# Higgs-Sector Predictions from Maximally Symmetric multi-Higgs Doublet Models

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Maximally Symmetric $n$-Higgs Doublet Models (MS-$n$HDMs) define very economic settings that enable sharp Higgs-sector predictions beyond the Standard Model (SM) potentially testable at high-energy colliders. The scalar potential of a MS-$n$HDM obeys an Sp($2n$) symmetry, which is softly broken by bilinear scalar masses and explicitly by hypercharge and Yukawa couplings through renormalisation-group effects. The Sp($2n$) also ensures natural SM alignment and allows for quartic coupling unification up to the Planck scale. As typical examples, we consider maximally symmetric realisations of the Type-II 2HDM and the Type-V 3HDM. We show how in terms of a few input parameters, definite predictions for the entire scalar mass spectrum of the MS-2HDM and MS-3HDM are obtained, including the SM-like Higgs-boson couplings to the gauge bosons and fermions.

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## 1. Introduction

An interesting class of Higgs-sector extensions is the one that augments the SM with $n$ Higgs doublets $\Phi_i^\mathsf{T} = (\Phi_i^+, \Phi_i^0)^\mathsf{T}$ (with $i = 2, \cdots, n$), usually called the $n$-Higgs Doublet Model ($n$HDM). The general $n$HDM potential may be conveniently expressed as follows [1]:

$$V = -\sum_{i,j=1}^n m_{ij}^2 (\Phi_i^\dagger \Phi_j) + \frac{1}{2} \sum_{i,j\,k,l=1}^n \lambda_{ijkl} (\Phi_i^\dagger \Phi_j)(\Phi_k^\dagger \Phi_l), \tag{1}$$

with $\lambda_{ijkl} = \lambda_{klij}$. In the $n$HDM, the couplings of the SM-like Higgs boson to the EW gauge bosons ($Z$, $W^\pm$) must resemble those predicted by the SM, so as to be in agreement with the current Higgs signals at the LHC. This is only possible within the so-called SM alignment limit [2–12].

The potential of $n$HDMs contains a large number of $SU(2)_L$-preserving accidental symmetries as subgroups of the symplectic group $Sp(2n)$. This maximal symmetry group plays an instrumental role in classifying accidental symmetries that may occur in the scalar potentials of $n$HDMs and $n$HDM-Effective Field Theories with higher-order operators [13–16]. Thus far, this classification has been done for: (i) the 2HDM [17, 18], (ii) the 2HDM Effective Field Theory for higher-order operators up to dimension-6 and dimension-8 [16], and (iii) the 3HDM [14, 19, 20]. From these symmetries, only a few possess the desirable property of natural SM alignment [13, 15]

$$\text{(i) } Sp(2n), \qquad \text{(ii) } SU(n)_{\text{HF}}, \qquad \text{(iii) } SO(n)_{\text{HF}} \times \text{CP}. \tag{2}$$

Evidently, the $Sp(2n)$-invariant $n$HDM is the most economic setting that realises naturally such an alignment from the general class of $n$HDMs. The other two class of models, based on $SU(n)_{\text{HF}}$ and $SO(n)_{\text{HF}} \times \text{CP}$ groups, require one and two extra parameters, respectively, as compared to the Maximally Symmetric $n$HDM (MS-$n$HDM) that implements the $Sp(2n)$ group. These additional theoretical parameters make these symmetric models less predictive, while spoiling the interesting feature of quartic coupling unification. In the following sections, after discussing the basic feature of Type II-2HDM and Type V-3HDM, we will focus on MS-$n$HDM with $n = 2, 3$ Higgs doublets.

## 2. The Higgs Spectrum of the Type-II 2HDM

The Higgs sector of the 2HDM is described by two scalar doublets $\Phi_1$ and $\Phi_2$. Performing the usual linear expansion of these scalar doublets about their VEVs, we may express them as

$$\Phi_i = \begin{pmatrix} \phi_i^+ \\ \frac{1}{\sqrt{2}}(v_i + \phi_i + i\chi_i) \end{pmatrix}, \tag{3}$$

with $i = 1, 2$. In the case of CP-conserving Type-II 2HDM, both scalar doublets receive real and nonzero vacuum expectation values (VEVs). In detail, we have $\langle \Phi_1^0 \rangle = v_1/\sqrt{2}$ and $\langle \Phi_2^0 \rangle = v_2/\sqrt{2}$, where $\tan\beta = v_2/v_1$ and $v \equiv (v_1^2 + v_2^2)^{1/2}$ forms the VEV of the SM Higgs doublet.

