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Singular Configuration Analysis for the Structure of Hybrid Grinding and Polishing Machine

Abstract: Blade is a key component in the energy power equipment. Researchers have made a lot of research on precision machining equipment and process for blade. In this paper, based on the designed hybrid grinding and polishing machine tool for blade finishing, we analyzed the singular configuration and designed the reasonable dimension for it, which help us avoid or predict the singular configuration. Firstly, we analyzed the singular configuration of parallel mechanism by using the motion condition of the singular configuration. We established the utmost position of model of the parallel mechanism by using geometric method in Matlab. Secondly, the parallel mechanism of the series and parallel hybrid structure was replaced by the model of series structure, which can simplify the structure of machine to a model of series structure. Finally, the whole structure of machine was analyzed by spinor method. The analysis results show that the designed machine tool for blade finishing has avoided the singular configuration.

Keywords: singular configuration; 3-RPS parallel mechanism; limit position; hybrid grinding and polishing machine

1 Introduction

Blade is a key and major component in the energy power equipment of turbine, marine propeller, aircraft engines and so on. It is characterized by complex surface structure shape, difficult processing and high precision. As a result, the finishing process of blade is one of the most important and difficult problems in the field of aerospace. In recent years, researchers have made a lot of research on precision machining equipment and process for blade. Such as MTS1000-6CNC the six axis belt grinding machine of IBS company, HS-196GC six-axis grinding center of Huffman company, five axis linkage high precision vertical machining center of C.B. Ferrari company, copy grinding machine of Japan's Okamoto machinery [1-4]. Regardless of any form of mechanism, singular configurations are always unavoidable [5]. It can be summed up that when the mechanism during exercise extreme point, uncontrolled movements, or changes occur freedom, the mechanism will lost a smooth, mechanical or kinematic

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properties of the phenomenon of instantaneous mutation. This makes the transfer mechanism motion and power capacity abnormal, in this case the structure in which the position of singularities. Singular configuration can't be avoided. The structure of machine early in the design should be calculated on the singular configurations and design a reasonable structure size, so we can avoid singular configurations or predict singular configurations and make compensation in control system [6]. One of the main concerns in the design of parallel mechanism is kinematic singularities. The notion of singularity in parallel mechanism kinematics refers to configurations in which a mechanism manipulator either loses or gains one or more degrees of freedom instantaneously. In other words, if a parallel mechanism is in a singular configuration, it will lose its designated motion and working capability. Hence, to design a parallel robot with a desired performance, e.g., high rigidity and manipulability, singularities must be excluded from its working area, if possible. As the singular configuration has an important influence on the performance of the mechanism, it has been paid more and more attention by researchers. Hunt apply the spinor theory to the singular configuration. Mer-let used Grassmann Geometry analyzed a special singular configuration of Stewart Platform. Kumar introduced a singular point because of drive joint selection. Gosselin and Angeles studied the singularity problem is generally parallel mechanism [7].

In this paper, the designed machine is series and parallel hybrid structure, which is shown in Figure 1 [8]. The machine is consisted by 3-RPS parallel mechanism and series structure. According to the processing requirement, the parallel mechanism just need rotate around the X axis and moving along the Z axis. Besides, the series structure include that moving along X axis, Y axis, rotating around Y axis. The parallel mechanism was analyzed by the motion condition of the singular configuration, which can reduce the calculation. When the parallel mechanism gets the result, we can convert the series and parallel hybrid structure to a model of series structure according to the result. Finally, the whole structure of machine can be analyzed by spinor method. The spinor method can carry out the results quickly. So the size of the machine can be designed according to the result of all the analysis and avoid the singular configuration.

2 Singular Configuration Analysis of Parallel Mechanism

3-RPS parallel mechanism is the most complex structure when we analyze serial hybrid grinding and polishing tool for singular configurations. There are many ways to analyze the singular configuration of parallel mechanisms such as Jacobi matrix, Grassmann law line geometry and kinematics condition [9], in which the kinematics condition is simple, so this paper we analyzes the singular configuration in kinematic condition [10]. The parallel mechanism model is shown in Figure 2.

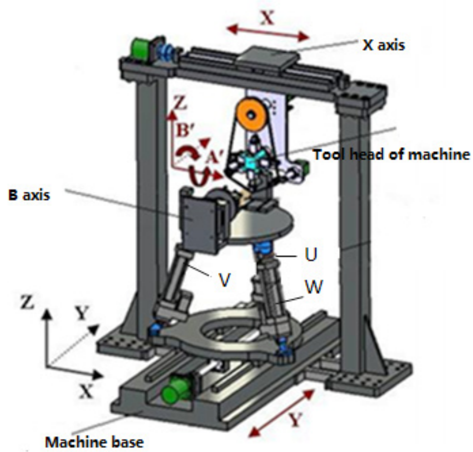


Figure 1. Structure of the machine.

