Beyond Passivity: Port-Hamiltonian Systems

Arjan van der Schaft

University of Groningen

In collaboration with Romeo Ortega, Alessandro Astolfi, Fernando Castanos, as well as with Bernhard Maschke, Ram Pasumarthy, Gerardo Escobar, Stefano Stramigioli, Alessandro Macchelli, ..

1. From passive systems to port-Hamiltonian systems
2. Examples and distributed-parameter components
3. Control by interconnection and Casimir generation
4. The dissipation obstacle
5. State feedback: IDA-PBC

A square nonlinear system

$$\Sigma : \begin{array}{rcl} \dot{x} & = & a(x) + b(x)u \\ \\ y & = & c(x) \end{array}$$

where $u, y \in \mathbb{R}^m$, and $x \in \mathbb{R}^n$ are local coordinates for an n-dimensional state space manifold $\mathcal{X}$, is **passive** if there exists a **storage function** $H : \mathcal{X} \to \mathbb{R}$ with $H(x) \geq 0$ for every x, such that

$$H(x(t_2)) - H(x(t_1)) \leq \int_{t_1}^{t_2} u^T(t)y(t)dt$$

for all solutions $(u(\cdot), x(\cdot), y(\cdot))$ and all time instants $t_1 \leq t_2$.

The system is **lossless** if $\leq$ is replaced by $=$.

If H is *differentiable* then 'passive' is equivalent to (Willems, Hill-Moylan)

$$
\begin{aligned}
\frac{\partial^T H}{\partial x}(x)a(x) &\leq 0 \\
c(x) &= b^T(x)\frac{\partial H}{\partial x}(x)
\end{aligned}
$$

and in the lossless case $\leq$ is replaced by $=$.

If additionally $H(x^*) = 0$ and $H(x) > 0$ for every $x \neq x^*$, then it follows that x^* is a **stable** equilibrium (which can *never* be asymptotically stable in the lossless case).

A *linear* system

$$
\begin{aligned}
\dot{x} &= Ax + Bu \\
y &= Cx
\end{aligned}
$$

with equilibrium $x^* = 0$ is passive if there exists a *quadratic* storage function $H(x) = \frac{1}{2}x^T Q x$ with $Q = Q^T \geq 0$ satisfying the LMIs

$$
A^T Q + QA \leq 0, \qquad C = B^T Q
$$

From passive systems to port-Hamiltonian systems

Every linear passive system with $H(x) = \frac{1}{2}x^T Qx$, satisfying

$$\ker Q \subset \ker A$$

can be rewritten as a **port-Hamiltonian system**

$$\begin{aligned} \dot{x} &= (J - R)Qx + Bu \\ y &= B^T Qx, \end{aligned}$$

with respect to the **Hamiltonian** $H(x) = \frac{1}{2}x^T Qx$.
The matrix $J = -J^T$ specifies the internal **interconnection structure** of the system.
The matrix $R = R^T \geq 0$ specifies the **energy-dissipation** (due to dampers, viscosity, resistors, etc.). In the lossless case $R = 0$.

Conversely, every port-Hamiltonian system with $Q = Q^T \geq 0$ is passive.

Mutatis mutandis 'most' nonlinear lossless systems can be written as a port-Hamiltonian system

$$\begin{aligned} \dot{x} &= J(x)\tfrac{\partial H}{\partial x}(x) + g(x)u \\ \\ y &= g^T(x)\tfrac{\partial H}{\partial x}(x) \end{aligned}$$

with $J(x) = -J^T(x)$. Here

$$\dot{x} = J(x)\frac{\partial H}{\partial x}(x)$$

is the internal Hamiltonian dynamics known from physics, which in classical mechanics can be written as

$$\begin{aligned} \dot{q} &= \tfrac{\partial H}{\partial p}(q,p) \\ \dot{p} &= -\tfrac{\partial H}{\partial q}(q,p) \end{aligned}$$

with the Hamiltonian H the total (kinetic + potential) energy.

