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A Competitive Newsvendor Problem with Product Substitution under the Carbon Cap-and-Trade System

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Abstract: This study investigates the competitive issues of the newsvendor problem with product substitution under the carbon cap-and-trade system. Building on existing research, this paper introduces the carbon cap-and-trade system under uncertain market demand and considers that the original equipment manufacturer (OEM) can choose to procure raw materials from the contract manufacturer (CM), with both final products being substitutable. Furthermore, we explore the different substitution relationships between OEM and CM products under both pure competitive and co-opetitive modes. For this problem, decision models are developed for various scenarios, and optimal solutions satisfying given conditions are provided. We find that in one-way substitution, under pure competition, an increase in the OEM's (or CM's) green investment and substitution rate only leads to an increase in OEM's (or CM's) yields, while an increase in the OEM's (or CM's) green investment does not necessarily reduce CM's (or OEM's) yields. In the co-opetitive mode, an increase in the substitution rate and green investments of both manufacturers may lead to an increase in the yields of both manufacturers. Furthermore, an increase in carbon trading prices does not necessarily inhibit the manufacturer's yields. Moreover, we find that under the same competition mode, under certain conditions, two-way substitution between OEM and CM can bring better profits to both manufacturers and the entire supply chain. When the two modes are in the same substitution scenario, and the CM cannot substitute for the OEM, the optimal decisions and total supply chain profits of the two modes are equal. Finally, through numerical analysis, we find that neither mode is necessarily optimal when CM can substitute for OEM. Additionally, it is observed that when the same mode is in different substitution scenarios, total supply chain profits may be enhanced in the presence of product substitution.

Keywords: carbon cap-and-trade system; competition; product substitution; low-carbon supply chain

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1. Introduction

With the increase in global greenhouse gas emissions, significant damage to the global climate has been observed [1]. This damage is primarily caused by excessive carbon emissions in industrial production processes [2,3]. Consequently, to effectively reduce carbon dioxide emissions, many countries have signed the Kyoto Protocol and The Paris Agreement, and have implemented various carbon emission reduction policies [1]. As carbon emission reduction policies are successively implemented, carbon-emitting enterprises and their stakeholders are facing immense pressure to reduce emissions [4].

Carbon cap-and-trade systems are considered one of the most effective emission reduction policies [2,5–7], and have been adopted by several countries, such as the European Union Emission Trading Scheme (EU-ETS), the California Scheme and China Carbon Emission Trade Exchange (CCETE) [8]. Under the carbon cap-and-trade system, at the beginning of each period, emission-reducing enterprises are allocated a certain amount of carbon allowances. During this period, if a company's carbon emissions do not exceed its carbon allowance, it can sell the surplus allowances on the carbon trading market to generate additional revenue. Similarly, if a company's carbon emissions exceed its carbon allowance, it can purchase allowances on the carbon trading market to fulfill its carbon quota obligation [4,6,8]. As the carbon cap-and-trade system may bring additional revenue to emission-reducing enterprises, their incentive to reduce emissions can be significantly enhanced [6].

Although the carbon cap-and-trade system can effectively enhance companies' enthusiasm for emission reduction, they still face threats from other risks during the emission reduction process. Since all companies within the same industry are required to reduce emissions, competitors of emission-reducing enterprises may offer similar or substitutable green products [9]. This phenomenon can lead to significant financial risks for emission-reducing enterprises that invest heavily but are unable to sell enough products [10]. However, co-operation among emission-reducing enterprises is also possible. For example, China Petrochemical Corporation is responsible for both oil extraction and refining, as well as wholesale and retail of finished oil products. On the other hand, China National Aviation Fuel Group Limited is only responsible for the procurement and sales of aviation fuel. China Petrochemical Corporation is both a competitor and a supplier of China National Aviation Fuel Group Limited. There is both competition and co-operation between the two entities. Inspired by this example, we will explore how an original equipment manufacturer (OEM) and a contract manufacturer (CM), two manufacturers under the carbon cap-and-trade system, can respond to demand uncertainty through competition and co-operation to achieve better supply chain performance.

Furthermore, during co-operation, both manufacturers sell substitutable products. When one manufacturer experiences a shortage, customers will shift their purchases to the other manufacturer. In other words, when the OEM (or CM) is unable to satisfy all of the customers' demands, there will be a fraction of unfulfilled demand that will be satisfied by the CM (or OEM). If the OEM's (or CM's) customers shift their purchases to CM (or OEM) because the OEM (or CM) is out of stock, the OEM (or CM) suffers a certain amount of lost sales, while the CM (or OEM) can earn additional sales revenue. The degree of product substitution is related to customer purchase loyalty [11]. Purchase loyalty can lead to greater market demand but does not result in higher product prices [12]. In this study, we adopt the same assumption as [11], namely that the degree of product substitution is related to the level of customer purchase loyalty. When customers have low purchase loyalty towards a manufacturer, their products are more easily substituted. When purchase loyalty is at zero, one-way substitution occurs [11,13].

Currently, there is limited research on the competitive issues considering demand uncertainty under the carbon cap-and-trade system. Given this context, we will investigate the following questions. Firstly, how does product substitution affect the decisions of supply chain members under the carbon cap-and-trade system? Secondly, what impact do carbon trading prices and manufacturers' green investments have on supply chain decisions? Finally, under different substitution scenarios, which competitive mode selection can bring better profits to the supply chain?

To address this issue, we first focus on the research of the purely competitive mode. Under this mode, we construct a newsvendor model for OEM purchasing from a third-party supplier in scenarios of no substitution, one-way substitution, and two-way substitution. Then, for the co-opetitive mode, we construct a newsvendor model for OEM purchasing from the CM in scenarios of no substitution, one-way substitution, and two-way substitution. Finally, we compare the optimal decisions and profits under both modes for the same substitution scenario.

This study contributes to the research on the competitive issues of the newsvendor problem considering product substitution under the carbon cap-and-trade system. Firstly, by constructing a newsvendor model under different scenarios, we derive optimal decisions satisfying given conditions for various situations. Next, we analyze the impact of substitution rates, green investments, and carbon trading prices on optimal decisions under different scenarios. Finally, this paper explores how supply chain members under the carbon cap-and-trade system can achieve better supply chain performance through competition and co-operative. Based on these findings, we find that when CM's products cannot substitute for OEM's products, the optimal decisions and total supply chain profits are the same under both purely competitive and co-opetitive modes. However, when CM's products can substitute for OEM's products, the co-opetitive mode may not necessarily lead to better supply chain performance. Additionally, we find that an increase in carbon trading prices does not necessarily suppress manufacturer's yields.

The structure of this paper is as follows. Section 2 provides a literature review of relevant studies. Section 3 defines variables and describes the models. Section 4 analyzes the optimal decisions of the purely competitive mode under different substitution scenarios and examines the impact of substitution rates, green investments, and carbon trading prices on manufacturers' optimal decisions. Section 5 analyzes the optimal decisions of the co-opetitive mode under different substitution scenarios and investigates the impact of substitution rates, green investments, and carbon trading prices on manufacturers' optimal ordering decisions. Section 6 presents a numerical analysis. Section 7 summarizes the conclusions.

2. Literature Review

With the increasing awareness of carbon emissions reduction worldwide, various carbon emission reduction policies have been introduced. Among these policies, the carbon cap-and-trade system, as one of the most effective carbon reduction measures [5,14], has received extensive attention from scholars. Previous studies have indicated that changes in carbon trading prices and carbon quotas can have significant impacts on emission reduction [2,6]. As research has progressed further, scholars have started integrating the carbon cap-and-trade system with supply chain management. For instance, against the backdrop of the carbon cap-and-trade system, scholars have investigated inventory management [15], financing [15–17], joint replenishment [4,18], dual-channel sales [19], supply disruptions [20], product configuration [21], green innovation [22], competition [4,17,23], supply chain coordination [24], and supply chain design [25], among other issues. Additionally, there have been studies comparing the effectiveness of different carbon reduction policies by comparing the carbon cap-and-trade system with carbon tax policies [5,26–29]. However, ref. [26] explored the influence of different carbon reduction policies on enterprise technology selection and capacity decisions. Ref. [5] analyzed the effects of carbon taxes and carbon quota and trading policies on enterprise profits and emission reduction. Ref. [27] compared the effects of the carbon cap-and-trade system and carbon tax policies on multi-product production planning for enterprises. Ref. [28] compared the effects of the carbon cap-and-trade system and carbon tax policies on trade credit. Ref. [29] compared the impact of two policies on emissions reductions from electric commercial vehicles (ECVs).

In the actual operation of supply chains, competition between supply chains can also threaten supply chain security. For instance, refs. [30,31] pointed out that manufacturers may be concerned about remanufacturing eroding their sales, leading to significant profit loss. Therefore, to address the security issues brought by competition, scholars have conducted research on competition between retailers [32,33], manufacturers [11,34], manufacturers and retailers [32,35] and supply chains [36]. Particularly concerning horizontal competition between manufacturers, researchers have employed various research methods to address such issues, such as the Cournot model [31,35], Nash equilibrium [37,38], and newsvendor model [11,39,40], among others. The difference lies in that the newsvendor model further considers the uncertainty of market demand and takes into account the substitutability of products between competing enterprises. When market demand is uncertain, although the above literature similarly uses the newsvendor model, ref. [39] studied the impact of sales commissions between two retailers on the optimal order quantity and profit. On the other hand, ref. [11] considered the effects of co-opetitive between two manufacturers under different substitution relationships.

As carbon emission reduction becomes increasingly emphasized, refs. [14,37,41–47] have further considered the issue of supply chain competition in a low-carbon environment. The difference among these studies lies in that ref. [41] did not specifically introduce carbon emission reduction policies while addressing the issue of interruptions combined with competition. Although refs. [37,46,47] all discuss financing issues, ref. [37] explores the impact of future carbon reduction policy stringency on green investment by competing firms, while refs. [46,47] only examine optimal financing strategies. Similar to ref. [41], refs. [37,46,47] also did not introduce specific carbon emission reduction policies. On the other hand, ref. [14] introduced carbon quota and trading policies and compared the profits and carbon emissions of supply chains under purely competitive and co-operative modes. Anand and ref. [42] compared the impacts of Carbon cap-and-trade system and carbon taxes on competition. Ref. [43] considered competition between multinational manufacturers and local manufacturers under carbon lock-in conditions. However, the aforementioned literature, due to its use of the Cournot model or Nash equilibrium, overlooks the uncertainty of market demand or the impact of product substitution on supply chain competition. In contrast to these studies, ref. [45] explores the factors influencing firms' green competitive advantage.

Unlike existing research, this paper considers the newsvendor competition problem under various substitution scenarios within a carbon cap-and-trade system. First, the paper explores the optimal decisions of two manufacturers under both purely competitive and co-opetitive modes. Then, we characterize the competition between manufacturers through different product substitution relationships and examine the impact of these relationships on the competing manufacturers. Third, we address the competition issue under market demand uncertainty by constructing a newsvendor model to analyze the problem. Finally, we incorporate the carbon cap-and-trade system and analyze the impact of low-carbon constraints on different scenarios to explore how the supply chain can achieve better benefits under various substitution situations.

3. Model Description

We will consider the competition between an original equipment manufacturer (OEM) and a contract manufacturer (CM) under the carbon cap-and-trade system. In this scenario, the final products manufactured by both the OEM and CM are sold in the same market, leading to a competitive relationship between them. Additionally, both the OEM and CM's final products generate carbon emissions, thereby subjecting both manufactures to carbon emission constraints. Furthermore, the CM has the capability to produce the raw materials required for the final products. Therefore, in this study, we will compare and analyze two competition modes as follows:

(1) Purely competitive mode: In this mode, the OEM procures raw materials from a third-party supplier (TS) for production and competes with the CM in the market for final products.

(2) Co-opetitive mode: In this mode, the OEM procures raw materials from the CM for production and competes with the CM in the market for final products.

At the beginning of the period, the enterprise acting as the supplier announces the wholesale prices of raw material w. Subsequently, OEM and CM, respectively, undertake their green investment k_o and k_c, and determine their yields q_o and q_c. Simultaneously, OEM orders raw materials from its supplier and pays wq_o for the supplier. At the end of the period, OEM and CM sell their respective final products at prices p and settle their carbon allowances through the carbon cap-and-trade system. If customers are unable to purchase products from OEM (or CM), they will attempt to purchase them from CM (or OEM). The parameters of this study are presented in Table 1.

Table 1. Parameters and definitions.

Parameter	Definition	Parameter	Definition
c	unit production cost of raw materials	D_i	the random demand of manufacturer i, $i = o, c$
p	unit selling price of the final product	p_e	carbon trading price
q_i	the yields of manufacturer i's final product, $i = o, c$	δ_i	the demand growth caused by the green investment of manufacturer i, $i = o, c$
w_j	wholesale price from supplier j, $j = s, c$	ε_i	the demand uncertainty of manufacturer i, $i = o, c$
r_i	the proportion of manufacturer j's remaining demand allocated to manufacturer i, $i \neq j, i, j = o, c$	R_i	manufacturer i's actual demand after substitution occurs, $i = o, c$
k_i	green investment of manufacturer i, $i = o, c$	e_i	the initial emission level of manufacturer i's final product, $i = o, c$
E_i	carbon allowances of manufacturer i, $i = o, c$	θ_i	the emission reduction rate per unit resulting from manufacturer i's green investment, $i = o, c$
Subscript			
s	third-party supplier (TS)	c	contract manufacturer (CM)
o	original equipment manufacturer (OEM)		
Superscript			
P	purely competitive mode	C	co-opetitive mode

We assume that the random demand of manufacturer i is D_i $(i = o, c)$ and is influenced by its green investment. Therefore, the demand function is $D_i = \delta_i\sqrt{k_i} + \varepsilon_i$ [48], in which δ_i represents the increase in demand per unit of green investment, and ε_i is a non-negative random variable. Its distribution function and density function on $[0, +\infty)$ are $F(\varepsilon_i)$ and $f(\varepsilon_i)$, respectively. Let $\bar{F}(\varepsilon_i) = 1 - F(\varepsilon_i)$ be the reliability function of ε_i. The increasing generalized failure rate of ε_i is $g(\varepsilon_i) = \varepsilon_i h(\varepsilon_i)$ [49], where $h(\varepsilon_i) = f(\varepsilon_i)/\bar{F}(\varepsilon_i)$ is the failure rate. Both $h(\varepsilon_i)$ and $g(\varepsilon_i)$ are monotonically increasing with ε_i [50]. We assume that D_o and D_c are mutually independent and follow the same distribution [11]. We also adhere to the product substitution assumptions outlined in [40], which relate the degree of product substitution to the level of customer purchase loyalty. When customer loyalty to a manufacturer is low, its products are more easily substituted. We assume that when one party cannot meet all the demand, a certain proportion of the remaining demand will be transferred to the competitor. When purchase loyalty is zero, one-way substitution will occur [13]. Therefore, when CM cannot satisfy its demand, the actual demand R_o for OEM is $R_o = D_o + r_o(D_c - q_c)^+$, where r_o is the proportion of CM's remaining demand allocated to OEM. Similarly, when OEM cannot satisfy its demand, the actual demand R_c for CM is r_c is the proportion of OEM's remaining demand allocated to CM. Additionally, we assume that the production cost of raw materials is c. The procurement cost of non-critical components and the production cost of final products are zero [11], where $c + p_e(e_c - \theta_c k_c) < p$. Furthermore, to simplify the problem, we assume that no carbon emissions occur during the production process of raw materials. The wholesale price w satisfies $c < w \leq p - p_e(e_o - \theta_o k_o)$.

4. Model Analysis of Purely Competitive Supply Chain

In this section, we first consider the scenario where OEM and CM are in a state of pure competition. The supply chain structure of the purely competitive mode is illustrated in Figure 1.

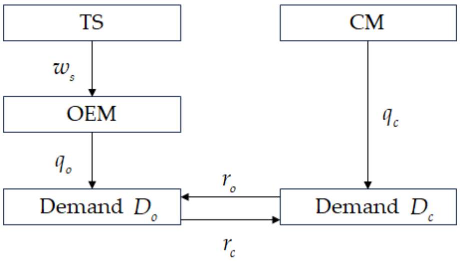


Figure 1. Supply chain structure of the purely competitive mode.

At the beginning of the period, TS announces the wholesale price of raw materials w_s. Then, OEM and CM undertake their respective green investments k_o and k_c, and determine their yields q_o and q_c. Meanwhile, OEM orders raw materials from TS and pays TS $w_s q_o$. After undertaking green investments, the unit carbon emissions of the final products of OEM and CM, respectively, decrease to $(e_o - \theta_o k_o)$ and $(e_c - \theta_c k_c)$. At the end of the period, the final products of OEM and CM are sold at prices p, and they, respectively, obtain carbon trading revenues of $p_e[E_o - (e_o - \theta_o k_o)q_o]$ and $p_e[E_c - (e_c - \theta_c k_c)q_c]$ through the carbon cap-and-trade system. The sequence of events is illustrated in Figure 2.