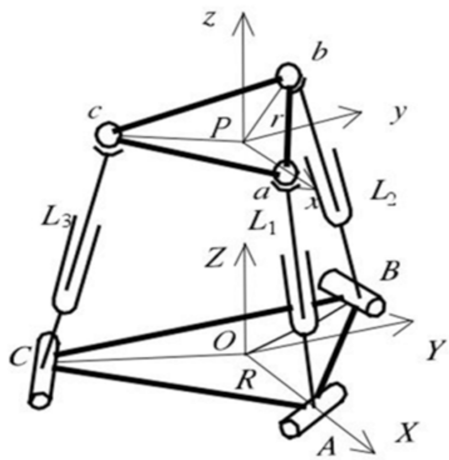


Figure 2. Parallel mechanism diagram.

According to the need of machining, the radius r of the moving platform is 250mm and the radius R of the static platform is 450mm. A, B, C of parallel mechanism is static platform triangle vertex respectively, in static platform coordinate system O-XYZ, the coordinates of three points are that: $A=(R,0,0)$ $B=(-R/2,\sqrt{3}R/2,0)$ $C=(-R/2,\ -\sqrt{3}R/2,0)$; a, b, c of parallel mechanism is moving platform triangle vertex respectively, in the moving platform coordinate system P-xyz, the coordinates of three points are that: $a=(r,0,0)$ $b=(-r/2,\sqrt{3}r/2,0)$ $c=(-r/2,\ -\sqrt{3}r/2,0)$.

P-xyz moving platform coordinate system relative to the static platform coordinate system conversion O-xyz relations can be expressed as the homogeneous matrix T_o. P-xyz coordinate system of the three main unit vector with respect to the coordinate system O-XYZ direction cosine were presented by $x_{m,}$ $y_{m,}$ z_m (m=k,j,i). $X_{p,}$ $Y_{p,}$ Z_p represent the position coordinates of the point P in the O-XYZ.

$$T_0 = \begin{bmatrix} x_i & y_i & z_i & X_p \\ x_j & y_j & z_j & Y_p \\ x_k & y_k & z_k & Z_p \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{1}$$

Direction and position of the parallel mechanism described by Euler angles [5]. The moving platform will rotating around the z,x,y axis, which are angle for α, γ, β. According to the requirement of movement of Serial hybrid grinding and polishing tool, 3 - RPS parallel mechanism of the rod L_3, L_2 have the same state of motion. The moving platform only rotates around the Y axis, so α=γ=0. Finally we get the value of T by (1).

In the coordinate system P-xyz, m_p is a point of the moving platform. In the coordinate system O-XYZ, m_p is converted to m_o.

$$T = \begin{bmatrix} \cos\beta & 0 & \sin\beta & X_p \\ 0 & 1 & 0 & Y_p \\ -\sin\beta & 0 & \cos\beta & Z_p \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \begin{bmatrix} m_o \\ 1 \end{bmatrix}_{XYZ} = T\begin{bmatrix} m_p \\ 1 \end{bmatrix}. \tag{2}$$

The three point a, b, c values into (2), so we can get the results.

$$\begin{cases} a_o = \left(r\times\cos\beta + X_p \quad Y_p \quad -r\times\sin\beta + Z_p\right) \\ b_o = \left(-r\times\cos\beta/2 + X_p \quad \sqrt{3}r/2 + Y_p \quad r\times\sin\beta/2 + Z_p\right) \\ c_o = \left(-r\times\cos\beta/2 + X_p \quad -\sqrt{3}r/2 + Y_p \quad r\times\sin\beta/2 + Z_p\right) \end{cases}. \tag{3}$$

Because of the structure characteristics of 3-RPS parallel mechanism, a, b and c of the three ball hinge centers of the moving platform must be in the plane of the three perpendicular to the static platform. In the static platform coordinate system O-XYZ, the projection equation of three planes is obtained.

$$Y = 0, \quad Y = -\sqrt{3}X, \quad Y = \sqrt{3}X. \tag{4}$$

The (3) are put into the (4) corresponding to the projection plane, so we can get the values of X_p and Y_p. Beside the r is 250mm.