Similarly, most nonlinear passive systems can be written as a **port-Hamiltonian system (with dissipation)**

$$\begin{aligned} \dot{x} &= [J(x) - R(x)]\tfrac{\partial H}{\partial x}(x) + g(x)u \\ y &= g^T(x)\tfrac{\partial H}{\partial x}(x) \end{aligned}$$

with $R(x) = R^T(x) \geq 0$ specifying the energy dissipation

$$\frac{d}{dt}H = -\frac{\partial^T H}{\partial x}(x)R(x)\frac{\partial H}{\partial x}(x) - u^T y$$

However, in (network) *modelling* it is the **other way around**[a]: one derives the system in port-Hamiltonian form, and if the Hamiltonian $H \geq 0$ then it is the storage function of a passive system.

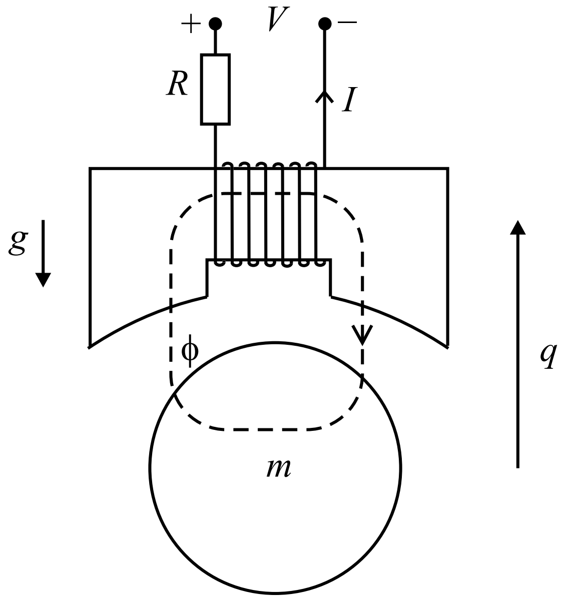
The matrix $J(x)$ corresponds to the internal power-conserving structure of physical systems:

- Oscillation between potential and kinetic energy components.
- Kinematic constraints in mechanical systems.
- Kirchhoff's laws
- Transformers, gyrators, etc.

Main message of this talk: **start with port-Hamiltonian models instead of passive models**, because more can be done this way, and one stays closer to the 'physics' of the controlled system.

[a] See e.g. the website of the EU-project Geoplex: www.geoplex.cc

Example: Electro-mechanical systems



$$\begin{bmatrix} \dot{q} \\ \dot{p} \\ \dot{\varphi} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -\frac{1}{R} \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial q} \\ \frac{\partial H}{\partial p} \\ \frac{\partial H}{\partial \varphi} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} V, \qquad I = \frac{\partial H}{\partial \varphi}(q, p, \phi)$$

Coupling electrical and mechanical domain via the Hamiltonian H.

Example: Mechanical systems with kinematic constraints

Constraints on the generalized velocities $\dot{q}$:

$$A^T(q)\dot{q} = 0.$$

This leads to **constrained** Hamiltonian equations

$$\begin{aligned}
\dot{q} &= \frac{\partial H}{\partial p}(q,p) \\
\dot{p} &= -\frac{\partial H}{\partial q}(q,p) + A(q)\lambda + B(q)u \\
0 &= A^T(q)\frac{\partial H}{\partial p}(q,p) \\
y &= B^T(q)\frac{\partial H}{\partial p}(q,p)
\end{aligned}$$

with $H(q,p)$ total energy, and λ the constraint forces.
By elimination of the constraints and constraint forces one derives a port-Hamiltonian model as before.

Can be extended to general *multi-body systems*.

Aside In modelling one often arrives at models with **constraints**, leading to DAEs and/or models with Lagrange multipliers. The theory of port-Hamiltonian systems can be naturally extended to such cases by replacing the matrix $J(x)$ (defining a Poisson structure) by a **Dirac structure**.

