1-a}} [s(p_1)]^{-a} - p_A^{\frac{1}{1-a}} [s(p_0)]^{-a} \right) \cdot \theta > 0. \tag{30}$$

Similarly to Case I, the gain from trade for job-stayers in sector X is proportional to agents' endowments of used talent, θ. The multiplier component is the same for all these agents. Thus, by imposing, on the returns from talent of job-stayers in sector X, an ad valorem tax rate of

$$t_{\pi X} = \frac{p_1^{\frac{1}{1-a}} [s(p_1)]^{-a} - p_0^{\frac{1}{1-a}} [s(p_0)]^{-a}}{p_1^{\frac{1}{1-a}} [s(p_1)]^{-a}}, \tag{31}$$

the policymaker can make the status of these individuals the same as that under autarky in the first-stage equilibrium.

Case III. Among job-switching individuals,—all those who are in the area $(p_0)^{\frac{2}{1-a}}\theta < \tau < \frac{g(p_1)}{g(p_0)} \cdot \theta$—gain, since $\pi_{X1}(p_1) > \pi_{Y0}(p_0)$ when $p_1 > p_0$. Specifically, the gain for those who have the talent vector, (θ,τ), is given by

$$\pi_{X1}(p_1) - \pi_{Y0}(p_0) = g(p_1) \cdot \theta - g(p_0) \cdot \tau > 0, \tag{32}$$

where

$$\begin{cases} g(p_1) = p_1^{\frac{1}{1-a}} r(p_1)^{\frac{-a}{1-a}} \left(a^{\frac{a}{1-a}} - a^{\frac{1}{1-a}} \right) \\ g(p_0) = p_0^{\frac{1}{1-a}} r(p_0)^{\frac{-a}{1-a}} \left(a^{\frac{a}{1-a}} - a^{\frac{1}{1-a}} \right) \end{cases}.$$

Unlike in Cases I and II, the gain for job-switching individuals is not proportional to their endowments of used talent, θ. Although $g(p_1)$ and $g(p_0)$ are the same for all these individuals and the policymaker can calculate $g(p_1)$ and $g(p_0)$, the gain, $g(p_1) \cdot \theta - g(p_0) \cdot \tau$, depends on both elements of the talent vector, (θ, τ), and this vector is not observed by the policymaker. The policymaker could recalculate the value of used talent, θ on the basis of profits from the production of X. However, the value of τ is not known by the policymaker. To understand this, suppose that the policymaker would like to impose an ad valorem tax rate of

$$t_{\pi X-Y} = \frac{g(p_1) \cdot \theta - g(p_0) \cdot \tau}{g(p_1) \cdot \theta} = 1 - \frac{g(p_0) \cdot \tau}{g(p_1) \cdot \theta} \tag{33}$$

to make these Case III individuals as well off as they were under autarky. However, the feasible tax rate to be imposed by the policymaker should be of the form $t_{\pi X-Y}(\pi_X(\theta))$ so that it depends only on the currently observable $\pi_X(\theta)$, which also depends on the currently used talent, θ.

Case IV. All other job-switching individuals—those in the area $\frac{g(p_1)}{g(p_0)} \cdot \theta < \tau < p^{\frac{2}{1-a}} \theta$—lose since $\pi_{X1}(p_1) < \pi_{Y0}(p_0)$ when $p_1 > p_0$. Specifically, the loss for those who have talent (θ, τ) is given by

$$-\left(\pi_{X1}(p_1) - \pi_{Y0}(p_0)\right) = g(p_0) \cdot \tau - g(p_1) \cdot \theta > 0. \tag{34}$$

This case is quite similar to Case III in terms of the loss for each individual and the subsidy rate. The (infeasible) subsidy rate that the policymaker would like to impose on this group is

$$s_{\pi X-Y} = \frac{g(p_0) \cdot \tau - g(p_1) \cdot \theta}{g(p_1) \cdot \theta} = \frac{g(p_0) \cdot \tau}{g(p_1) \cdot \theta} - 1, \tag{35}$$

whereas the feasible subsidy rate must depend on only θ and be in the form $s_{\pi X-Y}(\pi_X(\theta))$.

Case V. All job-staying individuals in sector Y—those who are in the area $p^{\frac{2}{1-a}} \theta < \tau$—lose since $\pi_{Y1}(p_1) < \pi_{Y0}(p_0)$ when $p_1 > p_0$. More specifically, the loss for those who have talent τ is given by

$$-\left(\pi_{Y1}(p_1) - \pi_{Y0}(p_0)\right) = \overline{K}^a (1-a) \cdot \left(p_0^{\frac{-1}{1-a}} [s(p_0)]^{-a} - p_1^{\frac{-1}{1-a}} [s(p_1)]^{-a} \right) \cdot \tau > 0. \tag{36}$$

Similarly to Cases I and II, the gain from trade liberalization for job-stayers in sector Y is proportional to their endowments of used talent, τ. The multiplier component is the same for all of these agents. Thus, by imposing, on the returns from talent of job-stayers in sector Y, an ad valorem subsidy rate of

$$s_{\pi Y} = \frac{p_0^{\frac{-1}{1-a}} [s(p_0)]^{-a} - p_1^{\frac{-1}{1-a}} [s(p_1)]^{-a}}{p_1^{\frac{-1}{1-a}} [s(p_1)]^{-a}}, \tag{37}$$

the policymaker can make the status of all the job-staying individuals in sector Y the same as it was under autarky in the first-stage equilibrium.

It is instructive to look at a best case outcome, even if in reality it is impossible to achieve. Consider the following hypothetical first-best scheme.

Case 1 *As a first-stage equilibrium, tax the winning groups (those covered by Cases I, II, and III) and subsidize the losing groups (those covered by Cases IV and V) in amounts equal to their gains and losses, so that every individual is in the same situation as he or she was in under autarky. Such tax and subsidy rates are represented by the equations (29), (31), (33), (35), and (37).*

This hypothetical first-best scheme would be rent neutral. However, while the taxation and subsidy schemes for Cases I, II and V are feasible, the determination of the tax and subsidy rates for the job-switchers, Cases III and IV, must be based on a combination of observable and unobservable variables. The government cannot distinguish between the groups in Cases III and IV because it cannot observe the relative values of (θ, τ) for each individual. The policymaker can observe only the profit from current production and thus can observe, when $p_1 > p_0$, only the profit from production in sector X. The policymaker cannot observe (and therefore cannot condition the taxation scheme on) the counterfactual profit from sector Y, which would be proportional to the agent's unused latent talent, τ. In terms of Figure 2, for instance, this means that the government cannot distinguish between points q and r because in equilibrium the individuals at these points earn the same profit and produce the same amount of product X. This leads to the following result.

Proposition 3 *Given the setup of the model, if the government is aiming to achieve a* ***Pareto improvement*** *over autarky, there is no* ***informationally feasible*** *first-stage compensated equilibrium that is* ***rent neutral****.*

By consulting the equations (28), (30), and (36), which represent the gains and losses for the various groups of individuals, we establish the taxation and subsidy rates for the following three groups of individuals and make them as well off as they were under the ex ante situation: (a) owners of the generic-factor K, at the rate (29); (b) job-stayers in sector X, at the rate (31); and (c) job-stayers in sector Y, at the rate (37). We can do this because these individuals' gains and losses are proportional to their factor returns (in terms of both residual profits and generic-factor returns), and thus also proportional to their employed talents (or factor endowments). In this case, a linear tax or subsidy system applies.

We now focus on job-switching individuals. From equations (32) and (34), individual gains or loss depend on relative amounts of used talent, θ and unused latent talent, τ. Because the policymaker does not have data on each individual—past profits and losses—the policymaker can base a taxation-subsidy scheme on currently observable variables only. In this case, the current profit from sector

X production is observable. In effect, the policymaker can observe θ but not τ. (The policymaker observes the profits of the individual agents. If profit is reported truthfully, the policymaker can infer the amount of talent being used.) Thus, the policymaker cannot make all job-switching individuals as well off as they were under autarky, except in a case that we examine later. Hence, we can conclude as follows.