$$X_p = 125(\cos\beta - 1), \quad Y_p = 0. \tag{5}$$

The motion condition of the singular configuration has the following performance in the 3-RPS parallel mechanism [5]. When moving platform motion, three ball joint center points a, b, c of the instantaneous velocity is V_1, V_2, V_3, which corresponding to the plane recorded as D_1, D_2, D_3. When these three planes hand in a point n and n is located in a plane which posed by points a, b, and c, the mechanism is located in the singular configuration. According to these conditions, we can get the trajectory equation P of the center point of the moving platform when the singular configuration occurs. Z_p has three possible conditions when the singular configuration occurs.

$$\begin{cases} 1.Z_p = -125\sin\beta \\ 2.Z_p = -(525+125\cos\beta)\times\sin\beta/(1+\cos\beta); \beta\in\left[-15^\circ, 15^\circ\right] \\ 3.Z_p = (325-125\cos\beta)\times\tan\beta \end{cases} \tag{6}$$

According to (5) and (6), the trajectory of the P with a singular configuration is obtained as shown in Figure 3 by Matlab, when the moving platform is rotated in $\beta\in[-15°, 15°]$. In the end, the singular configuration may occur at the maximum position at Z_1=85mm.

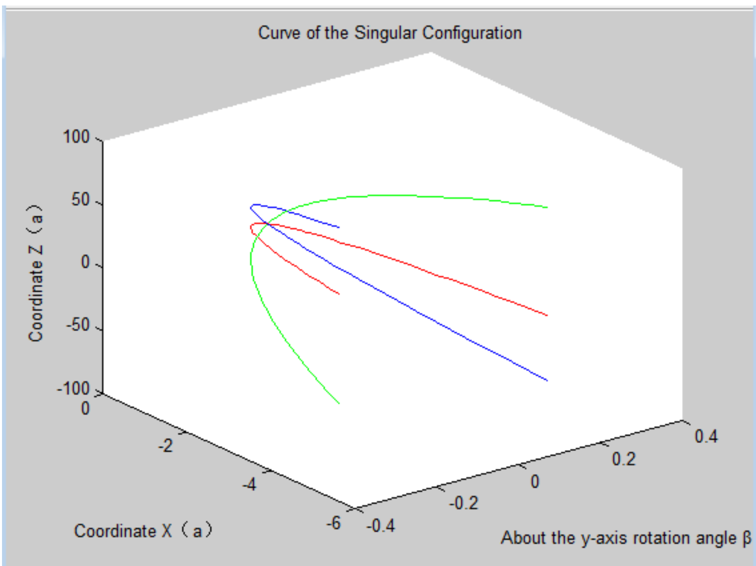


Figure 3. Singular configuration locus diagram.

3 Limit Position Analysis of Parallel Structure

According to the needs of machine tools, the initial length of the three electric cylinders L_1, L_2, L_3 of the 3-RPS parallel mechanism is set to be 620 mm and maximum elongation of electric cylinder is ΔL=160mm. In this dimension, the limit state of parallel mechanism is calculated. Because of electric cylinder L_2, L_3 running state is consistent, according to Figure 1 shows, the electric cylinder L_2, L_3 and BC and bc constant form an isosceles trapezoid, as shown in Figure 4.

Because of the characteristics of the 3-RPS parallel motion, aA and vertical center line hH of isosceles trapezoid constant positioned in the same plane, which constitute the plane hHaA, as shown in Figure 5. The geometric center P of moving platform is located in ah and we can get the results is that 2hP=aP by geometric properties of equilateral triangle. The spatial motion of the geometric center point P is converted to the model of plane motions.

The point H is coincident with the origin of the coordinate system O and Linear HA and Y axis coincidence. Suppose point h coordinates (x_1, y_1); point a coordinates (x_2, y_2); point P coordinates (x_3, y_3). The turning angle of the line segment ha with respect to the line segment HA is β. We can establish the boundary condition of the point P motion through the plane motion model.

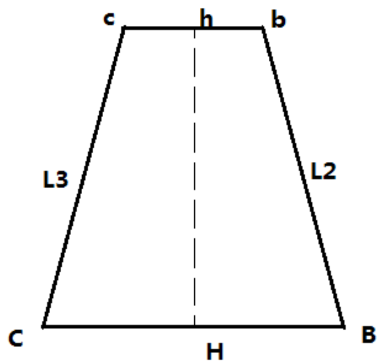


Figure 4. Isosceles trapezoid.