Main theorem: Any power-conserving interconnection of port-Hamiltonian systems is again a port-Hamiltonian system (with respect to a Dirac structure).

Example: distributed-parameter port-Hamiltonian systems

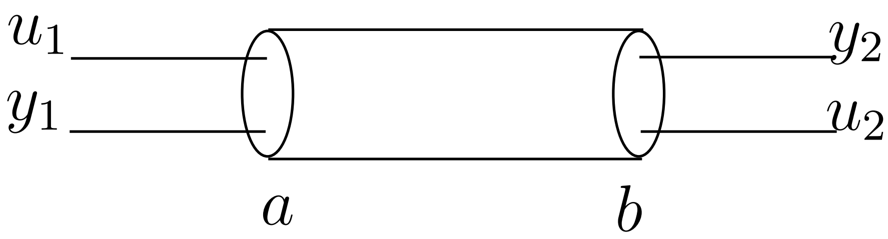


Figure 1: **Transmission line**

Telegrapher's equations define the boundary control system

$$\begin{aligned}
\frac{\partial Q}{\partial t}(z,t) &= -\frac{\partial}{\partial z} I(z,t) &&= -\frac{\partial}{\partial z}\frac{\phi(z,t)}{L(z)} \\
\frac{\partial \phi}{\partial t}(z,t) &= -\frac{\partial}{\partial z} V(z,t) &&= -\frac{\partial}{\partial z}\frac{Q(z,t)}{C(z)} \\
u_1(t) &= V(a,t), \quad y_1(t) &&= I(a,t) \\
y_2(t) &= V(b,t), \quad u_2(t) &&= I(b,t)
\end{aligned}$$

In this case, $J(x)$ is replaced by the differential operator $\frac{\partial}{\partial z}$.

Use of passivity for control

- The standard feedback interconnection of two passive systems is again passive, with storage function being the **sum** of the individual storage functions.

- Passive systems, with the minimum of the storage function being a stable equilibrium, can be asymptotically stabilized by **additional damping**, provided an **observability condition** is met:

$$\frac{d}{dt}H \leq u^T y$$

together with the additional damping $u = -y$ leads to

$$\frac{d}{dt}H \leq - \parallel y \parallel^2$$

(Jurdjevic-Quinn)

Beyond control via passivity: what can we do if the desired set-point is **not** a minimum of the storage function ??

Recall the proof of stability of the equilibrium $(\omega_1^*, 0, 0)$ of the Euler equations for the angular velocities $\omega_1, \omega_2, \omega_3$ of a rigid body

$$\begin{aligned} I_1 \dot{\omega}_1 &= (I_2 - I_3)\omega_2\omega_3 \\ I_2 \dot{\omega}_2 &= (I_3 - I_1)\omega_3\omega_1 \\ I_3 \dot{\omega}_3 &= (I_1 - I_2)\omega_1\omega_2 \end{aligned}$$

The total energy $K = \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2 + \frac{1}{2}I_3\omega_3^2$ has a minimum at $(0,0,0)$. Stability of $(\omega_1^*, 0, 0)$ is shown by taking as Lyapunov function a suitable combination of the total energy K and another **conserved quantity**, namely the total angular momentum

$$C = I_1\omega_1 + I_2\omega_2 + I_3\omega_3$$

In general, for any Hamiltonian dynamics

$$\dot{x} = J(x)\frac{\partial H}{\partial x}(x)$$

one may search for a special type of conserved quantities C, called **Casimirs**, as being solutions of

$$\frac{\partial^T C}{\partial x}(x)J(x) = 0$$

Then $\frac{d}{dt}C = 0$ for *every* H, and $H + C$ are candidate Lyapunov functions.