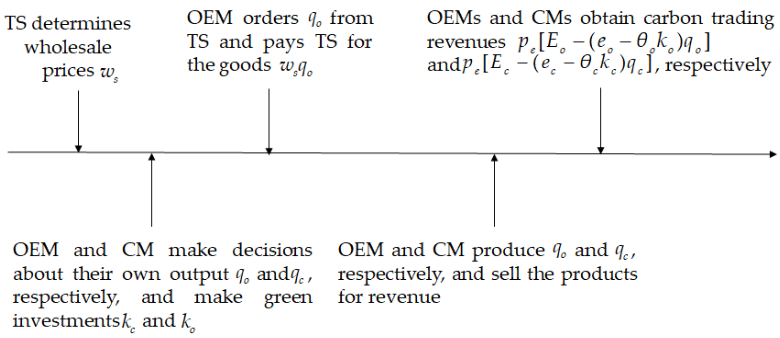


Figure 2. Sequence of events under purely competitive mode.

On this basis, we will analyze the purely competitive mode under different product substitution scenarios.

4.1. Purely Competitive Mode: Non-Substitutable (0, 0)

When there is no substitutability between the final products of OEM and CM, it satisfies $r_o = r_c = 0$. Therefore, the actual demand for OEM and CM equals their respective random demands, i.e., $R_o = D_o$, $R_c = D_c$. In this case, the expected sales revenue for OEM and CM at the end of the period are $pE[\min[D_o, q_o]]$ and $pE[\min[D_c, q_c]]$, respectively.

The profit functions for TS, OEM and CM in this scenario are as follows:

$$\pi_s^P = (w_s - c)q_o \tag{1}$$

$$\pi_o^P = pE[\min[D_o, q_o]] - w_s q_o + p_e[E_o - (e_o - \theta_o k_o)q_o] - k_o \tag{2}$$

$$\pi_c^P = pE[\min[D_c, q_c]] - cq_c + p_e[E_c - (e_c - \theta_c k_c)q_c] - k_c \tag{3}$$

Lemma 1. *Under the (0, 0) scenario in the purely competitive mode, the optimal decisions for TS, OEM and CM are satisfied by: (All proofs in Supplementary Materials)*

$$w_s^{P(0,0)} = pq_o^{P(0,0)} f(q_o^{P(0,0)} - \delta_o \sqrt{k_o}) + c$$

$$q_o^{P(0,0)} = F^{-1}(\frac{p - w_s^{P(0,0)} - p_e(e_o - \theta_o k_o)}{p}) + \delta_o \sqrt{k_o}$$

$$q_c^{P(0,0)} = F^{-1}(\frac{p - c - p_e(e_c - \theta_c k_c)}{p}) + \delta_c \sqrt{k_c}$$

From Lemma 1, we observe that under purely competitive mode, when there is no substitution between the products of OEM and CM, the optimal order quantities for both are independent of each other. Building upon Lemma 1, in Proposition 1, we provide the impact of carbon trading price p_e and the green investments k_o and k_c on the optimal decisions.

Proposition 1. *Under the (0, 0) scenario in the purely competitive mode, (1) $q_o^{P(0,0)}$ and $w_o^{P(0,0)}$ increase with the increase in k_o, while $q_c^{P(0,0)}$ remains unchanged with variations in k_o; (2) $q_c^{P(0,0)}$ increases with the increase in k_c, while $q_o^{P(0,0)}$ and $w_o^{P(0,0)}$ remains unchanged with variations in k_c; (3) $q_c^{P(0,0)}$, $q_o^{P(0,0)}$ and $w_o^{P(0,0)}$ decrease with the increase in p_e.*

In Proposition 1, we observe that $q_o^{P(0,0)}$ and $w_o^{P(0,0)}$ are unaffected by CM's green investment, while $q_c^{P(0,0)}$ is unaffected by OEM's green investment. This is because there is no substitution relationship between the final products of OEM and CM. The growth in green investment in one manufacturer does not affect the demand of the other manufacturer. Additionally, the manufacturers' green investment can increase the demand; hence, the optimal production quantities of OEM and CM increase with their respective green investments. For TS, an increase in OEM's green investment leads to higher demand, prompting TS to raise wholesale prices to enhance its revenue. However, the impact of carbon trading prices on optimal decisions differs. With an increase in carbon trading prices, OEM and CM reduce their production quantities to maximize carbon trading revenue. As OEM's production decreases, TS lowers wholesale prices to stimulate OEM to increase its order quantity and ensure its revenue.

To simplify subsequent analyses, we assume that ε_i $(i = o, c)$ follows a uniform distribution from 0 to 1 [51]. Under this assumption, we employ backward induction, and in Lemma 2, we provide the conditions for the optimal solutions of TS, OEM and CM.

Lemma 2. *Under the (0, 0) scenario in the purely competitive mode, when ε_i $(i = o, c)$ follows a uniform distribution from 0 to 1, the optimal decisions for TS, OEM and CM are satisfied by:*

$$w_s^{P(0,0)} = \frac{p(1 + \delta_o \sqrt{k_o}) + c - p_e(e_o - \theta_o k_o)}{2}$$

$$q_o^{P(0,0)} = \frac{p(1 + \delta_o \sqrt{k_o}) - c - p_e(e_o - \theta_o k_o)}{2p}$$

$$q_c^{P(0,0)} = 1 + \delta_c \sqrt{k_c} - \frac{c + p_e(e_c - \theta_c k_c)}{p}$$

4.2. Purely Competitive Mode: OEM Substitutable for CM (r, 0)

When OEM products partially substitute for CM products, we have $r_o = r \neq 0$ and $r_c = 0$. In this case, the actual demand for OEM is $R_o = D_o + r(D_c - q_c)^+$, while the actual demand for CM is equal to its random demand, i.e., $R_c = D_c$. Consequently, the expected sales revenue for OEM and CM at the end of the period are $pE[\min[D_o + r(D_c - q_c)^+, q_o]$

and $pE[\min[D_c, q_c]]$, respectively. In this scenario, the profit functions for TS, OEM and CM are as follows:

$$\pi_s^P = (w_s - c)q_o \tag{4}$$

$$\pi_o^P = pE[\min[D_o + r(D_c - q_c)^+, q_o] - w_s q_o + p_e[E_o - (e_o - \theta_o k_o)q_o] - k_o \tag{5}$$

$$\pi_c^P = pE[\min[D_c, q_c]] - cq_c + p_e[E_c - (e_c - \theta_c k_c)q_c] - k_c \tag{6}$$

Next, we denote $L = \frac{\partial \pi_s}{\partial w_s}$ and provide the conditions satisfied by the optimal solutions of TS, OEM and CM in Lemma 3.

Lemma 3. *In the case of (r, 0) under purely competitive mode,*

(1) Under any conditions, $q_c^{P(r,0)} = 1 + \delta_c\sqrt{k_c} - \frac{c+p_e(e_c-\theta_c k_c)}{p}$;

(2) When $e_o - \theta_o k_o < M_1$,

$$q_o^{P(r,0)} = \frac{1+\delta_o\sqrt{k_o}}{2} + \frac{r[c+p_e(e_c-\theta_c k_c)]^2}{4p^2} - \frac{c}{2p} - \frac{p_e(e_o-\theta_o k_o)}{2p}$$

$$w_s^{P(r,0)} = \frac{p(1+\delta_o\sqrt{k_o})}{2} + \frac{r[c+p_e(e_c-\theta_c k_c)]^2}{4p} + \frac{c}{2} - \frac{p_e(e_o-\theta_o k_o)}{2}$$

(3) When $M_1 \le e_o - \theta_o k_o < M_2$, $w_s^{P(r,0)}$ *satisfies* $L(w_s^{P(r,0)}) = 0$, *and* $q_o^{P(r,0)}$ *satifies*

$$q_o^{P(r,0)} = \sqrt{r^2[1 - \frac{c+p_e(e_c-\theta_c k_c)}{p}]^2 + 2r[1 - \frac{w_s^{P(r,0)} + p_e(e_o-\theta_o k_o)}{p}]} + \delta_o\sqrt{k_o} - r[1 - \frac{c+p_e(e_c-\theta_c k_c)}{p}]$$

(4) When $e_o - \theta_o k_o \ge M_2$, $q_o^{P(r,0)} = \delta_o\sqrt{k_o}$, $w_s^{P(r,0)} = p - p_e(e_o - \theta_o k_o)$.

where $M_1 = \frac{p}{p_e}\{\frac{r[c+p_e(e_c-\theta_c k_c)]^2}{2p^2} - \frac{c}{p} - \frac{2r[c+p_e(e_c-\theta_c k_c)]}{p} + 1 - \delta_o\sqrt{k_o}\}$, $M_2 = \frac{1}{p_e}[(p-c)(1-\delta_o\sqrt{k_o}) + \delta_o\sqrt{k_o}p_e(e_c-\theta_c k_c)]$.

Next, we first compare the optimal solutions for the (r, 0) and (0, 0) scenarios in the purely competitive mode in Proposition 2.

Proposition 2. *Comparing the (r, 0) and (0, 0) scenarios in the purely competitive mode, we obtain:*

(1) Under any conditions, $q_c^{P(r,0)} = q_c^{P(0,0)}$, $\pi_c^{P(r,0)} = \pi_c^{P(0,0)}$;

(2) When $e_o - \theta_o k_o < M_1$, $q_o^{P(r,0)} > q_o^{P(0,0)}$, $w_s^{P(r,0)} > w_s^{P(0,0)}$, $\pi_s^{P(r,0)} = \pi_s^{P(0,0)}$;

(3) When $e_o - \theta_o k_o \ge M_2$, $q_o^{P(r,0)} < q_o^{P(0,0)}$, $w_s^{P(r,0)} > w_s^{P(0,0)}$.

From Proposition 2, we observe that in the (r, 0) scenario, the optimal production quantity and profit of CM are the same as in the (0, 0) scenario. This is because CM's products cannot substitute for OEM's products, leading to no additional demand. Therefore, CM's decision-making process does not need to consider the influence of OEM. For OEM and TS, the optimal decisions in this scenario are influenced by the unit carbon emissions of OEM's products. When the carbon emissions of OEM's products are low ($e_o - \theta_o k_o < M_1$), compared to $q_o^{P(0,0)}$ and $w_s^{P(0,0)}$, both the production quantity of OEM and the wholesale price of TS will increase. This is because some demand that CM cannot fulfill will shift to OEM's market. Additionally, the increase in sales revenue due to increased production compensates for the loss of carbon trading revenue. In this case, raising the wholesale price by TS will not significantly reduce the ordering quantity of OEM; instead, it can bring more revenue. However, when the unit carbon emissions of OEM's products are sufficiently high ($e_o - \theta_o k_o \ge M_2$), OEM will produce at the minimum quantity to maximize profit. Similarly, TS will set the wholesale price at the maximum to ensure its profit.

Proposition 3. *In the (r, 0) scenario under purely competitive mode,* *(1) Under any conditions, $q_c^{P(r,0)}$ remains constant regardless of the change in r;*

(2) When $e_o - \theta_o k_o < M_1$, $q_o^{P(r,0)}$ and $w_s^{P(r,0)}$ increase with the increase in r;

(3) When $M_1 \leq e_o - \theta_o k_o < M_2$, $q_o^{P(r,0)}$ increases with the increase in r; if $rq_o^{P(r,0)}(1 - \frac{c+p_e(e_c-\theta_c k_c)}{p}) > [r(1 - \frac{c+p_e(e_c-\theta_c k_c)}{p}) + q_o^{P(r,0)} - \delta_o\sqrt{k_o}]^2$, $w_s^{P(r,0)}$ increases with the increase in r;

(4) When $e_o - \theta_o k_o \geq M_2$, $q_o^{P(r,0)}$ and $w_s^{P(r,0)}$ remain constant regardless of the change in r.

From Proposition 3, we observe that regardless of how the substitution rate r changes, the optimal production quantity for CM remains constant. This is because in this scenario, CM's actual demand equals its stochastic demand, and CM cannot substitute for the excess demand of OEM. Therefore, when making decisions, CM does not need to consider the influence of the substitution rate r. However, for OEM and TS, when the carbon emission per unit of OEM's product is sufficiently low ($e_o - \theta_o k_o < M_1$), the cost per unit for OEM is low, and increasing its production can lead to greater profits. Since the cost per unit for OEM is low, raising the wholesale price by TS not only does not result in a significant reduction in OEM's production but rather brings more profits to TS. But when the carbon emission per unit of OEM's product is sufficiently high ($e_o - \theta_o k_o \geq M_2$), the cost per unit for OEM is correspondingly high. To ensure its profit, OEM will always produce at the minimum quantity. For TS to ensure its profit, it will always set the wholesale price at the maximum. When the carbon emission per unit of OEM's product is at an intermediate level ($M_1 \leq e_o - \theta_o k_o < M_2$), there is still some profit margin for OEM, and increasing its production can yield greater returns. However, in this scenario, the wholesale price set by TS no longer monotonically changes with the substitution rate, as it is influenced by the carbon emission per unit of OEM and CM's products, as well as the carbon trading price.

Proposition 4. *In the (r, 0) scenario under purely competitive mode,*

(1) Under any conditions, $q_c^{P(r,0)}$ remains constant regardless of the change in k_c and decreases with the increase in p_e, and remains unchanged with the variation in k_o;

(2) When $e_o - \theta_o k_o < M_1$, $q_o^{P(r,0)}$ and $w_s^{P(r,0)}$ increase with the increase in k_o and decreases with the increase in k_c; if $p_e > \frac{1}{e_c-\theta_c k_c}[\frac{p(e_o-\theta_o k_o)}{r(e_c-\theta_c k_c)} - c]$, $q_o^{P(r,0)}$ and $w_s^{P(r,0)}$ increase with the increase in p_e;

(3) When $M_1 \leq e_o - \theta_o k_o < M_2$, $q_o^{P(r,0)}$ and $w_s^{P(r,0)}$ increase with the increase in k_o and decreases with the increase in k_c; if $\frac{e_o-\theta_o k_o}{(2q_o^{P(r,0)}-\delta_o\sqrt{k_o})(e_c-\theta_c k_c)} > \frac{q_o^{P(r,0)}-\delta_o\sqrt{k_o}+r[1-\frac{c+p_e(e_c-\theta_c k_c)}{p}]}{2q_o^{P(r,0)}-\delta_o\sqrt{k_o}+r[1-\frac{c+p_e(e_c-\theta_c k_c)}{p}]}$, $q_o^{P(r,0)}$ increase with the increase in p_e; $w_s^{P(r,0)}$ increase with the increase in k_o; if $k_c > \frac{p(q_o^{P(r,0)}-\delta_o\sqrt{k_o})^2}{\delta_o\sqrt{k_o} r p_e \theta_c} - \frac{p-c-p_e e_c}{p_e\theta}$, $w_s^{P(r,0)}$ increase with the increase in k_c; if $\frac{[1-\frac{c+p_e(e_c-\theta_c k_c)}{p}]\{r[1-\frac{c+p_e(e_c-\theta_c k_c)}{p}]-\delta_o\sqrt{k_o}\}}{q_o^{P(r,0)}-\delta_o\sqrt{k_o}+r[1-\frac{c+p_e(e_c-\theta_c k_c)}{p}]} > \frac{2}{r}[\frac{e_o-\theta_o k_o}{e_c-\theta_c k_c} - (q_o^{P(r,0)} - \delta_o\sqrt{k_o})]$, $w_s^{P(r,0)}$ increase with the increase in p_e;

(4) When $e_o - \theta_o k_o \geq M_2$, $q_o^{P(r,0)}$ increases with the increase in k_o and remains unchanged with changes in k_c and p_e; $w_s^{P(r,0)}$ increase with the increase in k_o, decreases with the increase in p_e, and remains unchanged with changes in k_c.

From Proposition 4, we can see that the impact of carbon trading price p_e and the green investment of both manufactures, k_o and k_c, on the optimal yields of CM is the same as in the case of a purely competitive mode (0, 0). This is because in this scenario, the actual demand of CM is equal to its stochastic demand. Additionally, the effect of OEM's green investment k_o on $q_o^{P(r,0)}$ and $w_s^{P(r,0)}$ is also similar to the purely competitive scenario (0, 0). This is because OEM's green investment k_o brings an increase in demand. Furthermore, an increase in OEM's green investment brings more demand to TS. To increase its revenue, TS raises the wholesale price. When OEM's carbon emissions are low enough ($e_o - \theta_o k_o < M_1$), increasing CM's green investment k_c can increase its yields, reducing the excess demand that cannot be satisfied. Consequently, OEM's optimal yields decrease. Additionally, because OEM's carbon emissions are low in this scenario, when carbon trading prices are low, OEM's carbon trading revenue will be severely affected. To obtain more carbon trading revenue, OEM will also reduce its yields. Due to the decrease in OEM's yields, TS will lower the wholesale price to ensure its profit. When OEM's carbon emissions are correspondingly high ($e_o - \theta_o k_o \geq M_2$), the cost per unit of OEM's product is high. Therefore, to ensure its profit, OEM produces at the minimum yields, and TS charges the highest wholesale price. When OEM's carbon emissions satisfy $M_1 \leq e_o - \theta_o k_o < M_2$, the impact of carbon trading price p_e on $q_o^{P(r,0)}$ and $w_s^{P(r,0)}$ is enough complex due to the influence of other parameters. This phenomenon indicates that for the manufacturer constrained by emission reduction, an increase in carbon trading price does not necessarily suppress his production capacity.

