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KOBAYASHI Keiichiro
RIETI

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There is No Natural Debt Limit with Consumption Tax*

KOBAYASHI Keiichiro

Research Institute of Economy, Trade and Industry

Abstract

In this paper, I demonstrate that the Laffer curve for a consumption tax increases monotonically and unboundedly in a closed economy in which the supply of one factor of production is fixed. Therefore, in this economy, an arbitrary amount of government debt can be made sustainable by choosing an appropriate tax rate. Tax revenue unboundedly increases due to a matter of accounting—tax revenue is transferred back to households as the redemption of government debt, which is used for the consumption, and is then taxed again.

Keywords: Natural debt limit, Laffer curve, Consumption tax.

JEL classification: E62, H20, H63

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Keio University/CIGS/RIETI/TKFD. E-mail: kobayasi@econ.keio.ac.jp

1 Introduction

One of the unchallenged notions in economics is that there should be a natural limit for government debt. The Laffer curve indicates that there are apparent upper limits for tax revenues from distortionary taxes such as labor income tax and capital income tax. It is usually assumed that the tax revenue from lump-sum tax is also bounded, as we implicitly assume that the government can take no more than the economy's total output as a lump-sum tax. Therefore, it seems natural to conclude that there is an upper limit for sustainable government debt, as it is financed by these tax instruments. For example, Davig, Leeper, and Walker (2010) called the upper bound of the revenue from labor and capital income taxes the "fiscal limit."

In this paper, I argue that a very ordinary tax instrument, consumption tax, can support an unbounded amount of government debt as long as the tax rate can be raised infinitely. On the premise that the Ponzi scheme is excluded, the debt should be no greater than the present value of the debtor's revenue in the future (see Aiyagari 1994). I demonstrate that, with an unbounded increase in consumption tax rate, government revenue can be raised unlimitedly so that the unlimited amount of the government debt can be supported by revenue from the consumption tax.

Trabandt and Uhlig (2011) estimate the Laffer curves for various taxes for the USA and EU. Nutahara (2013) estimates the Laffer curves for Japan. It was shown that, theoretically, the revenue from consumption tax is increasing in the tax rate, while the amount of tax revenue is bounded. In this paper, I demonstrate that it can be unbounded if the supply of a factor of production is fixed.

This paper is organized as follows. In the next section, I replicate Trabandt and Uhlig's (2011) finding that the Laffer curve of consumption tax is monotonically increasing and bounded in the standard neoclassical model. In Section 3, I demonstrate that the Laffer curve is unboundedly increasing in a neoclassical model in which the amount of one factor of production is fixed. Section 4 concludes and presents implications for the present findings.

2 Basic model with the bounded Laffer curve

In this section, I consider a standard neoclassical closed-economy model in which time is discrete and continues from 0 to infinity. There is a representative household that maximizes the following utility:

$$\sum_{t=0}^{\infty} \beta^t \{\ln c_t + \gamma \ln(1 - l_t)\},$$

subject to the budget constraint

$$(1 + \tau_{ct})c_t + b_{t+1} + k_{t+1} \leq w_t l_t + (1 + r_t)b_t + (r_t^k + 1 - \delta)k_t + s_t, \tag{1}$$

where β is the time discount factor, c_t is the consumption, l_t is the labor supply, τ_{ct} is the consumption tax rate, b_{t+1} is the government bond, k_{t+1} is the capital stock, w_t is the wage rate, r_t is the interest rate for the bonds, r_t^k is the rental rate of capital, δ is the depreciation rate, s_t is the lump-sum transfer from the government ($s_t \geq 0$). The household takes the variables $(\tau_{ct}, w_t, r_t, r_t^k, s_t)$ as given. There is also a representative firm that purchases the labor input and rents the capital input to maximize profit:

$$A_t k_t^{\alpha} l_t^{1-\alpha} - r_t^k k_t - w_t l_t,$$

where A_t is the productivity parameter and the consumption good is produced by the Cobb-Douglas production function: $y_t = A_t k_t^{\alpha} l_t^{1-\alpha}$. There is the government that decides the fiscal policy $\{\tau_{ct}, b_{t+1}, s_t, g_t\}_{t=0}^{\infty}$ subject to the budget constraint:

$$\tau_{ct} c_t + b_{t+1} = (1 + r_t)b_t + s_t + g_t y_t,$$

taking r_t and c_t as given, where $g_t y_t$ ($0 \leq g_t < 1$) represents government consumption, which is proportional to the total output. The government chooses $\{\tau_{ct}, b_{t+1}, g_t\}$, and then s_t is adjusted such that the government budget is satisfied. As the focus of this paper is the consumption tax, I assume that the government uses only consumption tax as the tax instrument. In the equilibrium, the following resource constraint must be satisfied:

