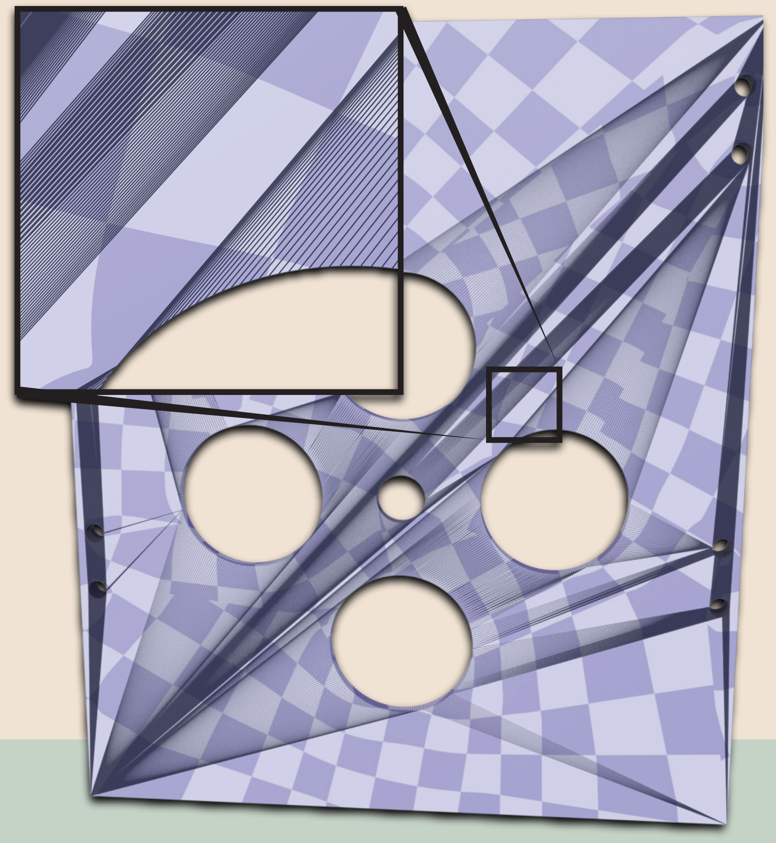

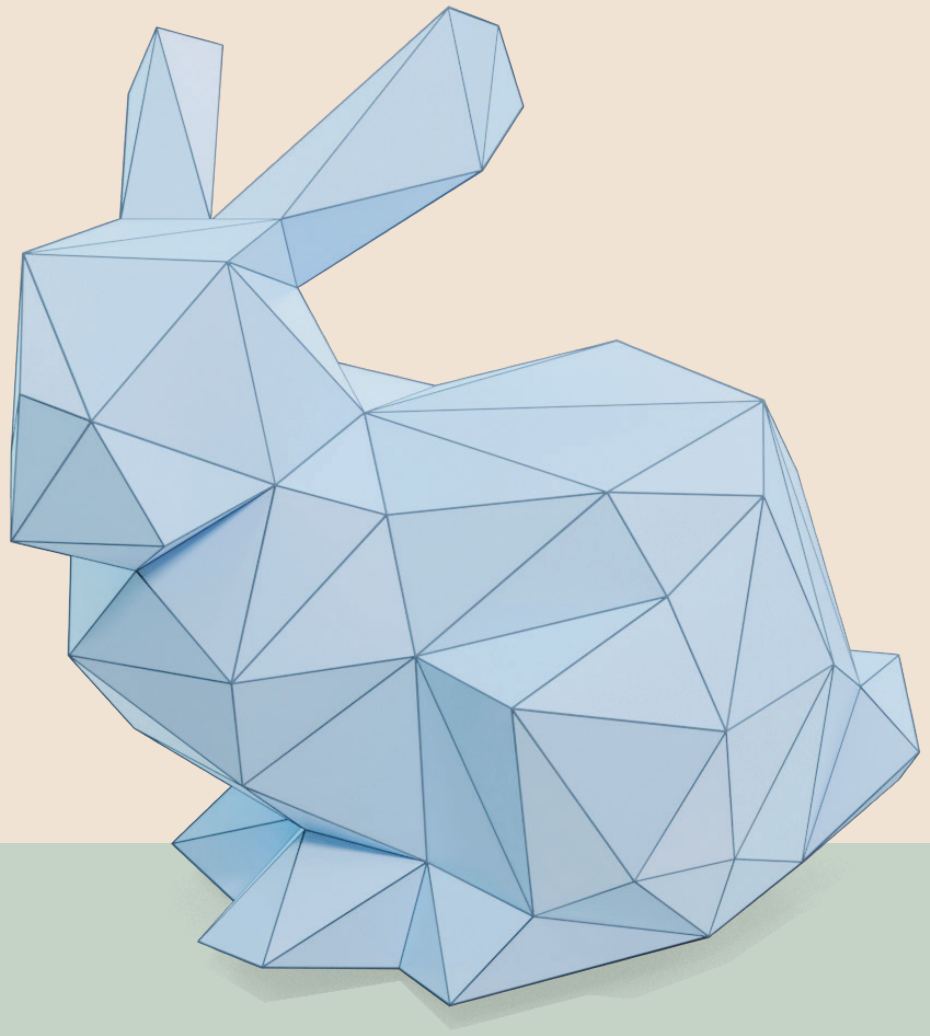

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Discrete Conformal Equivalence of Polyhedral Surfaces