After spontaneous symmetry breaking (SSB), the $Z$ and $W^\pm$ gauge bosons become massive after absorbing in the unitary gauge the three would-be Goldstone bosons $G^0$ and $G^\pm$, respectively [23]. Consequently, the CP-conserving 2HDM can account for only five physical scalar states: two CP-even scalars ($h,H$), one CP-odd scalar ($a$) and two charged bosons ($h^\pm$). The masses of the $a$ and $h^\pm$ scalars with $s_\beta \equiv \sin\beta$ and $c_\beta \equiv \cos\beta$ are given by

$$M_a^2 = M_{h^\pm}^2 + \frac{v^2}{2}(\lambda_4 - \lambda_5), \quad M_{h^\pm}^2 = \frac{m_{12}^2}{c_\beta s_\beta} - \frac{v^2}{2}(\lambda_4 + \lambda_5) + \frac{v^2}{2c_\beta s_\beta}(\lambda_6 c_\beta^2 + \lambda_7 s_\beta^2). \tag{4}$$

The masses of the two CP-even scalars, $h$ and $H$ may be obtained by diagonalising the CP-even mass matrix $\mathcal{M}_S^2$,

$$\overline{\mathsf{M}}_S^2 = \begin{pmatrix} M_H^2 & 0 \\ 0 & M_h^2 \end{pmatrix} = R(\alpha)\,\mathsf{M}_S^2\,R(\alpha)^\mathsf{T} = R(\alpha)\begin{pmatrix} A & C \\ C & B \end{pmatrix} R(\alpha)^\mathsf{T}. \tag{5}$$

The details of the above matrix are presented in [24–26]. Additionally, the mixing angle $\alpha$ may be determined by $\tan 2\alpha = 2C/A - B$.

To this extent, one may obtain the SM-normalised couplings of the CP-even scalars, $h$ and $H$, to the gauge bosons ($V = W^\pm, Z$) as follows:

$$g_{hVV} = \sin(\beta - \alpha), \qquad g_{HVV} = \cos(\beta - \alpha). \tag{6}$$

Therefore, there are two scenarios to realise the SM alignment limit: (i) SM-like $H$ scenario with $\cos(\beta - \alpha) \to 1$, and (ii) SM-like $h$ scenario $\sin(\beta - \alpha) \to 1$. Here, we consider the first scenario with $\cos(\beta - \alpha) \to 1$.

Our first step is to transform all scalar fields from a weak basis with generic choice of vacua to the so-called Higgs basis [21, 22], where only one Higgs doublet acquires the SM VEV. This can be achieved by virtue of a common orthogonal transformation that involves the mixing angles $\beta$, i.e.

$$\begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = \mathcal{R}(\beta)\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}, \qquad \begin{pmatrix} G^0 \\ a \end{pmatrix} = \mathcal{R}(\beta)\begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}, \qquad \begin{pmatrix} G^\pm \\ h^\pm \end{pmatrix} = \mathcal{R}(\beta)\begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix}, \tag{7}$$

where the two-dimensional rotational matrix is

$$R(\beta) = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix}. \tag{8}$$

In the Higgs basis, the CP-even mass matrix takes on the form

$$\widehat{\mathcal{M}}_S^2 = \begin{pmatrix} \widehat{A} & \widehat{C} \\ \widehat{C} & \widehat{B} \end{pmatrix} = R(\beta)\mathcal{M}_S{}^2 R(\beta)^\mathsf{T}, \tag{9}$$

with

$$\begin{aligned}
\widehat{A} &= 2v^2\left[c_\beta^4\lambda_1 + s_\beta^2 c_\beta^2\lambda_{345} + s_\beta^4\lambda_2 + 2c_\beta s_\beta\left(c_\beta^2\lambda_6 + s_\beta^2\lambda_7\right)\right], \\
\widehat{B} &= M_a^2 + \lambda_5 v^2 + 2v^2\left[s_\beta^2 c_\beta^2\left(\lambda_1 + \lambda_2 - \lambda_{345}\right) - c_\beta s_\beta\left(c_\beta^2 - s_\beta^2\right)\left(\lambda_6 - \lambda_7\right)\right], \\
\widehat{C} &= v^2\left[s_\beta^3 c_\beta\left(2\lambda_2 - \lambda_{345}\right) - c_\beta^3 s_\beta\left(2\lambda_1 - \lambda_{345}\right) + c_\beta^2\left(1 - 4s_\beta^2\right)\lambda_6 + s_\beta^2\left(4c_\beta^2 - 1\right)\lambda_7\right].
\end{aligned} \tag{10}$$

Thereby, in the SM alignment limit $\beta = \alpha$ the mass parameter $\widehat{A}$ becomes equal to $M_H^2$, while the parameters $\widehat{C}$ vanishes. Thus, in the limit $\widehat{C} \to 0$, we obtain the following conditions:

$$\lambda_1 = \lambda_2 = (\lambda_3 + \lambda_4 + \lambda_5)/2, \quad \lambda_6 = \lambda_7 = 0, \tag{11}$$

where by imposing symmetries identified in (2) with $n = 2$ the above relationships between the quartic couplings may be fulfilled naturally.

