# Chun-xiang XU\*, Wei SHI, Dong-mei CHEN, Xiang-wei WU **Coordinate Transformation on CNC Machining of Quasi-Hypoid-Gear**

**Abstract:** The structure model of mechanical shaking table machine tools and CNC gear milling machine was analyzed and the corresponding machine coordinate system was established in this paper. Under the condition of ensuring constant relative motion and relative position between the work piece and the tool of the shaking type machine tool. Using the principle of coordinate transformation, the function of instantaneous moving position was deduced as NC milling machine processing to program of CNC machining.

**Keywords:** coordinate transformation; quasi-hypoid-gear; CNC machining model

# **1 Introduction**

Now the production of the gears, especially the helical bevel gears occupied the important position in modern manufacturing industry, particularly a considerable proportion in automobiles, tractors and machine tools. Quasi-hypoid-gear is more complex in the form of the helical bevel gear and the overlapped coefficient is greater than the bevel gear with the intersecting shafts. So it is more stable in motion. Just because of this, it is almost universally used in cars [1].

The extensive use of multi-axis machine tools has brought about great advances in manufacturing technology of helical bevel gear, changing the tedious adjustment situation in machine producing of traditional shaking table machines.

Using the principle of the special coordinate transformation, the mechanical shaking desktop machine tool for quasi-hypoid gears can be equivalently switched to the environment of CNC machine tool.

# **2 CNC transformation model**

Figure 1 is a conception model of traditional shaking desktop machine tool [2], as can be seen, the number of the shaft is up to ten in traditional shaking desktop machine tool, and most of them were used for machine adjustment before working [4]. In the

**<sup>\*</sup>Corresponding author: Chun-xiang XU,** Zhengzhou Institute of Technology, Zhengzhou, China, E-mail: xchx\_2002@126.com

**Wei SHI, Dong-mei CHEN, Xiang-wei WU,** Zhengzhou Institute of Technology, Zhengzhou, China

process of working, only the cradle shaft and the piece spindle are in linkage to form generating motion which achieved by gear train [3]. Various adjustment and different change gears are used for manufacturing different gears, so the transmission chain is complex. While 5shafts linkage can be realized at Phoenix Free-form machine as working by NC system [5]. The result is shorter transmission chain, more exact motion and wider manufacturing range.



**Figure 1.** Traditional Mechanical Shaking Table Machine Tool

The essence of transformation from the traditional shaking desktop machine to free machining machine tool is to reproduce the processing principle of traditional machine tool on new machine tool. This needs to ensure any position while processing, the relative position of the cutter and the work piece is the same with traditional machine tool.

To reappear motion relationship between the tool coordinate system St and the work piece coordinate system Sp of the machine, the transition matrix of St and Sp is firstly studied.

- *1)* Suppose that there is a 4X4 coordinate transformation matrix of *Mpt(k)*(k=C, G), where the superscript C and G respectively represent CNC gear milling machine and mechanical gear milling machine.
- *2)* If there is the equation (1) of  $M_{p}^{(C)} = M_{p}^{(G)}$ , there can guarantee the tool and the work piece are in the same relative motion direction and position when generating the tooth surface of two kinds of the machine tools.

#### **2.1 The traditional mechanical shaking table machine coordinate system**

Set machine tool machining coordinate system according to the model of structure of traditional mechanical shaking table machine. The following is to take manufacturing a pinion on a hobbing machine for example to establish the machine tool coordinate system.

The following coordinate system is set: the fixed coordinate system Sm1 and Sq which rigidly consolidated with the gear cutting machine is shown in Figure 2 and Figure 3; respectively correspond to the moving coordinate system Sc and S1 of the cutting machine cradle and the pinion rigid consolidation [6]. During working, the cradle and Sc rotate around the axis Zm1 at the angular velocity of  $\ddot{\vec{Q}}^{(c)}$ , while the pinion and S1 rotate around the axis Xq at the angular velocity of  $\vec{\omega}^{(1)}$ .



**Figure 2.** Sm1, Sc and S b Coordinate System

The cutter disc mounted on the shaking table rotates with it. Coordinate system St1 rigidly consolidated to the shaking table. In order to describe the tool relative to the direction of the shaking table, use the coordinate system of Sb to achieve the following steps: Firstly consolidate coordinate system Sb to St1, then turn them as one rigid body around the axis of Zc to a rotation angle j; secondly,, incline the tool disc and St1 system to an angle of i around Yb axis. The tool disc can rotate around its axis of Zt1.