Mark Gillespie

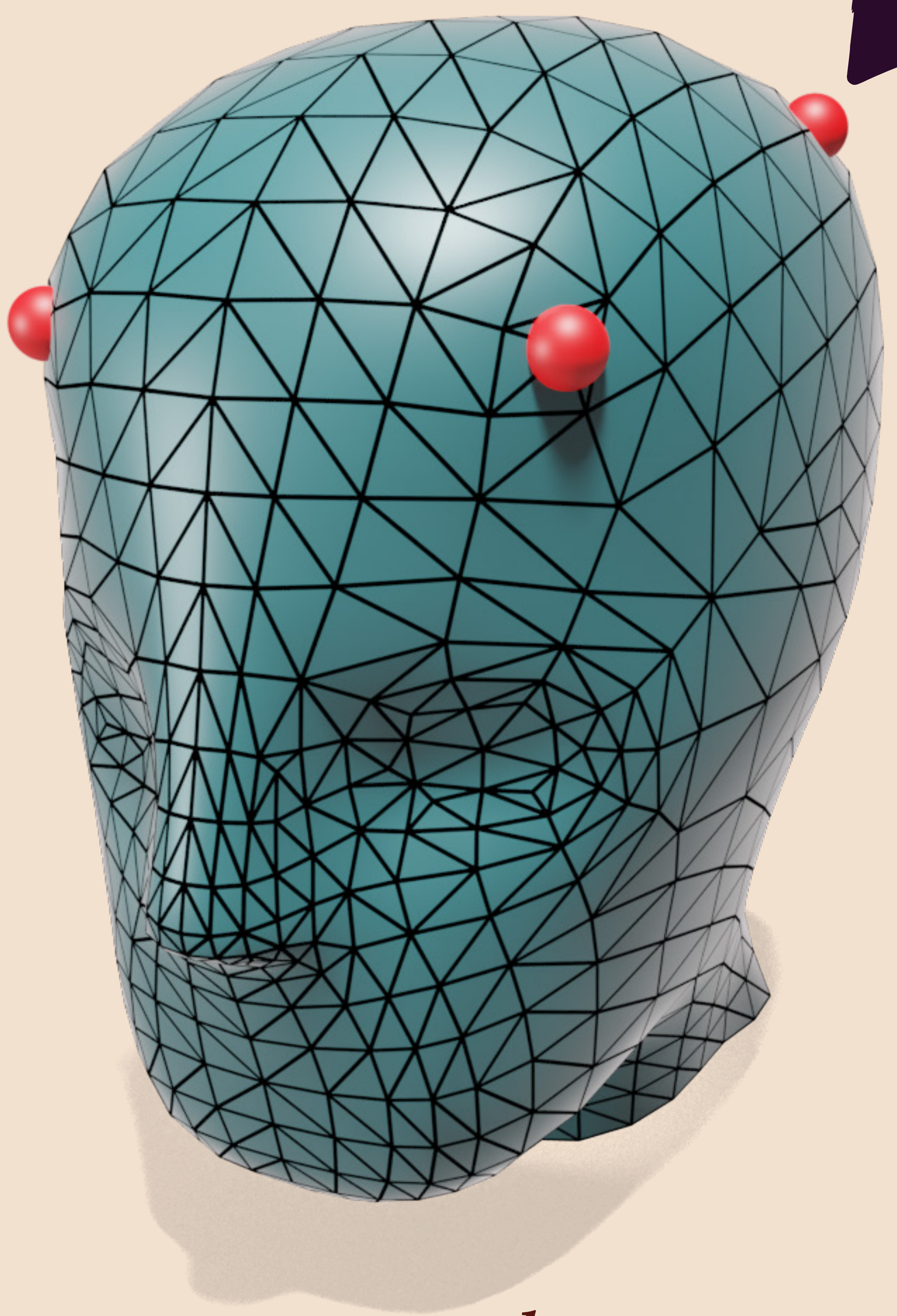
Boris Springborn

Keenan Crane

Goal: high-quality surface parameterization

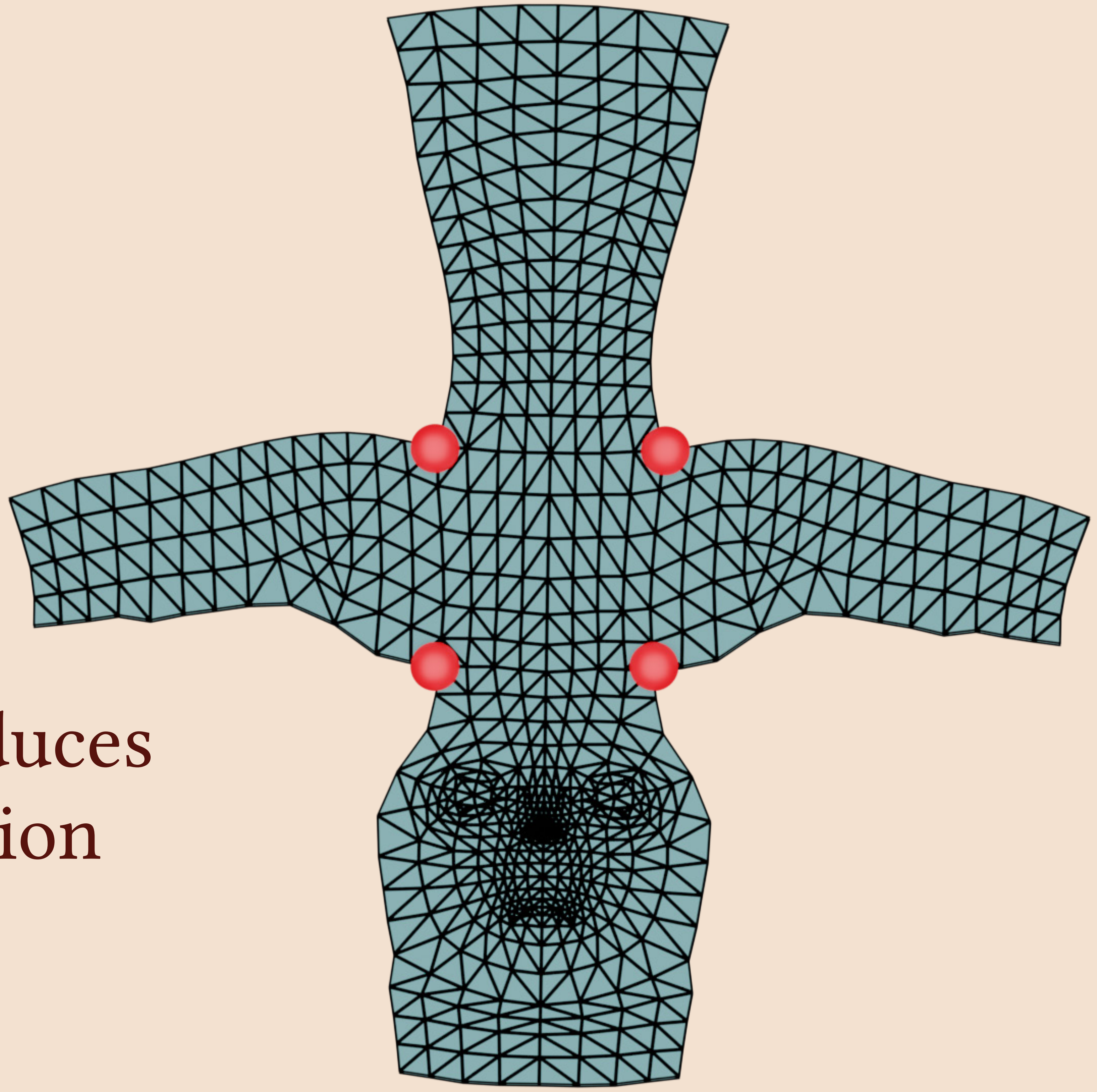
use *cone flattening*

Cones reduce area distortion



input mesh

Conformal reduces angle distortion



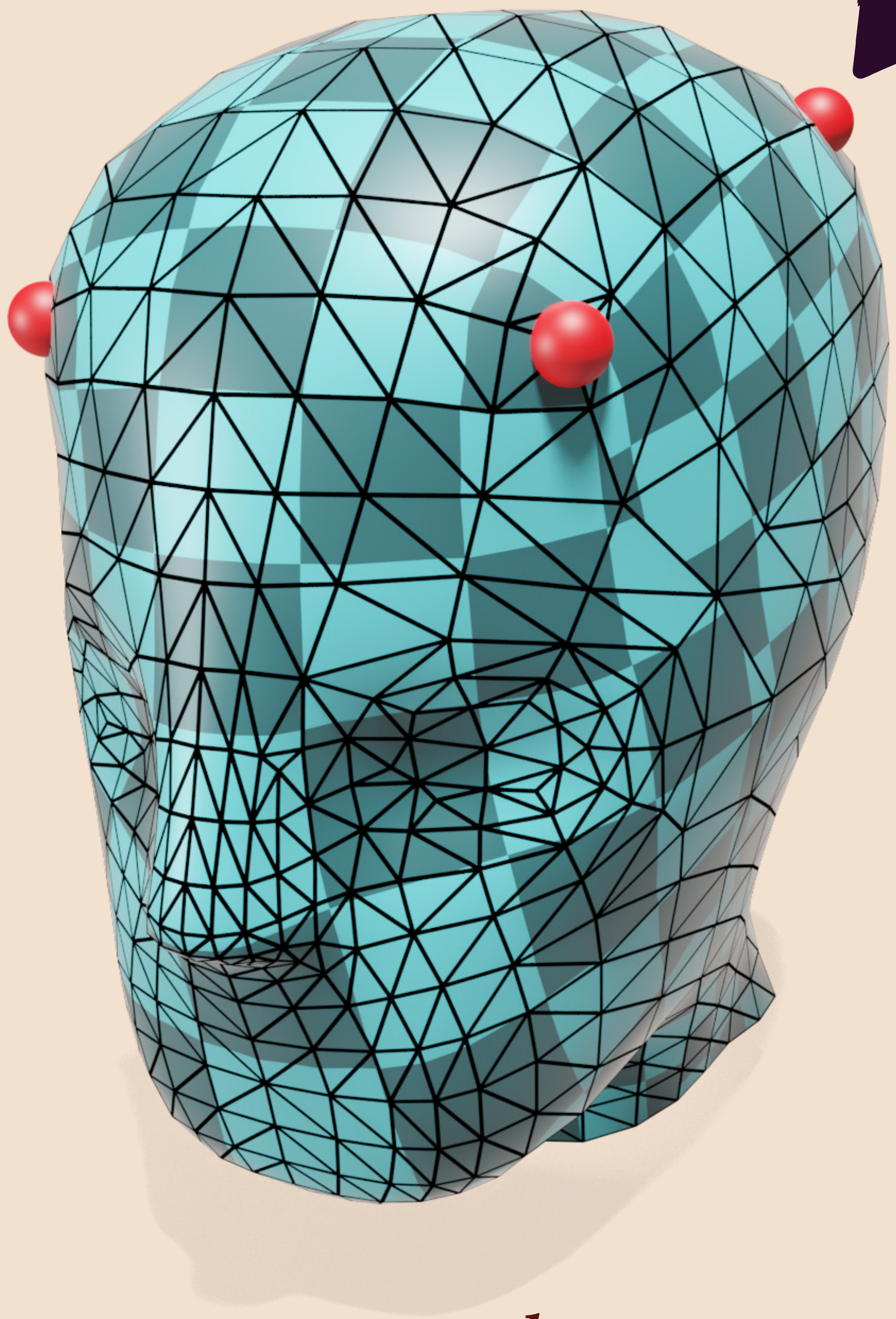