Moreover, the deviation of the $H$-boson couplings from their SM values in terms of the light-to-heavy scalar-mixing parameter $\theta_S \equiv \widehat{C}/\widehat{B}$ are given by

$$g_{HVV} \simeq 1 - \theta_S^2/2, \qquad g_{hVV} \simeq -\theta_S. \tag{12}$$

where the mixing parameter $\theta_S$ vanishes in the exact SM alignment limit as $\alpha \to \beta$. Accordingly, we may derive approximate analytic expressions for the $h$- and $H$-boson couplings to up- and down-type quarks to leading order in $\theta_S$, as

$$g_{Huu} \simeq 1 + t_\beta^{-1} \theta_S, \qquad g_{Hdd} \simeq 1 - \theta_S t_\beta, \tag{13}$$
$$g_{huu} \simeq -\theta_S + t_\beta^{-1}, \qquad g_{hdd} \simeq -\theta_S - t_\beta.$$

In the SM alignment limit, we have $g_{Huu} \to 1$ and $g_{Hdd} \to 1$. Obviously, any deviation of these couplings from their SM values is governed by quantities $\tan\beta$ and $\theta_S$.

## 3. The Higgs Spectrum of the Type-V 3HDM

In the case of CP-conserving Type-V 3HDM, the VEVs of scalar doublets, $\Phi_i$ ($i = 1, 2, 3$) may be given by

$$v_1 = v \cos\beta_1 \cos\beta_2, \quad v_2 = v \sin\beta_1 \cos\beta_2, \quad v_3 = v \sin\beta_2. \tag{14}$$

Equivalently, the VEVs may be defined by the two ratios, $\tan\beta_1 = v_2/v_1$ and $\tan\beta_2 = v_3/\sqrt{v_1^2 + v_2^2}$, given the constraint that $v \equiv \sqrt{v_1^2 + v_2^2 + v_3^2}$ is the SM VEV.

For later convenience, we proceed as in [27] and define the three-dimensional rotational matrices about the individual axes $z$, $y$ and $x$ as follows:

$$R_{12}(\alpha) = \begin{pmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}, \; R_{13}(\beta) = \begin{pmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{pmatrix}, \; R_{23}(\gamma) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & \sin\gamma \\ 0 & -\sin\gamma & \cos\gamma \end{pmatrix}. \tag{15}$$

In the case of 3HDM, the two mixing angles $\beta_1$ and $\beta_2$ are required to transform all scalar fields to the Higgs basis with $\mathcal{O}_\beta \equiv R_{13}(\beta_2) R_{12}(\beta_1)$, i.e.

$$\begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix} = \mathcal{O}_\beta \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}, \qquad \begin{pmatrix} G^0 \\ \eta_1 \\ \eta_2 \end{pmatrix} = \mathcal{O}_\beta \begin{pmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \end{pmatrix}, \qquad \begin{pmatrix} G^\pm \\ \eta_1^\pm \\ \eta_2^\pm \end{pmatrix} = \mathcal{O}_\beta \begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \\ \phi_3^\pm \end{pmatrix}. \tag{16}$$

Therefore, after SSB the model exhibits nine scalar mass eigenstates: (i) three CP-even scalars $(H, h_1, h_2)$, (ii) two CP-odd scalars $(a_1, a_2)$, and (iii) four charged scalars $(h_1^\pm, h_2^\pm)$.

In the Higgs basis, spanned by $\{\eta_{1,2}\}$ and $\{\eta_{1,2}^\pm\}$, the CP-odd and charged scalar mass matrices reduce to the $2 \times 2$ matrices given by

$$\mathcal{M}_P^2 = \begin{pmatrix} M_{P,22}^2 & M_{P,23}^2 \\ M_{P,32}^2 & M_{P,33}^2 \end{pmatrix}, \qquad \mathcal{M}_\pm^2 = \begin{pmatrix} M_{\pm,22}^2 & M_{\pm,23}^2 \\ M_{\pm,32}^2 & M_{\pm,33}^2 \end{pmatrix}, \tag{17}$$

where the details of all elements are given in [20]. Upon diagonalisation of the mass matrices given in (17), the masses of the two CP-odd scalars $(a_1, a_2)$ and the four charged Higgs bosons $(h_1^\pm, h_2^\pm)$ may be computed as

$$M_{a_1, a_2}^2 = \frac{1}{2} \left[ M_{P,22}^2 + M_{P,33}^2 \mp \left[ (M_{P,22}^2 - M_{P,33}^2)^2 + 4M_{P,23}^4 \right]^{1/2} \right], \tag{18}$$

$$M_{h_1^\pm, h_2^\pm}^2 = \frac{1}{2} \left[ M_{\pm,22}^2 + M_{\pm,33}^2 \mp \left[ (M_{\pm,22}^2 - M_{\pm,33}^2)^2 + 4M_{\pm,23}^4 \right]^{1/2} \right]. \tag{19}$$

In addition, the mixing angles $\rho$ and $\sigma$ may be given by

$$\tan 2\rho = \frac{2\,M_{\text{P},23}^2}{M_{\text{P},22}^2 - M_{\text{P},33}^2}, \quad \tan 2\sigma = \frac{2\,M_{\pm,23}^2}{M_{\pm,22}^2 - M_{\pm,33}^2}. \tag{20}$$