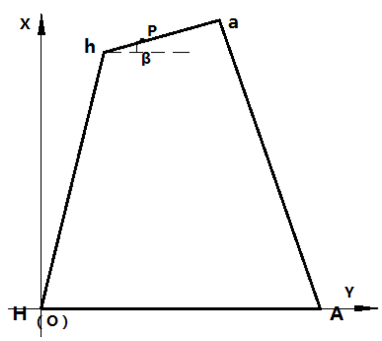


Figure 5. Point P motion plane model.

$$\begin{cases} x_1^{\ 2} + y_1^{\ 2} = l_5^{\ 2} & l_5 \in [595, 761] \\ (x_2 - 675)^2 + y_2^{\ 2} = l_1^{\ 2} & l_1 \in [595, 761] \\ (x_1 - x_2)^2 + (y_1 - y_2)^2 = 375^2 & \\ (y_1 - y_2)/(x_1 - x_2) = \tan\beta & \beta \in [-15^\circ, 15^\circ] \\ 0 \leqslant x_1 \leqslant 675 & \\ 0 \leqslant y_1 \leqslant 761 & \\ 0 \leqslant x_2 \leqslant 675 & \\ 0 \leqslant y_2 \leqslant 780 & \end{cases} \quad (7)$$

The point P vertical coordinate y_3 is the movement range of the moving platform in the Z direction.

$$y_3 = (y_2 + 2y_1)/3. \quad (8)$$

In the MATLAB with the search method [5] to calculate the range of y_3 value: 554mm~754mm. According to (7) and (8). Because the minimum value of the point P is 554mm, which is far greater than the maximum position 85mm of the singular configuration, so the design of the parallel mechanism is reasonable and has avoided the singular configuration.

4 Analysis of the Singular Configuration of whole Machine Tool

The mixed structure should be split, before the analysis of the singular configuration of the series and parallel hybrid structure. More complex structure should be analyzed separately, then a simplified alternative model for finding complex structures. The simplified model into the hybrid structure, and then establish a hybrid structure model, finally on the overall analysis. The machine tool is a series and parallel hybrid structure. The parallel structure has been analyzed in the above and there is no singular configuration in the required range of motion [7]. It can be reduced to a mobile vice and a revolute pair, which are converted into the series and parallel hybrid structure, so we can get a simplified model as shown in Figure 6.

O-XYZ is the global inertial coordinate system; θ1 is a mobile vice and moving along the X axis, beside its initial length is L1; θ2 is a mobile vice and moving along the Y axis, beside its initial length is L2; θ3 is a mobile vice and moving along the Z axis, beside its initial length is L3; θ4 is a revolute pair and rotate around the X axis; θ_5 is a revolute pair and rotate around the Y axis; θ_3 and θ_4 are the equivalent substitution model of parallel mechanism.

The hybrid structure of the machine tool is simplified as a series structure. It is shown that the singular configuration of the Jacobi matrix is the occurrence of reduced rank, and the linear correlation between the various kinematic pairs. We calculate the Jacobi matrix by spinor method [11].

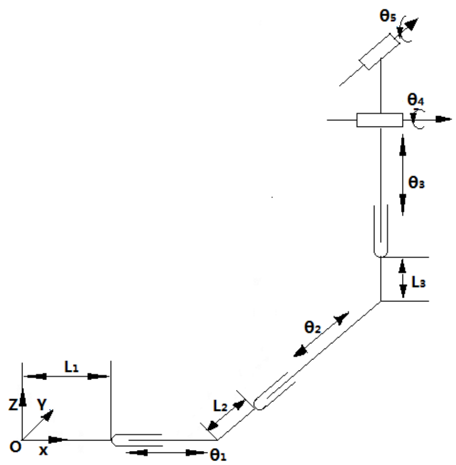


Figure 6. Schematic diagram of machine tool.

We can get the formula of the spinor method.

$$\$ = (s \quad ; \quad s^0) = (s \quad ; \quad r \times s + hs). \tag{9}$$

The s is the unit vector in the direction of the axis of spinor; the r is a point on the spinor axis; the s^0 is dual of the spinor vector; the h is pitch. We can get Jacobi matrix.

$$J(\theta) = (\$_1 \quad \$_2 \quad \$_3 \quad \cdots \quad \$_i \quad \cdots \quad \$_n). \tag{10}$$

$\theta_{1,}$ $\theta_{2,}$ $\theta_{3,}$ $\theta_{4,}$ θ_5 are substituted into (9), so we can get $\$_1$, $\$_2$, $\$_3$, $\$_4$, $\$_5$.So the $\$_1$, $\$_2$,$\$_3$,$\$_4$,$\$_5$ are substituted into (10).