The parameters of installation and adjustment of the pinion follows.

Em: Offset of the wheel blank on the machine tool, Γm: installation angle of the wheel blank, ΔB: moving distance of slider, ΔA: Distance from the machine center to supporting surface of the blank.

The parameter of installation and adjustment of the tool disc follows.

SR: Radial installation adjustment value, j: swivel angle, i: Cutter tilt

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**Figure 3.** Machine Coordinate System as Manufacturing Pinion

According to Figure 3, the following number sentence can be gotten through coordinate transformation of matrix [1].

$$
M_{p}^{(G)} = M_{1q} M_{q} M_{m1} M_{m1c} M_{b} M_{b1} = \begin{bmatrix} a_{1} a_{2} a_{3} a_{4} \\ a_{2} a_{2} a_{3} a_{4} \\ a_{3} a_{2} a_{3} a_{4} \\ a_{4} a_{4} a_{4} a_{4} \end{bmatrix}
$$

Among them:

$$
a_{11} = \cos i \cos \gamma_m \sin (q - j) - \sin i \sin \gamma_m
$$
  
\n
$$
a_{12} = -\cos (q - j) \cos \gamma_m
$$
  
\n
$$
a_{13} = \sin i \cos \gamma_m \sin (q - j) + \cos i \sin \gamma_m
$$
  
\n
$$
a_{14} = S_R \cos q \cos \gamma_m - \Delta B \sin \gamma_m - \Delta A
$$
  
\n
$$
a_{21} = \cos i \sin \gamma_m \sin \varphi_1 \sin (q - j) + \cos i \cos (q - j) \cos \varphi_1
$$
  
\n
$$
+ \sin i \cos \gamma_m \sin \varphi_1
$$
  
\n
$$
a_{22} = -\cos (q - j) \sin \gamma_m \sin \varphi_1 + \sin (q - j) \cos \varphi_1
$$
  
\n
$$
a_{23} = \sin i \sin \gamma_m \sin \varphi_1 \sin (q - j) + \sin i \cos (q - j) \cos \varphi_1
$$
  
\n
$$
- \cos i \cos \gamma_m \sin \varphi_1
$$
  
\n
$$
a_{24} = -S_R (\sin q \cos \varphi_1 - \cos q \sin \gamma_m \sin \varphi_1) + E_m \cos \varphi_1
$$
  
\n
$$
+ \Delta B \cos \gamma_m \sin \varphi_1
$$
  
\n
$$
a_{31} = -\cos i \sin \gamma_m \cos \varphi_1 \sin (q - j) + \cos i \cos (q - j) \sin \varphi_1
$$
  
\n
$$
- \sin i \cos \gamma_m \cos \varphi_1
$$

$$
a_{32} = \sin \gamma_m \cos \varphi_1 \cos(q-j) + \sin(q-j) \sin \varphi_1
$$
  
\n
$$
a_{33} = -\sin i \sin \gamma_m \sin(q-j) \cos \varphi_1 + \sin i \cos(q-j) \sin \varphi_1
$$
  
\n
$$
+ \cos i \cos \gamma_m \cos \varphi_1
$$
  
\n
$$
a_{34} = -S_R (\sin q \sin \varphi_1 + \cos q \sin \gamma_m \cos \varphi_1) + E_m \sin \varphi_1
$$
  
\n
$$
- \Delta B \cos \gamma_m \cos \varphi_1
$$
  
\n
$$
a_{31} = a_{32} = a_{33} = 0, a_{34} = 1
$$

#### **2.2 CNC Machine Tool Coordinate System**

The coordinate system of the Phoenix Free-form machine is shown in Figure 4.

As to seen in Figure 4, coordinate system St and Sp is respectively consolidated to the tool and the work piece, Sh and Sm is respectively consolidated to the cutter base and the sliding base. Coordinate axis Sh is parallel to coordinate axis Sm. (x,y,z) is used to represent the position of Sh relative to the position of Sm. Coordinate St rotates around the axis of zh relative to Sh. Coordinate system Se and Sd is used to describe coordinate transformation from Sm to Sp.