output parameterization

Goal: high-quality surface parameterization

Put the presenter's video feed here!

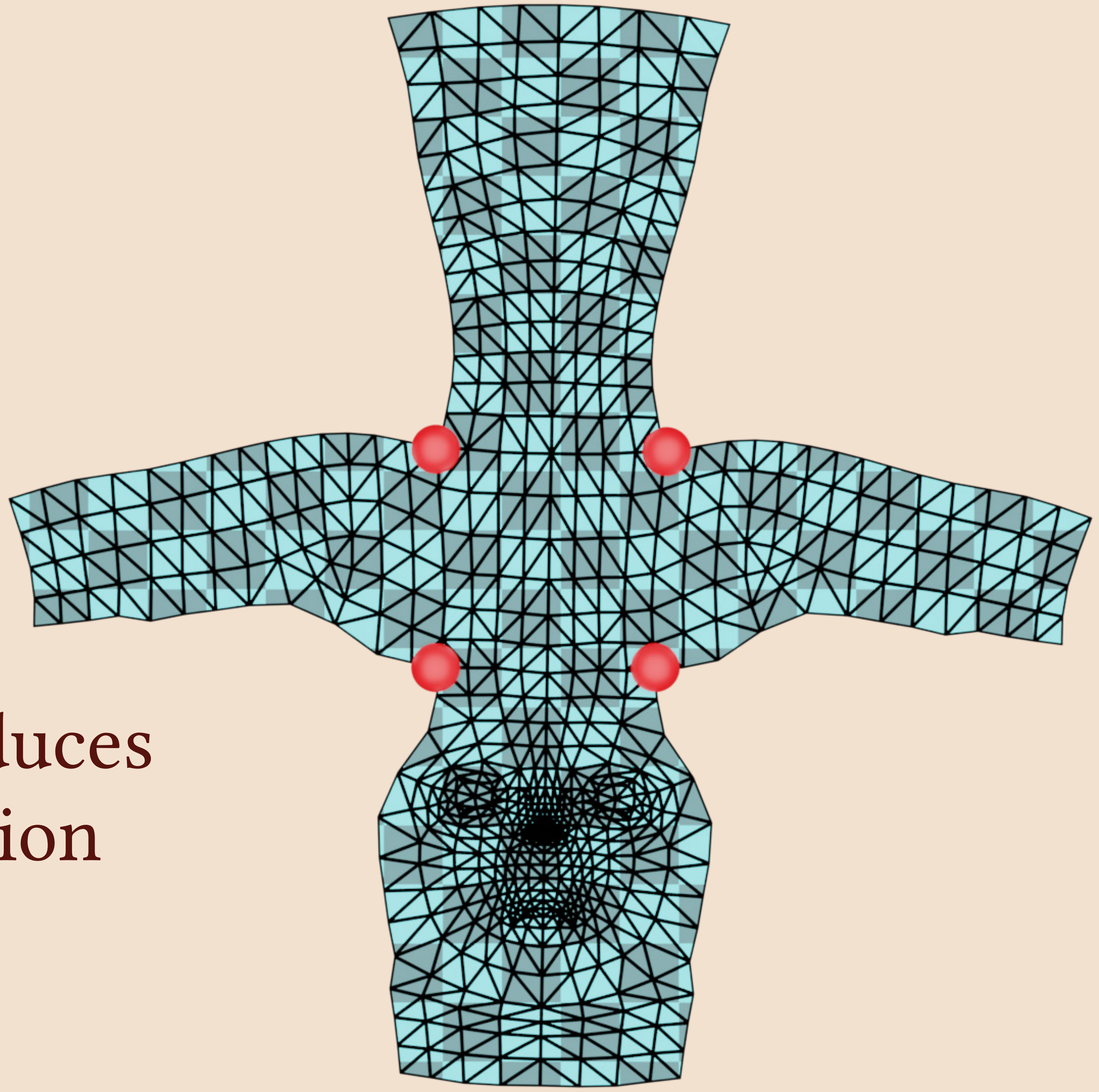
use *cone flattening*

Cones reduce area distortion



input mesh

Conformal reduces angle distortion



output parameterization

Why is this hard?