4.3. Purely Competitive Mode: CM Substitutable for OEM (0, r)

When CM products partially substitute for OEM products, it satisfies $r_c = r \neq 0$ and $r_o = 0$. In this case, the actual demand for OEM equals its random demand, i.e., $R_o = D_o$. Meanwhile, CM's actual demand is represented by $R_c = D_c + r(D_o - q_o)^+$. Consequently, the expected sales revenue for OEM and CM at the end of the period are $pE[\min[D_o, q_o]]$ and $pE[\min[D_c + r(D_o - q_o)^+, q_c]$, respectively. The profit functions for TS, OEM and CM under this scenario are as follows:

$$\pi_s^P = (w_s - c)q_o \tag{7}$$

$$\pi_o^P = pE[\min[D_o, q_o]] - w_s q_o + p_e[E_o - (e_o - \theta_o k_o)q_o] - k_o \tag{8}$$

$$\pi_c^P = pE[\min[D_c + r(D_o - q_o)^+, q_c] - cq_c + p_e[E_c - (e_c - \theta_c k_c)q_c] - k_c \tag{9}$$

Lemma 4. *In the case of (0, r) under purely competitive mode,*

(1) Under any conditions,

$$q_o^{P(0,r)} = \frac{p(1 + \delta_o \sqrt{k_o}) - c - p_e(e_o - \theta_o k_o)}{2p}$$

$$w_s^{P(0,r)} = \frac{p(1 + \delta_o \sqrt{k_o}) + c - p_e(e_o - \theta_o k_o)}{2}$$

(2) When $e_c - \theta_c k_c \leq H_1$,

$$q_c^{P(0,r)} = \frac{r[p(1 + \delta_o \sqrt{k_o}) + c + p_e(e_o - \theta_o k_o)]}{2p} + 1 + \delta_c \sqrt{k_c} - \sqrt{\frac{2r[c + p_e(e_c - \theta_c k_c)]}{p}}$$

(3) When $H_1 < e_c - \theta_c k_c < H_2$,

$$q_c^{P(0,r)} = 1 + \delta_c \sqrt{k_c} - \frac{c + p_e(e_c - \theta_c k_c)}{p} + \frac{r}{2}[\frac{p(1 + \delta_o \sqrt{k_o}) + c + p_e(e_o - \theta_o k_o)}{2p}]^2$$

(4) When $e_c - \theta_c k_c \geq H_2$,

$$q_c^{P(0,r)} = \sqrt{r^2[\frac{p(1 - \delta_o \sqrt{k_o}) - c - p_e(e_o - \theta_o k_o)}{2p}]^2 + 2r[1 - \frac{c + p_e(e_c - \theta_c k_c)}{p}]} + \delta_c \sqrt{k_c} - \frac{r}{2p}[p(1 - \delta_o \sqrt{k_o}) - c - p_e(e_o - \theta_o k_o)]$$

where $H_1 = \frac{r[p(1+\delta_o \sqrt{k_o}) + c + p_e(e_o - \theta_o k_o)]^2}{8 p_e p} - \frac{c}{p_e}$, $H_2 = \frac{r}{2p_e}[p(1 - \delta_o \sqrt{k_o}) - c - p_e(e_o - \theta_o k_o)] + (1 - r)\frac{p}{p_e} + \frac{r[p(1+\delta_o \sqrt{k_o}) + c + p_e(e_o - \theta_o k_o)]^2}{8 p_e p} - \frac{c}{p_e}$.

Proposition 5. *Comparing the optimal decisions under purely competitive for the (0, r) and (0, 0) scenarios, we obtain: (1) Under any conditions,* $q_o^{P(0,r)} = q_o^{P(0,0)}$, $w_s^{P(0,r)} = w_s^{P(0,0)}$, $\pi_o^{P(0,r)} = \pi_o^{P(0,0)}$, $\pi_s^{P(r,0)} = \pi_s^{P(0,0)}$*; (2) When* $e_c - \theta_c k_c < H_2$, $q_c^{P(0,r)} > q_c^{P(0,0)}$*; otherwise,* $q_c^{P(0,r)} < q_c^{P(0,0)}$.

From Proposition 5, we observe that the optimal decisions and profits of OEM and TS in this scenario are the same as those in the purely competitive mode under the (0, 0) scenario. This is because OEM's products cannot substitute for CM's products, resulting in no additional demand. Therefore, OEM and TS do not need to consider the influence of CM when making decisions. However, CM's decision is no longer uniform. Similar to the decision-making process of OEM in the purely competitive mode under the (r, 0) scenario, CM needs to consider the carbon emissions of its products in this scenario. When the carbon emissions per unit of CM's product satisfy $e_c - \theta_c k_c < H_2$, CM's order quantity will accept some of OEM's excess demand. In this case, CM's optimal production quantity will exceed the optimal production quantity under the (0, 0) scenario in the purely competitive mode. However, when the carbon emissions per unit of CM's product satisfy $e_c - \theta_c k_c \geq H_2$, the cost of CM's unit product becomes sufficiently high. In this case, CM will no longer accept some of OEM's excess demand. To ensure its profit, CM's production quantity will be smaller than the optimal production quantity under the (0, 0) scenario in the purely competitive mode.

Proposition 6. *In the purely competitive mode under the (r, 0) scenario,* $\frac{\partial q_o^{P(0,r)}}{\partial r} = 0$, $\frac{\partial w_s^{P(0,r)}}{\partial r} = 0$, $\frac{\partial q_c^{P(0,r)}}{\partial r} > 0$.

Proposition 6 illustrates that under any conditions, $q_o^{P(0,r)}$ and $w_s^{P(0,r)}$ are not influenced by the substitution rate r. Since OEM's actual demand equals its random demand, and only part of the excess demand from OEM is substituted by CM, OEM and TS do not need to consider this factor in their decision-making. As for CM, according to Proposition 5, when CM's product carbon emissions are sufficiently high ($e_c - \theta_c k_c \geq H_2$), its optimal yields $q_c^{P(0,r)}$ will be less than $q_c^{P(0,0)}$. Therefore, as CM's unit product carbon emissions increase, its optimal yields may decrease. For instance, when $e_c - \theta_c k_c \geq H_2$, CM's optimal yields are less than in the other two stages. However, within each stage, its optimal yields increase with the substitution rate r. This is because CM produces its raw materials and does not need to order from another supplier, and its unit production cost remains constant. Thus, regardless of how much CM's unit product carbon emissions increase, there will always be a certain profit margin per unit product. Additionally, an increase in the substitution rate r can bring more demand to CM. Therefore, CM's optimal yields increase with the increase in the substitution rate r. This phenomenon illustrates that when a manufacturer can independently produce raw materials, the higher the substitution rate of its products with those of competitors, the higher its optimal yields will be.

Proposition 7. *In the purely competitive mode under the (r, 0) scenario,*

(1) Under any conditions, $\frac{\partial q_o^{P(0,r)}}{\partial k_o} > 0$, $\frac{\partial w_s^{P(0,r)}}{\partial k_o} > 0$, $\frac{\partial q_o^{P(0,r)}}{\partial p_e} < 0$, $\frac{\partial w_s^{P(0,r)}}{\partial p_e} < 0$, $\frac{\partial q_c^{P(0,r)}}{\partial k_c} > 0$*; if* $k_o < (\frac{p\delta_o}{2p_e\theta_o})^2$, $\frac{\partial q_c^{P(0,r)}}{\partial k_o} > 0$*;*

(2) When $e_c - \theta_c k_c \leq H_1$*, if* $p_e > \frac{2p(e_c - \theta_c k_c)}{r} - \frac{c}{e_c - \theta_c k_c}$, $\frac{\partial q_c^{P(0,r)}}{\partial p_e} > 0$*;*

(3) When $H_1 < e_c - \theta_c k_c < H_2$*, if* $p_e > \frac{1}{e_o - \theta_o k_o}[\frac{2(e_c - \theta_c k_c)}{r(e_o - \theta_o k_o)} - p(1 + \delta_o\sqrt{k_o}) - c]$, $\frac{\partial q_c^{P(0,r)}}{\partial p_e} > 0$*;*

(4) When $e_c - \theta_c k_c \geq H_2$*, if* $p_e[(e_o - \theta_o k_o) - (e_c - \theta_c k_c)] > \frac{2p(e_c - \theta_c k_c)^2}{r(e_o - \theta_o k_o)^2} - p\delta_o\sqrt{k_o}$, $\frac{\partial q_c^{P(0,r)}}{\partial p_e} > 0$.

Proposition 7 indicates that $q_o^{P(0,r)}$ and $w_o^{P(0,0)}$ are unaffected by CM's green investment. This is because OEM's products cannot substitute for CM's products. However, manufacturer's green investments can lead to an increase in demand, thus causing optimal production quantities for both OEM and CM to increase with their respective green investments. Regarding TS, an increase in OEM's green investment results in greater demand. To enhance revenue, TS raises wholesale prices. As carbon trading prices increase, OEM and CM reduce their production to maximize carbon trading revenue. With a decrease in OEM's production, TS lowers wholesale prices to stimulate increased ordering by OEM and maintain revenue. In addition, for CM, regardless of its carbon emissions stage, the optimal yields of CM increase with an increase in k_o only when OEM's green investment is relatively low. This is because when OEM's green investment is low, it fails to satisfy the excess demand. CM will increase its yields to satisfy a greater substitutable demand. Furthermore, when the carbon emissions per unit product of CM are low ($e_c - \theta_c k_c < H_2$), a low carbon trading price will harm CM's carbon trading revenue. CM will reduce production to obtain more carbon trading revenue. When the carbon emissions per unit product of CM are sufficiently high ($e_c - \theta_c k_c \geq H_2$), the impact of carbon trading price on CM's yields need to be evaluated in conjunction with the carbon emissions per unit product of both CM and OEM.

4.4. Purely Competitive Mode: CM and OEM Symmetrically Substitutable (r, r)

When CM and OEM products can be symmetrically substituted, we have $r_c = r_o = r \neq 0$. In this scenario, the actual demands for OEM and CM are, respectively, $R_o = D_o + r(D_c - q_c)^+$ and $R_c = D_c + r(D_o - q_o)^+$. In the end, the expected sales revenue for OEM and CM are $pE[\min[D_o + r(D_c - q_c)^+, q_o]$ and $pE[\min[D_c + r(D_o - q_o)^+, q_c]$, respectively. In this case, the profit functions for TS, OEM, and CM are as follows:

$$\pi_s^P = (w_s - c)q_o \tag{10}$$

$$\pi_o^P = pE[\min[D_o + r(D_c - q_c)^+, q_o] - w_s q_o + p_e[E_o - (e_o - \theta_o k_o)q_o] - k_o \tag{11}$$

$$\pi_c^P = pE[\min[D_c + r(D_o - q_o)^+, q_c] - cq_c + p_e[E_c - (e_c - \theta_c k_c)q_c] - k_c \tag{12}$$

Due to the complexity of solving this scenario, to simplify the problem, we will consider the special case of $r = 1$, where there is complete product substitution between OEM and CM. In this case, market demand that OEM (or CM) cannot meet will be completely transferred to CM (or OEM). This situation also exists in reality. We provide the conditions for the optimal solutions of TS, OEM, and CM in Lemma 5.

Lemma 5. *In the scenario of a purely competitive mode under (1, 1) conditions,*

(1) When $\frac{w_s + p_e(e_o - \theta_o k_o)}{p} \leq \frac{1}{2}[\frac{c + p_e(e_c - \theta_c k_c)}{p}]^2$, $q_c^{P(1,1)} = 1 + \delta_c\sqrt{k_c} - \frac{c + p_e(e_c - \theta_c k_c)}{p}$*, the optimal wholesale price satisfies* $L(w_s^{P(1,1)}) = 0$*, and optimal yield for OEM satisfies:*

$$q_o^{P(1,1)} = 1 + \delta_o\sqrt{k_o} + \frac{c + p_e(e_c - \theta_c k_c)}{p} - \sqrt{\frac{2[w_s^{P(1,1)} + p_e(e_o - \theta_o k_o)]}{p}}$$

(2) When $\frac{1}{2}[\frac{c + p_e(e_c - \theta_c k_c)}{p}]^2 < \frac{w_s + p_e(e_o - \theta_o k_o)}{p} < \sqrt{\frac{2[c + p_e(e_c - \theta_c k_c)]}{p}}$*, if* $q_c - \delta_c\sqrt{k_c} + \frac{q_o - \delta_o\sqrt{k_o}}{r} > 1$*, the optimal wholesale price satisfies* $L(w_s^{P(1,1)}) = 0$*, and the optimal yields, respectively, satisfy:*

$$8[1 + \delta_c\sqrt{k_c} - q_c^{P(1,1)} - \frac{c + p_e(e_c - \theta_c k_c)}{p}] = -\{\frac{2[w_s^{P(1,1)} + p_e(e_o - \theta_o k_o)]}{p} - [1 - (q_c^{P(1,1)} - \delta_c\sqrt{k_c})]^2\}^2$$

$$8[1+\delta_o\sqrt{k_o}-q_o^{P(1,1)}-\frac{w_s^{P(1,1)}+p_e(e_o-\theta_o k_o)}{p}]=-\{\frac{2[c+p_e(e_c-\theta_c k_c)]}{p}-[1-(q_o^{P(1,1)}-\delta_o\sqrt{k_o})]^2\}^2$$

(3) When $\frac{1}{2}[\frac{c+p_e(e_c-\theta_c k_c)}{p}]^2 < \frac{w_s+p_e(e_o-\theta_o k_o)}{p} < \sqrt{\frac{2[c+p_e(e_c-\theta_c k_c)]}{p}}$, *if* $q_c-\delta_c\sqrt{k_c}+\frac{q_o-\delta_o\sqrt{k_o}}{r}\le 1$, *the optimal wholesale price satisfies* $L(w_s^{P(1,1)})=0$, *and the optimal yields, respectively, satisfy:*

$$\frac{q_c^{P(1,1)}-\delta_c\sqrt{k_c}}{2}+\frac{p-c-p_e(e_c-\theta_c k_c)}{p(q_c^{P(1,1)}-\delta_c\sqrt{k_c})}=\sqrt{(q_c^{P(1,1)}-\delta_c\sqrt{k_c})^2+2[1-\frac{w_s^{P(1,1)}+p_e(e_o-\theta_o k_o)}{p}]}$$

$$\frac{q_o^{P(1,1)}-\delta_o\sqrt{k_o}}{2}+\frac{p-w_s^{P(1,1)}-p_e(e_o-\theta_o k_o)}{p(q_o^{P(1,1)}-\delta_o\sqrt{k_o})}=\sqrt{(q_o^{P(1,1)}-\delta_o\sqrt{k_o})^2+2[1-\frac{c+p_e(e_c-\theta_c k_c)}{p}]}$$

(4) When $\frac{w_s+p_e(e_o-\theta_o k_o)}{p}\ge\sqrt{\frac{2[c+p_e(e_c-\theta_c k_c)]}{p}}$, *if* $\delta_o\sqrt{k_o}+\frac{p_e(e_o-\theta_o k_o)+c}{p}\ge 1$, $q_c^{P(1,1)}=2+\delta_c\sqrt{k_c}-\sqrt{\frac{2[c+p_e(e_c-\theta_c k_c)]}{p}}$, $q_o^{P(1,1)}=\delta_o\sqrt{k_o}$, $w_s^{P(1,1)}=p-p_e(e_o-\theta_o k_o)$;

(5) When $\frac{w_s+p_e(e_o-\theta_o k_o)}{p}\ge\sqrt{\frac{2[c+p_e(e_c-\theta_c k_c)]}{p}}$, *if* $\delta_o\sqrt{k_o}+\frac{p_e(e_o-\theta_o k_o)+c}{p}<1$,

$$q_c^{P(1,1)}=1+\delta_c\sqrt{k_c}-\sqrt{\frac{2[c+p_e(e_c-\theta_c k_c)]}{p}}+\frac{1}{2p}[p(1+\delta_o\sqrt{k_o})+c+p_e(e_o-\theta_o k_o)]$$

$$q_o^{P(1,1)}=\frac{1}{2p}[p(1+\delta_o\sqrt{k_o})-c-p_e(e_o-\theta_o k_o)]$$

$$w_s^{P(1,1)}=\frac{1}{2}[p(1+\delta_o\sqrt{k_o})+c-p_e(e_o-\theta_o k_o)]$$

Through Lemma 5, we find that when $r=1$, the problem of (r, r) in purely competitive mode can be simplified into five cases. However, the optimal solution in the (1, 1) scenario ceases to be an implicit function only when $\frac{w_s+p_e(e_o-\theta_o k_o)}{p}\ge\sqrt{\frac{2[c+p_e(e_c-\theta_c k_c)]}{p}}$. Therefore, building upon Lemma 5, we present in Proposition 8 a comparison of certain (1, 1) scenarios and the (0, 0) optimal solution under the purely competitive mode.

