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A Kind of Self-tuning Kalman Filter for the High Maneuvering Target Tracking System

Abstract: For the high maneuvering target tracking system, in order to solve the filtering problem with unknown noise statistics, based on the jerk model, a kind of self-tuning filter is presented in this paper. System identification method is used to estimate the noise statistics information which is substituted into the optimal Kalman filter to get the self-tuning Kalman filter. It's proved that this self-tuning Kalman filter converges to the optimal Kalman filter in this paper. A simulation example shows the effectiveness of this kind self-tuning filter.

Keywords: Jerk model; unknown noise statistics information; self-tuning Kalman filter

1 Introduction

Kalman filter has been proved to be the best method to deal with the filtering problem in the target tracking field [1]. In order to get the optimal convergence of the filter, noise statistics information should be precisely known. But in the practical applications, especially in the early stage of the filtering process, noise statistics information is always unknown [2]. In order to solve the filtering problem with unknown noise statistics, self-tuning filter was presented in [3]. Its basic principle is to estimate the unknown parameters and noise statistics information by system identification method. The estimations could be used to replace the real values in the Kalman filter [4]. Based on the Dynamic Error System Analysis (DESA) method [5], it has been proved that the self-tuning filter has good convergence to the optimal Kalman filter. Deng presented self-tuning α - β filter [6] and α - β - γ filter [7], which could deal with the models with unknown parameters and noise statistics information. In these filters, the target state vector includes position, velocity and acceleration. But when the target has high maneuverability, the tracking ability of these filters will be declined. Jerk model is known to be a accurate model with high order in these years. To get more accurate estimations, the acceleration rate was considered in the Jerk model filter [8], which could deal with the system with acceleration rate. This paper try to give a kind

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of self-tuning filter based on Jerk model to deal with the vector with acceleration rate. The convergence of this kind self-tuning filter is proved in this paper. An example shows the good convergence of this kind self-tuning Kalman filter.

2 Problem formulation

Consider the time-invariant system

$$X(t+1) = FX(t) + \Gamma w(t) \tag{1}$$

$$y(t) = HX(t) + v(t) \tag{2}$$

where t is the discrete time, $X(t)$ is state vector, $X(t) = [x(t), \dot{x}(t), \ddot{x}(t), \overset{\cdot}{\ddot{x}}(t)]^T$, $x(t)$ is the position of the target, $\dot{x}(t)$ is the speed of the target, $\ddot{x}(t)$ is the acceleration of the target, $\overset{\cdot}{\ddot{x}}(t)$ is the acceleration rate(jerk $j(t)$) of the target. F , Γ and H are constant matrices. T is sampling period.

Assumption 1. $w(t)$ and $v_i(t)$ are uncorrelated white noises with zeros mean and variances σ_w^2 and $\sigma_{v_i}^2$ respectively, the values of σ_w^2 and $\sigma_{v_i}^2$ are both unknown.

Assumption 2. The observational processes $y(t)$ is bounded, $|y(t)| \leq c_1$.

The jerk's self-correlation function

$$r_j(\tau) = E[j(t)j(t+\tau)] = \sigma_j^2 \exp(-\alpha|\tau|) \tag{3}$$

τ is the interval of two discrete time, σ_j^2 is the variance of $j(t)$, α is the constant of the self-correlation function, when α is very small, $\alpha \rightarrow 0$, $j(t)$ can be treated as a constant value, F has the form

$$\lim_{\alpha \rightarrow 0} F = \begin{pmatrix} 1 & T & T^2/2 & T^3/6 \\ 0 & 1 & T & T^2/2 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{4}$$

$$\Gamma = [T^3/6 \quad T^2/2 \quad T \quad 1]^T \tag{5}$$

The covariance of $w(t)$ is

$$Q = \Gamma \sigma_w^2 \Gamma^T = \begin{pmatrix} T^6/36 & T^5/12 & T^4/6 & T^3/6 \\ T^5/12 & T^4/4 & T^3/2 & T^2/2 \\ T^4/6 & T^3/2 & T^2 & T \\ T^3/6 & T^2/2 & T & 1 \end{pmatrix} \sigma_w^2 \tag{6}$$

$$H = [1 \ 0 \ 0 \ 0] \quad (7)$$

σ_v^2 is recorded as R. With the assumption 1-2, the objective is to get the self-tuning Kalman filter and its convergence analysis.