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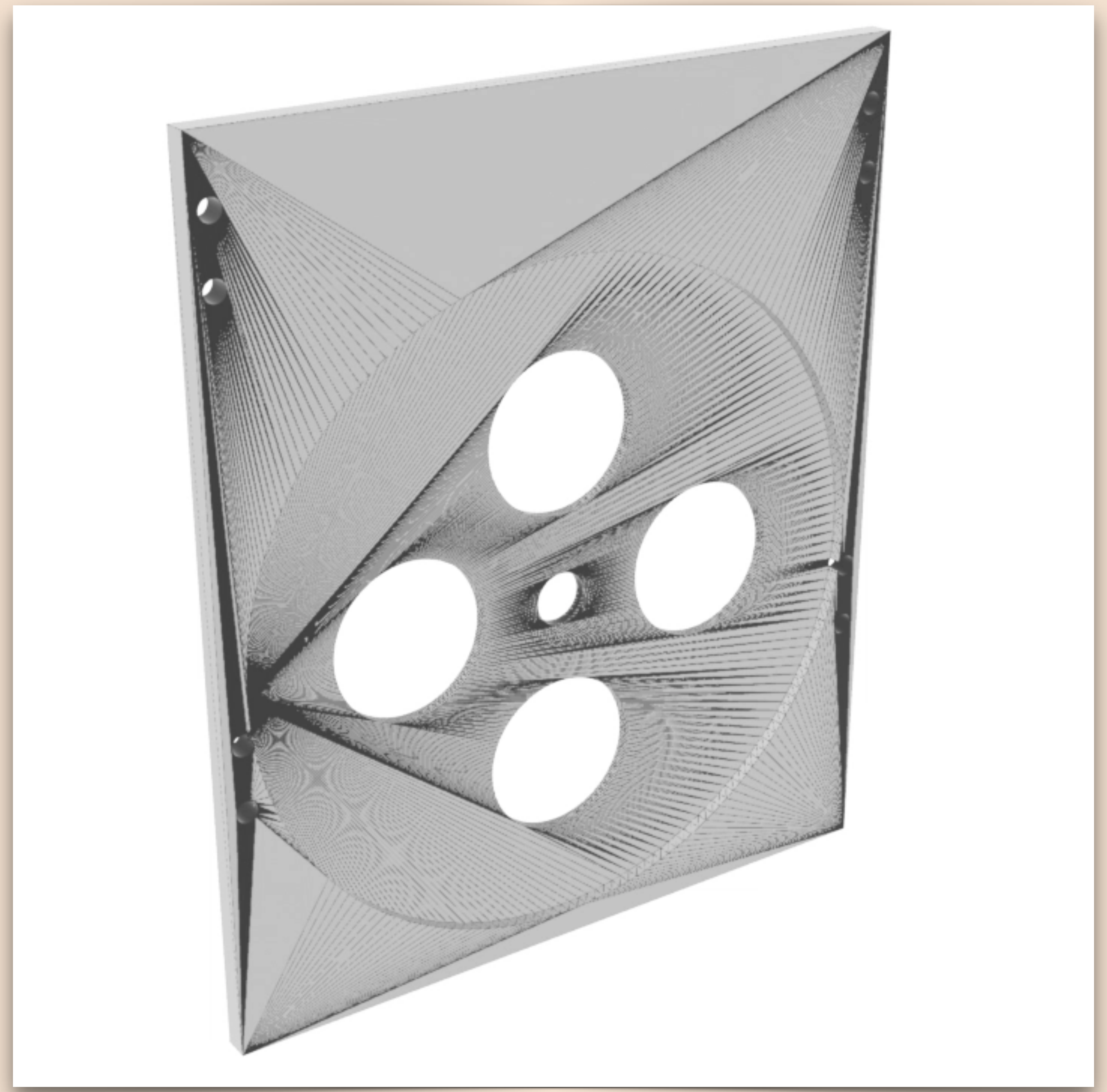