Finally, the masses for the three CP-even scalars, $H$, $h_1$ and $h_2$ with mass ordering $M_H \leq M_{h_1} \leq M_{h_2}$ can be evaluated by diagonalising the squared mass matrix $\mathcal{M}_S^2$, expressed in the general weak basis $\{\phi_{1,2,3}\}$, by employing the orthogonal matrix $\mathcal{O} = R_{23}(\alpha)\,R_{13}(\alpha_2)\,R_{12}(\alpha_1)$, as

$$\overline{\mathcal{M}}_S^2 = \begin{pmatrix} M_H^2 & 0 & 0 \\ 0 & M_{h_1}^2 & 0 \\ 0 & 0 & M_{h_2}^2 \end{pmatrix} = \mathcal{O}\,\mathcal{M}_S^2\,\mathcal{O}^{\mathsf{T}} = \mathcal{O} \begin{pmatrix} A & C_1 & C_2 \\ C_1 & B_1 & C_3 \\ C_2 & C_3 & B_2 \end{pmatrix} \mathcal{O}^{\mathsf{T}}. \tag{21}$$

where the analytic form of all its entries is given in [20].

In the CP-conserving 3HDM, the SM-normalised couplings of the SM-like Higgs boson to the EW gauge bosons ($V = Z, W^{\pm}$) are calculated to be:

$$g_{HVV} = \cos\alpha_2 \cos\beta_2 \cos(\beta_1 - \alpha_1) + \sin\alpha_2 \sin\beta_2, \tag{22}$$

$$g_{h_1VV} = \cos\alpha \, \cos\beta_2 \sin(\beta_1 - \alpha_1) + \sin\alpha \left[ \cos\alpha_2 \sin\beta_2 - \cos\beta_2 \sin\alpha_2 \cos(\beta_1 - \alpha_1) \right], \tag{23}$$

$$g_{h_2VV} = \cos\alpha \left[ \cos\alpha_2 \sin\beta_2 - \cos\beta_2 \sin\alpha_2 \cos(\beta_1 - \alpha_1) \right] - \sin\alpha \, \cos\beta_2 \sin(\beta_1 - \alpha_1). \tag{24}$$

Evidently, there are three possible scenarios for which the 125-GeV resonance can be identified with the SM-like Higgs boson [20]. Here, we consider the so-called *canonical* SM-like Higgs scenario i.e. $\beta_1 = \alpha_1$ and $\beta_2 = \alpha_2$, where $M_H \approx 125$ GeV with the coupling strength $g_{HVV} = 1$ and $g_{h_{1,2}VV} = 0$.

In the Higgs basis $\{H_{1,2,3}\}$, the $3 \times 3$ mass matrix of the CP-even scalars is given by

$$\widehat{\mathcal{M}}_S^2 = \begin{pmatrix} \widehat{A} & \widehat{C}_1 & \widehat{C}_2 \\ \widehat{C}_1 & \widehat{B}_1 & \widehat{C}_3 \\ \widehat{C}_2 & \widehat{C}_3 & \widehat{B}_2 \end{pmatrix} = \mathcal{O}_\beta\,\mathcal{M}_S^2\,\mathcal{O}_\beta^{\mathsf{T}}. \tag{25}$$

The details of the above matrix are presented in [20].

In the SM alignment limit $\beta_1 = \alpha_1$ and $\beta_2 = \alpha_2$ under consideration, the mass parameter $\widehat{A}$ becomes equal to $M_H^2$, while the parameters $\widehat{C}_1$ and $\widehat{C}_2$ vanish. Thus, taking the limit $\widehat{C}_{1,2} \to 0$, the following relationships between the quartic couplings may be derived:

$$\lambda_{11} = \lambda_{22} = \lambda_{33} = (\lambda_{1122} + \lambda_{1221} + \lambda_{1212})/2, \quad \lambda_{1122} = \lambda_{1133} = \lambda_{2233},$$
$$\lambda_{1221} = \lambda_{1331} = \lambda_{2332}, \quad \lambda_{1212} = \lambda_{1313} = \lambda_{2323}, \tag{26}$$

while the remaining quartic couplings are zero. Note that by virtue of symmetries identified in (2) with $n = 3$ the above relationships between the quartic couplings can be met.

The reduced $H$-boson couplings to the EW gauge bosons in a power expansion of $\theta_{S_{1,2}} \equiv \widehat{C}_{1,2}/\widehat{B}_{1,2}$ and $\theta_{S_{12}} \equiv \widehat{C}_1/\widehat{B}_2$, are given by the following approximate analytic expressions:

$$g_{HVV} \simeq 1 - \theta_{S_1}^2/4,$$
$$g_{h_1VV} \simeq c_\alpha c_{\beta_2} \theta_{S_1} + s_\alpha \theta_{S_2} (1 + \theta_{S_1})/4,$$
$$g_{h_2VV} \simeq -s_\alpha c_{\beta_2} \theta_{S_1} + c_\alpha \theta_{S_2} (1 + \theta_{S_1})/4. \tag{27}$$