Set-point stabilization problem:

Consider a (lossless) Hamiltonian plant system P

$$\begin{aligned}\dot{x} &= J(x)\frac{\partial H}{\partial x}(x) + g(x)u \\ y &= g^T(x)\frac{\partial H}{\partial x}(x)\end{aligned}$$

where the desired set-point x^* is **not** a minimum of the Hamiltonian H, while the Hamiltonian dynamics $\dot{x} = J(x)\frac{\partial H}{\partial x}(x)$ does not possess useful Casimirs.

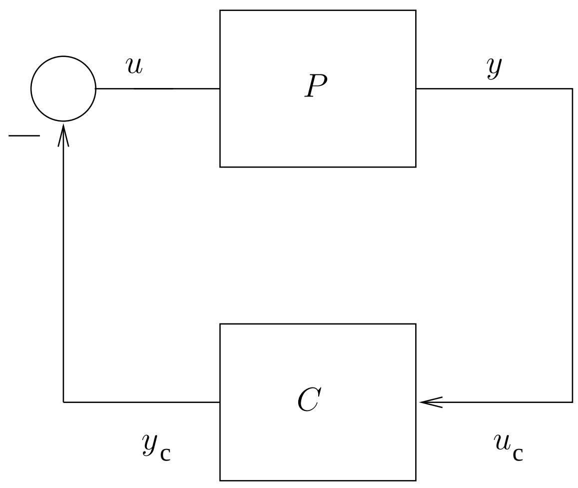
How to (asymptotically) stabilize x^* ?

Strategy: Consider a *controller* port-Hamiltonian system

$$C: \begin{array}{rcl} \dot{\xi} & = & J_c(\xi)\frac{\partial H_c}{\partial \xi}(\xi) + g_c(\xi)u_c, \quad \xi \in \mathcal{X}_c \\ y_c & = & g^T(\xi)\frac{\partial H_c}{\partial \xi}(\xi) \end{array}$$

via the standard feedback interconnection

$$u = -y_c, \quad u_c = y$$



Then the closed-loop system is the port-Hamiltonian system

$$\begin{bmatrix} \dot{x} \\ \dot{\xi} \end{bmatrix} = \begin{bmatrix} J(x) & -g(x)g_c^T(\xi) \\ g_c(\xi)g^T(x) & J_c(\xi) \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial x}(x) \\ \frac{\partial H_c}{\partial \xi}(\xi) \end{bmatrix}$$

with state space $\mathcal{X} \times \mathcal{X}_c$, and total Hamiltonian $H(x) + H_c(\xi)$.

Main idea: design the controller system in such a manner that the closed-loop system has useful Casimirs !

Thus we look for functions $C(x, \xi)$ satisfying

$$\begin{bmatrix} \frac{\partial^T C}{\partial x}(x, \xi) & \frac{\partial^T C}{\partial \xi}(x, \xi) \end{bmatrix} \begin{bmatrix} J(x) & -g(x) g_c^T(\xi) \\ g_c(\xi) g^T(x) & J_c(\xi) \end{bmatrix} = 0$$

such that

$$V(x, \xi) := H(x) + H_c(\xi) + C(x, \xi)$$

has a minimum at (x^*, ξ^*) for some (or a set of) $\xi^* \Rightarrow$ stability.

Remark: The set of such *achievable* closed-loop Casimirs $C(x, \xi)$ can be fully characterized.

Subsequently, one may add extra damping (directly or in the dynamics of the controller) to achieve *asymptotic* stability.

Example: the ubiquitous pendulum

Consider the mathematical pendulum with Hamiltonian

$$H(q,p) = \frac{1}{2}p^2 + (1 - \cos q)$$

actuated by a torque u, with output $y = p$ (angular velocity). Suppose we wish to stabilize the pendulum at a non-zero angle q^* and $p^* = 0$.

Apply the nonlinear integral control

$$\begin{aligned} \dot{\xi} &= y = p \\ u &= -y_c = -\frac{\partial H_c}{\partial \xi}(\xi) \end{aligned}$$

which is a port-Hamiltonian system with $J_c = 0$.