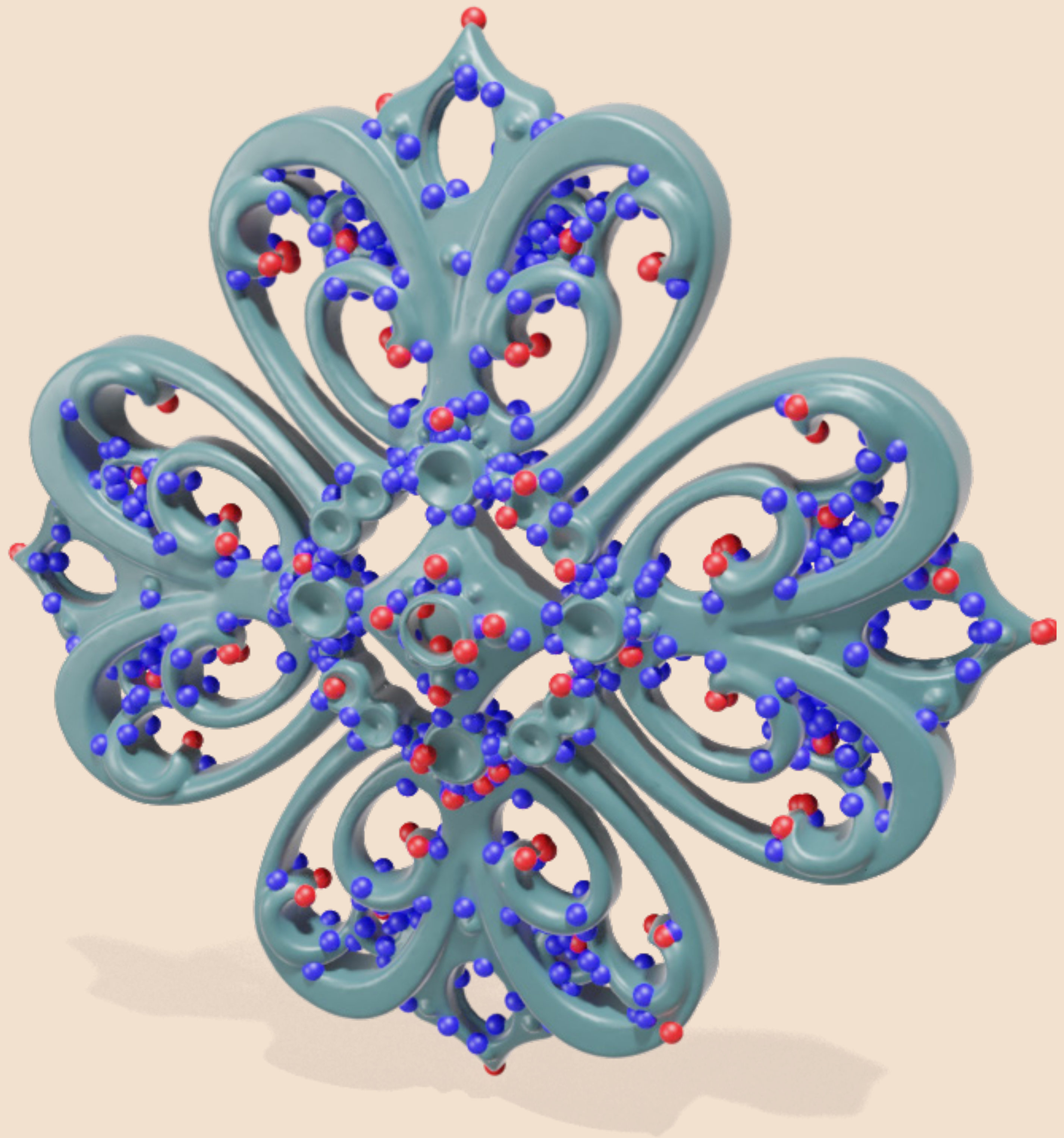

Reliable surface parameterization

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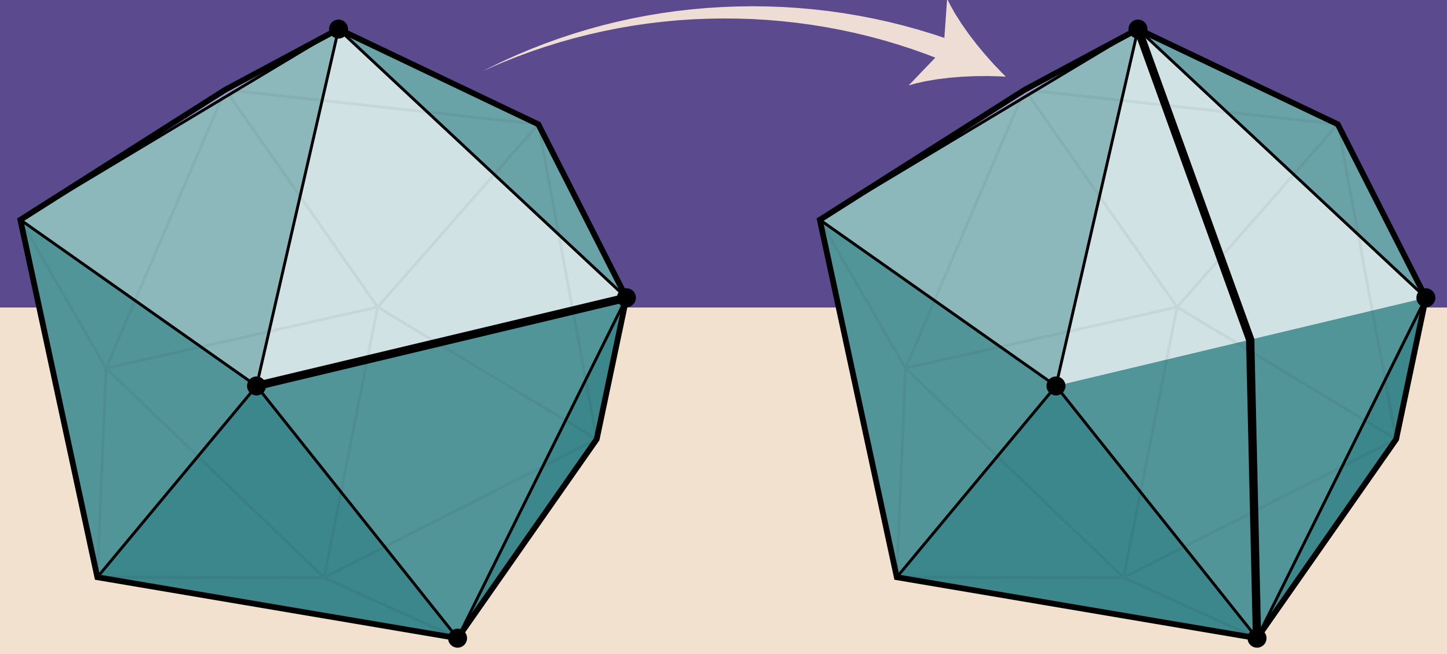
via the *discrete uniformization theorem*