3 Noise Variance Estimation

Define backward shift operator q^{-1} and $q^{-1}X(t+1) = X(t)$, From (1) we have

$$X(t) = (I - Fq^{-1})^{-1} \Gamma w(t-1) \quad (8)$$

Substituting (8) into (2), we can get

$$y(t) = H(I - Fq^{-1})^{-1} \Gamma w(t-1) + v(t) \quad (9)$$

Assume $I - Fq^{-1}$ is non-singular matrix, we can get

$$\det(I - Fq^{-1})y(t) = H \text{adj}(I - Fq^{-1}) \Gamma w(t-1) + \det(I - Fq^{-1})v(t) \quad (10)$$

From (3)-(5), we can get

$$\det(I - Fq^{-1}) = (1 - q^{-1})^4 \quad (11)$$

From (10), we can get

$$\begin{aligned} & (2q^{-7} - 14q^{-6} + 42q^{-5} - 70q^{-4} + 70q^{-3} - 42q^{-2} + 14q^{-1} - 2)y(t) = \\ & (-T^3q^{-6} + 3T^3q^{-5} - 2T^3q^{-4} - \frac{3}{2}T^3q^{-3} + \frac{19}{6}T^3q^{-2} - \frac{5}{6}T^3q^{-1} + \frac{1}{6}T^3)w(t-1) + \\ & (2q^{-7} - 14q^{-6} + 42q^{-5} - 70q^{-4} + 70q^{-3} - 42q^{-2} + 14q^{-1} - 2)v(t) \end{aligned} \quad (12)$$

According to the Assumption 1, we define

$$\begin{aligned} r(t) &= \det(I - Fq^{-1})y(t) \\ &= H \text{adj}(I - Fq^{-1}) \Gamma w(t-1) + \det(I - Fq^{-1})v(t) \end{aligned} \quad (13)$$

$r(t)$ has the relationship

$$r(t) = (2q^{-7} - 14q^{-6} + 42q^{-5} - 70q^{-4} + 70q^{-3} - 42q^{-2} + 14q^{-1} - 2)y(t) \quad (14)$$

We define

$$M(q^{-1}) = -T^3q^{-6} + 3T^3q^{-5} - 2T^3q^{-4} - \frac{3}{2}T^3q^{-3} + \frac{19}{6}T^3q^{-2} - \frac{5}{6}T^3q^{-1} + \frac{1}{6}T^3 \quad (15)$$

$$N(q^{-1}) = 2q^{-7} - 14q^{-6} + 42q^{-5} - 70q^{-4} + 70q^{-3} - 42q^{-2} + 14q^{-1} - 2 \quad (16)$$

$M(q^{-1})$ and $N(q^{-1})$ are both the polynomial matrices with the form

$$X(q^{-1}) = x_0 + x_1q^{-1} + \dots + x_{n_x}q^{-n_x} \quad (17)$$

n_x is the highest order of $X(q^{-1})$. From (13)-(16) we can get

$$r(t) = M(q^{-1})w(t-1) + N(q^{-1})v(t) \quad (18)$$

The correlation function of $r(t)$ could be estimated by sampling method

$$\hat{R}_r^t(k) = \frac{1}{t} \sum_{u=1}^t r(u)r(u-k) \quad (19)$$

Which has the recursive formula

$$\hat{R}_r^t(k) = R_r^{t-1}(k) + \frac{1}{t} [r(t)r(t-k) - R_r^{t-1}(k)] \quad (20)$$

From (18) we can get the correlation function

$$\hat{R}_r^t(k) = \sum_{i=k}^{n_m} m_i Q m_{n_m-i} + \sum_{i=k}^{n_n} n_i R n_{n_n-i}, k = 0, 1, \dots, 7 \quad (21)$$

Equation (21) can be treated as linear functions with unknown variables Q and R , this linear functions are compatible, so the estimations $\hat{Q}(t)$ and $\hat{R}(t)$ at time t could be obtained.