Proposition 8. *Comparing the optimal decisions in the (1, 1) and (0, 0) scenarios under the purely competitive mode, we obtain:*

(1) When $\frac{w_s+p_e(e_o-\theta_o k_o)}{p}\ge\sqrt{\frac{2[c+p_e(e_c-\theta_c k_c)]}{p}}$ *and* $\delta_o\sqrt{k_o}+\frac{p_e(e_o-\theta_o k_o)+c}{p}\ge 1$ *are satisfied,* $q_c^{P(1,1)}>q_c^{P(0,0)}$, $q_o^{P(1,1)}\le q_o^{P(0,0)}$, $w_s^{P(1,1)}\ge w_s^{P(0,0)}$;

(2) When $\frac{w_s+p_e(e_o-\theta_o k_o)}{p}\ge\sqrt{\frac{2[c+p_e(e_c-\theta_c k_c)]}{p}}$ *and* $\delta_o\sqrt{k_o}+\frac{p_e(e_o-\theta_o k_o)+c}{p}<1$ *are satisfied,* $q_o^{P(1,1)}=q_o^{P(0,0)}$, $w_s^{P(1,1)}=w_s^{P(0,0)}$; *if* $e_o-\theta_o k_o>\chi$, $q_c^{P(1,1)}>q_c^{P(0,0)}$.
where $\chi=\frac{1}{p_e}[\sqrt{8p[c+p_e(e_c-\theta_c k_c)]}-2p_e(e_c-\theta_c k_c)-p(1+\delta_o\sqrt{k_o})-3c]$.

When $\frac{w_s+p_e(e_o-\theta_o k_o)}{p}\ge\sqrt{\frac{2[c+p_e(e_c-\theta_c k_c)]}{p}}$ is satisfied, it indicates that the profit per unit of OEM products is less than that of CM products. If $\delta_o\sqrt{k_o}+\frac{p_e(e_o-\theta_o k_o)+c}{p}\ge 1$ is satisfied, then the carbon emissions per unit of OEM products are relatively high, further reducing the profit margin of OEM products. To ensure its own profit, the OEM will produce at the minimum quantity, while TS sets the wholesale price to the maximum. Since the unmet demand by the OEM will be entirely transferred to the CM, the optimal production quantity for the CM increases. If $\delta_o\sqrt{k_o}+\frac{p_e(e_o-\theta_o k_o)+c}{p}<1$ is satisfied, then the carbon emissions per unit of OEM products are relatively low, giving OEM products some profit margin. However, since the profit per unit of OEM products is still less than that of CM products, the OEM will not produce excessively to completely substitute for the OEM's products. To ensure its own profit, the OEM will produce at the (0, 0) level, leading TS to also adopt the wholesale price under the (0, 0) condition. For CM, its production quantity will only increase when the carbon emissions per unit of OEM products are high, as the increased carbon emissions make CM's complete substitution of OEM's demand more likely. The results of Proposition 8 indicate that when the products of two manufacturers are completely substitutable, the optimal decisions of all members are not necessarily worse than the optimal decisions when no substitution exists.

Proposition 9. *In the scenario of a purely competitive mode under (1, 1) conditions,*

(1) When $\frac{w_s+p_e(e_o-\theta_o k_o)}{p} \leq \frac{1}{2}[\frac{c+p_e(e_c-\theta_c k_c)}{p}]^2$, $w_s^{P(1,1)}$ *increases with the increase in both* k_o *and* p_e*, while decreasing with the increase in* k_c*;* $q_o^{P(1,1)}$ *increases with the increase in* k_o *and decreases with the increase in* k_c*; if* $\frac{e_c-\theta_c k_c}{e_o-\theta_o k_o} > \frac{2\sqrt{2p[w_s^{p(1,1)}+p_e(e_o-\theta_o k_o)]}}{w_s^{p(1,1)}+p_e(e_o-\theta_o k_o)+c}$, $q_o^{P(1,1)}$ *increases with the increase in* p_e*;* $q_c^{P(1,1)}$ *increases with the increase in* k_c *and decreases with the increase in* p_e*, remaining unchanged with the variation in* k_o*;*

(2) When $\frac{w_s+p_e(e_o-\theta_o k_o)}{p} \geq \sqrt{\frac{2[c+p_e(e_c-\theta_c k_c)]}{p}}$*, if* $\delta_o\sqrt{k_o} + \frac{p_e(e_o-\theta_o k_o)+c}{p} < 1$, $q_o^{P(1,1)}$ *and* $w_s^{P(1,1)}$ *increases with the increase in both* k_o *and decreases with the increase in* p_e*, remaining unchanged with the variation in* k_c*;* $q_c^{P(1,1)}$ *increases with the increase in* k_c*; if* $k_o < (\frac{p\delta_o}{2p_e\theta_o})^2$, $q_c^{P(1,1)}$ *increases with the increase in* k_o*; if* $p_e > \frac{p}{2(e_o-\theta_o k_o)^2} - \frac{c}{e_c-\theta_c k_c}$, $q_c^{P(1,1)}$ *increases with the increase in* p_e*;*

(3) When $\frac{w_s+p_e(e_o-\theta_o k_o)}{p} \geq \sqrt{\frac{2[c+p_e(e_c-\theta_c k_c)]}{p}}$*, if* $\delta_o\sqrt{k_o} + \frac{p_e(e_o-\theta_o k_o)+c}{p} \geq 1$, $w_s^{P(1,1)}$ *increases with the increase in both* k_o *and decreases with the increase in* p_e*, remaining unchanged with the variation in* k_c*;* $q_o^{P(1,1)}$ *increases with the increase in both* k_o*, remaining unchanged with the variation in* k_c *and* p_e*;* $q_c^{P(1,1)}$ *increases with the increase in* k_c *and decreases with the increase in* p_e*, remaining unchanged with the variation in* k_o*.*

From Proposition 9, we observe that an increase in manufacturer's green investments leads to an increase in their yields. This is because manufacturer's green investments can bring about an increase in demand. Additionally, the increase in OEM green investments promotes TS to raise wholesale prices. This is because the increase in OEM green investments brings more demand to TS. To enhance their profits, TS raises wholesale prices. However, the impact of manufacturers' green investments on competitors varies with the situation. When $\frac{w_s+p_e(e_o-\theta_o k_o)}{p} \leq \frac{1}{2}[\frac{c+p_e(e_c-\theta_c k_c)}{p}]^2$, an increase in CM's green investments increases its yields, satisfying more demand. Conversely, OEM's demand decreases, resulting in a reduction in its production. To ensure its profit, TS lowers wholesale prices to encourage OEMs to increase production. Furthermore, since OEM's yields are large enough, there is no excess demand. Therefore, CM's production remains unchanged and does not vary with k_o. When $\frac{w_s+p_e(e_o-\theta_o k_o)}{p} \geq \sqrt{\frac{2[c+p_e(e_c-\theta_c k_c)]}{p}}$ and $\delta_o\sqrt{k_o} + \frac{p_e(e_o-\theta_o k_o)+c}{p} < 1$, CM's yields are large enough, and there is no excess demand. Therefore, the optimal decisions of OEM and TS do not vary with k_c. However, only when OEM's green investment is low do CM's optimal yields increase with the increase of k_o. This is because when OEM's green investment is low, there is more excess demand. Thus, CM increases yields to satisfy larger substitution demand. When $\delta_o\sqrt{k_o} + \frac{p_e(e_o-\theta_o k_o)+c}{p} \geq 1$, OEM's unit production cost is high, so it will produce at the minimum level. At this time, CM increases yields to fully satisfy OEM's uncertain demand. Therefore, the yields of CM (or OEM) only increase with the increase in CM (or OEM) green investments.

Furthermore, we find that an increase in carbon trading prices always leads to a decrease in wholesale prices. This is because the increase in carbon trading prices increases the unit production cost of OEM products. To prevent significant yield cuts by the OEM, TS lowers wholesale prices. Additionally, when the manufacturer's yields are sufficiently large, the increase in carbon trading prices significantly raises the cost of products, leading to a reduction in their yields. However, if yields are not sufficient, an increase in carbon trading prices can promote an increase in yields under specific circumstances. This phenomenon also illustrates that an increase in carbon trading prices does not necessarily suppress the yields of manufacturing enterprises.

5. Model Analysis of Co-Opetitive Supply Chain

In this section, we consider the scenario where OEM and CM are in a co-opetitive relationship. In this scenario, CM will become a supplier to OEM and compete with OEM in the final market. The supply chain structure under co-opetitive mode is illustrated in Figure 3.

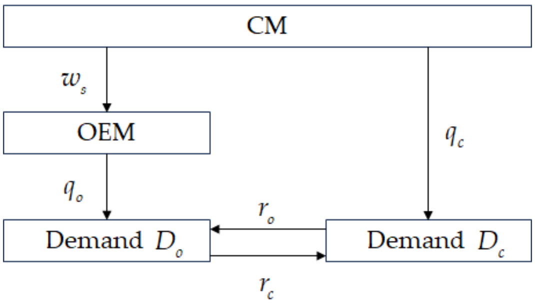


Figure 3. Supply chain structure under co-opetitive mode.

At the beginning of the period, CM announces the wholesale price of raw materials, denoted as w_c. Then, OEM and CM, respectively, conduct their green investments, k_o and k_c, and determine their yields q_o and q_c. Meanwhile, OEM orders raw materials from CM and pays $w_c q_o$ for CM. After the green investments, the unit carbon emissions of the final products of OEM and CM decrease to $(e_o - \theta_o k_0)$ and $(e_c - \theta_c k_c)$, respectively. At the end of the period, the final products of OEM and CM are sold at price p, and they, respectively, obtain carbon trading profits of $p_e[E_o - (e_o - \theta_o k_o)q_o]$ and $p_e[E_c - (e_c - \theta_c k_c)q_c]$ through carbon cap-and-trade system. The sequence of events is illustrated in Figure 4.

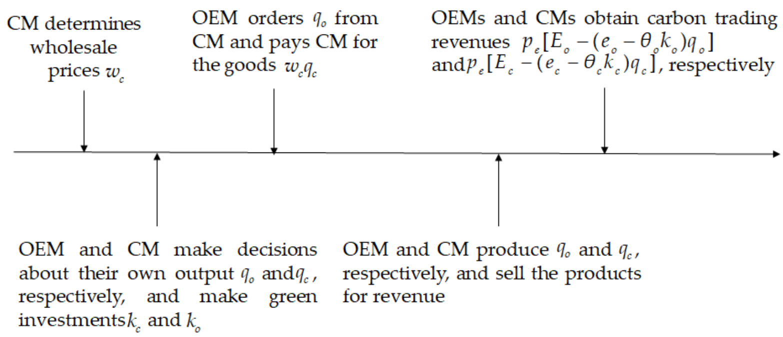


Figure 4. Sequence of events in the co-opetitive mode.

On this basis, we will analyze the co-opetitive mode under different product substitution scenarios.

5.1. Co-Opetitive Mode: Non-substitutable (0, 0)

First, we consider the scenario where there is no substitutability between the final products of OEM and CM. In this case, the profit functions of OEM and CM are, respectively, given by:

$$\pi_o^C = pE[\min[D_o, q_o]] - w_c q_o + p_e[E_o - (e_o - \theta_o k_o)q_o] - k_o \tag{13}$$

$$\pi_c^C = pE[\min[D_c, q_c]] - cq_c + p_e[E_c - (e_c - \theta_c k_c)q_c] - k_c + (w_c - c)q_o \tag{14}$$

We solve this problem using backward induction and in Proposition 9, comparing it with the optimal solution of the (0, 0) scenario under a purely competitive mode.

Proposition 10. *In the (0, 0) scenario,* $w_c^{C(0,0)} = w_s^{P(0,0)}$, $q_c^{C(0,0)} = q_c^{P(0,0)}$, $q_o^{C(0,0)} = q_o^{P(0,0)}$, $\pi_o^{C(0,0)} = \pi_o^{P(0,0)}$, $\pi_c^{C(0,0)} = \pi_s^{P(0,0)} + \pi_c^{P(0,0)}$.

Proposition 10 reveals that in the (0, 0) scenario, whether in a purely competitive or co-opetitive mode, the optimal solutions and OEM's optimal profit remain unchanged. Despite CM assuming the role of supplier, OEM and CM operate independently of each other, so decisions made by one manufacture do not influence the other. Therefore, the optimal solutions remain constant in both scenarios. Since the optimal yields for OEM remain unchanged, OEM's profit remains unaffected. In the co-opetitive mode, where CM acts as the supplier to OEM, CM's optimal profit equals the sum of CM's profit with TS in the purely competitive mode.

5.2. Co-Opetitive Mode: OEM Substitutable for CM (r, 0)

When OEM products partially substitute for CM products, the profit functions for OEM and CM, respectively, are:

$$\pi_o^C = pE[\min[D_o + r(D_c - q_c)^+, q_o] - w_c q_o + p_e[E_o - (e_o - \theta_o k_o)q_o] - k_o \tag{15}$$

$$\pi_c^C = pE[\min[D_c, q_c]] - cq_c + p_e[E_c - (e_c - \theta_c k_c)q_c] - k_c + (w_c - c)q_o \tag{16}$$

In Proposition 11, we demonstrate a comparison between this optimal solution and the optimal solution for the (r, 0) scenario under a purely competitive mode.

Proposition 11. *In the (r, 0) scenario,* $w_c^{C(r,0)} = w_s^{P(r,0)}$, $q_c^{C(r,0)} = q_c^{P(r,0)}$, $q_o^{C(r,0)} = q_o^{P(r,0)}$, $\pi_o^{C(r,0)} = \pi_o^{P(r,0)}$, $\pi_c^{C(r,0)} = \pi_s^{P(r,0)} + \pi_c^{P(r,0)}$.

Similar to Proposition 10, Proposition 11 also indicates that in the (r, 0) scenario, both under purely competitive and co-opetitive modes, the optimal solution and the optimal profit for OEM remain unchanged. This is because in the co-opetitive mode, CM's products cannot substitute for OEM's products, so yields and wholesale price decisions remain independent of each other. Additionally, in the co-opetitive mode, when OEM makes optimal ordering quantity decisions, it only needs to consider the overflow of CM demand. Therefore, in the (r, 0) scenario, both modes exhibit the same optimal decisions. Since OEM's optimal production quantity remains unchanged, its profit remains unaffected. Furthermore, in the co-opetitive mode, where CM acts as a supplier to OEM, CM's optimal profit will be equal to the sum of CM's profit under purely competitive modes and TS's profit.

Corollary 1. *When CM cannot substitute for OEM, the total profit and carbon emissions in the supply chain are the same under both co-opetitive and purely competitive modes.*

Corollary 1 indicates that when CM's products cannot substitute for OEM's products, collaborating with CM or not will not affect the overall performance of the supply chain. Moreover, in this scenario, both modes have an equal impact on carbon emission reduction.

5.3. Co-Opetitive Mode: CM Substitutable for OEM (0, r)

When CM's products partially substitute for OEM's products, the profit functions for OEM and CM, respectively, are:

$$\pi_o^C = pE[\min[D_o, q_o]] - w_s q_o + p_e[E_o - (e_o - \theta_o k_o)q_o] - k_o \tag{17}$$

$$\pi_c^C = pE[\min[D_c + r(D_o - q_o)^+, q_c] - cq_c + p_e[E_c - (e_c - \theta_c k_c)q_c] - k_c + (w_c - c)q_o \quad (18)$$

We define $G = \frac{\partial \pi_c}{\partial w_c}$, and provide the conditions for the optimal solutions of OEM and CM in Lemma 6.