Contributions



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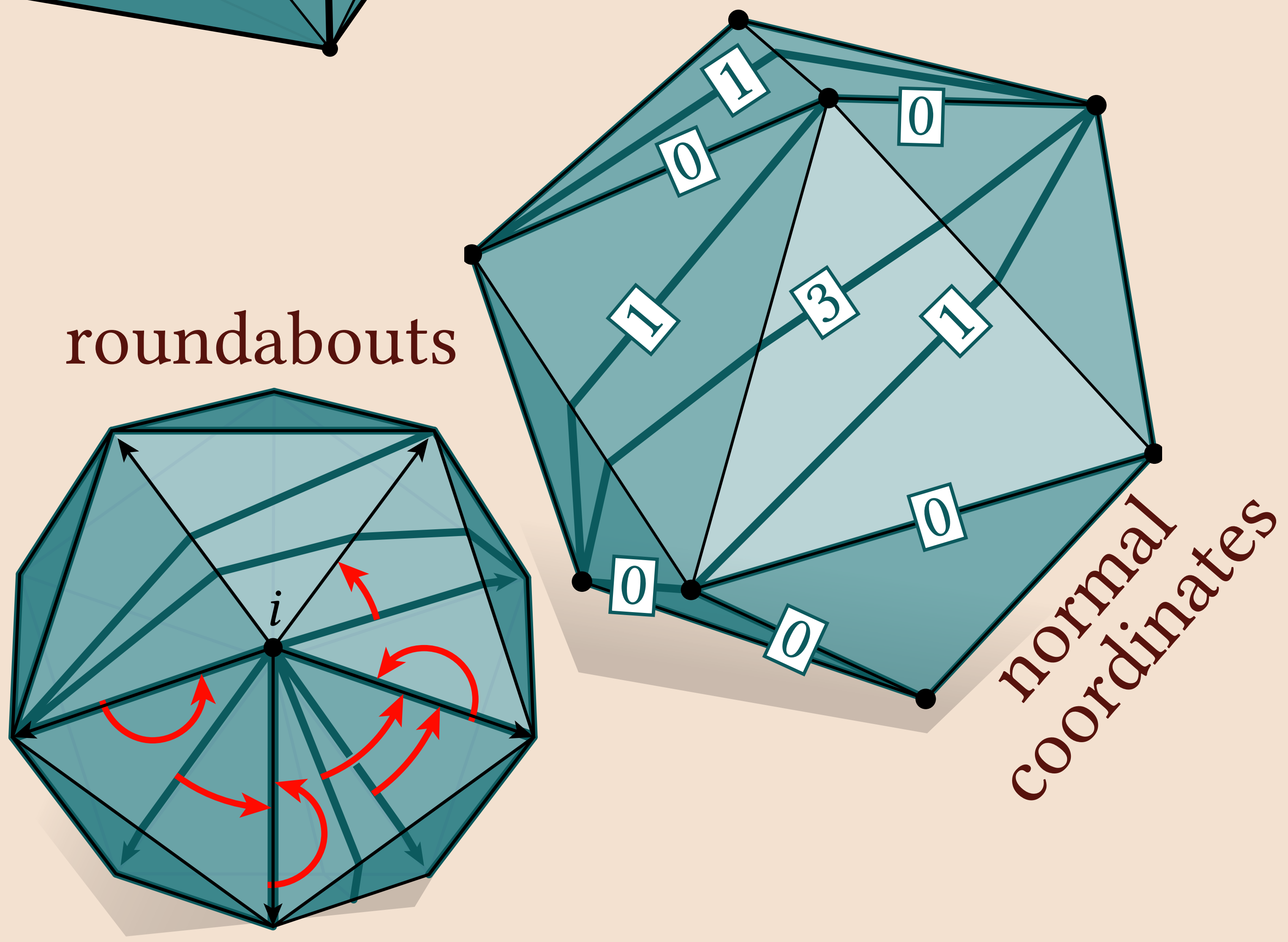
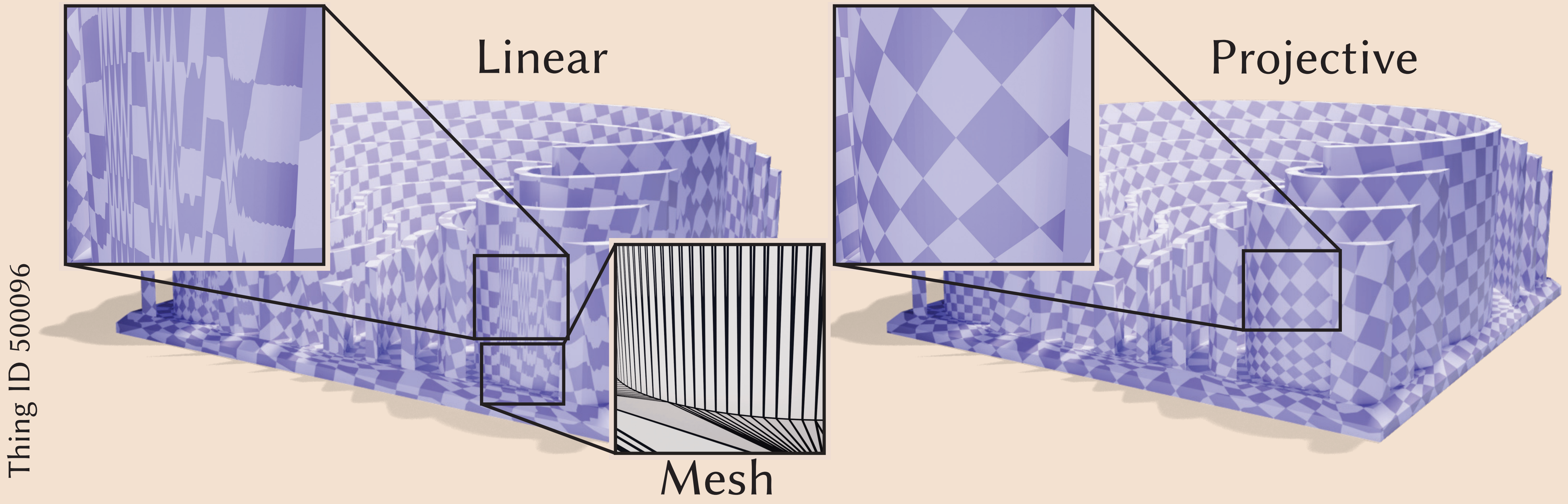
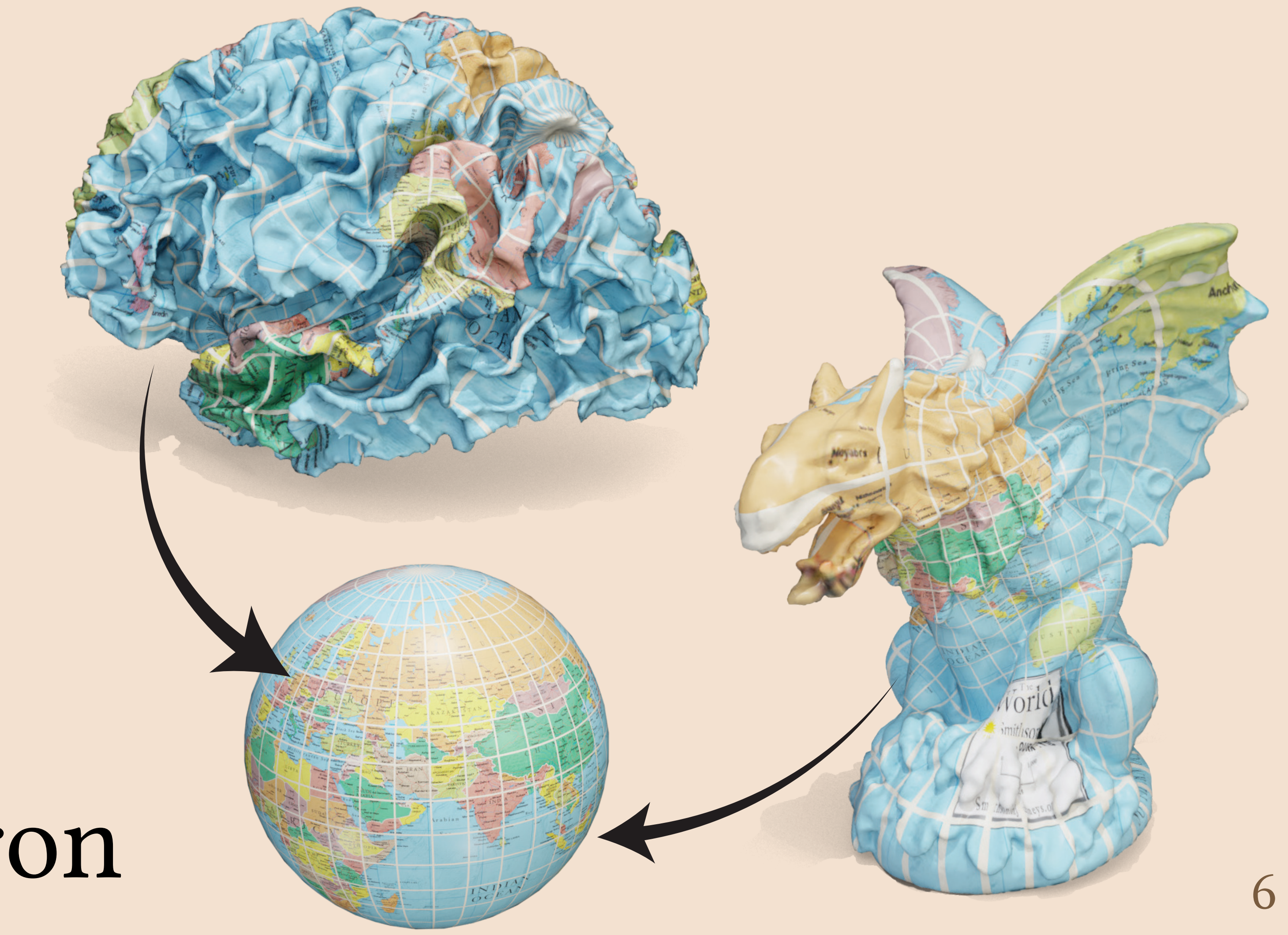
- Generalize CETM [Springborn+ 2008]
- Change mesh connectivity ⟶ use Ptolemy flips
  - Ensures that we find a valid parameterization
- Correspondence ⟶ normal coordinates & roundabouts
- Interpolation ⟶ calculate in the hyperboloid model