Proposition 4 *Given the set-up of the model, if the government is aiming to achieve a* ***Pareto improvement****, an* ***informationally feasible*** *post-liberalization compensation scheme must* ***overcompensate*** *job-switching individuals in its first-stage equilibrium.*

If the policymaker's most pressing concern is to ensure a Pareto improvement over the ex ante situation (such as autarky), then the informationally feasible scheme must overcompensate the group of job-switching individuals. The preceding analysis shows that the policymaker can tax and subsidize job stayers in a rent-neutral manner but cannot do so for job-switchers simply because the policymakers can observe their levels of θ but not of τ.

We return temporarily to Figure 2, which has a unit-square support for the joint distribution of talents. The left-hand side of the figure contains lines that represent the same percentage change in the gain or loss from trade. The right-hand side contains lines indicating that those individuals are making the same amount of residual profit. The iso-percentage gain/loss lines are rays from the origin, and the iso-current-profit lines for X producers are parallel vertical lines.

Although this first-best scheme requires a linear taxation/subsidy system to be imposed along the iso-percentage gain/loss lines, the policymaker observes only the differences between individuals along the iso-current-profit lines. This is because job-switching individuals appear the same when they are earning the same profit, and hence are represented by the same iso-current-profit line.

Of those who earn the same profit, it is individuals on the upper end of the iso-current profit line who gain least (lose most) from the change. Since the policymaker cannot distinguish among individuals on the same iso-profit line, the policymaker must compensate all individuals on the same profit line at the same level as the least fortunate of those individuals, who is on the upper end of that line. Apart from the least fortunate individual, however, individuals receiving the same amount of compensation from the policymaker will obtain positive rents because their iso-percentage gain/loss lines are higher than that of the individual on the upper end.

Let us examine two points q and r in Figure 2, which are on the same iso-current-profit line. Thus, although they appear the same to the policymaker, q represents a loser and r represents a winner. Nevertheless, compensation must be the same for both. Even though the individual at r is a winner, he or she receives the same amount of subsidy (as opposed to paying a tax) as the individual at point q. Hence, a government aiming for a Pareto improvement inevitably overcompensates job-switching individuals. The formal proof of Proposition 4 is in Appendix A.5.

These overcompensation results lead to the following proposition.

Proposition 5 *An* ***informationally feasible*** *post-liberalization compensation policy that achieves* ***weak Pareto improvement*** *may or may not be* ***self-financing****, depending on the joint distribution of individual talents.*

According to Ohyama (1972), a Pareto-improving compensation scheme is self-financing when the set of aggregate consumption possibilities is larger than that under autarky. In this model, however, when we impose the condition of informational feasibility, a compensation scheme without a lump-sum transfer may or may not be self-financing. This is because overcompensating job-switching individuals may cancel out the positive aggregate rents arising from trade. Whether the amount of overcompensation is large depends on the shape of the joint distribution of talents. In particular, if the total mass of job-switching individuals is large, then the total amount of overcompensation is high. Some parameter values then imply that the total compensation scheme is not self-financing. A detailed analysis of the unit-square case is discussed in Appendix A.6.

5.2 Anticipated Schemes

In the previous section, a compensation program was implemented after the shock (i.e., trade liberalization). The introduction of the program is assumed to have been a surprise. This might have been the case in the 1960s, but it may not accurately describe more recent situations. Once a compensation scheme is in place, individual agents take its existence into account. They change their behavior because the program affects their incentives.[19] In this section, we analyze an *anticipated compensation scheme.*

To begin, let us consider the situation in which individual agents expect the compensation program to exist and behave accordingly. In the previous section, some agents switched occupations before knowing whether there would be a compensation scheme. In this section, we posit that some individual agents who had changed their jobs under that scenario (without compensation) may not switch their occupations when they expect compensation only if they remain in a declining industry. This is inevitable, since any compensation scheme must specify the tax and subsidy rates not just for job switchers but also for job-stayers. When job-stayers stay in their own industry, policymakers cannot tell whether they are counterfactual job-switchers (who would have switched without compensation). With this difficulty in mind, we analyze an anticipated compensation scheme.

[19] The argument is analogous to the Friedman–Phelps hypothesis about the natural rate of unemployment. Policymakers who try to take advantage of the Phillips curve by choosing higher inflation to reduce unemployment succeed in reducing unemployment only temporarily. High inflation shifts the augmented Phillips curve upwards because expected inflation at the natural rate of unemployment rises. Thus, policymakers must wait for a long time before they can take advantage of surprise inflation. By a similar logic, the policymaker cannot take advantage of an unanticipated compensation scheme for long.

We use the same approach as before. In the first-stage equilibrium, the policymaker tries to make agents at least as well off as they were ex ante.[20] Any non-negative revenues that accrue to the government can be returned to agents in the second stage. Let us consider the following tax scheme for the producers of X under the ex ante situation (autarky).

1. For those who stay in industry X, there is a linear tax rate of

$$t_{ant} = \frac{\pi_{X1} - \pi_{X0}}{\pi_{X1}} = \frac{p_1^{\frac{1}{1-a}} \left[s(p_1)\right]^{-a} - p_0^{\frac{1}{1-a}} \left[s(p_0)\right]^{-a}}{p_1^{\frac{1}{1-a}} \left[s(p_1)\right]^{-a}}.$$

   This tax rate can make job-stayers in X indifferent between compensation (ex post) and autarky (ex ante).

2. For those who switch from industry X to industry Y, there is a linear tax rate of

$$t^*_{ant} > \frac{\pi_{X1} - \pi_{Y0}}{\pi_{X1}} = \frac{p_1^{\frac{1}{1-a}} \left[s(p_1)\right]^{-a} - p_0^{\frac{-1}{1-a}} \left[s(p_0)\right]^{-a}}{p_1^{\frac{1}{1-a}} \left[s(p_1)\right]^{-a}}.$$

   In practice, there will be no job-switchers in this direction because of the change in the terms of trade.

Thus, all members of the Θ_{XX} group stay in industry X, and all must pay the amount of tax that makes them indifferent between the ex post and ex ante situations. No agent switches from X to Y, since paying tax at the rate t^*_{ant} makes no sense.

Now, to ensure that those in group Θ_{YY} are at least as well off as they were under the ex ante situation, we consider the following subsidy scheme for the ex ante producers of Y.

3. Any producer of Y ex ante who chooses to stay in industry Y under trade liberalization is granted a positive subsidy, which is proportional to his or her occupational return in producing Y. The linear subsidy rate is

$$s_{ant} = \frac{\pi_{Y0} - \pi_{Y1}}{\pi_{Y1}} = \frac{p_0^{\frac{-1}{1-a}} \left[s(p_0)\right]^{-a} - p_1^{\frac{-1}{1-a}} \left[s(p_1)\right]^{-a}}{p_1^{\frac{-1}{1-a}} \left[s(p_1)\right]^{-a}}.$$

   This offer by the government guarantees that no one is made worse off by trade liberalization, which holds because the ex ante producers of Y have the option of staying in the same industry and earning the same return as before.

[20] It may be necessary to provide some positive surplus for informational reasons.

The government specifies the tax-subsidy scheme for those who switch from sector Y to sector X—namely, the group Θ_{YX}. For a more rigorous analysis, let us consider Figure 4, in which there is a unit-square support.

We partition the unit square into five regions. As well as natural job stayers—the groups Θ_{XX} and Θ_{YY}—there are three new groups of counterfactual job-switchers. These are (1) D, individuals who were job-switchers under liberalization without compensation but who remain in industry Y with compensation; (2) L, winning job-switchers under liberalization without compensation but whose current profits are indistinguishable from those of losing job-switchers; and (3) H, winning job-switchers under liberalization without compensation whose current profits exceed those of losing job-switchers.

With respect to group D, the government cannot do better than to implement the above subsidy scheme, targeting those who stay in industry Y. If the agents in D decide to stay in sector Y, they are indistinguishable from natural stayers in that sector. Therefore, the tax scheme targets two groups primarily: L and H. This entails the following.