Lemma 6. *In the case of (0, r) in the co-opetitive mode,*

(1) When $e_c - \theta_c k_c + \frac{p(2-r)}{rp_e}\sqrt{\frac{2[c+p_e(e_c-\theta_c k_c)]}{rp}} \le U$, *if* $e_c - \theta_c k_c > V$,

$$q_o^{C(0,r)} = \frac{(1-r)[p(1+\delta_o\sqrt{k_o}) - c] + rp_e(e_c - \theta_c k_c) - p_e(e_o - \theta_o k_o)}{p(2-r)}$$

$$q_c^{C(0,r)} = 1 + \delta_c\sqrt{k_c} + \frac{r[p(1+\delta_o\sqrt{k_o}) + (1-r)c - rp_e(e_c - \theta_c k_c) + p_e(e_o - \theta_o k_o)]}{p(2-r)} - \sqrt{\frac{2r[c + p_e(e_c - \theta_c k_c)]}{p}}$$

$$w_c^{C(0,r)} = \frac{p(1+\delta_o\sqrt{k_o}) + (1-r)c - (1-r)p_e(e_o - \theta_o k_o) - rp_e(e_c - \theta_c k_c)}{2-r}$$

(2) When $e_c - \theta_c k_c + \frac{p(2-r)}{rp_e}\sqrt{\frac{2[c+p_e(e_c-\theta_c k_c)]}{rp}} \le U$, *if* $e_c - \theta_c k_c \le V$, $q_o^{C(0,r)} = \delta_o\sqrt{k_o}$, $q_c^{C(0,r)} = 1 + \delta_c\sqrt{k_c} + r - \sqrt{\frac{2r[c+p_e(e_c-\theta_c k_c)]}{p}}$, $w_c^{C(0,r)} = p - p_e(e_o - \theta_o k_o)$;

(3) When $e_c - \theta_c k_c + \frac{p(2-r)}{rp_e}\sqrt{\frac{2[c+p_e(e_c-\theta_c k_c)]}{rp}} > U$, *if* $q_o^{C(0,r)} - \delta_o\sqrt{k_o} + \frac{q_c^{C(0,r)} - \delta_c\sqrt{k_o}}{r} > 1$, *the optimal wholesale price satisfies* $G(w_c^{C(0,r)}) = 0$, *and the optimal yields, respectively, satisfy:*

$$q_o^{C(0,r)} = 1 + \delta_o\sqrt{k_o} - \frac{w_c^{C(0,r)} + p_e(e_o - \theta_o k_o)}{p}$$

$$q_c^{C(0,r)} = 1 + \delta_c\sqrt{k_c} - \frac{c + p_e(e_c - \theta_c k_c)}{p} + \frac{r}{2}\left[\frac{w_c^{C(0,r)} + p_e(e_o - \theta_o k_o)}{p}\right]^2$$

(4) When $e_c - \theta_c k_c + \frac{p(2-r)}{rp_e}\sqrt{\frac{2[c+p_e(e_c-\theta_c k_c)]}{rp}} \le U$, *if* $q_o^{C(0,r)} - \delta_o\sqrt{k_o} + \frac{q_c^{C(0,r)} - \delta_c\sqrt{k_o}}{r} \le 1$, *the optimal wholesale price satisfies* $G(w_c^{C(0,r)}) = 0$, *and the optimal yields, respectively, satisfy:*

$$q_o^{C(0,r)} = 1 + \delta_o\sqrt{k_o} - \frac{w_c^{C(0,r)} + p_e(e_o - \theta_o k_o)}{p}$$

$$q_c^{C(0,r)} = \sqrt{r^2\left[1 - \frac{w_c^{C(0,r)} + p_e(e_o - \theta_o k_o)}{p}\right]^2 + 2r\left[1 - \frac{c + p_e(e_c - \theta_c k_c)}{p}\right]} - r\left[1 - \frac{w_c^{C(0,r)} + p_e(e_o - \theta_o k_o)}{p}\right] + \delta_c\sqrt{k_c}$$

where $U = \frac{1}{rp_e p}[p(1+\delta_o\sqrt{k_o}) + (1-r)c + p_e(e_o - \theta_o k_o)]$, $V = \frac{1}{rp_e}[p\delta_o\sqrt{k_o} - (1-r)(p - c) + p_e(e_o - \theta_o k_o)]$.

Proposition 12. *In the (0, r) scenario, when* $e_c - \theta_c k_c \le \min[H_1, U - \frac{p(2-r)}{rp_e}\sqrt{\frac{2[c+p_e(e_c-\theta_c k_c)]}{rp}}]$,

(1) When $e_c - \theta_c k_c > V$, *if* $e_c - \theta_c k_c < \alpha$, $w_c^{C(0,r)} > w_s^{P(0,r)}$; *if* $e_c - \theta_c k_c > \beta$, $q_o^{C(0,r)} > q_o^{P(0,r)}$; *if* $e_c - \theta_c k_c < \gamma$, $q_c^{C(0,r)} > q_c^{P(0,r)}$;

(2) When $e_c - \theta_c k_c \le V$, $w_c^{C(0,r)} > w_s^{P(0,r)}$, $q_o^{C(0,r)} < q_o^{P(0,r)}$; *if* $e_o - \theta_o k_o > \frac{p(1-\delta_o\sqrt{k_o}) - c}{p_e}$, $q_c^{C(0,r)} > q_c^{P(0,r)}$.

where $\alpha = \frac{1}{2p_e}[p(1+\delta_o\sqrt{k_o}) + c - p_e(e_o - \theta_o k_o)]$, $\beta = \frac{1}{2p_e}[p(1+\delta_o\sqrt{k_o}) - c + p_e(e_o - \theta_o k_o)]$, $\gamma = \frac{1}{2p_e}[p(1+\delta_o\sqrt{k_o}) + c + p_e(e_o - \theta_o k_o)]$.

Since the optimal solutions for the three scenarios in the co-opetitive mode are implicit functions, Proposition 12 only compares the optimal solutions when the carbon emissions of CM products are lower. From Proposition 12, we observe that even with a reduction in CM carbon emissions, the comparison between the optimal decisions in the co-opetitive mode and the purely competitive mode still varies with the carbon emissions of CM and OEM unit products.

Proposition 13. *Comparing the (0, r) and (0, 0) scenarios in the co-opetitive mode, we obtain:*

(1) When $e_c - \theta_c k_c + \frac{p(2-r)}{rp_e}\sqrt{\frac{2[c+p_e(e_c-\theta_c k_c)]}{rp}} \le U$ *and* $e_c - \theta_c k_c > V$ *are satisfied, if* $e_o - \theta_o k_o > \varphi$, $q_c^{C(0,r)} > q_c^{C(0,0)}$; *if* $e_c - \theta_c k_c > \kappa$, $q_o^{C(0,r)} > q_o^{C(0,0)}$; *if* $e_o - \theta_o k_o < \varsigma$, $w_c^{C(0,r)} > w_c^{C(0,0)}$;

(2) When $e_c - \theta_c k_c + \frac{p(2-r)}{rp_e}\sqrt{\frac{2[c+p_e(e_c-\theta_c k_c)]}{rp}} \le U$ *and* $e_c - \theta_c k_c \le V$ *are satisfied,* $q_c^{C(0,r)} > q_c^{C(0,0)}$, $q_o^{C(0,r)} \le q_o^{C(0,0)}$, $w_c^{C(0,r)} \ge w_c^{C(0,0)}$.

where $\varphi = \frac{1}{p_e}[(2-r)[\sqrt{\frac{2p[c+p_e(e_c-\theta_c k_c)]}{r}} - \frac{c+p_e(e_c-\theta_c k_c)}{r}] - p(1+\delta_o\sqrt{k_o}) - (1-r)c + rp_e(e_c - \theta_c k_c)]$, $\kappa = \frac{1}{2p_e}[p(1+\delta_o\sqrt{k_o}) - c + p_e(e_o - \theta_o k_o)]$, $\varsigma = \frac{1}{p_e}[p(1+\delta_o\sqrt{k_o}) - c - 2p_e(e_c - \theta_c k_c)]$.

From the results of Proposition 13, we observe that when the carbon emissions per unit of CM products are relatively high ($e_c - \theta_c k_c > V$), the optimal decisions in the (0, r) scenario are favorable under specific conditions compared to the optimal decisions in the (0, 0) scenario. This indicates that increasing production quantity is advantageous when the competitor's carbon emissions per unit are high. However, when the carbon emissions per unit of CM products are relatively low ($e_c - \theta_c k_c \le V$), the optimal decision for CM in the (0, r) scenario is better, but the optimal decision for OEM is better in the (0, 0) scenario. This suggests that the carbon emissions per unit of both products have a significant impact on optimal decisions. Moreover, when the CM can substitute for excess demand from the OEM, it does not necessarily harm the optimal decisions.

Proposition 14. *In the co-opetitive mode with the (0, r) scenario,*

(1) When $e_c - \theta_c k_c + \frac{p(2-r)}{rp_e}\sqrt{\frac{2[c+p_e(e_c-\theta_c k_c)]}{rp}} \le U$ *and* $e_c - \theta_c k_c > V$, *if* $e_c - \theta_c k_c > \beta$, $w_c^{C(0,r)}$ *decreases with the increase in r,* $q_o^{C(0,r)}$ *increases with the increase in r; if* $\frac{2}{p(2-r)^2}[p(1+\delta_o\sqrt{k_o}) + p_e(e_o - \theta_o k_o)] + \frac{c}{p} > \frac{r(4-r)}{p(2-r)^2}p_e(e_c - \theta_c k_c) + \sqrt{\frac{c+p_e(e_c-\theta_c k_c)}{rp}}$, $q_c^{C(0,r)}$ *increases with the increase in r;*

(2) When $e_c - \theta_c k_c + \frac{p(2-r)}{rp_e}\sqrt{\frac{2[c+p_e(e_c-\theta_c k_c)]}{rp}} \le U$ *and* $e_c - \theta_c k_c \le V$, $w_c^{C(0,r)}$ *and* $q_o^{C(0,r)}$ *do not change with the variation in r; if* $r > \frac{c+p_e(e_c-\theta_c k_c)}{2p}$, $q_c^{C(0,r)}$ *increases with the increase in r;*

(3) When $e_c - \theta_c k_c + \frac{p(2-r)}{rp_e}\sqrt{\frac{2[c+p_e(e_c-\theta_c k_c)]}{rp}} > U$ *and* $q_o^{C(0,r)} - \delta_o\sqrt{k_o} + \frac{q_c^{C(0,r)} - \delta_c\sqrt{k_o}}{r} > 1$, $w_c^{C(0,r)}$ *and* $q_o^{C(0,r)}$ *increases with the increase in r, while* $q_c^{C(0,r)}$ *decreases with the increase in r.*

Proposition 14 demonstrates that the impact of the substitution rate r on the optimal decisions varies under different conditions. This differs from the results presented in Proposition 6. This is because CM acts as the supplier to OEM. When CM can substitute for the excess demand of OEM, CM can influence OEM's decisions by controlling the wholesale price. Therefore, in the co-opetitive mode, the effect of the substitution rate r on the optimal decisions differs from that in the purely competitive mode.

Proposition 15. *In the co-opetitive mode with the (0, r) scenario,*

(1) When $e_c - \theta_c k_c + \frac{p(2-r)}{rp_e}\sqrt{\frac{2[c+p_e(e_c-\theta_c k_c)]}{rp}} \le U$ *and* $e_c - \theta_c k_c > V$, $w_c^{C(0,r)}$ *increases with the increase in* k_c *and* k_o, *and decreases with the increase in* p_e; $q_o^{C(0,r)}$ *increases with the increase in* k_o, *and decreases with the increase in* k_c; *if* $r > \frac{e_o-\theta_o k_o}{e_c-\theta_c k_c}$, $q_o^{C(0,r)}$ *increases with the*

increase in p_e; $q_c^{C(0,r)}$ increases with the increase in k_c; if $k_o < (\frac{p\delta_o}{2p_e\theta_o})^2$, $q_c^{C(0,r)}$ increases with the increase in k_o; if $p_e > \frac{p(2-r)^2(e_c-\theta_c k_c)}{2[(e_o-\theta_o k_o)-r(e_c-\theta_c k_c)]^2} - \frac{c}{e_c-\theta_c k_c}$, $q_c^{C(0,r)}$ increases with the increase in p_e;

(2) When $e_c - \theta_c k_c + \frac{p(2-r)}{rp_e}\sqrt{\frac{2[c+p_e(e_c-\theta_c k_c)]}{rp}} \le U$ and $e_c - \theta_c k_c \le V$, $w_c^{C(0,r)}$ increases with the increase in k_o and decreases with the increase in p_e, but remains unchanged with the increase in k_c; $q_o^{C(0,r)}$ increases with the increase in k_o, but remains unchanged with the increase in k_c and p_e; $q_c^{C(0,r)}$ increases with the increase in k_c and decreases with the increase in p_e, but remains unchanged with the increase in k_o;

(3) When $e_c - \theta_c k_c + \frac{p(2-r)}{rp_e}\sqrt{\frac{2[c+p_e(e_c-\theta_c k_c)]}{rp}} > U$ and $q_o^{C(0,r)} - \delta_o\sqrt{k_o} + \frac{q_c^{C(0,r)}-\delta_c\sqrt{k_o}}{r} > 1$, $w_c^{C(0,r)}$ decreases with the increase in p_e and remains unchanged with the increase in k_c; if $\frac{\delta_o}{2\sqrt{k_o}p_e\theta_o} < \vartheta$, $w_c^{C(0,r)}$ increases with the increase in k_o; $q_o^{C(0,r)}$ decreases with the increase in p_e and remains unchanged with the increase in k_c; if $2p_e\theta_o\sqrt{k_o} - \delta_o p^2 < \frac{\delta_o p^2}{\vartheta}$, $q_o^{C(0,r)}$ increases with the increase in k_o; $q_c^{C(0,r)}$ increases with the increase in k_c; if $\frac{\delta_o p}{2p_e\theta_o\sqrt{k_o}} < \vartheta$, $q_c^{C(0,r)}$ increases with the increase in k_o; if $[w_c^{C(0,r)} + p_e(e_o-\theta_o k_o)](e_o-\theta_o k_o) > (e_c-\theta_c k_c)\vartheta$, $q_c^{C(0,r)}$ increases with the increase in p_e.
where $\vartheta = \frac{3r^2}{2p^3}[w_c^{C(0,r)} + p_e(e_o-\theta_o k_o)]^2 + \frac{2}{p} - \frac{r^2}{p}[w_c^{C(0,r)} + p_e(e_o-\theta_o k_o)]$.

From Proposition 15, we have identified a new phenomenon. In the case of $e_c - \theta_c k_c + \frac{p(2-r)}{rp_e}\sqrt{\frac{2[c+p_e(e_c-\theta_c k_c)]}{rp}} > U$ and $q_o^{C(0,r)} - \delta_o\sqrt{k_o} + \frac{q_c^{C(0,r)}-\delta_c\sqrt{k_o}}{r} > 1$, the increase in OEM's green investment does not necessarily lead to an increase in its production. This is because CM acts as OEM's supplier and can also substitute for OEM's excess demand. Therefore, under specific conditions, CM can make the OEM's green investment unable to improve the OEM's production by adjusting the wholesale price. Additionally, Proposition 13 also indicates that the increase in carbon trading prices does not necessarily suppress manufacturers' production.

5.4. Co-Opetitive Mode: CM and OEM Symmetrically Substitutable (r, r)

When CM and OEM products are symmetrically substitutable, the profit functions for OEM and CM are as follows:

$$\pi_o^C = pE[\min[D_o + r(D_c - q_c)^+, q_o] - w_s q_o + p_e[E_o - (e_o - \theta_o k_o)q_o] - k_o \tag{19}$$

$$\pi_c^C = pE[\min[D_c + r(D_o - q_o)^+, q_c] - cq_c + p_e[E_c - (e_c - \theta_c k_c)q_c] - k_c + (w_c - c)q_o \tag{20}$$

Similar to Section 4.4, the complexity of this problem is exceptionally high. Therefore, to simplify the problem, we consider the special case of $r = 1$, where there is complete product substitution between OEM and CM. In this case, market demand that OEM (or CM) cannot satisfy will be entirely transferred to CM (or OEM). This situation also exists in reality. We provide the conditions for the optimal solutions of OEM and CM in Lemma 7.