Casimirs $C(q,p,\xi)$ are found by solving

$$\begin{bmatrix} \frac{\partial C}{\partial q} & \frac{\partial C}{\partial p} & \frac{\partial C}{\partial \xi} \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} = 0$$

leading to Casimirs $C(q,p,\xi) = K(q-\xi)$, and candidate Lyapunov functions

$$V(q,p,\xi) = \frac{1}{2}p^2 + (1-\cos q) + H_c(\xi) + K(q-\xi)$$

with the functions H_c and K to be determined.

For a local minimum, determine K and H_c such that

Equilibrium assignment

$$
\begin{aligned}
&\sin q^* + \tfrac{\partial K}{\partial z}(q^* - \xi^*) = 0 \\
&-\tfrac{\partial K}{\partial z}(q^* - \xi^*) + \tfrac{\partial H_c}{\partial \xi}(\xi^*) = 0
\end{aligned}
$$

Minimum condition

$$
\begin{bmatrix}
\cos q^* + \frac{\partial^2 K}{\partial z^2}(q^* - \xi^*) & 0 & -\frac{\partial^2 K}{\partial z^2}(q^* - \xi^*) \\
0 & 1 & 0 \\
-\frac{\partial^2 K}{\partial z^2}(q^* - \xi^*) & 0 & \frac{\partial^2 K}{\partial z^2}(q^* - \xi^*) + \frac{\partial^2 H_c}{\partial \xi^2}(\xi^*)
\end{bmatrix} > 0
$$

Many possible solutions.

Example: stabilization of the shallow water equations

The dynamics of the water in a canal can be described by

$$\partial_t \begin{bmatrix} h \\ v \end{bmatrix} + \begin{bmatrix} v & h \\ g & v \end{bmatrix} \partial_z \begin{bmatrix} h \\ v \end{bmatrix} = 0$$

with $h(z,t)$ the height of the water at position z, and $v(z,t)$ its velocity (and g the gravitational constant).

By recognizing the total energy

$$H(h,v) = \int_a^b \mathcal{H} dz = \int_a^b \frac{1}{2}[hv^2 + gh^2] dz$$

this can be written (similarly to the telegrapher's equations) as a port-Hamiltonian system

$$
\begin{aligned}
\frac{\partial h}{\partial t}(z,t) &= -\frac{\partial}{\partial z}\frac{\partial \mathcal{H}}{\partial v}(h,v) \\
\frac{\partial v}{\partial t}(z,t) &= -\frac{\partial}{\partial z}\frac{\partial \mathcal{H}}{\partial h}(h,v)
\end{aligned}
$$

with four boundary variables

$$
\begin{aligned}
& hv_{|a,b} \\
& -(\tfrac{1}{2}v^2 + gh)_{|a,b},
\end{aligned}
$$

denoting respectively the **mass flow** and the **Bernoulli function**[a]. Two of these variables can be taken as inputs and two as outputs.

(Note that the product $hv \cdot (\frac{1}{2}v^2 + gh)$ again equals *power*.)

---

[a] Daniel Bernoulli, born in 1700 in Groningen as son of Johann Bernoulli, professor in Mathematics at the local university and founder of the Calculus of Variations.

Suppose we want to control the water level h to a desired height h^*.

The obvious 'physical' controller is to add to one side of the canal an infinite water reservoir of height h^*, corresponding to the port-Hamiltonian 'source' system

$$\begin{aligned} \dot{\xi} &= u_c \\ y_c &= \tfrac{\partial H_c}{\partial \xi} (= gh^*) \end{aligned}$$