4. Tax Exemption for group L. Those who are in this group are natural gainers from trade. Therefore, despite the subsidy for job-stayers in sector Y, the agents find it profitable to switch occupations, conditional on the tax exemption in the new sector.

5. Group H is taxed at the same rate as in the post-trade unanticipated scheme:

$$t_{ant}^{**}(\pi(\theta^*)) = \frac{\pi_{X1} - \pi_{Y0}\mid_{\tau=1}}{\pi_{X1}} = \frac{g(p_1)\cdot\theta^* - g(p_0)}{g(p_1)\cdot\theta^*} - \delta(\theta^*).$$

   Then, all except those who have $\tau = 1$ gain a positive rent. Thus, this tax rate is incentive compatible for those who are in group H. The term $\delta(\theta^*)$ is a very small number and has the same property, as explained in Appendix A.6.

This scheme satisfies all three conditions: it has informational feasibility, it delivers weak Pareto improvement, and it is self-financing. It is informationally feasible since all tax and subsidy rates are incentive compatible. It is weakly Pareto improving since every agent is at least as well off as ex ante (under autarky). If there are aggregate gains from trade, the tax revenues from this scheme exceed the costs of subsidy. The net government revenues brought in by the job-staying individuals in both sectors X and Y are likely to be positive. With respect to the job-switchers, who created an overcompensation problem in the unanticipated case, this scheme either taxes some or exempts some from tax; hence, the policymaker generates strictly positive tax revenue. Although there are some positive rents, and hence overcompensation in the form of smaller taxes for group H, this overcompensation does not negatively affect the government budget since it takes the form of a lower-than-ideal tax rate (rather than a wasteful give-away subsidy).

Nevertheless, the allocation achieved in this scheme is not without costs. Although the scheme satisfies informational feasibility, delivers weak Pareto improvement, and is self-financing, it generates aggregate-level inefficiency in the form of a smaller aggregate consumption possibility set when evaluated at the new world price. These smaller aggregate gains are a result of there being fewer job-switchers.

Proposition 6 *There is an **anticipated (ex ante)** compensation program that is **informationally feasible, weakly Pareto improving** and **self-financing**. The aggregate consumption possibilities set is smaller than that of the unanticipated (ex post) scheme.*

Furthermore, in the context of the current TAA program, we find a striking result. Noting that our model does not have frictional costs for occupation switching, it proposes taxing at a positive rate or exempting from tax those who switch occupations. This contradicts the results in Feenstra and Lewis (1994), where a relocation subsidy for job-switchers is proposed. Our optimal scheme suggests, to the contrary, that the policymaker should give no subsidy to job-switchers. We propose that the subsidy be given only to job-stayers who remain in a declining industry. Given that the model has no frictional cost for moving between sectors, it is not surprising to obtain this negative result for the current TAA, which provides a poll subsidy to occupation switchers.

Proposition 7 *The poll subsidy for those who have changed industries creates a disincentive. It induces an inefficient allocation of individuals.*

Given the setup of the model in this paper, the minimum subsidy for job-switching individuals must be non-positive; that is, it must contain a tax exemption for group L and a positive tax for group H. By giving a positive subsidy to job-switching individuals, some job-stayers in sector Y (particularly those closer to the zero-gain line, OZ) may find it profitable to move to sector X. However, while this positive subsidy is successful in inducing some counterfactual job-switchers to move to a more efficient sector (in the post-liberalization world), it also creates a huge side effect. Because the policymaker cannot distinguish between counterfactual job switchers and natural (winning) job-switchers, a positive subsidy overcompensates job-switchers who are on the same iso-current-profit lines. In the extreme case, the policymaker must offer exactly the same tax-subsidy rates that were applied in the unanticipated post-liberalization compensation scheme if the government maximizes the number of job-switchers. This subsidy generates overcompensation and makes self-financing questionable.

When the policymaker assigns balancing the budget the highest priority, taxing job-switchers (at a small rate) may be another policy option.[21] Taxing job-switchers, but not too heavily, may induce some natural job-switchers to change their occupations. Since these job-switchers pay tax, this policy will help

[21] I thank Professor Eiichi Miyagawa for pointing out the possibility of this type of policy.

to balance the budget problem but may induce fewer individuals to switch to an efficient industry. More individuals will remain in a declining industry. Thus, the trade-off between the government's budget and aggregate gains remains.

The preceding analysis has shown that, in the case of an anticipated compensation scheme in which the government aims to attain a Pareto improvement after the change, there is a trade-off between the aggregate production gains from trade and the amount of overcompensation.

6 Conclusion

This paper presented a two-dimensional version of a model of occupational choice. In this paper, a model that predicts aggregate production gains from trade was developed. We attempted to model a realistic situation in which individual agents often find themselves. We assume that individual agents must choose one job at a time and that they are endowed with different levels of talent in different sectors. That is, productivity is assumed to differ between agents. This setup creates winners and losers from trade, but the gains and losses are based on the talents that agents use relative to their hidden latent talents. When the government chooses to impose a realistic tax-subsidy scheme on current factor prices and profits, policymakers face a trade-off between Pareto improvement and overcompensation. In other words, if policymakers do achieve a Pareto improvement, then the compensation scheme necessarily overcompensates some individuals; we showed here that these are job-switching individuals. If, instead, policymakers rigorously avoid overcompensation because they care about a balanced budget, then the compensation program cannot be Pareto improving.

Additionally, when a compensation scheme is anticipated by individual agents, there is another trade-off, which is between overcompensation and aggregate production gains. Although most policymakers are aware of these trade-offs, few studies of this issue exist. Thus, in this paper, a theoretical framework was developed to explain the trade-offs that governments face when trying to implement compensating redistribution schemes.

We also provided an explanation of the difficulty in distinguishing winners from losers when an economy opens to trade. Such distinctions have been made in the context of basic trade models, such as the Heckscher–Ohlin model and the specific-factors model. Feenstra and Lewis (1994) noted the difficulty of identification in their imperfectly mobile factors model, which they developed to investigate heterogeneous adjustment costs. While Feenstra and Lewis assumed positive adjustment costs for their imperfectly mobile factors, our model reveals cases in which the adjustment costs for some job-switching agents may be negative; that is, some job-switching agents will gain. Thus, the poll subsidy for job-switching individuals (as proposed by Feenstra and Lewis) may not be desirable in the context of our model. Furthermore, any observation of current profits does not reflect actual gains or losses from opening to trade. This makes it difficult for any government to implement a reliably Pareto-improving compensation scheme that bases taxes and subsidies on currently taxable variables.

This paper has provided a model of individuals' occupational choices and welfare changes when the economy faces a change in the terms of trade, particularly in the case of trade liberalization. We found that there are both winners and losers among job-switchers. However, although this paper's analysis can explain individuals' long-run gains and losses from moving to a new sector, the model does not take into account short-run costs of labor adjustment. (We implicitly assumed that frictional unemployment costs are zero.) Therefore, the paper's chief theoretical result—that no positive subsidy should be given to job-switching individuals under a self-financing compensation scheme—should not be taken too literally. Indeed, the compensation provided by the United States Department of Labor through its TAA program involves a relocation subsidy for those who move to a new location when switching jobs in response to trade changes. Such a program may be justified to the extent that there are short-run frictional costs associated with job switching.

A simplifying assumption made in this paper is that occupational talents are exogenously given for each individual. In reality, people may invest much of their time in expanding their skills. We have omitted the possibility of such dynamic development of individual talents through human-capital investment. Grossman and Shapiro (1982) analyzed the determinants of individual talent training when the individual agents are identical ex ante. An interesting extension of this paper's model would be to incorporate dynamic formation of specific factors by allowing for investment in individual occupational talents. This is a promising avenue for future research and Ichida (2011) is one of the first attempts toward the avenue.

A Appendix

This appendix contains proofs of some of the assertions in the paper.