Lemma 7. *In the (1, 1) scenario under the co-opetitive mode:*

(1) When $\frac{w_c+p_e(e_o-\theta_o k_o)}{p} \le \frac{1}{2}[\frac{c+p_e(e_c-\theta_c k_c)}{p}]^2$, $q_c^{C(1,1)} = 1 + \delta_c\sqrt{k_c} - \frac{c+p_e(e_c-\theta_c k_c)}{p}$, the optimal wholesale price satisfies $G(w_c^{C(0,r)}) = 0$, and the optimal yields for OEM satisfy:

$$q_o^{C(1,1)} = 1 + \delta_o\sqrt{k_o} + \frac{c + p_e(e_c - \theta_c k_c)}{p} - \sqrt{\frac{2[w_c^{C(1,1)} + p_e(e_o - \theta_o k_o)]}{p}}$$

(2) When $\frac{1}{2}[\frac{c+p_e(e_c-\theta_c k_c)}{p}]^2 < \frac{w_c+p_e(e_o-\theta_o k_o)}{p} < \sqrt{\frac{2[c+p_e(e_c-\theta_c k_c)]}{p}}$, if $q_c-\delta_c\sqrt{k_c}+q_o-\delta_o\sqrt{k_o}>1$, the optimal wholesale price satisfies $G(w_c^{C(0,r)})=0$, and the optimal yields, respectively, satisfy:

$$8[1+\delta_c\sqrt{k_c}-q_c^{C(1,1)}-\frac{c+p_e(e_c-\theta_c k_c)}{p}]=-\{\frac{2[w_s^{C(1,1)}+p_e(e_o-\theta_o k_o)]}{p}-[1-(q_c^{C(1,1)}-\delta_c\sqrt{k_c})]^2\}^2$$

$$8[1+\delta_o\sqrt{k_o}-q_o^{C(1,1)}-\frac{w_s^{C(1,1)}+p_e(e_o-\theta_o k_o)}{p}]=-\{\frac{2[c+p_e(e_c-\theta_c k_c)]}{p}-[1-(q_o^{C(1,1)}-\delta_o\sqrt{k_o})]^2\}^2$$

(3) When $\frac{1}{2}[\frac{c+p_e(e_c-\theta_c k_c)}{p}]^2 < \frac{w_c+p_e(e_o-\theta_o k_o)}{p} < \sqrt{\frac{2[c+p_e(e_c-\theta_c k_c)]}{p}}$, if $q_c-\delta_c\sqrt{k_c}+q_o-\delta_o\sqrt{k_o}\leq 1$, the optimal wholesale price satisfies $G(w_c^{C(0,r)})=0$, and the optimal yields, respectively, satisfy:

$$\frac{q_c^{C(1,1)}-\delta_c\sqrt{k_c}}{2}+\frac{p-c-p_e(e_c-\theta_c k_c)}{p(q_c^{C(1,1)}-\delta_c\sqrt{k_c})}=\sqrt{(q_c^{C(1,1)}-\delta_c\sqrt{k_c})^2+2[1-\frac{w_c^{C(1,1)}+p_e(e_o-\theta_o k_o)}{p}]}$$

$$\frac{q_o^{C(1,1)}-\delta_o\sqrt{k_o}}{2}+\frac{p-w_s^{C(1,1)}-p_e(e_o-\theta_o k_o)}{p(q_o^{C(1,1)}-\delta_o\sqrt{k_o})}=\sqrt{(q_o^{C(1,1)}-\delta_o\sqrt{k_o})^2+2[1-\frac{c+p_e(e_c-\theta_c k_c)}{p}]}$$

(4) When $\frac{w_c+p_e(e_o-\theta_o k_o)}{p}\geq\sqrt{\frac{2[c+p_e(e_c-\theta_c k_c)]}{p}}$, if $\delta_o\sqrt{k_o}\geq\frac{p_e(e_c-\theta_c k_c)}{p}$, $q_c^{C(1,1)}=2+\delta_c\sqrt{k_c}-\sqrt{\frac{2[c+p_e(e_c-\theta_c k_c)]}{p}}$, $q_o^{C(1,1)}=\delta_o\sqrt{k_o}$, $w_c^{C(1,1)}=p-p_e(e_o-\theta_o k_o)$;

(5) When $\frac{w_c+p_e(e_o-\theta_o k_o)}{p}\geq\sqrt{\frac{2[c+p_e(e_c-\theta_c k_c)]}{p}}$, if $\delta_o\sqrt{k_o}<\frac{p_e(e_c-\theta_c k_c)}{p}$,

$$q_c^{C(1,1)}=1+\delta_c\sqrt{k_c}-\sqrt{\frac{2[c+p_e(e_c-\theta_c k_c)]}{p}+\frac{p(1+\delta_o\sqrt{k_o})-p_e[(e_c-\theta_c k_c)-(e_o-\theta_o k_o)}{p}}$$

$$q_o^{C(1,1)}=\frac{p_e[(e_c-\theta_c k_c)-(e_o-\theta_o k_o)]}{p}$$

$$w_c^{C(1,1)}=p(1+\delta_o\sqrt{k_o})-p_e(e_c-\theta_c k_c)$$

Proposition 16. *In the (1, 1) scenario under the co-opetitive mode:*

(1) When both modes satisfy $\frac{w+p_e(e_o-\theta_o k_o)}{p}\leq\frac{1}{2}[\frac{c+p_e(e_c-\theta_c k_c)}{p}]^2$, $w_c^{C(1,1)}=w_s^{P(1,1)}$, $q_c^{C(1,1)}=q_c^{P(1,1)}$, $q_o^{C(1,1)}=q_o^{P(1,1)}$, $\pi_o^{C(1,1)}=\pi_o^{P(1,1)}$, $\pi_c^{C(1,1)}=\pi_c^{P(1,1)}+\pi_s^{P(1,1)}$;

(2) When both modes satisfy $\frac{w+p_e(e_o-\theta_o k_o)}{p}\geq\sqrt{\frac{2[c+p_e(e_c-\theta_c k_c)]}{p}}$ and $\delta_o\sqrt{k_o}\geq\max[\frac{p_e(e_c-\theta_c k_c)}{p}, 1-\frac{p_e(e_o-\theta_o k_o)+c}{p}]$, $w_c^{C(1,1)}=w_s^{P(1,1)}$, $q_c^{C(1,1)}=q_c^{P(1,1)}$, $q_o^{C(1,1)}=q_o^{P(1,1)}$, $\pi_o^{C(1,1)}=\pi_o^{P(1,1)}$, $\pi_c^{C(1,1)}=\pi_c^{P(1,1)}+\pi_s^{P(1,1)}$;

(3) When both modes satisfy $\frac{w+p_e(e_o-\theta_o k_o)}{p}\geq\sqrt{\frac{2[c+p_e(e_c-\theta_c k_c)]}{p}}$ and $\delta_o\sqrt{k_o}<\min[\frac{p_e(e_c-\theta_c k_c)}{p}, 1-\frac{p_e(e_o-\theta_o k_o)+c}{p}]$, $q_c^{C(1,1)}+q_o^{C(1,1)}=q_c^{P(1,1)}+q_o^{P(1,1)}$; if $e_c-\theta_c k_c<\frac{1}{2p_e}[p(1+\delta_o\sqrt{k_o})-c+p_e(e_o-\theta_o k_o)]$, $w_c^{C(1,1)}>w_s^{P(1,1)}$, $q_c^{C(1,1)}>q_c^{P(1,1)}$, $q_o^{C(1,1)}<q_o^{P(1,1)}$.

Due to the complexity of the (1, 1) scenario, Proposition 16 only allows for partial comparisons between the two modes. From Proposition 16, it is evident that when both modes satisfy $\frac{w+p_e(e_o-\theta_o k_o)}{p}\leq\frac{1}{2}[\frac{c+p_e(e_c-\theta_c k_c)}{p}]^2$, there is no difference in the optimal solutions between the two modes. This is because the OEM's production is sufficiently large, eliminating any excess demand. Even with higher wholesale prices in the co-opetitive mode, CM cannot attract additional demand. Consequently, the profits of the OEM remain the same under both modes, and in the co-opetitive mode, CM sets its optimal decision to be the same as in the purely competitive mode. Additionally, as CM serves as the OEM's supplier in the co-opetitive mode, CM's profits in this mode are the sum of those in the purely competitive mode and the profits of the TS. Furthermore, since the optimal decisions are the same under both modes, the carbon emissions are also identical at this point. When both modes satisfy $\frac{w+p_e(e_o-\theta_o k_o)}{p} \geq \sqrt{\frac{2[c+p_e(e_c-\theta_c k_c)]}{p}}$, if $\delta_o\sqrt{k_o} < \min[\frac{p_e(e_c-\theta_c k_c)}{p}, 1-\frac{p_e(e_o-\theta_o k_o)+c}{p}]$, the OEM will produce at the minimum capacity. In this scenario, all uncertain demands of the OEM cannot be satisfied. Consequently, CM will increase its yields to fulfill all uncertain demands of the OEM. Additionally, as the supplier to the OEM, CM sets the wholesale price to maximize its revenue. Therefore, the optimal decisions are the same under both modes, and in the co-opetitive mode, CM's profit is the sum of the profit in the purely competitive mode and that of the TS. However, if $\delta_o\sqrt{k_o} < \min[\frac{p_e(e_c-\theta_c k_c)}{p}, 1-\frac{p_e(e_o-\theta_o k_o)+c}{p}]$, the relationship between the optimal decisions under both modes is influenced by the CM's unit product carbon emissions. This is because, in the co-opetitive mode, CM can adjust its demand by controlling the wholesale price, effectively mitigating the risk of greater losses from excessive production when the CM's unit product carbon emissions are high. Furthermore, we observe that the total yields of both manufacturers remain unchanged under both modes. This phenomenon indicates that the market satisfaction rate remains constant regardless of the mode chosen.

Proposition 17. *Comparing the (1, 1) and (0, 0) scenarios in the co-opetitive mode, we obtain:*

(1) When $\frac{w_c+p_e(e_o-\theta_o k_o)}{p} \geq \sqrt{\frac{2[c+p_e(e_c-\theta_c k_c)]}{p}}$ *and* $\delta_o\sqrt{k_o} < \frac{p_e(e_c-\theta_c k_c)}{p}$ *are satisfied, if* $e_o-\theta_o k_o > \psi$, $q_c^{C(1,1)} > q_c^{C(0,0)}$; *if* $e_c-\theta_c k_c > \iota$, $q_o^{C(1,1)} > q_o^{C(0,0)}$, *if* $e_o-\theta_o k_o < \upsilon$, $w_c^{C(1,1)} > w_c^{C(0,0)}$;

(2) When $\frac{w_c+p_e(e_o-\theta_o k_o)}{p} \geq \sqrt{\frac{2[c+p_e(e_c-\theta_c k_c)]}{p}}$ *and* $\delta_o\sqrt{k_o} \geq \frac{p_e(e_c-\theta_c k_c)}{p}$ *are satisfied,* $q_c^{C(1,1)} > q_c^{C(0,0)}$, $q_o^{C(1,1)} \leq q_o^{C(0,0)}$, $w_c^{C(1,1)} \geq w_c^{C(0,0)}$.

where $\psi = \frac{1}{p_e}[\sqrt{2p[c+p_e(e_c-\theta_c k_c)]} - p(1+\delta_o\sqrt{k_o}) - c]$, $[\iota = \frac{1}{2p_e}[p(1+\delta_o\sqrt{k_o}) - c + p_e(e_o-\theta_o k_o)]$, $\upsilon = \frac{1}{p_e}[p(1+\delta_o\sqrt{k_o}) - c - 2p_e(e_c-\theta_c k_c)]$.

When $[\frac{w_c+p_e(e_o-\theta_o k_o)}{p} \geq \sqrt{\frac{2[c+p_e(e_c-\theta_c k_c)]}{p}}$ is satisfied, it indicates that the profit per unit of OEM products is less than that of CM products. From Proposition 17, we observe that when the carbon emissions per unit of CM products are relatively high ($\delta_o\sqrt{k_o} < \frac{p_e(e_c-\theta_c k_c)}{p}$), both CM and OEM's optimal decisions in the (1, 1) scenario may be superior to the optimal decisions in the (0, 0) scenario. This result suggests that increasing production quantity is advantageous when the competitor's carbon emissions per unit are high. When the carbon emissions per unit of CM products are relatively low ($\delta_o\sqrt{k_o} \geq \frac{p_e(e_c-\theta_c k_c)}{p}$), CM in the (1, 1) scenario can completely substitute for more of OEM's demand. To maximize profits, CM in the (1, 1) scenario will set the wholesale price to the maximum, leading OEM to produce at the minimum quantity. Therefore, the results of Proposition 17 indicate that when the products of two manufacturers are completely substitutable, the optimal decisions of all members are not necessarily worse than the optimal decisions when no substitution exists.

Proposition 18. *In the (1, 1) scenario under the co-opetitive mode:*

(1) When $\frac{w_c+p_e(e_o-\theta_o k_o)}{p} \leq \frac{1}{2}[\frac{c+p_e(e_c-\theta_c k_c)}{p}]^2$, $w_c^{C(1,1)}$ *increases with the increase in* k_o *and* p_e, *and decreases with the increase in* k_c; $q_o^{C(1,1)}$ *increases with the increase in* k_o *and* k_c; *if* $\frac{e_c-\theta_c k_c}{e_o-\theta_o k_o} > \frac{2\sqrt{2p[w_c^{C(1,1)}+p_e(e_o-\theta_o k_o)]}}{w_c^{C(1,1)}+p_e(e_o-\theta_o k_o)+c}$, $q_o^{C(1,1)}$ *increases with the increase in* p_e; $q_c^{C(1,1)}$ *increases with the increase in* k_c *and decreases with the increase in* p_e, *without varying with* k_o;

(2) When $\frac{w_c+p_e(e_o-\theta_o k_o)}{p} \geq \sqrt{\frac{2[c+p_e(e_c-\theta_c k_c)]}{p}}$ *and* $\delta_o\sqrt{k_o} < \frac{p_e(e_c-\theta_c k_c)}{p}$, $w_c^{C(1,1)}$ *increases with the increase in* k_o *and* k_c, *and decreases with the increase in* p_e; $q_c^{C(1,1)}$ *increases with the*

increase in k_c and decreases with the increase in k_o; if $\frac{e_o-\theta_o k_o}{e_c-\theta_c k_c} > 1 + \frac{p}{\sqrt{2p[c+p_e(e_c-\theta_c k_c)]}}$, $q_c^{C(1,1)}$ increases with the increase in p_e; $q_o^{C(1,1)}$ increases with the increase in k_o and decreases with the increase in k_c; if $e_c - \theta_c k_c > e_o - \theta_o k_o$, $q_o^{C(1,1)}$ increases with the increase in p_e;

(3) When $\frac{w_c+p_e(e_o-\theta_o k_o)}{p} \geq \sqrt{\frac{2[c+p_e(e_c-\theta_c k_c)]}{p}}$ and $\delta_o\sqrt{k_o} \geq \frac{p_e(e_c-\theta_c k_c)}{p}$, $w_c^{C(1,1)}$ increases with the increase in k_o and decreases with the increase in p_e, without changing with k_c; $q_o^{C(1,1)}$ increases with the increase in k_o, without changing with k_c and p_e; $q_c^{C(1,1)}$ increases with the increase in k_c and decreases with the increase in p_e, without changing with k_o.

Similarly, due to the complexity of the problem in the (1, 1) scenario, in Proposition 18, we can only demonstrate the influence of green investment and carbon trading prices on optimal decisions in some cases. The results of Proposition 18 also indicate that in the co-opetitive mode, when CM can replace OEM, carbon trading prices may not necessarily suppress the production of manufacturers.

6. Numerical Analysis

In this section, we continue to conduct a numerical analysis and explore new findings by consulting the carbon trading prices from the China Carbon Emission Trade Exchange (CCETE) and using partial parameter settings from ref. [11]. We assume that p is 5, c is 1, p_e is 4, r is 0.5, e_i $(i = o, c)$ are both 1, θ_i $(i = o, c)$ are both 0.1, δ_i $(i = o, c)$ are both 0.01, and E_i $(i = o, c)$ are both 10. Next, we set CM's green investment k_c to be 3.5 and 6.5, respectively, and use OEM's green investment k_o as the horizontal axis to compare and analyze the profits of OEM π_o, CM π_c, and the total profit π of the supply chain under different scenarios.

First, we compare the three scenarios under purely competitive: (0, 0), (r, 0), (0, r) and (1, 1), and present the corresponding results in Figures 5–7. From Figures 5 and 6, we observe that $\pi_o^{P(r,0)} = \pi_o^{P(0,0)}$ and $\pi_c^{P(0,r)} = \pi_c^{P(0,0)}$ always exist. This further validates the results analyzed in Propositions 2 and 5. Additionally, we observe that whether $k_c = 3.5$ or $k_c = 6.5$ is satisfied, when k_o is relatively small, $\pi_o^{P(r,0)}$ is always optimal. However, as k_o increases, $\pi_o^{P(1,1)}$ becomes optimal. This is because in the (r, 0) scenario of the purely competitive mode, the CM cannot substitute for the excess demand of the OEM, whereas in the (1, 1) scenario, the CM can fully substitute for the demand of the OEM. Therefore, when k_o is small, the unmet demand by the OEM is significant, and in the (r, 0) scenario, this excess demand of the OEM does not transfer to the CM. In contrast, in the (1, 1) scenario, the excess demand of the OEM is fully transferred to the CM. At this point, the OEM's sales revenue in the (1, 1) scenario significantly decreases, leading to $\pi_o^{P(r,0)} > \pi_o^{P(1,1)}$. However, when k_o is large, the OEM can substitute for the unmet excess demand of the CM. In the (r, 0) scenario, the OEM can only partially substitute for the excess demand of the CM, while in the (1, 1) scenario, the OEM can fully substitute for the excess demand of the CM. At this point, the OEM can achieve higher sales revenue in the (1, 1) scenario, leading to $\pi_o^{P(1,1)} > \pi_o^{P(r,0)}$.