Likewise, to order $\widehat{C}_{1,2}^2/\widehat{B}_{1,2}^2$, the SM-normalised couplings of the $H$ boson to fermions are dictated by the following approximate analytic formulae:

$$
\begin{aligned}
g_{Hd\bar{d}} &\simeq 1 - (t_{\beta_1} + t_{\beta_2})\,\theta_{\mathcal{S}_1} - (1 - t_{\beta_1}t_{\beta_2})\,\theta_{\mathcal{S}_2}^2, \\
g_{Hu\bar{u}} &\simeq 1 - t_{\beta_1}^{-1}\,\theta_{\mathcal{S}_1} + \frac{t_{\beta_2}}{2}(1 - t_{\beta_1}^{-1}\,\theta_{\mathcal{S}_1})\,\theta_{\mathcal{S}_2} - \frac{1}{8}(4\theta_{\mathcal{S}_1}^2 + \theta_{\mathcal{S}_2}^2), \\
g_{Hl\bar{l}} &\simeq 1 - 2\theta_{\mathcal{S}_2}t_{\beta_2}^{-1}(4 + \theta_{\mathcal{S}_1}\,\theta_{\mathcal{S}_{12}})^{-1} - 2\,\theta_{\mathcal{S}_2}^2(4 + \theta_{\mathcal{S}_1}\,\theta_{\mathcal{S}_{12}})^{-2}.
\end{aligned}
\tag{28}
$$

We note that the analytic expressions of the reduced $H$-boson couplings to the gauge bosons and fermions go to the SM value 1, when the exact SM alignment limit $\widehat{C}_{1,2} \to 0$ is considered, or when the new-physics mass scales $\widehat{B}_{1,2} \sim M_{h_{1,2}}^2$ are taken to infinity. Since we are interested in the former possibility which in turn implies a richer collider phenomenology, we will study scenarios in which SM alignment limit is accomplished by virtue of Sp(6) symmetry.

## 4. Maximally Symmetric *n*HDMs

MS-*n*HDMs are the most economic settings that realise naturally SM alignment from the general class of *n*HDMs. The Sp($2n$)-invariant potential has the same form as the SM potential,

$$
V_{\text{SM}} = -m^2\,|\Phi|^2 + \lambda\,|\Phi|^4\,,
\tag{29}
$$

that contains a single mass term and a single quartic coupling. In the case of the 2HDM Sp(4)/$Z_2$-invariant potential, the so called MS-2HDM is given by

$$
V_{\text{MS-2HDM}} = -m^2\left(|\Phi_1|^2 + |\Phi_2|^2\right) + \lambda\left(|\Phi_1|^2 + |\Phi_2|^2\right)^2,
\tag{30}
$$

where the non-zero parameters have the following relations,

$$
m^2 = m_{11}^2 = m_{22}^2, \qquad \lambda = \lambda_1 = \lambda_2 = \lambda_3/2.
$$

Likewise, in the case of Sp(6)/$Z_2$-invariant potential, the so called MS-3HDM we have

$$
V_{\text{MS-3HDM}} = -m^2\left(|\Phi_1|^2 + |\Phi_2|^2 + |\Phi_3|^2\right) + \lambda\left(|\Phi_1|^2 + |\Phi_2|^2 + |\Phi_3|^2\right)^2,
\tag{31}
$$

with the following relationship between non-zero parameters:

$$
m^2 = m_{11}^2 = m_{22}^2 = m_{33}^2, \qquad \lambda = \lambda_{11} = \lambda_{22} = \lambda_{33} = \lambda_{1122}/2 = \lambda_{1133}/2 = \lambda_{2233}/2,
$$

In particular, in the MS-3HDM one can realize the mixing angles that diagonalise the heavy sectors of the CP-even, CP-odd and charged- scalar mass matrices are all equal, i.e. $\alpha = \rho = \sigma$, that is a distinct and unique feature of the MS-3HDM.

Moreover, after SSB and in the Born approximation, the MS-*n*HDM predicts one CP-even scalar $H$ with non-zero squared mass $M_H^2 = 2\lambda v^2$, while all other scalars are massless pseudo-Goldstone bosons with sizeable gauge and Yukawa interactions. Therefore, we need to consider two main sources that break the Sp($2n$) symmetry of the theory: (i) the RG effects of the gauge and Yukawa couplings on the potential parameters, and (ii) soft symmetry-breaking bilinear masses, rendering all pseudo-Goldstone fields sufficiently heavy in agreement with current LHC data and other low-energy experiments. Hence, the complete MS-*n*HDM potential is [7, 13, 26, 28]