$$J(\theta) = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \cos\theta_4 \\ 0 & 0 & 0 & 0 & \sin\theta_4 \\ 1 & 0 & 0 & 0 & (L_2+\theta_2)\sin\theta_4 - (L_3+\theta_3)\cos\theta_4 - L \\ 0 & 1 & 0 & L_3+\theta_3 & -(L_1+\theta_1)\sin\theta_4 \\ 0 & 0 & 1 & -L_2-\theta_2 & (L_1+\theta_1)\cos\theta_4 \end{bmatrix}. \tag{11}$$

Because the J(θ) is the 6 row and 5 column type, the analysis of the rank need to be discussed. When the rank of the velocity Jacobi matrix is 5, it can ensure that the spinor of each pair of motion is linearly independent. At this time, the series mechanism has no singular configuration.

When cosθ4•sinθ4≠0,Jacobi matrix can be simplified.

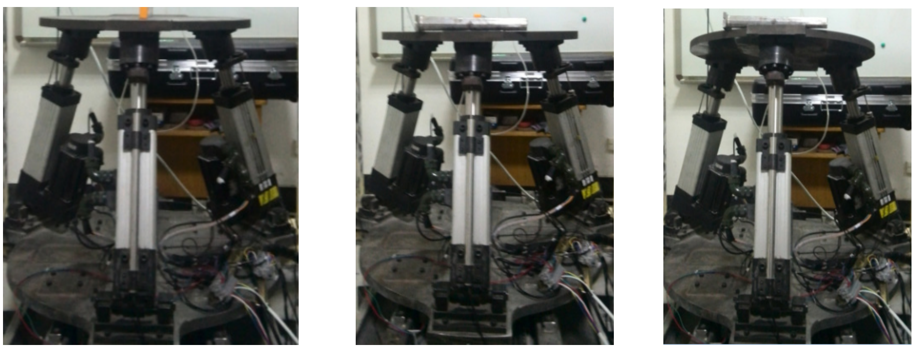
$J(\theta)=[0,0,0,1,0;0,0,0,0,\sin\theta_4;1,0,0,0,0;0,1,0,0,0;0,0,1,0,0]$.

Rank constant was 5, there was no reduction in rank; When $\cos\theta_4=0$, speed Jacobi matrix can be simplified as $J(\theta)=[0,0,0,1,0;\ 0,0,0,0,\sin\theta_4;1,0,0,0,0;0,1,0,0,0;0,0,1,0,0]$.

Rank constant was 5, there was no reduction in rank; When $\sin\theta_4=0$, speed Jacobi matrix can be simplified as $J(\theta)=[0,0,0,1,0;\ 0,0,0,0,\sin\theta_4;1,0,0,0,0;0,1,0,0,0;0,0,1,0,0]$. Rank constant was 5, there was no reduction in rank. So the series mechanism has no singular configuration.

5 Experiment

In order to test the analysis about singular configuration, We design these experiments. When moving and static platform of the 3-RPS parallel mechanism are coincident, the moving is stopped, which is a singular configuration [9]. When the length of the three electric cylinder is equal, the parallel mechanism is in the initial position. It shows in Figure 7a. It is non-singular configuration through the motion condition of the singular configuration. This position will be locked in the electric brake cylinder, the moving platform of the applied force, as shown in Figure 7b, which did not appear the phenomenon of structural instability. In the range of motion of the machine tool, the parallel mechanism rotates one angle around the X axis, and the phenomenon of instability of parallel mechanism is not appeared, as shown in Figure 7c. So the theoretical analysis is correct. In the same way, through the machine tool overall motion experiment, the structure is stable, does not appear the movement instability phenomenon.



(a) Initial position (b) Exerting force (c) Rotating 15 degree

Figure 7. Non-singular configuration of experiment of parallel mechanism.

6 Conclusion

According to the experiment, the theory of singular configuration is correct. In the 3-RPS parallel mechanism minimum value of the point P is 554mm, which is far greater than the maximum position 85mm of the singular configuration, so the design of the parallel mechanism has avoided the singular configuration. To sum up the overall speed of the machine tool the rank of Jacobi matrix is constant 5, so there is no singular configuration. The machine should be designed according to the processing requirements of machine tools. In the future we will analysis that if the mechanism has singular configurations, how to avoid the singular configurations by using the method of control.

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