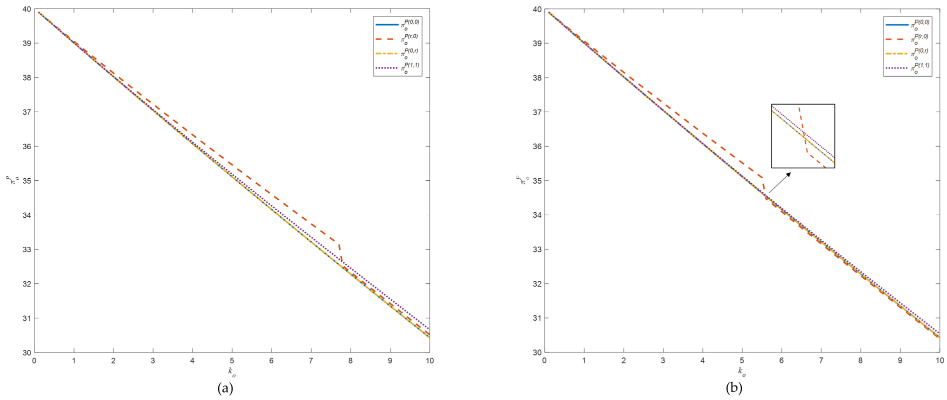


Figure 5. Comparison of π_o^P in the scenarios of (0, 0), (r, 0), (0, r) and (1, 1) under purely competitive mode. (**a**) $k_c = 3.5$. (**b**) $k_c = 6.5$.

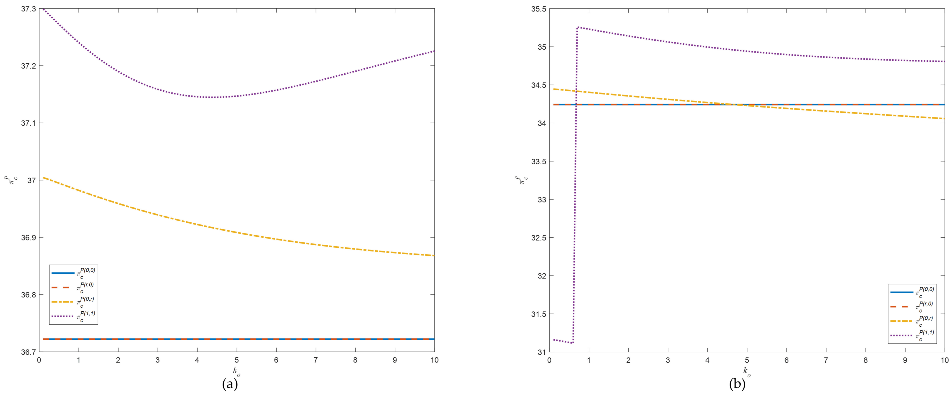


Figure 6. Comparison of π_c^P in the scenarios of (0, 0), (r, 0), (0, r) and (1, 1) under purely competitive mode. (**a**) $k_c = 3.5$. (**b**) $k_c = 6.5$.

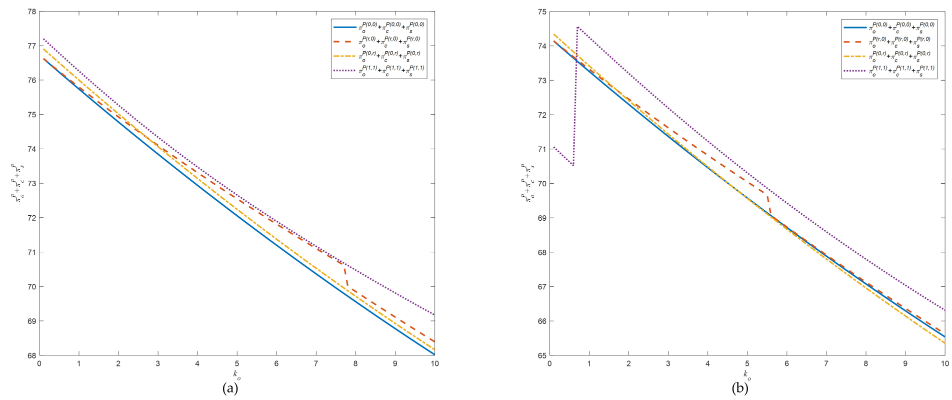
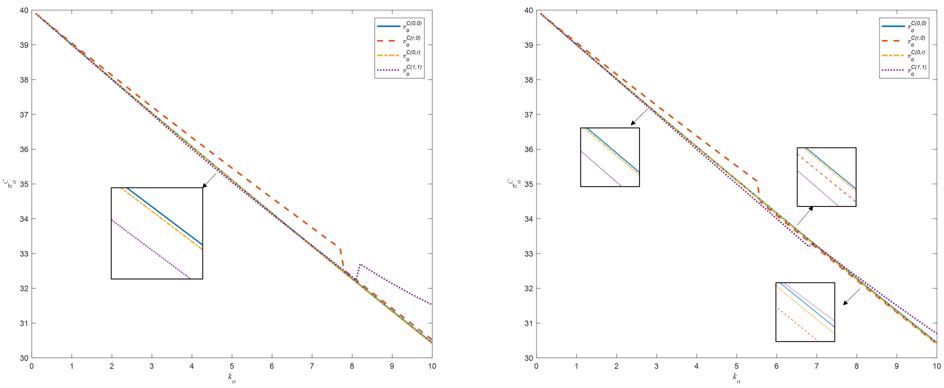


Figure 7. Comparison of π^P in the scenarios of (0, 0), (r, 0), (0, r) and (1, 1) under purely competitive mode. (**a**) $k_c = 3.5$. (**b**) $k_c = 6.5$.

From Figure 6, we observe that when $k_c = 3.5$ is satisfied, $\pi_c^{P(1,1)}$ is always optimal. However, when $k_c = 6.5$ is satisfied, if k_o is relatively small, $\pi_c^{P(1,1)}$ is no longer optimal. As k_o increases, $\pi_c^{P(1,1)}$ becomes optimal. This is because when k_c is small, in the (1, 1) scenario of the purely competitive mode, the CM can fully substitute for the excess demand of the OEM. The additional revenue CM gains from substitution can offset the carbon emission costs incurred during the production process. Therefore, $\pi_c^{P(1,1)}$ is optimal. However, when k_c is large and k_o is small, in the (1, 1) scenario, the CM can fully substitute for more excess demand of the OEM. At this point, the additional revenue from substitution does not offset the carbon emission costs incurred during the production process, making $\pi_c^{P(1,1)}$ not optimal. As k_o increases, the excess demand of the OEM decreases, and the additional revenue CM gains from substitution can again offset the carbon emission costs incurred during the production process. Thus, $\pi_c^{P(1,1)}$ becomes optimal again.

For the entire supply chain, we observe from Figure 7 that when k_c is relatively small, $\pi^{P(1,1)}$ is always optimal. However, when k_c is large, if k_o is small, $\pi^{P(1,1)}$ is no longer optimal. As k_o increases, $\pi^{P(1,1)}$ becomes optimal. This is because when k_c is small, in the (1, 1) scenario of the purely competitive mode, the additional revenue CM gains from substitution can not only offset its own carbon emission costs incurred during the production process but also compensate for the losses of OEM and TS. Therefore, $\pi^{P(1,1)}$ is optimal. However, when k_c is large and k_o is small, the additional revenue CM gains from substitution in the (1, 1) scenario cannot offset its own carbon emission costs or the losses incurred by OEM and TS, making $\pi^{P(1,1)}$ not optimal. As k_o increases, the unmet excess demand of OEM decreases, and at this point, the additional revenue CM gains from substitution can again offset its own carbon emission costs and compensate for the losses incurred by OEM and TS. Therefore, $\pi^{P(1,1)}$ becomes optimal again. This result indicates that in the purely competitive mode, product substitution can bring better profits to the supply chain.

Next, we compare the co-opetitive mode under scenarios (0, 0), (r, 0), (0, r) and (1, 1), and present the corresponding results in Figures 8–10. From Figure 8, we observe that regardless of the value of k_c, $\pi_o^{C(0,r)}$ is never optimal and consistently lower than $\pi_o^{C(0,0)}$. This is because, in the co-opetitive mode, CM serves as the manufacturer for OEM. When only CM can substitute for OEM's products, CM will set higher wholesale prices to suppress OEM's production, resulting in $\pi_o^{C(0,r)} < \pi_o^{C(0,0)}$. Propositions 9 and 10 indicate that in both scenarios (0, 0) and (r, 0), $\pi_o^{C(0,0)} = \pi_o^{P(0,0)}$ and $\pi_o^{C(r,0)} = \pi_o^{P(r,0)}$ always exist. Therefore, $\pi_o^{C(0,0)}$ and $\pi_o^{C(r,0)}$ yield the same results as in Figure 5. However, unlike Figure 5, when k_o is small, $\pi_o^{C(1,1)}$ is the minimum in the (1, 1) scenario under the co-opetitive mode. This is because, in the (1, 1) scenario, CM can not only fully substitute for OEM's excess demand but also manipulate the wholesale price, leading to a further decrease in OEM's production. Consequently, the sales revenue of OEM in the (1, 1) scenario significantly decreases, resulting in $\pi_o^{C(1,1)}$ being the minimum.



(a) (b)

Figure 8. Comparison of π_o^C in the scenarios of (0, 0), (r, 0), (0, r) and (1, 1) under co-opetitive mode. (**a**) $k_c = 3.5$. (**b**) $k_c = 6.5$.

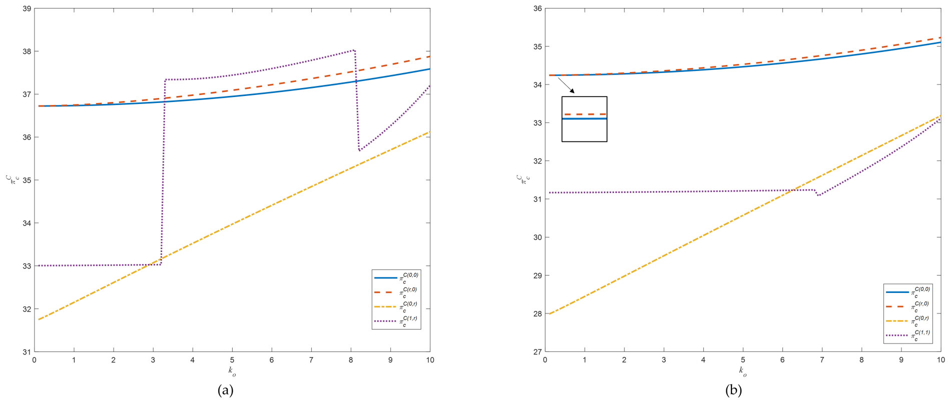


Figure 9. Comparison of π_c^C in the scenarios of (0, 0), (r, 0), (0, r) and (1, 1) under co-opetitive mode. (**a**) $k_c = 3.5$. (**b**) $k_c = 6.5$.

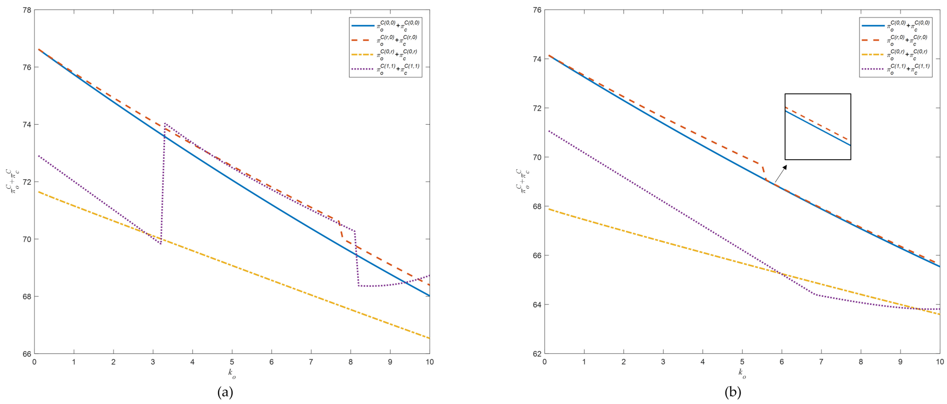


Figure 10. Comparison of π^C in the scenarios of (0, 0), (r, 0), (0, r) and (1, 1) under co-opetitive mode. (**a**) $k_c = 3.5$. (**b**) $k_c = 6.5$.

From Figure 9, we find that $\pi_c^{C(1,1)}$ is not necessarily optimal. Specifically, when $k_c = 6.5$ is satisfied, $\pi_c^{C(r,0)}$ is optimal. This is because when k_c is small, if k_o is also small, the demand that the OEM cannot satisfy is high. In this case, under the (1, 1) scenario, the CM can fully substitute for the OEM's excess demand. However, the revenue gained by the CM through substitution cannot offset the carbon emission costs incurred during its production process. Therefore, $\pi_c^{C(1,1)}$ is not optimal. As k_o increases, the unmet demand by the OEM gradually decreases. In the (1, 1) scenario, the revenue gained by the CM through substitution can offset the carbon emission costs incurred during its production process, and as a supplier, the CM can further suppress the OEM's production by raising the wholesale price, making $\pi_c^{C(1,1)}$ optimal. However, as k_o further increases, the OEM's production significantly increases. In the (1, 1) scenario, the OEM can fully substitute for the CM's excess demand, leading to a decrease in the CM's sales revenue, making $\pi_c^{C(1,1)}$ not optimal. When k_c is large, in the (1, 1) scenario, the revenue gained by the CM through substitution can never offset the costs incurred from its green investments. Moreover, the CM, in an attempt to gain more excess demand, will raise the wholesale price to suppress the OEM's production. However, a sufficiently high wholesale price leads to a significant reduction in the OEM's production, resulting in a substantial decrease in the CM's profit from selling raw materials. Therefore, $\pi_c^{C(1,1)}$ is not optimal. Conversely, in the (r, 0) scenario, OEM can satisfy CM's excess demand, prompting OEM to increase its production. Consequently, CM earns more revenue by selling more raw materials, making $\pi_c^{C(r,0)}$ optimal.

In Figure 10, we similarly observe that $\pi^{C(1,1)}$ is not necessarily optimal. Specifically, when $k_c = 6.5$ is satisfied, $\pi^{C(r,0)}$ is optimal, for reasons similar to those in Figure 9. When k_c is small, if k_o is also small, the unmet demand from the OEM is high. In this case, under the (1, 1) scenario, the CM can fully substitute for the OEM's excess demand. However, the revenue gained by the CM through substitution cannot offset the carbon emission costs incurred during its production process. Therefore, $\pi^{C(1,1)}$ is not optimal. As k_o increases, the unmet demand from the OEM gradually decreases. In the (1, 1) scenario, the revenue gained by the CM through substitution can offset the carbon emission costs incurred during its production process. Moreover, as a supplier, the CM can further suppress the OEM's production by raising the wholesale price, making $\pi^{C(1,1)}$ optimal. However, as k_o further increases, the OEM's production significantly increases. In the (1, 1) scenario, the OEM can fully substitute for the CM's excess demand, leading to a decrease in the CM's sales revenue, making $\pi^{C(1,1)}$ not optimal. When k_c is large, in the (1, 1) scenario, the revenue gained by the CM through substitution can never offset the costs incurred from its green investments. Additionally, the CM, in an attempt to gain more excess demand, will raise the wholesale price to suppress the OEM's production. However, a sufficiently high wholesale price leads to a significant reduction in the OEM's production, resulting in a substantial decrease in the CM's profit from selling raw materials. Therefore, $\pi^{C(1,1)}$ is not optimal. In the (r, 0) scenario, the OEM can meet the CM's excess demand, leading the OEM to increase its production. In this case, the CM gains more revenue by selling more raw materials, making $\pi^{C(r,0)}$ optimal. Additionally, this result indicates that under the co-opetitive mode, product substitution can potentially lead to better supply chain profits.

Next, we compare the corresponding profits of the two modes under the (0, r) scenario. Firstly, from Figure 11, we observe that regardless of the values of k_o and k_c, $\pi_o^{P(0,r)} > \pi_o^{C(0,r)}$ always exists. This is because in the co-opetitive mode, CM acts as the supplier to OEM. In this mode, CM increases wholesale prices to suppress OEM's production to gain more demand. Consequently, OEM's yields decrease, leading to reduced sales revenue and ultimately lower profits.