Coordinate system Se achieves rotation around the axis of ym. The coordinate axis of coordinate system Sd parallels to coordinate axis of Se; ΔX =C is used to confirm the original point position of Od relative to Oe. Coordinate system Sp achieves rotation around the axis of xd relative to Sd.



**Figure 4.** The Coordinate System of Phoenix Free-form

According to Figure 4, the following number sentence can be gotten through coordinate transformation of matrix as the following formula (3).

$$
M_{pl}^{(c)} = M_{pd} M_{de} M_{em} M_{mh} M_{hl} = \begin{bmatrix} b_{11}b_{12}b_{13}b_{14} \\ b_{21}b_{22}b_{23}b_{24} \\ b_{31}b_{32}b_{33}b_{34} \\ b_{41}b_{42}b_{43}b_{44} \end{bmatrix}
$$

 $b_{11} = \cos \mu \cos \varphi$ ;  $b_{12} = -\sin \mu \cos \varphi$ ;  $b_{13} = \sin \varphi$ ;  $b_{14} = \Delta X - x \cos \varphi - z \sin \varphi$  $b_{21} = -\cos \mu \sin \varphi \sin \psi + \sin \mu \cos \psi;$  $b_{22}$  = sin  $\mu$  sin  $\varphi$  sin  $\psi$  + cos  $\mu$  cos  $\psi$ ;  $b_{23}$  = cos  $\varphi$  sin  $\psi$ ;  $b_{24} = x \sin \varphi \sin \psi - y \cos \psi - z \cos \varphi \sin \psi;$  $b_{31} = -\cos \mu \sin \varphi \cos \psi - \sin \mu \sin \psi;$  $b_{32}$  = sin  $\mu$  sin  $\varphi$  cos  $\psi$  - cos  $\mu$  sin  $\psi$ ;  $b_{33}$  = cos  $\varphi$  cos  $\psi$ ;  $b_{34}$  = x sin  $\varphi$  cos  $\psi$  + y sin  $\psi$  - z cos  $\varphi$  cos  $\psi$ ;  $b_{41} = b_{42} = b_{43} = 0, b_{44} = 1$ 

According to formula (2) and (3), solving the equation (1) can be used to determine the NC model of Phoenix Free-form machine tool by adjusting parameter of traditional shaking table milling machine.

### **3 Parameter Determination**

*1)* Determine the installation angle  $\varphi$  of the pinion of the work piece,  $\sin \varphi = a_{\pi}$ ; *2)* Determine the work piece spindle rotation angle  $\psi$ ,  $\cos\varphi \cos\psi = a_3$ ;

*3)* Determine the cutter spindle rotation angle  $\mu$ ,  $\cos \mu \cos \phi = a_1$ ,  $-\sin \mu \cos \phi = a_2$ ; *4)* Determine the displacement of the three axes of X, Y, Z, and solve the system of linear equation contained three unknown numbers of x, y, z

$$
\begin{cases} a_{\mathfrak{u}} = b_{\mathfrak{u}} \\ a_{\mathfrak{u}} = b_{\mathfrak{u}} \\ a_{\mathfrak{u}} = b_{\mathfrak{u}} \end{cases}
$$

In summary, the six-axis linkage instantaneous movement position can be obtained when machining hypoid pinion by the face hobbing method on CN milling machine, instantaneous movement position of each axis can be expressed by a function to realize the machine tool programming and machining. It also provides the reference for setting the parameters of the Quasi-Hypoid-Gear CNC machining.

# **4 Conclusion**

The machine model of the mechanical shaking table milling machine and CN milling machine was analyzed when processing the quasi-hypoid-gear and the machining coordinate system was established in the paper. According to the working principle, coordinate transformation was completed as machining of CN milling machine. The instantaneous movement position of each axis was got when CN milling gear by coordinate transformation. It can be applied in simulation study on CN milling machine tool of making quasi-hypoid-gear and the parameter transformation between the mechanical machine tool and the CN machine tool and has theoretical and practical value. The method provides the practical and theoretical value in the coordinate transformation and calculation of the multi-axial linkage machine, the working simulation of CNC machining of Quasi-Hypoid-Gear and the parameter settings of multi-axial CNC machines.

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