$$
V = V_{\text{MS-}n\text{HDM}} + \sum_{i,j=1}^{n} m_{ij}^2\,(\Phi_i^\dagger\Phi_j)\,.
\tag{32}
$$
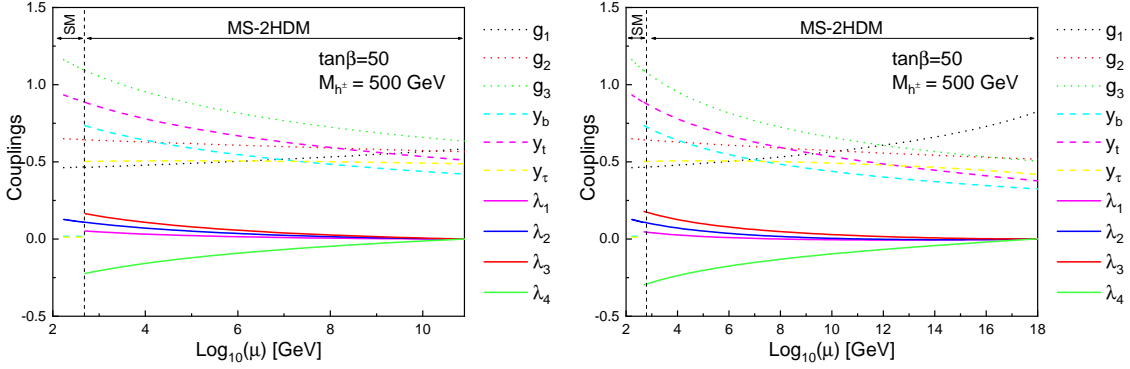
**Figure 1:** *In the left (right), the RG evolution of the quartic couplings in MS-3HDM from the threshold scale $M_{h^\pm} = 500$ GeV up to their first (second) quartic coupling unification scale $\mu_X^{(1)} \sim 10^{11}$ GeV ($\mu_X^{(2)} \sim 10^{18}$ GeV) for $\tan\beta = 50$.*

Henceforth, with this minimal addition, the scalar-boson masses in MS-2HDM are given by

$$M_H^2 = 2\lambda_2 v^2, \qquad M_h^2 = M_a^2 = M_{h^\pm}^2 = \frac{\text{Re}(m_{12}^2)}{c_\beta s_\beta}, \tag{33}$$

where this degeneracy will be broken by RG effect.

In the case of MS-3HDM, the scalar masses are found to be:

$$M_H^2 \simeq 2\lambda_{22} v^2, \quad M_{h_1,h_2}^2 \simeq M_{a_1,a_2}^2 \simeq M_{h_1^\pm,h_2^\pm}^2 \simeq \frac{1}{2}\left[ S_1 + S_2 \mp \left[ (S_1 - S_2)^2 + 4S_3^2 \right]^{1/2} \right], \quad (34)$$

with

$$S_1 = \frac{m_{12}^2 + t_{\beta_2}\left(m_{13}^2 s_{\beta_1}^3 + m_{23}^2 c_{\beta_1}^3\right)}{c_{\beta_1} s_{\beta_1}}, \quad S_2 = \frac{m_{13}^2 c_{\beta_1} + m_{23}^2 s_{\beta_1}}{c_{\beta_2} s_{\beta_2}}, \quad S_3 = \frac{m_{13}^2 s_{\beta_1} - m_{23}^2 c_{\beta_1}}{c_{\beta_2}}. \quad (35)$$

Therefore, with the introduction of soft symmetry-breaking bilinears, all pseudo-Goldstone bosons, $h_{1,2}$, $a_{1,2}$ and $h_{1,2}^\pm$, receive appreciable masses at the tree level.

## 4.1 Quartic Coupling Unification

The interesting feature of MS-$n$HDM is quartic coupling unification, where all quartic couplings can unify to a single value $\lambda$ at very high-energy scales $\mu_X$. As the highest scale of unification, we take the values at which $\lambda(\mu_X) = 0$. We will see that there are two such conformally invariant unification points in the MS-$n$HDM which we distinguish them as $\mu_X^{(1,2)}$, with $\mu_X^{(1)} \le \mu_X^{(2)}$.