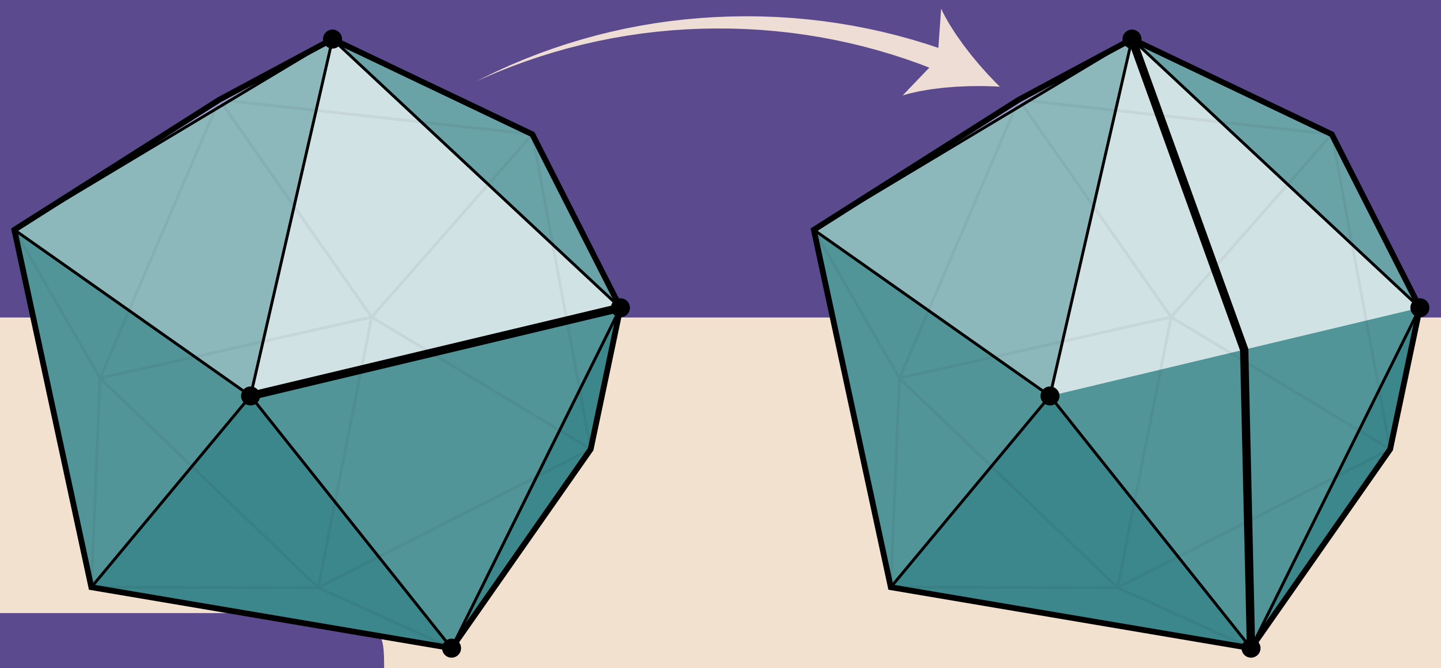




- Spherical case (guaranteed)
  - Discrete conformal map to convex, sphere-inscribed polyhedron