A.1 Proof that $V'_\theta(p) > 0$ and $V'_\tau(p) < 0$

Here, we prove that this holds for the case of a unit-square space $\Theta = [0,1]^2$:

$$\iint_{\Theta_X} \theta dF(\theta,\tau) = \int_0^1 \left[\int_0^{p^{\frac{2}{1-a}}\theta} f(\theta,\tau)d\tau \right] d\theta \equiv V_\theta(p).$$

Now, consider increasing the value of p infinitesimally. Let us define

$$G(\theta) \equiv \int_0^{p^{\frac{2}{1-a}}\theta} f(\theta,\tau)d\tau,$$

and then we can say that $V'_\theta(p) > 0$ where

$$V'_\theta(p) = \lim_{h\to 0} \frac{V_\theta(p+h) - V_\theta(p)}{h}$$

because

$$
\begin{aligned}
V_\theta(p+h) - V_\theta(p) &= \int_0^1 \left[\int_0^{(p+h)^{\frac{2}{1-a}}\theta} f(\theta,\tau)d\tau \right] d\theta - \int_0^1 \left[\int_0^{p^{\frac{2}{1-a}}\theta} f(\theta,\tau)d\tau \right] d\theta \\
&= \int_0^1 \left[\int_{p^{\frac{2}{1-a}}\theta}^{(p+h)^{\frac{2}{1-a}}\theta} f(\theta,\tau)d\tau \right] d\theta
\end{aligned}
$$

and

$$
\begin{aligned}
\lim_{h\to 0} \frac{V_\theta(p+h) - V_\theta(p)}{h} &= \lim_{h\to 0} \frac{1}{h} \int_0^1 \left[\int_{p^{\frac{2}{1-a}}\theta}^{(p+h)^{\frac{2}{1-a}}\theta} f(\theta,\tau)d\tau \right] d\theta \\
&= G'()\left[(p+h)\theta - p\theta\right]\frac{1}{h} = f(\theta,\tau)\left[(p+h)\theta - p\theta\right]\frac{1}{h}
\end{aligned}
$$

hold. Therefore, we can conclude that

$$
\lim_{h\to 0} \frac{V_\theta(p+h) - V_\theta(p)}{h} = f(\theta,\tau)\theta > 0.
$$

$V_\tau'(p) < 0$ can be proved in a similar manner. QED

A.2 Proof of Lemma 1.

>From (14), we can rewrite the GNI

$$
r(p)\cdot \overline{K} + \iint_{\Theta_X} \pi_X(\cdot)dF(\theta,\tau) + \iint_{\Theta_Y} \pi_Y(\cdot)dF(\theta,\tau)
$$

with

$$
r(p)\cdot \overline{K} + (r(p))^{\frac{-a}{1-a}} \cdot \left[a^{\frac{a}{1-a}} - a^{\frac{1}{1-a}} \right] \cdot s(p)
$$

and use (13) in the form

$$
s(p) = \left(r(p) \cdot \overline{K}^{1-a}/a \right)^{\frac{1}{1-a}},
$$

from which we can get (15). QED

A.3 Proof of Proposition 1

When the price of X is given as p, the profit for an agent $(\theta,\tau) \in \Theta_X(p)$ will be written as

$$
\pi_X(\theta,\tau;p) = \left[p^{\frac{1}{1-a}} \cdot (r(p))^{\frac{-a}{1-a}} \cdot \left(a^{\frac{a}{1-a}} - a^{\frac{1}{1-a}} \right) \right] \cdot \theta. \tag{38}
$$

The equilibrium general factor price r can be written using p,

$$
r(p) = a \cdot \overline{K}^{a-1} \left[s(p) \right]^{1-a}, \tag{39}
$$

which can be substituted into the equation (38) to obtain

$$\pi_X(\theta,\tau;p) = \overline{K}^a(1-a)\cdot p^{\frac{1}{1-a}}\,[s(p)]^{-a}\cdot\theta. \tag{40}$$

Because both $\overline{K}^a(1-a)$ and θ are non-negative, the sign of the derivative of $p^{\frac{1}{1-a}}\,[s(p)]^{-a}$ with respect to p,

$$\frac{d\left(p^{\frac{1}{1-a}}\,[s(p)]^{-a}\right)}{dp} = s^{-a}\cdot p^{\frac{a}{1-a}}\cdot a\cdot\left(\frac{1}{a(1-a)}-\frac{p\cdot s'(p)}{s(p)}\right),$$

will be the same as the sign of the derivative of the profit for an agent. By assumption, $0<a<1$ and $p>0$, and

$$s^{-a}\cdot p^{\frac{a}{1-a}}\cdot a>0 \text{ and } \frac{1}{a(1-a)}>0$$

are clear. Additionally, when $p>p_0$ holds, $s'(p)<0$ must also hold. Therefore, we can say that

$$\left(\frac{1}{a(1-a)}-\frac{p\cdot s'(p)}{s(p)}\right)>0.$$

This proves that job-stayers in sector X will gain from an increase in price p. The analysis for the sector-Y job-stayers can be conducted in a similar manner. QED

A.4 Analysis of Profit Taxation System

Assume that the production function is

$$x = X(k,\theta), \tag{41}$$

where x is the quantity of output, k is the amount of the generic factor employed by the firm, and θ is the specific occupational factor, which is indivisible and embodied in the individual agent. Let $X(k,\theta)$ be increasing on both arguments, strictly concave, and infinitely continuously differentiable with constant returns to scale.

Let p be the output price of x. Let r be the market price for the generic factor, k. The agent's profit-maximization program is

$$\max_k \pi(k,\theta;p,r) = p\cdot X(k,\theta) - r\cdot k. \tag{42}$$

Note that the only choice variable for the agent is k because θ is intrinsic and indivisible. The regular first-order condition is

$$\frac{\partial\pi}{\partial k}=0 \Longleftrightarrow p\cdot\frac{\partial X}{\partial k}=r. \tag{43}$$

Strict concavity of the production function $X(\cdot,\cdot)$ guarantees that the second-order condition for the regular problem (42) holds, with strict inequality.

$$\frac{\partial^2 \pi}{\partial k^2} < 0 \tag{44}$$

Now, consider a tax on the profits of the agent, given equation (42). If the ad valorem tax rate is t, then the profit-maximization program is

$$\max_k (1-t)\{p \cdot X(k,\theta) - r \cdot k\}. \tag{45}$$

When t does not depend on k or θ, then the profit-maximization problem faced by an individual is unchanged. Hence, the first-order condition is (43).

A.4.1 Tax Rate Proportional to Profit

Now, let $1 - t = T(\pi)$ be the profit-tax schedule. The rate of tax depends on the observed profit of the individual. The program is now

$$\max_k \{T(\pi) \cdot \pi\} = T(\pi)\{p \cdot X(k,\theta) - r \cdot k\}. \tag{46}$$

The first-order condition for (46) is

$$\frac{\partial T}{\partial \pi} \cdot \frac{\partial \pi}{\partial k} \cdot \pi + T \cdot \frac{\partial \pi}{\partial k} = \frac{\partial \pi}{\partial k} \cdot \left\{ \frac{\partial T}{\partial \pi} \cdot \pi + T \right\} = 0. \tag{47}$$

Condition (47) implies that $\frac{\partial \pi}{\partial k} = 0$, except when

$$\frac{\partial T}{\partial \pi} \cdot \pi + T = T\left(1 + \frac{\partial T}{\partial \pi} \cdot \frac{\pi}{T}\right) = T(1+\varepsilon) = 0,$$

with $\varepsilon \equiv \frac{\partial T/T}{\partial \pi/\pi}$ as the elasticity of the tax rate with respect to profit. Thus, unless $\varepsilon = -1$, the first-order condition (47) implies the same condition as (43).