In Figures 1 and 2, we display the RG evolution of all quartic couplings for MS-2HDM and MS-3HDM, with a low charged Higgs mass $M_{h^\pm} = M_{h_1^\pm} = 500$ GeV. From the left panels of these figures, we observe that in the case of MS-3HDM (MS-2HDM) the quartic coupling $\lambda_{22}$ ($\lambda_2$), which determines the SM-like Higgs-boson mass $M_H$, decreases at high RG scales, due to the running of the top-Yukawa coupling $y_t$. The coupling $\lambda_{22}$ ($\lambda_2$) turns negative just above the quartic coupling unification scale $\mu_X^{(1)} \sim 10^{13}$ GeV ($\mu_X^{(1)} \sim 10^{11}$ GeV), at which all quartic couplings vanish. Below $\mu_X^{(1)}$, the MS-3HDM (MS-2HDM) quartic couplings exhibit different RG runnings, and especially the couplings $\lambda_{ijij}$ (with $i \ne j$) take on non-zero values. From the right panels of Figures 1 and 2,
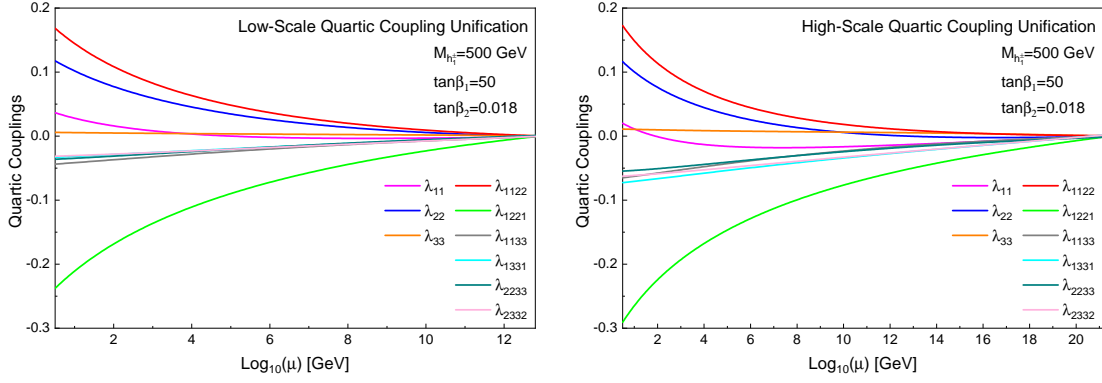
**Figure 2:** *In the left (right), the RG evolution of the quartic couplings in MS-3HDM from the threshold scale $M_{h_1^\pm} = 500$ GeV up to their first quartic coupling unification scale $\mu_X^{(1)} \sim 10^{13}$ GeV ($\mu_X^{(2)} \sim 10^{21}$ GeV) for $\tan\beta_1 = 50$ and $\tan\beta_2 = 0.018$.*
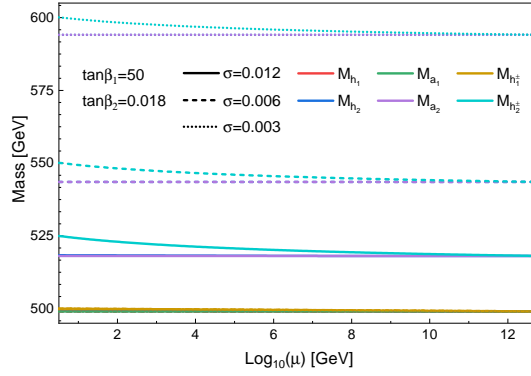


**Figure 3:** The scalar mass spectrum of the MS-3HDM with $\tan\beta_1 = 50$, $\tan\beta_2 = 0.018$, $M_{h_1^\pm} = 500$ GeV and $M_{H_2^\pm} - M_{H_1^\pm} = 25, 50$ and $100$ GeV relevant to the mixing angles $\sigma = 0.012, 0.006, 0.003$, respectively.

we observe that in addition to the conformal unification point $\mu_X^{(1)}$, there is in general a second and higher conformal point $\mu_X^{(2)} \sim 10^{21}$ GeV ($\mu_X^{(2)} \sim 10^{18}$ GeV) in the MS-3HDM (MS-2HDM).

In Figure 3, we give numerical estimates of the mass spectrum of the MS-3HDM, for which a low charged Higgs mass $M_{h_1^\pm} = 500$ GeV and the first unification point $\mu_X = \mu_X^{(1)}$ are chosen. At this scale, the masses of all heavy Higgs bosons, $h_{1,2}$, $a_{1,2}$ and $h_{1,2}^\pm$, are mainly determined by the soft-breaking mass terms, which are: $m_{12}^2 \approx 75^2$ GeV$^2$, $m_{23}^2 \approx 70^2$ GeV$^2$, $m_{13}^2 \approx 20^2$ GeV$^2$, and the VEV ratios $\tan\beta_1 = 50$ and $\tan\beta_2 = 0.018$. The heavy mass spectrum of the MS-3HDM becomes degenerate at the unification point $\mu_X$, clustering about the different charged Higgs masses, $M_{h_1^\pm} = 500$ GeV and $M_{h_2^\pm} = 525, 550, 600$ GeV, with the mixing angles $\sigma$ [rad] = 0.012, 0.006, 0.003 for the $h_1^\pm - h_2^\pm$ system. As shown in Figure 3, RG effects will break these mass degeneracies from a few MeV, for $M_{h_1} - M_{a_1}$ and $M_{h_2} - M_{a_2}$, up to about 30 GeV, for $M_{h_2^\pm} - M_{a_2}$.

We have already seen how at the conformal points, $\mu_X^{(1)}$ and $\mu_X^{(2)}$, all quartic couplings vanish simultaneously, leading to an exact SM alignment. Nevertheless, for lower RG scales, the maximal symmetry is broken, resulting in calculable non-vanishing misalignment predictions. In Figure 4, we present our numerical estimates of the predicted deviations for the SM-like Higgs-boson coupling $HXX$ (with $X = W^\pm, Z, t, b, \tau$) from its respective SM value in MS-2HDM (left panel) and in MS-

