Contents

Introduction —— VII

1 The Schwarz lemma and Riemann surfaces —— 1
1.1 The Schwarz–Pick lemma —— **2**
1.2 The Poincaré distance —— **8**
1.3 The upper half-plane —— **14**
1.4 Fixed points of automorphisms —— **19**
1.5 Multipoint Schwarz–Pick lemmas —— **28**
1.6 Riemann surfaces —— **36**
1.7 Hyperbolic Riemann surfaces and the Montel theorem —— **54**
1.8 Boundary behavior of the universal covering map —— **67**
1.9 The Poincaré metric —— **73**
1.10 The Ahlfors lemma —— **84**
1.11 Bloch domains —— **88**

2 Boundary Schwarz lemmas —— 96
2.1 The Julia lemma —— **97**
2.2 Stolz regions and nontangential limits —— **113**
2.3 The Julia–Wolff–Carathéodory theorem —— **125**
2.4 The Lindelöf theorem —— **136**
2.5 The Wolff lemma —— **143**
2.6 The automorphism group of hyperbolic Riemann surfaces —— **147**
2.7 The Burns–Krantz theorem —— **153**

3 Discrete dynamics on Riemann surfaces —— 158
3.1 The fixed-point case —— **159**
3.2 The Wolff–Denjoy theorem —— **166**
3.3 The Heins theorem —— **169**
3.4 Stability of the Wolff point —— **182**
3.5 Models on Riemann surfaces —— **184**
3.6 Random iteration on Bloch domains —— **192**
3.7 Random iteration of small perturbations —— **201**

4 Discrete dynamics on the unit disk —— 212
4.1 Elliptic dynamics —— **213**
4.2 Superattracting dynamics —— **217**
4.3 Hyperbolic dynamics —— **221**
4.4 Parabolic dynamics —— **226**
4.5 Models on the unit disk —— **234**
4.6 The hyperbolic step —— **240**

4.7 Parabolic type and boundary smoothness — **251**
4.8 Boundary fixed points — **260**
4.9 Backward dynamics — **266**
4.10 Commuting functions — **286**

5 Continuous dynamics on Riemann surfaces — 294
5.1 Algebraic semigroup homomorphisms — **295**
5.2 One-parameter semigroups — **297**
5.3 One-parameter semigroups on Riemann surfaces — **300**
5.4 The infinitesimal generator — **304**
5.5 The continuous Wolff–Denjoy theorem — **313**
5.6 The Berkson–Porta formula — **315**
5.7 One-parameter semigroups on the unit disk — **320**

A Appendix — 325
A.1 The Hurwitz theorems — **325**
A.2 The Fatou uniqueness theorem — **326**
A.3 Holomorphic functions with nonnegative real part — **328**
A.4 Sequences — **331**
A.5 Topological groups — **334**

Bibliography — 337

Index — 353