The second-order condition for the profit-maximization is

$$\frac{\partial^2 \pi}{\partial k^2} \cdot \left\{ \frac{\partial T}{\partial \pi} \cdot \pi + T \right\} + \frac{\partial \pi}{\partial k} \cdot \frac{\partial}{\partial k} \left\{ \frac{\partial T}{\partial \pi} \cdot \pi + T \right\} \equiv SOC < 0. \tag{48}$$

The second term of SOC is

$$\frac{\partial \pi}{\partial k} \cdot \left\{ \frac{\partial^2 T}{\partial \pi^2} \cdot \frac{\partial \pi}{\partial k} \cdot \pi + 2\left(\frac{\partial T}{\partial \pi} \cdot \frac{\partial \pi}{\partial k}\right) \right\}.$$

This is evaluated around the optimum point, where $\frac{\partial \pi}{\partial k} = 0$. Thus, given (44), it follows that the relevant condition for the program's second-order condition is

$$\frac{\partial T}{\partial \pi} \cdot \pi + T = T(1+\varepsilon) > 0.$$

Since $T > 0$, the condition can also be written as

$$\varepsilon = \frac{\partial T/T}{\partial \pi/\pi} > -1. \tag{49}$$

So, unless the profit-tax rate decreases by more than 1% as the profit simultaneously increases by 1%, the agent maximizes profit even after profit has been taxed.

A.4.2 Tax Rate Proportional to Output

Now, let $1 - t = T(x)$ be a new profit-tax schedule. The rate of tax depends on the observed output of the individual. The program is now

$$\max_k \{T(x) \cdot \pi\} = T(x) \{p \cdot X(k,\theta) - r \cdot k\}. \tag{50}$$

The first-order condition is

$$\frac{\partial T}{\partial x} \cdot \frac{\partial X}{\partial k} \cdot \pi + T \cdot \left\{ p \cdot \frac{\partial X}{\partial k} - r \right\} = \frac{\partial X}{\partial k} \cdot \left\{ \frac{\partial T}{\partial x} \cdot \pi + pT \right\} - rT = 0. \tag{51}$$

Note that the optimal level of k is smaller than in the no-tax case (42) because

$$\frac{\partial T}{\partial x} \cdot \frac{\partial X}{\partial k} \cdot \{p \cdot X(k,\theta) - r \cdot k\} < 0$$

together with $r > 0$ and $T > 0$ implies that

$$\left\{ p \cdot \frac{\partial X}{\partial k} - r \right\} > 0.$$

Thus, the profit-tax system based on observed output is inevitably distortionary.

A.5 Proof of Proposition 4

To explore Proposition 4 more thoroughly, let us define the iso-current-profit set, $I^{CP}(\theta^*)$.

Definition 8 *The iso-current-profit set, $I^{CP}(\theta^*)$, is the set of all those job-switching individuals who have the same talent, θ^*:*

$$I^{CP}(\theta^*) \equiv \{(\theta, \tau) \in \Theta_{YX} : \theta = \theta^*\},$$

where Θ_{YX} is a subset of job-switchers; that is,

$$\Theta_{YX} \equiv \left\{ (\theta, \tau) \in \Theta : \; p_0^{\frac{2}{1-a}} \theta < \tau < p_1^{\frac{2}{1-a}} \theta \right\}.$$

Note that $I^{CP}(\theta^*)$ is a linear one-dimensional subspace of $\mathbb{R}^2$. Let $\underline{\tau(\theta^*)}$ be the lower bound for the value of the component τ in a set $I^{CP}(\theta^*)$, and let $\overline{\tau(\theta^*)}$ be the upper bound for the same subspace. Note that $\underline{\tau(\theta^*)}$ is equal to $p_0^{\frac{2}{1-a}}\theta^*$, whereas $\overline{\tau(\theta^*)}$ depends on the value of θ^*. In particular,

$$\overline{\tau(\theta^*)} = \sup\left\{p_1^{\frac{2}{1-a}}\theta^*, \overline{\Theta^\tau(\theta^*)}\right\},$$

where $\overline{\Theta^\tau(\theta^*)}$ is an upper bound for the component τ in the whole Θ space when $\theta = \theta^*$. In the case of a unit-square support for the joint distribution, $\overline{\Theta^\tau(\theta^*)} = 1$.

Because all individuals in the set $I^{CP}(\theta^*)$ are job-switchers from sector Y to sector X, they are currently producing output X. Since all members of the set $I^{CP}(\theta^*)$ have the same talent, θ^*, their profit is the same: $\pi_X(p, r(p), \theta^*)$. Their individual gains or losses, however, differ because they have different latent talents, τ. Given (32) and (34), the individual gains or losses can be expressed as $|g(p_1)\cdot\theta^* - g(p_0)\cdot\tau|$. Whether the individual who has the talent θ^* gains or loses, and what the gain or loss is, depends on the value of τ. Among those who belong to the set $I^{CP}(\theta^*)$, there are many individuals who have the latent talent τ in the interval $\left[\underline{\tau(\theta^*)}, \overline{\tau(\theta^*)}\right]$. The policymaker, however, cannot distinguish between them.

A policymaker who wants to ensure Pareto gains from the economic change must be sure to make the least well-off individual as well off as he or she was under the ex ante situation. Note also that this least well-off individual must have had the most talent in the previous sector, Y, and hence must have been the one with the most latent talent, $\overline{\tau(\theta^*)}$. Therefore, for all individuals, $(\theta^*, \tau) \in I^{CP}(\theta^*)$, the subsidy or tax must be $\left|g(p_1)\cdot\theta^* - g(p_0)\cdot\overline{\tau(\theta^*)}\right|$. The ad valorem rate for any individual with the profit $\pi(\theta^*)$ is

$$t_{\pi X-Y}(\pi(\theta^*)) = \left|\frac{g(p_1)\cdot\theta^* - g(p_0)\cdot\overline{\tau(\theta^*)}}{g(p_1)\cdot\theta^*}\right|. \tag{52}$$

If $g(p_1)\cdot\theta^* - g(p_0)\cdot\overline{\tau(\theta^*)} > 0$, then equation (52) represents a tax rate. If $g(p_1)\cdot\theta^* - g(p_0)\cdot\overline{\tau(\theta^*)} < 0$, then it represents a subsidy rate. With the exception of the individual at the point $(\theta^*, \overline{\tau(\theta^*)})$, which acts as zero, all individuals in the set $I^{CP}(\theta^*)$ are overcompensated, since the inequality

$$g(p_1)\cdot\theta^* - g(p_0)\cdot\overline{\tau(\theta^*)} < g(p_1)\cdot\theta^* - g(p_0)\cdot\tau \tag{53}$$

must hold for all of those with latent talent $\tau \in \left[\underline{\tau(\theta^*)}, \overline{\tau(\theta^*)}\right)$.

>From (53), it follows that

$$\begin{aligned}&\int_{\underline{\tau(\theta^*)}}^{\overline{\tau(\theta^*)}} \left\{g(p_1)\cdot\theta^* - g(p_0)\cdot\overline{\tau(\theta^*)}\right\} f(\theta^*, \tau)d\tau \\ &< \int_{\underline{\tau(\theta^*)}}^{\overline{\tau(\theta^*)}} \left\{g(p_1)\cdot\theta^* - g(p_0)\cdot\tau\right\} f(\theta^*, \tau)d\tau.\end{aligned}$$

Integrating over all job-switching individuals yields

$$\int_{\Theta_{YX}} \int_{\underline{\tau(\theta^*)}}^{\overline{\tau(\theta^*)}} \left\{ g(p_1) \cdot \theta^* - g(p_0) \cdot \overline{\tau(\theta^*)} \right\} f(\theta, \tau) d\tau d\theta^* < \\ \int_{\Theta_{YX}} \int_{\underline{\tau(\theta^*)}}^{\overline{\tau(\theta^*)}} \left\{ g(p_1) \cdot \theta^* - g(p_0) \cdot \tau \right\} f(\theta, \tau) d\tau d\theta^*, \tag{54}$$

with the integration being over θ^* for all job-switching individuals. The difference between the right- and left-hand sides of the inequality (54) relates to the total amount of overcompensation for job-switching individuals. QED

A.6 Unit-square Case: Unanticipated

We now consider an example in which the support of the joint distribution is a unit square. Figure 3 illustrates the scheme for this case. For this unit-square case, we introduce a further partition of the group Θ_{YX} into two groups: a group of absolute gainers and a group of mixed gainers and losers, using only the observable variables for determination. For clarification, consider the following:

(i) generic-factor owners, as in Case I above;

(ii) all individuals in Θ_{XX}, as in Case II;

(iii) those individuals in Θ_{YX} who meet the condition $\theta > \frac{g(p_0)}{g(p_1)}$;

(iv) those individuals in Θ_{YX} who meet the condition $\theta < \frac{g(p_0)}{g(p_1)}$; and

(v) all individuals in Θ_{YY}, as in Case V.