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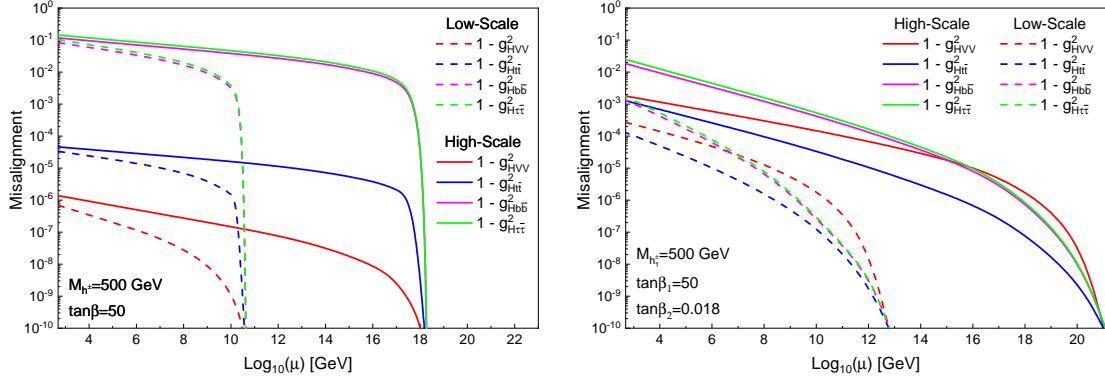
**Figure 4:** *Numerical estimates of the misalignment parameter $1 - g_{HXX}^2$ pertinent to the HXX-coupling (with $X = V, t, b, \tau$ and $V = W^{\pm}, Z$) as functions of the RG scale $\mu$, for the low-scale and the high-scale quartic coupling unification scenarios in MS-2HDM (left panel) and MS-3HDM (right panel).*

3HDM (right panel). Specifically, Figure 4 exhibits the dependence of the misalignment parameter $1 - g_{HXX}^2$ (with $g_{H_{\rm SM}XX} = 1$) as functions of the RG scale $\mu$, for both low- and high-scale quartic coupling unification scenarios. We observe that the normalised couplings $g_{HVV}$ and $g_{Htt}$ reach their SM values $g_{H_{\rm SM}VV} = g_{H_{\rm SM}tt} = 1$ at the two quartic coupling unification points, $\mu_X^{(1)}$ and $\mu_X^{(2)}$. Moreover, the deviation of $g_{Hbb}$ and $g_{H\tau\tau}$ from their SM values get larger for the higher-scale unification scenario and can be fitted to the observed data within the 68% CL [29, 30].

## 5. Conclusions

We have analysed the basic low-energy structure of the general 2HDM and 3HDM. Our study was focused on the canonical SM-like Higgs scenario of the Type-II 2HDM and the Type-V 3HDM, for which conditions on these models parameters for achieving exact SM alignment were derived. Interestingly enough, there are three continuous symmetries which, when imposed on the $n$HDM scalar potential, are sufficient to ensure SM alignment: (i) Sp($2n$), (ii) SU($n$)$_{\rm HF}$, and (iii) SO($n$)$_{\rm HF}$. Amongst these symmetries, the most economic setting is the Maximally Symmetric multi-Higgs Doublet Model (MS-$n$HDM), whose potential obeys an Sp($2n$) symmetry. This symmetry is softly broken by bilinear masses $m_{ij}^2$ (with $i \neq j$), as well as explicitly by hypercharge and Yukawa couplings through RG effects, whilst the theory exhibits quartic coupling unification up to the Planck scale. In particular, we have shown that all quartic couplings in the MS-3HDM (MS-2HDM) can unify at high-energy scales $\mu_X$ and vanish simultaneously at two distinct conformal points, that are denoted by $\mu_X^{(1,2)}$ with $\mu_X^{(1)} \lesssim 10^{13}$ GeV ($\mu_X^{(1)} \lesssim 10^{11}$ GeV) and $\mu_X^{(2)} \gtrsim 10^{21}$ GeV ($\mu_X^{(2)} \gtrsim 10^{18}$ GeV).

The MS-$n$HDM is a remarkably predictive scenario, as it only depends on a few theoretical parameters when compared to the large number of independent parameters that are required in the general $n$HDM. For example, in the MS-2HDM the only additional parameters are: (i) the charged Higgs mass $M_{h^{\pm}}$ and (ii) $\tan\beta$. In the case of MS-3HDM, besides the ratios of the Higgs-doublet VEVs, $\tan\beta_{1,2}$, the model is mainly governed by only three input parameters: the masses of the two charged Higgs bosons, $M_{h_{1,2}^{\pm}}$, and their mixing angle $\sigma$. Most notably, with the help of these input parameters, we have obtained misalignment predictions for the entire scalar mass spectrum of the theory, including the interactions of all Higgs particles to the SM fields. The present results for the MS-$n$HDM [20, 26], demonstrate the high predictive power of maximally symmetric settings in $n$HDMs. Such settings not only can naturally provide the experimentally favoured SM alignment, but also allow us to obtain sharp predictions for the entire scalar mass spectrum of the theory.

In conclusion, the MS-*n*HDM provides a unique framework that can be tested or even falsified at the Large Hadron Collider (LHC) and at upcoming high-energy colliders, such as the projected Future Circular Collider (FCC) at CERN.

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