$$c_t + k_{t+1} = (1 - g_t)A_t k_t^{\alpha} l_t^{1-\alpha} + (1 - \delta)k_t.$$

The equilibrium is determined by the first-order conditions for the household and the firm, the budget constraint for the government, and the resource constraint. The steady state in which the variables $\{\tau_{ct}, A_t, c_t, l_t, k_{t+1}, b_{t+1}, r_t, r_t^k, w_t, s_t, g_t\}$ are all time-invariant and $s_t = 0$ is determined by

$$\gamma \frac{(1+\tau_c)c}{1-l} = (1-\alpha)A\left(\frac{k}{l}\right)^{\alpha}, \tag{2}$$

$$\beta^{-1} = \alpha A\left(\frac{l}{k}\right)^{1-\alpha} + 1 - \delta, \tag{3}$$

$$\beta^{-1} = 1 + r, \tag{4}$$

$$\tau_c c = rb + gAk^{\alpha}l^{1-\alpha}, \tag{5}$$

$$c = (1-g)Ak^{\alpha}l^{1-\alpha} - \delta k. \tag{6}$$

The steady-state consumption is

$$c = \frac{Z}{(1+\tau_c)X + Y},$$

where

$$X = \gamma[(1-g)\alpha^{-1}(\beta^{-1} - 1 + \delta) - \delta) - \delta],$$
$$Y = (1-\alpha)A\left(\frac{\beta^{-1} - 1 + \delta}{\alpha A}\right),$$
$$Z = [(1-g)\alpha^{-1}(\beta^{-1} - 1 + \delta) - \delta) - \delta](1-\alpha)A\left(\frac{\alpha A}{\beta^{-1} - 1 + \delta}\right)^{\frac{\alpha}{1-\alpha}}.$$

Apparently, the consumption tax revenue, $\tau_c c$, is increasing in τ_c and bounded:

$$\tau_c c < \frac{Z}{X}.$$

This was demonstrated by Trabandt and Uhlig (2011). Note that $\lim_{\tau_c \to \infty} \tau_c c = \frac{Z}{X}$, whereas $\lim_{\tau_c \to \infty} c = \lim_{\tau_c \to \infty} k = \lim_{\tau_c \to \infty} l = 0$. The consumption tax revenue remains positive while the output converges to zero, because the tax revenue is transferred back to the household as the redemption of the government bond or the lump-sum subsidy, which are in turn used in purchasing consumption goods. As Trabandt and Uhlig (2011) argue, this is a matter of "accounting."

3 The model with a fixed factor and the unbounded Laffer curve

In this section, we modify the model such that the supply of capital is fixed over time:

$$k_t = 1, \quad \forall t. \tag{7}$$

We can regard k_t as representing the land in reality. The consumption good is produced by the following technology: $y_t = A_t k_t^{\alpha} m_t^{\nu} l_t^{1-\alpha-\nu}$, where m_t is the material or variable capital, which is the consumption good that is invested in the previous period. Thus, the household's budget constraint changes from (1) to

$$(1+\tau_{ct})c_t + b_{t+1} + m_{t+1} + q_t k_{t+1} \le w_t l_t + r_t^m m_t + (1+r_t)b_t + (r_t^k + q_t)k_t + s_t,$$

where r_t^m is the gross rate of return on m_t and q_t is the price of capital. The representative firm chooses inputs, $\{k_t, m_t, l_t\}$, to maximize

$$A_t k_t^{\alpha} m_t^{\nu} l_t^{1-\alpha-\nu} - r_t^k k_t - w_t l_t - r_t^m m_t.$$

The resource constraints are (7) and

$$c_t + m_{t+1} = (1-g_t) A_t m_t^{\nu} l_t^{1-\alpha-\nu},$$

where $g_t A_t m_t^{\nu} l_t^{1-\alpha-\nu}$ is government consumption. The competitive equilibrium is defined in a standard way. The steady state in which $s_t = 0$ and $g_t = g$, where $0 \le g < 1 - \beta\nu$, is determined by

$$\gamma \frac{(1+\tau_c)c}{1-l} = (1-\alpha-\nu) A l^{-\alpha} \left(\frac{m}{l}\right)^{\nu}, \tag{8}$$

$$\beta^{-1} = \nu A m^{-\alpha} \left(\frac{l}{m}\right)^{1-\alpha-\nu}, \tag{9}$$