Note that in Figure 3, the dotted line, OZ, denotes the zero-gain line: $\theta = \frac{g(p_0)}{g(p_1)} \cdot \tau$. This categorization uses only observable variables because the distinction between group (iii) and group (iv) is based on θ only, which can be inferred from individuals' current profits. Given this new categorization, let us propose a revised post-liberalization compensation scheme.

Case 2 *As a first-stage equilibrium, tax (i), (ii), and (iii), and subsidize (iv) and (v). Note, in particular, that the tax and subsidy rates are represented by the following equations: (29) for (i); (31) for (ii); (52) for groups (iii) and (iv); and (37) for (v).*

Since this scheme is based on observable variables, it is feasible. However, it is not an optimal scheme because groups (iii) and (iv) are overcompensated. This is inevitable given that winners and losers in this category are indistinguishable from one another.

To find the appropriate tax-subsidy rates, we obtain the minimum subsidy rate and the maximum tax rate for each group such that together they satisfy the requirements shown in (24) to ensure weak Pareto improvement. Because our model uses a price normalization system that causes nominal income (using the parameter p) to be equal to real income, it is easy to find the tax-subsidy rates, for all groups, at which everyone is as well off as they were ex ante. Note that the tax-subsidy rates must be based on observable variables (or variables that are easily calculated). Thus, the features of the tax-subsidy rates for each group are the following:

(i) (linear) factor (commodity) tax on generic factors;

(ii) (linear) profits tax on the occupation rewards for job-staying producers of output X;

(iii) (nonlinear) profits tax on the occupation rewards for job-switching producers of output X;

(iv) (nonlinear) profits subsidy on the occupation rewards for job-switching producers of output X; and

(v) (linear) profits subsidy on the occupation rewards for job-staying producers of output Y.

The linear factor tax for generic-factor owners is the same as that in the best case. Now, we focus on the individual heterogeneity of talents. Given the above categorization, we create a finer partition of the ability vector space, as follows.

1. $\Theta_{XX} \equiv \left\{(\theta,\tau) \in \Theta : \tau < (p_0)^{\frac{2}{1-a}}\theta\right\}$

2. $H = \Theta_{YX}^{H} \equiv \left\{(\theta,\tau) \in \Theta : p_1^{\frac{2}{1-a}}\theta > \tau > (p_0)^{\frac{2}{1-a}}\theta \text{ and } 1 > \frac{g(p_1)}{g(p_0)} \cdot \theta\right\}$

3. $M = \Theta_{YX}^{M} \equiv \left\{(\theta,\tau) \in \Theta : p_1^{\frac{2}{1-a}}\theta > \tau > (p_0)^{\frac{2}{1-a}}\theta \text{ and } 1/(p_1^{\frac{2}{1-a}}) < \theta < \frac{g(p_0)}{g(p_1)}\right\}$

4. $L = \Theta_{YX}^{L} \equiv \left\{(\theta,\tau) \in \Theta : p_1^{\frac{2}{1-a}}\theta > \tau > (p_0)^{\frac{2}{1-a}}\theta \text{ and } 0 < \theta < 1/(p_1^{\frac{2}{1-a}})\right\}$

5. $\Theta_{YY} \equiv \left\{(\theta,\tau) \in \Theta : p_1^{\frac{2}{1-a}}\theta < \tau\right\}$

The groups of job-stayers, Θ_{XX} and Θ_{YY}, face the same linear tax-subsidy scheme as in the best case. Thus, we focus on the job-switchers, H, M, and L, all of whom are currently producing the output X. Because the government cannot distinguish among those earning the same profit from their production of X, the policymaker must take from (respectively, give to) each individual the same tax (subsidy) as that taken from (given to) the individual who gains the least (loses the most) among those earning the same profit. For a given profit, those who gain the least are those who have the most latent ability to produce Y. For the groups H and M, those who gain the least (lose the most) are the individuals with $\overline{\tau(\theta^*)} = 1$. For group L, they are $\overline{\tau(\theta^*)} = p_1^{\frac{2}{1-a}}\theta^*$.

Next, we check the optimal tax rate for those who have an ability vector $(\theta^*, 1)$, where $1 \geq \theta^* > 1/(p_1^{\frac{2}{1-a}})$, and the optimal tax rate for those with a vector $(\theta^*, p_1^{\frac{2}{1-a}}\theta^*)$, where $0 < \theta^* < 1/(p_1^{\frac{2}{1-a}})$. Thus, the individuals in group H who earn $\pi(\theta^*)$ are taxed at a rate

$$t_H(\pi(\theta^*)) = \frac{g(p_1) \cdot \theta^* - g(p_0)}{g(p_1) \cdot \theta^*} - \delta(\theta^*),$$

while the individuals in group M who earn $\pi(\theta^*)$ are given a subsidy of

$$s_M(\pi(\theta^*)) = \frac{g(p_1) - g(p_0) \cdot \theta^*}{g(p_1) \cdot \theta^*} + \delta(\theta^*),$$

where $\delta(\theta^*) > 0$ represents an arbitrarily small number for which $\delta'(\theta^*) > 0$. The purpose of this additional term is to avoid violating the condition $\varepsilon = \frac{\partial T/T}{\partial \pi/\pi} > -1$, which was discussed in Remark 1. Without the term $\delta(\theta^*)$, the condition is $\varepsilon = -1$. (For a formal proof, see Appendix A.4.) The group-L individuals have the linear subsidy

$$s_L = \frac{g(p_0) \cdot p_1^{\frac{2}{1-a}}\theta^* - g(p_1) \cdot \theta^*}{g(p_1) \cdot \theta^*} = \frac{g(p_0) \cdot p_1^{\frac{2}{1-a}} - g(p_1)}{g(p_1)}.$$

This completes the description of the tax-subsidy scheme for the first-stage equilibrium in the unit-square case.

References

Atkinson, A. B., and J. E. Stiglitz (1980): *Lectures on Public Economics*. McGraw-Hill, New York.

Brander, J. A., and B. J. Spencer (1994): "Trade Adjustment Assistance Welfare and Incentive Effects of Payments to Displaced Workers," *Journal of International Economics*, 36(3-4), 239–261.

Brecher, R. A., and E. U. Choudhri (1994): "Pareto Gains from Trade, Reconsidered: Compensating for Jobs Lost," *Journal of International Economics*, 36(3-4), 223–238.

Davidson, C., and S. J. Matusz (2006): "Trade Liberalization and Compensation," *International Economic Review*, 47(3), 723–747.

Dixit, A., and V. Norman (1980): *Theory of International Trade*. Cambridge University Press, Cambridge, UK.

——— (1986): "Gains from Trade Without Lump-Sum Compensation," *Journal of International Economics*, 21(1-2), 111–122.

Falvey, R., D. Greenaway, and J. Silva (2006): "Trade, Human Capital and Labour Market Adjustment," Research Paper Series, University of Nottingham.

Feenstra, R. C., and T. R. Lewis (1994): "Trade Adjustment Assistance and Pareto Gains from Trade," *Journal of International Economics*, 36(3-4), 201–222.

Grossman, G. M. (1983): "Partially Mobile Capital: A General Approach to Two-Sector Trade Theory," *Journal of International Economics*, 15(1-2), 1–17.

——— (2004): "The Distribution of Talent and the Pattern and Consequences of International Trade," *Journal of Political Economy*, 112(1), 209–239.

Grossman, G. M., and C. Shapiro (1982): "A Theory of Factor Mobility," *Journal of Political Economy*, 90(5), 1054–1069.

Guesnerie, R. (1995): *A Contribution to the Pure Theory of Taxation*, vol. 25 of *Econometric Society Monographs*. Cambridge University Press, Cambridge, UK.