with Hamiltonian $H_c(\xi) = gh^* \xi$, by the feedback interconnection

$$u_c = y = hv_b, \quad y_c = -u = (\frac{1}{2}v^2 + gh)|_b$$

This yields a closed-loop port-Hamiltonian system with total Hamiltonian

$$\int_0^l \frac{1}{2}[hv^2 + gh^2]dz + gh^* \xi$$

By mass balance,

$$\int_a^b h(z,t)dz + \xi + c$$

is a Casimir for the closed-loop system. Thus we may take as Lyapunov function

$$\begin{aligned} V(h,v,\xi) &:= \tfrac{1}{2}\int_a^b [hv^2 + gh^2]dz + gh^*\xi - gh^*(\int_a^b h(z,t)dz + \xi) + \tfrac{1}{2}g(b-a)(h^*)^2 \\ &= \tfrac{1}{2}\int_a^b [hv^2 + g(h-h^*)^2]dz \end{aligned}$$

showing stability of the equilibrium $(h^*, v^* = 0, \xi^*)$, with ξ^* arbitrary.

Set-point stabilization of port-Hamiltonian systems with dissipation ($R \neq 0$)

Surprisingly, the presence of dissipation may pose a problem !

$C(x)$ is a Casimir for the Hamiltonian dynamics with dissipation

$$\dot{x} = [J(x) - R(x)] \frac{\partial H}{\partial x}(x), \qquad J = J^T, R = R^T \geq 0$$

iff

$$\frac{\partial^T C}{\partial x}[J - R] = 0 \Rightarrow \frac{\partial^T C}{\partial x}[J - R]\frac{\partial C}{\partial x} = 0 \Rightarrow \frac{\partial^T C}{\partial x} R \frac{\partial C}{\partial x} = 0 \Rightarrow \frac{\partial^T C}{\partial x} R = 0$$

and thus C is a Casimir iff

$$\frac{\partial^T C}{\partial x}(x) J(x) = 0, \qquad \frac{\partial^T C}{\partial x}(x) R(x) = 0$$

The physical reason for the dissipation obstacle is that with a passive controller only equilibria where **no** energy-dissipation takes place may be stabilized.

Similarly, if $C(x,\xi)$ is a Casimir for the closed-loop port-Hamiltonian system then it must satisfy

$$\begin{bmatrix} \frac{\partial^T C}{\partial x}(x,\xi) & \frac{\partial^T C}{\partial \xi}(x,\xi) \end{bmatrix} \begin{bmatrix} R(x) & 0 \\ 0 & R_c(\xi) \end{bmatrix} = 0$$

implying by semi-positivity of $R(x)$ and $R_c(x)$

$$\begin{aligned} \frac{\partial^T C}{\partial x}(x,\xi)R(x) = 0 \\ \frac{\partial^T C}{\partial \xi}(x,\xi)R_c(\xi) = 0 \end{aligned}$$

This is the **dissipation obstacle**, which implies that one cannot shape the Lyapunov function in the coordinates that are directly affected by energy dissipation.

Remark: For shaping the potential energy in mechanical systems this is **not** a problem since dissipation enters in the differential equations for the momenta.

To overcome the dissipation obstacle

Suppose one can find a mapping $C : \mathcal{X} \to \mathbb{R}^m$, with its (transposed) Jacobian matrix $K^T(x) := \frac{\partial C}{\partial x}(x)$ satisfying

$$[J(x) - R(x)]K(x) + g(x) = 0$$

Construct now the interconnection and dissipation matrix of an *augmented system* as

$$J_{aug} := \begin{bmatrix} J & JK \\ K^T J & K^T JK \end{bmatrix}, \quad R_{aug} := \begin{bmatrix} R & RK \\ K^T R & K^T RK \end{bmatrix}$$

By construction

$$[K^T(x) \mid -I]J_{aug} = [K^T(x) \mid -I]R_{aug} = 0$$

implying that the components of C are Casimirs for the Hamiltonian dynamics

$$\begin{bmatrix} \dot{x} \\ \dot{\xi} \end{bmatrix} = [J_{aug} - R_{aug}] \begin{bmatrix} \frac{\partial H}{\partial x}(x) \\ \frac{\partial H_c}{\partial \xi}(\xi) \end{bmatrix}$$

Furthermore, since $[J(x) - R(x)]K(x) + g(x) = 0$

$$\begin{aligned} J_{aug} - R_{aug} &= \begin{bmatrix} J - R & [J - R]K \\ K^T[J - R] & K^T JK - K^T RK \end{bmatrix} \\ &= \begin{bmatrix} J - R & -g \\ [g - 2RK]^T & K^T JK - K^T RK \end{bmatrix} \end{aligned}$$

Thus the augmented system is a closed-loop system for a **different output** !