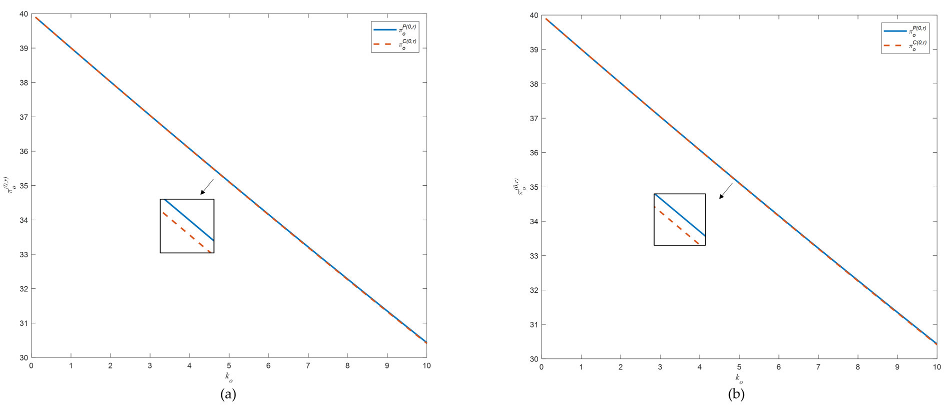


Figure 11. Comparison of π_o under the (0, r) scenario. (**a**) $k_c = 3.5$. (**b**) $k_c = 6.5$.

From Figure 12, we observe that $\pi_c^{C(0,r)}$ always being less than $\pi_c^{P(0,r)}$. This is because under the co-opetitive mode, CM increases wholesale prices to suppress OEM production to obtain more excess demand. However, excessively high wholesale prices lead to a significant reduction in CM's profits from selling raw materials due to the substantial reduction in OEM production. At the same time, the excess demand obtained by CM cannot bring in a sufficiently large profit, leading to $\pi_c^{C(0,r)}$ always being less than $\pi_c^{P(0,r)}$.

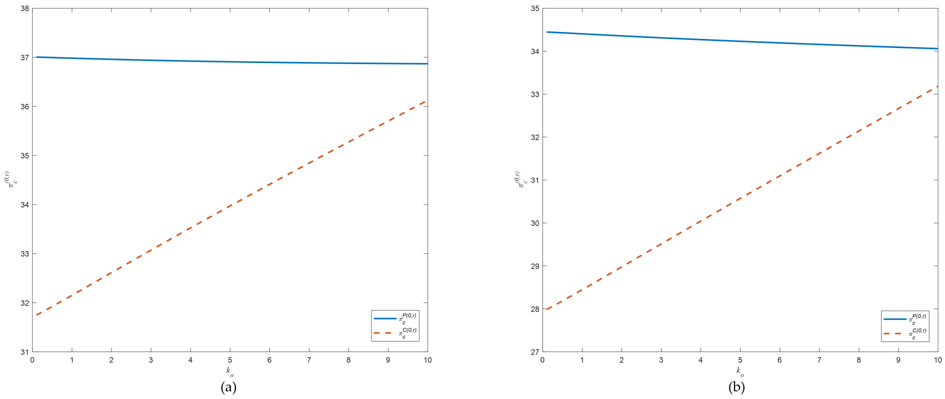


Figure 12. Comparison of π_c under the (0, r) scenario. (**a**) $k_c = 3.5$. (**b**) $k_c = 6.5$.

Similarly to the results from Figures 11 and 12, we observe from Figure 13 that $\pi^{C(0,r)} < \pi^{P(0,r)}$. This is also due to the co-opetitive mode, where CM increases wholesale prices to suppress OEM production to obtain more excess demand. However, excessively high wholesale prices lead to a significant reduction in OEM production, resulting in a substantial decrease in CM's profits from selling raw materials. Additionally, the excess demand obtained by CM cannot bring in a sufficiently large profit, resulting in $\pi^{C(0,r)} < \pi^{P(0,r)}$. This result indicates that when only CM can substitute for OEM, the co-opetitive mode may not bring better profits to the supply chain.

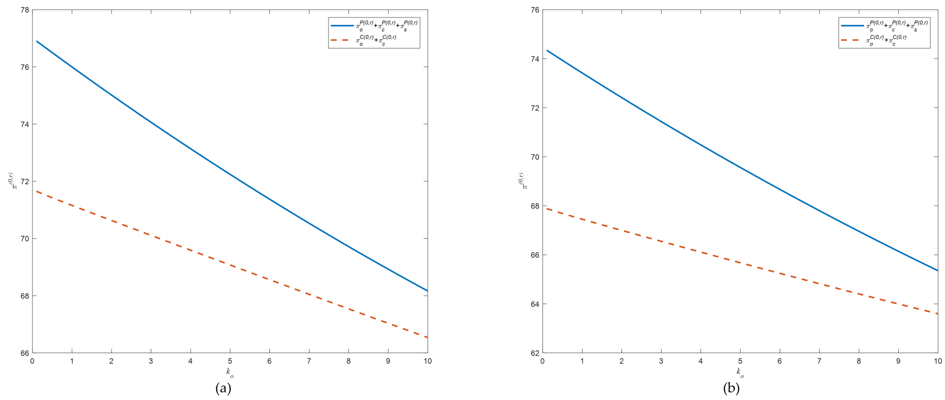


Figure 13. Comparison of π under the (0, r) scenario. (**a**) $k_c = 3.5$. (**b**) $k_c = 6.5$.

Finally, we compare the respective profits of the two modes under the (1, 1) scenario. Firstly, Figure 14 illustrates that with the increase in k_o, $\pi_o^{C(1,1)} > \pi_o^{P(1,1)}$ emerges. This is because, in this scenario, the products of CM and OEM can substitute for each other. At this point, as k_o increases, OEM can fulfill more excess demand that CM cannot satisfy. In the co-opetitive mode, CM, acting as a supplier to OEM, offers lower wholesale prices to stimulate OEM to increase orders to gain more profit. Consequently, OEM can obtain more sales revenue at a lower ordering cost, leading to $\pi_o^{C(1,1)} > \pi_o^{P(1,1)}$.

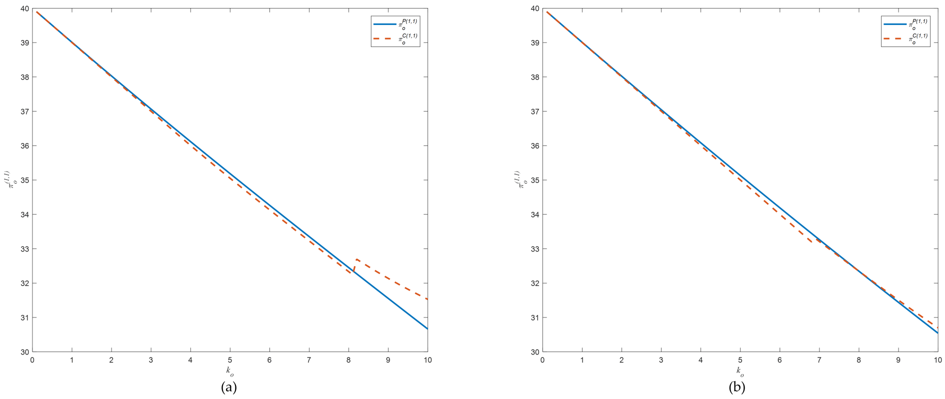


Figure 14. Comparison of π_o under the (1, 1) scenario. (**a**) $k_c = 3.5$. (**b**) $k_c = 6.5$.

In Figure 15, we observe that when k_c is small, with the increase in k_o, we will sequentially encounter $\pi_c^{C(1,1)} < \pi_c^{P(1,1)}$, $\pi_c^{C(1,1)} > \pi_c^{P(1,1)}$ and $\pi_c^{C(1,1)} < \pi_c^{P(1,1)}$. However, when k_c is large, only when k_o is small, $\pi_c^{C(1,1)} > \pi_c^{P(1,1)}$ appears. This is because when k_c and k_o are small, in the co-opetitive mode, CM increases the wholesale price to expand its substitutable demand. However, the significant increase in the wholesale price results in a substantial reduction in OEM's production, leading to greater losses, thus resulting in $\pi_c^{C(1,1)} < \pi_c^{P(1,1)}$. As k_o increases, the excess demand that OEM cannot satisfy decreases. In this scenario, CM increases the wholesale price to boost its additional demand for sales revenue. At this point, the extra sales revenue obtained by CM far exceeds the loss caused by the increase in the wholesale price, leading to $\pi_c^{C(1,1)} > \pi_c^{P(1,1)}$. Nevertheless, as k_o continues to increase, the excess demand that OEM cannot satisfy decreases significantly. At this point, CM reduces the wholesale price to stimulate OEM to further increase production. However, the loss caused by the decrease in the wholesale price is sufficiently significant, resulting in $\pi_c^{C(1,1)} < \pi_c^{P(1,1)}$. When k_c is large, CM's demand that can be satisfied is substantial. If k_o is small, CM in the co-opetitive mode increases the wholesale price to obtain more substitutable demand. At this point, CM in the co-opetitive mode can obtain more sales revenue, resulting in $\pi_c^{C(1,1)} > \pi_c^{P(1,1)}$. However, as k_o increases, CM in the co-opetitive mode increases the wholesale price without obtaining more substitutable demand, but instead significantly reduces its revenue from selling raw materials. Therefore, $\pi_c^{C(1,1)} < \pi_c^{P(1,1)}$ occurs.

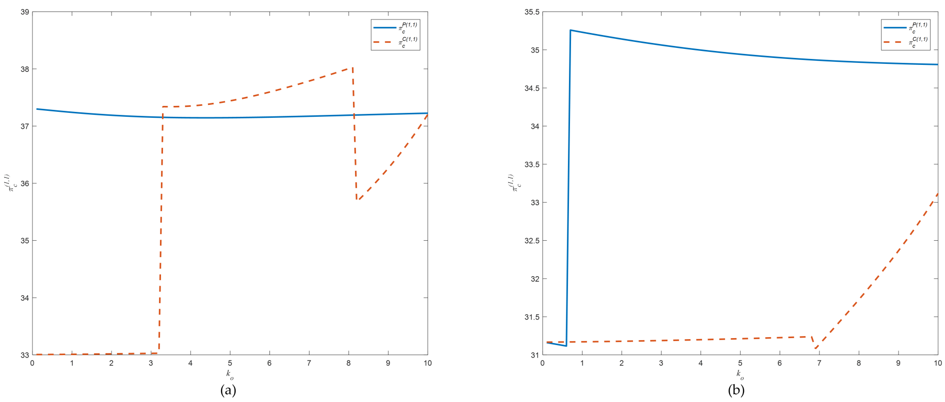


Figure 15. Comparison of π_c under the (1, 1) scenario. (**a**) $k_c = 3.5$. (**b**) $k_c = 6.5$.

In Figure 16, we observe that when k_c is small, $\pi^{C(1,1)} < \pi^{P(1,1)}$ always exists. When k_c is large, $\pi^{C(1,1)} > \pi^{P(1,1)}$ appears. This is because when k_c is small, in the co-opetitive mode, CM seeks to maximize its profits by manipulating the wholesale price. Although in certain situations, manipulating the wholesale price can bring more profits to CM, it also causes greater damage to OEM's profits; hence, $\pi^{C(1,1)} < \pi^{P(1,1)}$ always exists. When k_c is large, in the co-opetitive mode, although CM seeks to maximize its profits by manipulating the wholesale price, this operation may cause less damage to OEM's profits. Therefore, $\pi^{C(1,1)} > \pi^{P(1,1)}$ appears. This result indicates that the co-opetitive mode may also bring more profits to the supply chain.

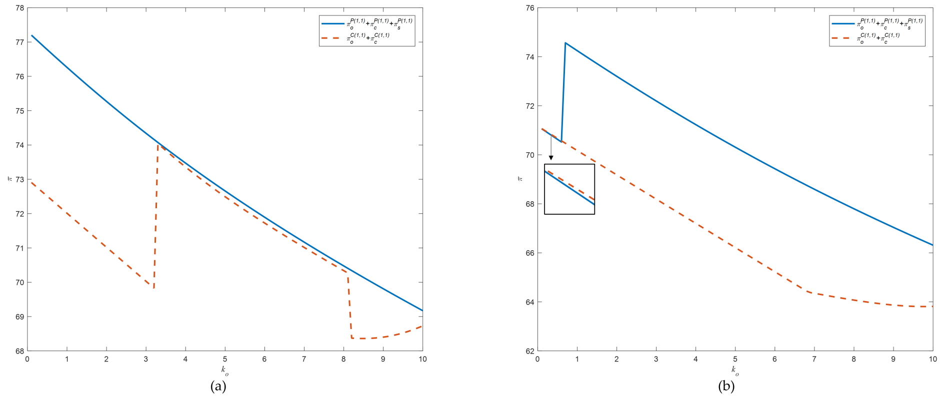


Figure 16. Comparison of π under the (1, 1) scenario. (**a**) $k_c = 3.5$. (**b**) $k_c = 6.5$.

7. Discussion and Conclusions

This study investigates the competitive newsvendor problem with product substitution under the carbon cap-and-trade system. Unlike studies such as those by refs. [23,43], our research introduces a carbon cap-and-trade system under conditions of market demand uncertainty. Additionally, we consider the scenario where the OEM has the option to procure raw materials from the CM or from a third-party supplier TS, and both of their final products are substitutable. Both OEM and CM products result in carbon emissions, which are offset through the carbon cap-and-trade system. Based on this problem, we consider different substitution relationships between OEM and CM products under both purely competitive and co-opetitive modes. Subsequently, we develop decision models for different scenarios and provide corresponding optimal solutions under these conditions. This approach enhances our understanding of the competitive issues when product substitution is considered.

Based on the obtained optimal solutions, we first analyze the impact of substitution rates, green investments, and carbon trading prices on the optimal decisions of OEM and CM under both purely competitive and co-opetitive modes. Unlike studies such as refs. [31,35] which did not consider product substitution and market demand uncertainty, we find some new findings based on the consideration of these factors. In purely competitive mode, when only one-way substitution exists, an increase in the substitution rate and green investment of OEM (or CM) leads to an increase in OEM's (or CM's) yields, while an increase in the green investment of OEM (or CM) does not necessarily reduce yields of CM (or OEM). In co-opetitive mode, when only one-way substitution exists, an increase in the substitution rate and green investments of both manufactures may lead to an increase in yields of both manufactures. Moreover, when only one-way substitution exists, an increase in carbon trading prices in the purely competitive mode may only lead to an increase in yields of the substitutable manufacturer, while an increase in carbon trading prices in the co-opetitive mode may lead to an increase in yields of both manufactures. Next, by comparing the two modes under the same substitution conditions, we find that when CM cannot substitute for OEM, the optimal decisions and total profits of the supply chain are the same in both modes. Finally, through numerical analysis, we further discovered that under the same competitive mode, under specific conditions, when the products of the two manufacturers can substitute for each other, it can bring better profits to both manufacturers and the entire supply chain. In contrast to the findings of ref. [11], we found that under different competitive modes, when CM can substitute for OEM, neither mode is necessarily optimal for the profits of both manufacturers and the total profit of the supply chain. Furthermore, by comparing different substitution scenarios within the same mode, we found that under certain conditions, the existence of product substitution can enhance the total profit of the supply chain.

This study also provides new insights into operational decision-making for manufacturers in competitive environments. Firstly, under specific conditions, the existence of product substitution not only helps increase the production and profits of manufacturers but also enhances the total profit of the supply chain. Secondly, the study points out that under specific conditions, an increase in carbon trading prices can prompt manufacturers to increase their production. Therefore, manufacturers in competitive environments can alleviate the inhibitory effect of carbon emissions reduction on their production by participating in carbon cap-and-trade systems in specific environments. Furthermore, when only OEM products can substitute for CM products, whether or not the two manufacturers co-operate does not affect the total profit of the supply chain. Lastly, when competing manufacturers participate in the carbon cap-and-trade system, under specific conditions, the blind increase in wholesale prices by CM in the co-opetitive mode to seize the market will lead to a lose-lose situation. Therefore, CM in the co-opetitive mode should not blindly increase wholesale prices to seize the market.

The present study still has certain limitations. Firstly, it assumes that the green investments of OEM and CM are predetermined. However, in the actual operation of the supply chain, the green investments of OEM and CM can influence the optimal decisions of supply chain members. Therefore, treating the green investments of OEM and CM as decision variables for both parties simultaneously could pose an interesting new research question. Additionally, the study overlooks the flexibility of the carbon cap-and-trade system, as well as the uncertainty of carbon trading prices. It only assumes that OEM and CM settle their carbon quotas through the carbon cap-and-trade system at the end of the period. Hence, exploring carbon trading by OEM and CM throughout the period could be another intriguing research avenue.

Supplementary Materials: The following supporting information can be downloaded at: https://www.mdpi.com/article/10.3390/systems12060201/s1, Supplementary Materials: All Proofs.

Author Contributions: Conceptualization, Y.R.; methodology, Y.R.; software, Y.R. and H.L.; writing—original draft preparation, Y.R.; writing—review and editing, H.L. and W.B.; visualization, Y.R. and Y.J.; project administration and funding acquisition, W.B. All authors have read and agreed to the published version of the manuscript.

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