Hammond, P. J. (1979): "Straightforward Individual Incentive Compatibility in Large Economies," *Review of Economic Studies*, 46(2), 263–282.

Ichida, T. (2004): "Occupational Choice and International Trade," Ph.D. dissertation, Columbia University, Department of Economics.

——— (2005): "Occupational Choice and Compensation for Losers from International Trade," mimeo, Columbia University and Waseda University.

——— (2011): "A Model of Multi-Dimensional Human Capital Investment: Specific vs. general investments under uncertainty," *RIETI Discussion Paper Series*, 11-E-056, 1–37.

Jones, R. W. (1971): "A Three-Factor Model in Theory, Trade and History," in *Trade, the Balance of Payments, and Growth: Papers in Honor of Charles P. Kindleberger*, ed. by J. N. Bhagwati, R. W. Jones, R. A. Mundell, and J. Vanek, pp. 3–21. North Holland, Amsterdam.

Kemp, M. C., and H. Y. Wan (1986): "Gains from Trade With and Without Lump-Sum Compensation," *Journal of International Economics*, 21(1-2), 99–110.

Mas-Colell, A., M. D. Whinston, and J. R. Green (1995): *Microeconomic Theory*. Oxford University Press, New York.

Matsuyama, K. (1992): "A Simple Model of Sectoral Adjustment," *Review of Economic Studies*, 59(2), 375–387.

MAYER, W. (1974): "Short-Run and Long-Run Equilibrium for a Small Open Economy," *Journal of Political Economy*, 82(5), 955–967.

MURPHY, K. M. (1986): "Specialization and Human Capital," Ph.D. thesis, University of Chicago, Department of Economics.

MUSSA, M. (1982): "Imperfect Factor Mobility and the Distribution of Income," *Journal of International Economics*, 12(1-2), 125–141.

NEARY, J. P. (1978a): "Dynamic Stability and the Theory of Factor Market Distortions," *American Economic Review*, 68(4), 671–682.

——— (1978b): "Short-Run Capital Specificity and the Pure Theory of International Trade," *Economic Journal*, 88(351), 488–510.

OHNSORGE, F., AND D. TREFLER (2004): "Sorting It Out: International Trade and Protection with Heterogeneous Workers," mimeographed. IMF and University of Tronto.

——— (2007): "Sorting It Out: International Trade with Heterogeneous Workers," *Journal of Political Economy*, 115(5), 868–892.

OHYAMA, M. (1972): "Trade and Welfare in General Equilibiurm," *Keio Economic Studies*, 9(2), 37–73.

ROSEN, S. (1978): "Substitution and Division of Labour," *Economica*, 45(179), 235–250.

ROY, A. D. (1951): "Some Thoughts on the Distribution of Earnings," *Oxford Economic Papers*, 3(2), 135–146.

RUFFIN, R. J. (1988): "The Missing Link: The Ricardian Approach to the Factor Endowments Theory of Trade," *American Economic Review*, 78(4), 759–772.

——— (2001): "Quasi-specific factors: worker comparative advantage in the two-sector production model," *Journal of International Economics*, 53(2), 445–461.

RUFFIN, R. J., AND R. W. JONES (1977): "Protection and Real Wages: The Neoclassical Ambiguity," *Journal of Economic Theory*, 14(2), 337–348.

SAMUELSON, P. A. (1962): "The Gains from International Trade Once Again," *Economic Journal*, 72(288), 820–829.

SATTINGER, M. (1975): "Comparative Advantage and the Distributions of Earnings and Abilities," *Econometrica*, 43(3), 455–468.

SPECTOR, D. (2001): "Is It Possible to Redistribute the Gains from Trade Using Income Taxation?," *Journal of International Economics*, 55(2), 441–460.

WEYMARK, J. A. (1979): "A Reconciliation of Recent Results in Optimal Taxation Theory," *Journal of Public Economics*, 12(2), 171–189.