Port-Hamiltonian systems with **feedthrough term** take the form

$$\begin{aligned} \dot{x} &= [J(x) - R(x)]\frac{\partial H}{\partial x}(x) + g(x)u \\ y &= (g(x) + 2P(x))^T \frac{\partial H}{\partial x}(x) + [M(x) + S(x)]u, \end{aligned}$$

with M skew-symmetric and S symmetric, while

$$\begin{bmatrix} R(x) & P(x) \\ P^T(x) & S(x) \end{bmatrix} \geq 0$$

The augmented system is thus the feedback interconnection of the nonlinear integral controller

$$\begin{aligned} \dot{\xi} &= u_c \\ y_c &= \frac{\partial H_c}{\partial \xi}(\xi) \end{aligned}$$

with the plant port-Hamiltonian system with **modified** output with feedthrough term

$$\begin{aligned} \dot{x} &= [J(x) - R(x)]\frac{\partial H}{\partial x}(x) + g(x)u \\ y_{mod} &= [g(x) - 2R(x)K(x)]^T \frac{\partial H}{\partial x}(x) + [-K^T(x)J(x)K(x) + K^T(x)R(x)K(x)]u \end{aligned}$$

Remark: In electrical circuits the conversion from y to y_{mod} has the interpretation of Thévenin-Norton equivalence.

Generalization to feedback interconnection with state-modulation.

Recall that $K^T(x) := \frac{\partial C}{\partial x}(x)$ is a solution to $[J(x) - R(x)]K(x) + g(x) = 0$. This can be generalized to

$$[J(x) - R(x)]K(x) + g(x)\beta(x) = 0$$

with $\beta(x)$ an $m \times m$ design matrix.

The same scheme as above works if we extend the standard feedback interconnection $u = -y_c, u_c = y$ to the state-modulated feedback

$$u = -\beta(x)y_c, \quad u_c = \beta^T(x)y$$

Furthermore, $K(x)$ is a solution for some $\beta(x)$ iff

$$g^{\perp}(x)[J(x) - R(x)]K(x) = 0$$

(In fact, $\beta(x) := -(g^T(x)g(x))^{-1}g^T(x)[J(x) - R(x)]K(x)$ does the job.)

Further possibilities to generate Lyapunov functions

Recall that the set of storage functions H of a passive system generally has a minimal and maximal element (Willems, 1972):

$$S_a(x) \leq H(x) \leq S_r(x), \quad \text{for all } x.$$

where the available storage $S_a(x)$ at x is given as

$$S_a(x) = \sup_{u, T \geq 0} - \int_0^T u^T(t) y(t) dt$$

while the required supply $S_r(x)$ to reach x at $t = 0$ starting from x_0 equals

$$S_r(x) = \inf_{u, T \geq 0, x(-T) = x_0} \int_{-T}^0 u^T(t) y(t) dt$$

In the lossless case $S_a = S_r$, and thus H is unique.

Let $\tilde{H}$ be a different storage function, then there exist $\tilde{J}(x)$ and $\tilde{R}(x)$ such that

$$[J(x) - R(x)]\frac{\partial H}{\partial x}(x) = [\tilde{J}(x) - \tilde{R}(x)]\frac{\partial \tilde{H}}{\partial x}(x)$$

Hence, the same story as before can be repeated for the new data.