$$q = \beta \left\{ \alpha A m^{\nu} l^{1-\alpha-\nu} + q \right\}, \tag{10}$$

$$\beta^{-1} = 1 + r, \tag{11}$$

$$\tau_c c = rb + g A m^{\nu} l^{1-\alpha-\nu}, \tag{12}$$

$$c + m = (1-g) A m^{\nu} l^{1-\alpha-\nu}. \tag{13}$$

We define the variable $x \equiv l/m$. The steady-state variables are given by

$$
\begin{aligned}
c &= \{(1-g)(\beta\nu)^{-1} - 1\}(\beta\nu A)^{\frac{1}{\alpha}} x^{\frac{1-\alpha-\nu}{\alpha}}, \\
l &= (\beta\nu A)^{\frac{1}{\alpha}} x^{\frac{1-\nu}{\alpha}}, \\
m &= (\beta\nu A)^{\frac{1}{\alpha}} x^{\frac{1-\alpha-\nu}{\alpha}}, \\
x &= \frac{1}{\{(1+\tau_c)\Gamma + \Omega\}^{\frac{\alpha}{1-\nu}}},
\end{aligned}
$$

where

$$
\begin{aligned}
\Gamma &= \frac{(1-g-\beta\nu)\gamma(\beta\nu A)^{\frac{1}{\alpha}}}{1-\alpha-\nu}, \\
\Omega &= (\beta\nu A)^{\frac{1}{\alpha}}.
\end{aligned}
$$

The Laffer curve in this model is determined by the tax revenue:

$$
\tau_c c = \frac{\{(1-g)(\beta\nu)^{-1} - 1\}\Omega\tau_c}{\{(1+\tau_c)\Gamma + \Omega\}^{\frac{1-\alpha-\nu}{1-\nu}}},
$$

which is asymptotically proportional to $\tau_c^{\frac{\alpha}{1-\nu}}$, as $\tau_c \to \infty$. Thus, the tax revenue is increasing in τ_c and is unbounded: $\lim_{\tau_c \to \infty} \tau_c c = \infty$. Note that there is a tax distortion that reduces consumption, output, and employment, as in the model of the previous section: $\lim_{\tau_c \to \infty} c = \lim_{\tau_c \to \infty} k = \lim_{\tau_c \to \infty} l = 0$. The tax revenue increases as the tax rate increases, while the output converges to zero, due to the same reason as in the model of the previous section.

Why is the Laffer curve unbounded in the model where the supply of k_t is fixed? In the neoclassical model presented in Section 2, the increase in τ_c reduces the amount of k and l proportionately, which then implies that c decreases in τ_c proportionately. In the model with a fixed supply of k_t, the increase in τ_c reduces l, but it cannot reduce the amount of k, and therefore c decreases in τ_c to a lesser extent than in the case where the supply of k is varied. This difference causes the unboundedness of the tax revenue in the model presented in the current section.

It is clearly demonstrated that, in this model, any amount of government debt can be made sustainable. For an arbitrarily large value b_0 (> 0), suppose the initial amount of government debt equals b_0 (> 0). The government can set $\{\tau_c, b_t, s_t, g\}_{t=0}^{\infty}$ as follows:

$$b_t = b_0,$$

$g < 1 - \beta\nu$, and τ_c and s_t are determined such that

$$(\beta^{-1} - 1)b_0 \leq \frac{\{(1-g)(\beta\nu)^{-1} - 1\}\Omega\tau_c - g(\beta\nu)^{-1}\Omega}{\{(1+\tau_c)\Gamma + \Omega\}^{\frac{1-\alpha-\nu}{1-\nu}}},$$

$$s_t = \tau_c c_t - r_t b_0 - g y_t.$$

Given this fiscal policy, the equilibrium is the steady state where $s_t \geq 0$ and $r_t = \beta^{-1} - 1$ and the government debt stays at $b_t = b_0$ forever. Thus, for all $b_0 > 0$, the debt b_0 is sustainable.

4 Conclusion

We demonstrate that a natural debt limit for the government may not exist under a certain condition. The Laffer curve for consumption tax is monotonically and unboundedly increasing in a closed economy in which the supply of one factor of production is fixed. In this economy, therefore, an arbitrary amount of government debt can be made sustainable by appropriate choice of consumption tax rate. The tax revenue is increasing in the tax rate because of a matter of accounting: tax revenue is transferred back to the household as a redemption of government debt, used for consumption, which is subsequently taxed again. A higher tax rate can support a larger amount of debt, which then influences higher tax distortion, which substantially reduces consumption, output, and employment. This exercise implies, therefore, that any amount of sovereign debt can be sustainable in a closed economy, as long as the people withstand the pains from tax distortion.

We should note two caveats for this surprising result. First, the choice of the model is too simplistic. If tax evasion by home production is available for consumers, they would shift their consumption from market products to home products if the consumption tax rate is very high. In this case, the consumption tax revenue should be bounded. Second, the amount of the government bond is exogenous in our model and we do not tell any reason why the government bonds are maintained. The government bond can be accumulated if there is a political economy interactions, moral hazard, or intergenerational transfers. Gaining realistic policy implications necessitates a deep understanding of the cause of the government bond accumulations.

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