Figure 1:
Unit Square and Occupational Choice Partitions of Type Space

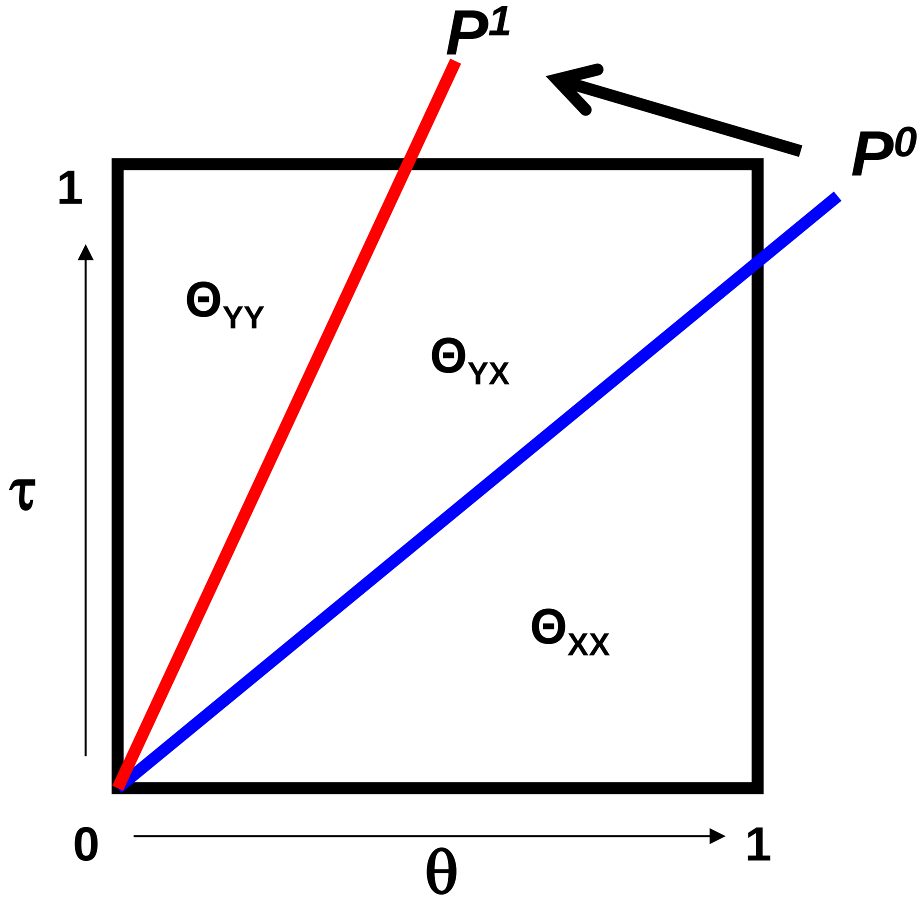


Figure 2: Gain and Loss

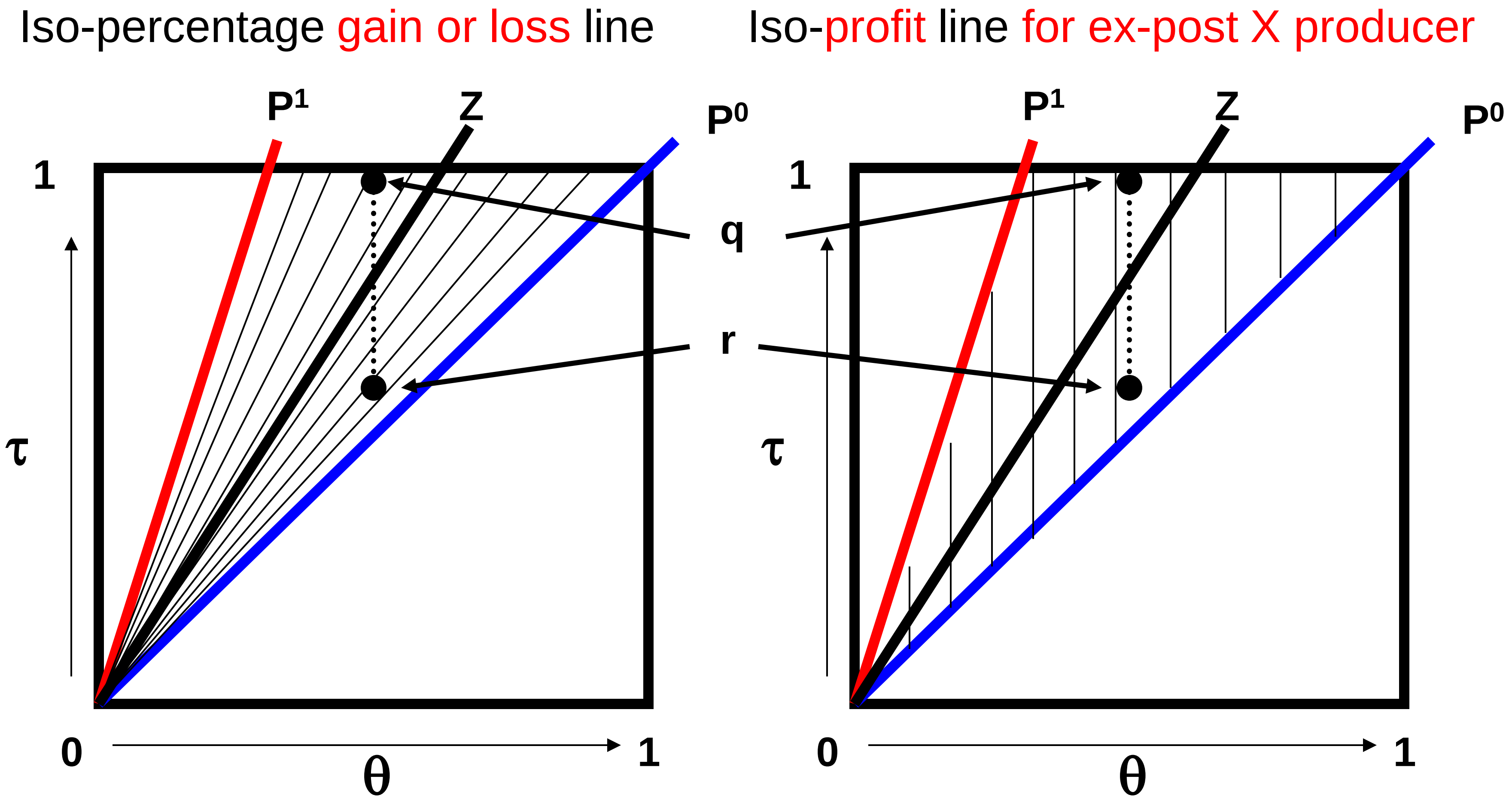
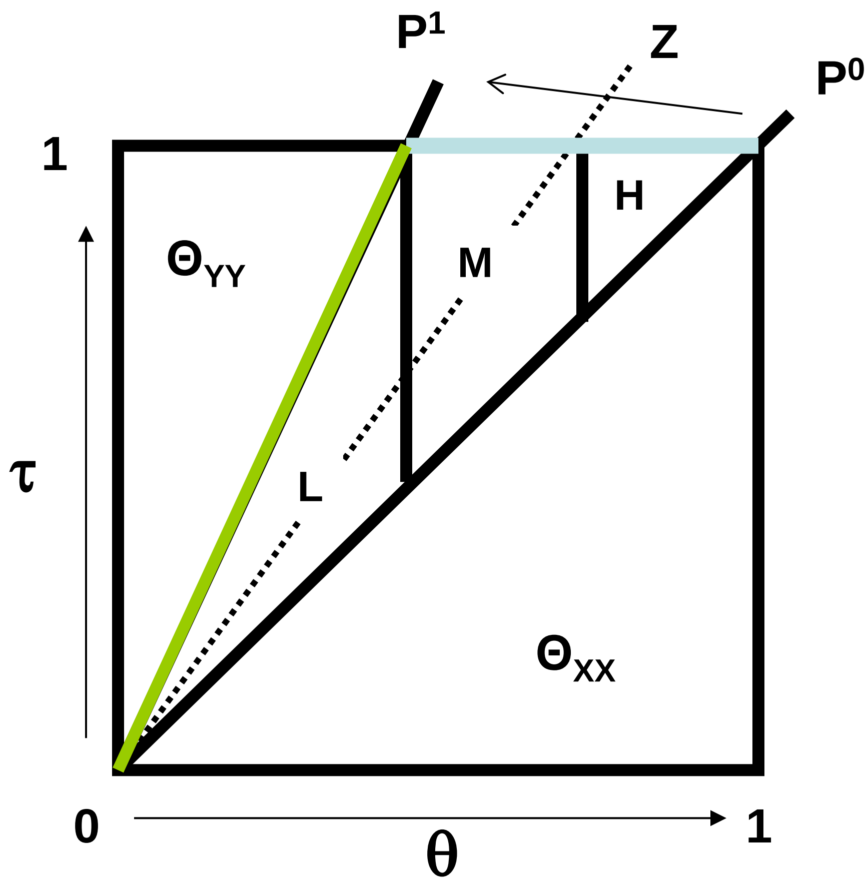


Figure 3: The **feasible** post-change (unanticipated) compensation

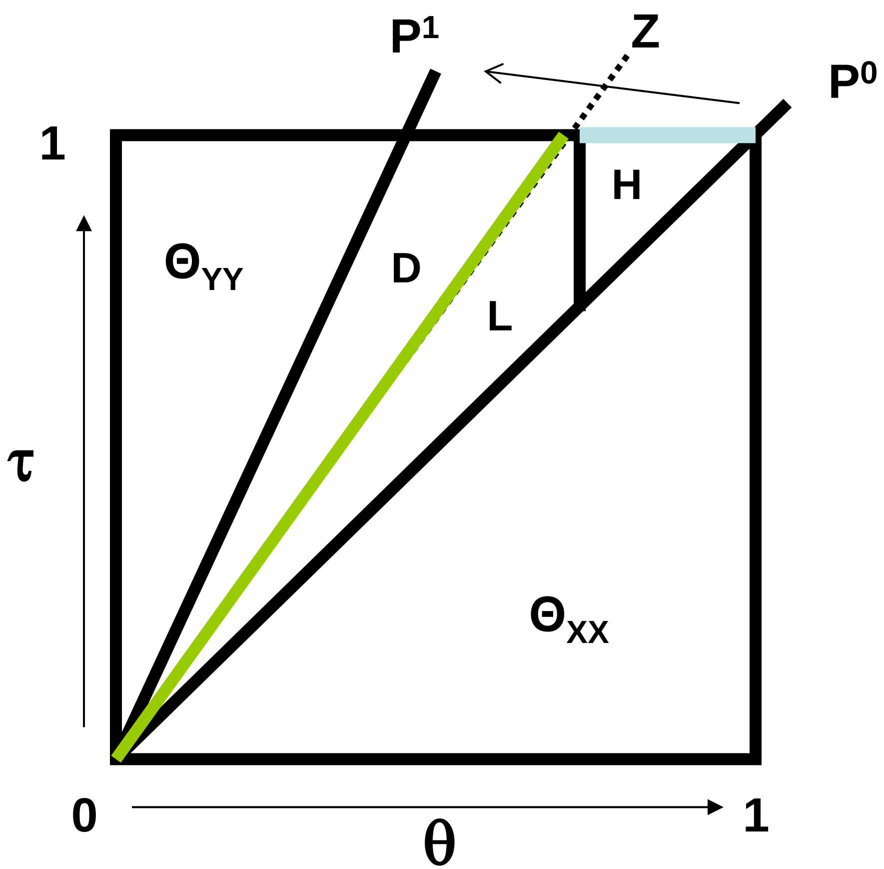


H = all gainers (tax)

M = mixture of gainers and losers (subsidy)

L = mixture of gainers and losers (subsidy)

Figure 4: The feasible ex ante (anticipated) compensation program



H = all winners (positive tax)

L = the group of individuals who were winning job-switchers ex ante but whose current profits are indistinguishable from those of losing job-switchers (tax exempt)

D = the group of individuals who were job switchers ex ante but who will stay in industry Y (They will undertake same subsidy as Θ_{YY} group)