Remark: An effective characterization of the class of possible storage functions $\tilde{H}$, together with a characterization of the achievable Casimirs corresponding to $\tilde{J}(x)$ and $\tilde{R}(x)$ seems to be lacking currently.

State feedback interpretation of Casimir generation in the plant-controller state space: Shaping the Hamiltonian H

Restrict (without much loss of generality) to Casimirs of the form

$$C(x, \xi) = \xi_j - G_j(x)$$

It follows that for all time instants

$$\xi_j = G_j(x) + c_j, \qquad c_j \in \mathbb{R}$$

Suppose that in this way all control state components ξ_i can be expressed as function

$$\xi = G(x)$$

of the plant state x. Then the dynamic feedback reduces to a **state feedback**, and the Lyapunov function $H(x) + H_c(\xi) + C(x, \xi)$ reduces to the **shaped** Hamiltonian

$$H(x) + H_c(G(x))$$

A direct state feedback perspective: Interconnection-Damping Assignment (IDA)-PBC control

A direct way to generate candidate Lyapunov functions H_d is to look for state feedbacks $u = \hat{u}_{IDA}(x)$ such that

$$[J(x) - R(x)]\frac{\partial H}{\partial x}(x) + g(x)\hat{u}_{IDA}(x) = [J_d(x) - R_d(x)]\frac{\partial H_d}{\partial x}(x)$$

where J_d and R_d are newly assigned interconnection and damping structures.

Remark: For mechanical systems IDA-PBC control is equivalent to the theory of Controlled Lagrangians (Bloch, Leonard, Marsden, ..).

For $J_d = J$ and $R_d = R$ (so-called *Basic IDA-PBC* this reduces to

$$[J(x) - R(x)]\frac{\partial(H_d - H)}{\partial x}(x) = g(x)\hat{u}_{BIDA}(x)$$

and thus in this case, there exists an $\hat{u}_{BIDA}(x)$ if and only if

$$g^{\perp}(x)[J(x) - R(x)]\frac{\partial(H_d - H)}{\partial x}(x) = 0$$

which is the same equation as obtained for stabilization by Casimir generation with a state-modulated nonlinear integral controller !

Conclusion: Basic IDA-PBC $\Leftrightarrow$ State-modulated Control by Interconnection.

For further connections: see the paper.

Conclusions

- Unified framework for *modeling, analysis, simulation, and control* of nonlinear multi-physics systems.
- Inclusion of distributed-parameter components. Merging with semi-group approach to infinite-dimensional systems, and boundary control of PDEs.
- Control by interconnection and Casimir generation, IDA-PBC control.

◦ Suggests new control paradigms for nonlinear systems:

Example: Energy transfer control

Consider two port-Hamiltonian systems Σ_i

$$
\begin{aligned}
\dot{x}_i &= J_i(x_i)\frac{\partial H_i}{\partial x_i}(x_i) + g_i(x_i)u_i \\
y_i &= g_i^T(x_i)\frac{\partial H_i}{\partial x_i}(x_i), \qquad i = 1, 2
\end{aligned}
$$

Suppose we want to transfer the energy from the port-Hamiltonian system Σ_1 to the port-Hamiltonian system Σ_2, while keeping the total energy $H_1 + H_2$ constant.

This can be done by using the output feedback

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 & -y_1 y_2^T \\ y_2 y_1^T & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

It follows that the closed-loop system is energy-preserving.
However, for the individual energies

$$\frac{d}{dt} H_1 = -y_1^T y_1 y_2^T y_2 = -||y_1||^2 ||y_2||^2 \leq 0$$

implying that H_1 is decreasing as long as $||y_1||$ and $||y_2||$ are different from 0. On the other hand,

$$\frac{d}{dt} H_2 = y_2^T y_2 y_1^T y_1 = ||y_2||^2 ||y_1||^2 \geq 0$$

implying that H_2 is increasing at the same rate. Has been successfully applied to *tracking control* of mechanical systems (Duindam & Stramigioli).

THANK YOU !