4 Self-tuning Kalman filter

Substituting $\hat{Q}(t)$ and $\hat{R}(t)$ (\hat{Q} and \hat{R}) into the optimal Kalman filter, we have the self-tuning Kalman filter

$$\hat{x}(t|t) = \hat{\Psi}(t)x(t-1|t-1) + K(t)y(t) \quad (22)$$

In (22), we can get Riccati function

$$\Sigma = F[\Sigma - \Sigma H'(H\Sigma H' + \hat{R})^{-1} H\Sigma]F' + \hat{Q} \quad (23)$$

When t is renewed, the values of the variables are renewed in (22)-(23).

5 The Convergence analysis

Applying the ergodicity of stationary stochastic process, we can get

$$\hat{R}_r^t(k) \rightarrow R_r(k), t \rightarrow \infty, w.p.1 \quad (24)$$

$w.p.1$ is in short for “with probability 1”, In(19), the estimations of Q and R are consistent, they converge to the real values, we can get

$$\hat{Q}(t) \rightarrow Q, \hat{R}(t) \rightarrow R, t \rightarrow \infty, w.p.1 \quad (25)$$

$$\hat{K}(t) \rightarrow K, \Psi(t) \rightarrow \Psi, t \rightarrow \infty, w.p.1 \quad (26)$$

We can get $\|\hat{K}(t)\| \rightarrow \|K\|, t \rightarrow \infty, w.p.1$, there must be a constant c_2 to satisfy $\|\hat{K}(t)\| \leq c_2, \forall t$, according to the Assumption2, we can get $|y(t)| \leq c_1$. Dynamic Error System Analysis method [9,10] can be used to get the below Eq. 27.

$$\|\hat{x}(t|t) - x(t|t)\| \rightarrow 0, t \rightarrow \infty, w.p.1 \quad (27)$$

So self-tuning Kalman filter could converge to the optimal Kalman filter.

6 Simulation Example

We take sampling period $T = 1$ s, input noise variance $\sigma_w^2 = 0.8$, measurement noise variance $\sigma_v^2 = 0.2$. The self-tuning Kalman filter presented in this paper is used, Figure 1 is the comparison curve of self-tuning Kalman filter and optimal Kalman filter. In Figure 1, the curved line denotes the optimal Kalman filter, spots denote self-tuning Kalman filter. It can be seen that self-tuning Kalman filter converges to the optimal filter. Figure 2 is the error curve, the curved line means the difference between the self-tuning Kalman filter and optimal filter. It has convergence to 0 as (27) is satisfied.

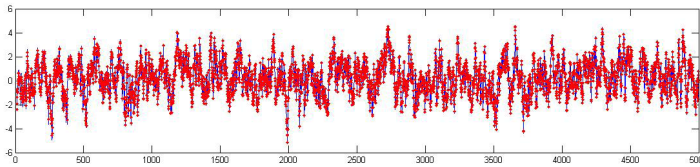


Figure 1. The self-tuning Kalman filter and optimal Kalman filter

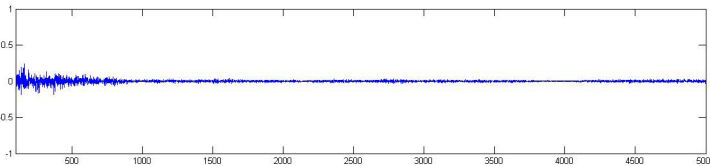


Figure 2. The difference between the self-tuning Kalman filter and optimal filter

7 Conclusion

This paper presents a kind of self-tuning Kalman filter, which could deal with the target tracking model with accelerate rate. Jerk model is used to fit the state equations. The convergence of this algorithm is proved. The simulation example shows the good convergence of this algorithm. This kind of self-tuning Kalman filter could improve

the tracking ability of the high maneuvering target, but the disadvantage is that the calculation is complicated. In the future, we can use the dimension reduction method to deal with the matrices to reduce the amount of computation.

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