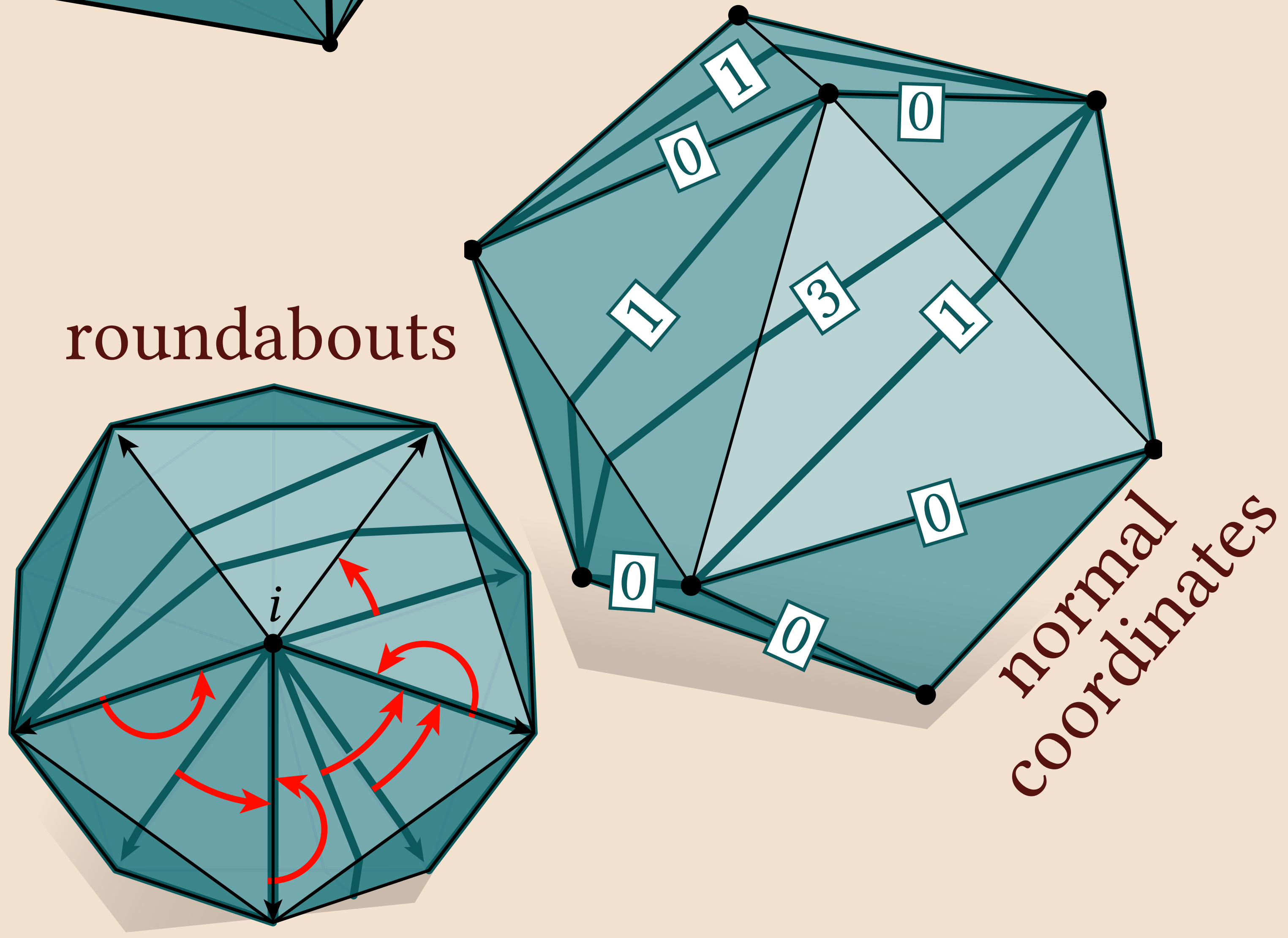


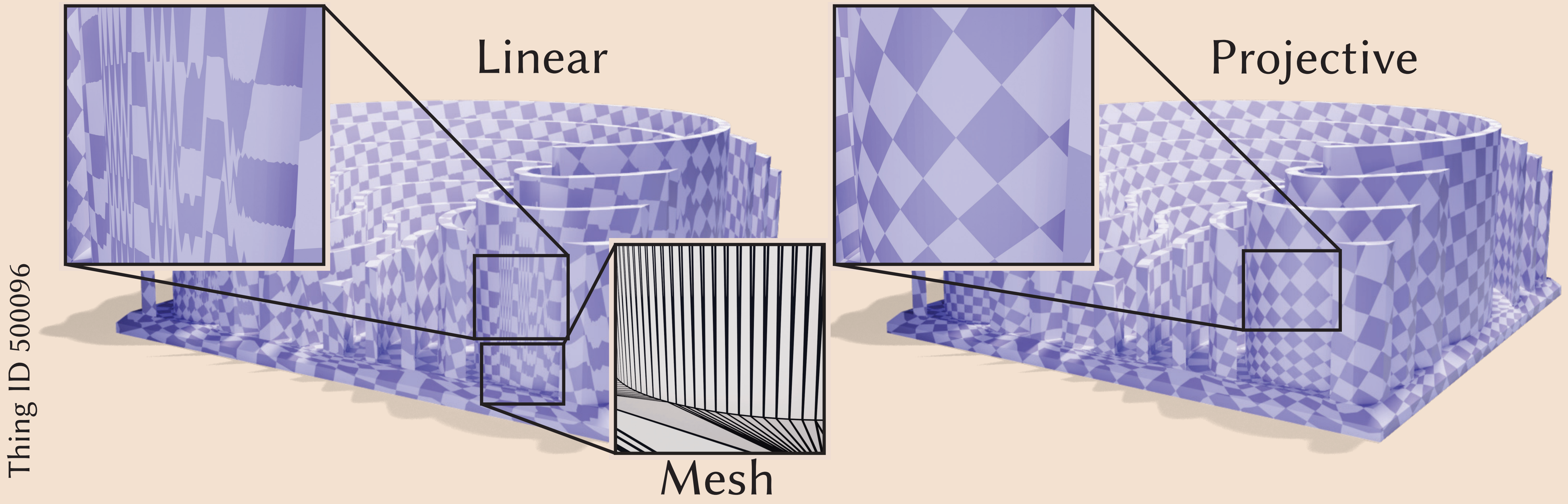
Contributions



Put the presenter's video feed here!

- Generalize CETM [Springborn+ 2008]
- Change mesh connectivity ⟶ use Ptolemy flips
  - Ensures that we find a valid parameterization
- Correspondence ⟶ normal coordinates & roundabouts
- Interpolation ⟶ calculate in the light cone





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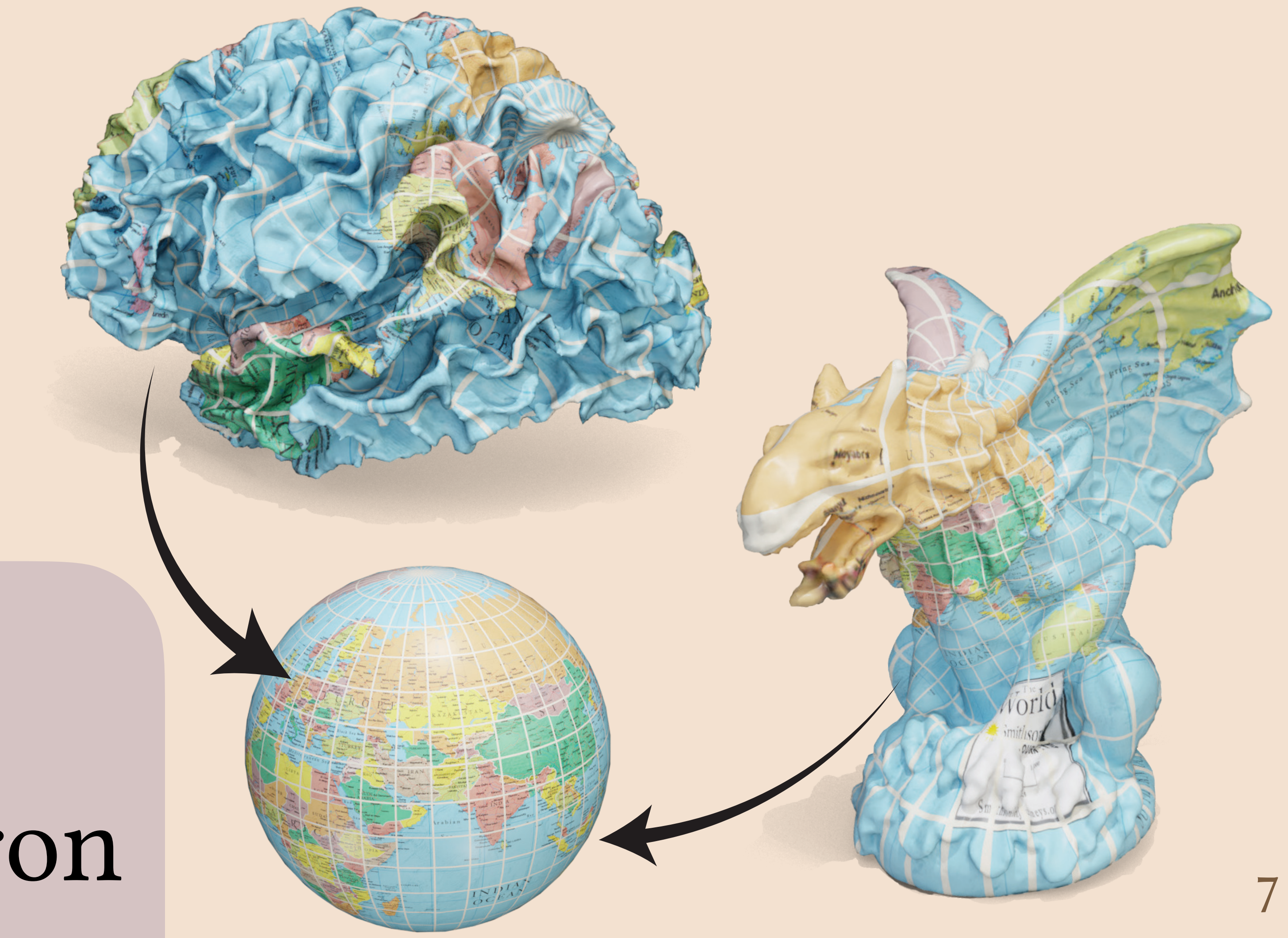

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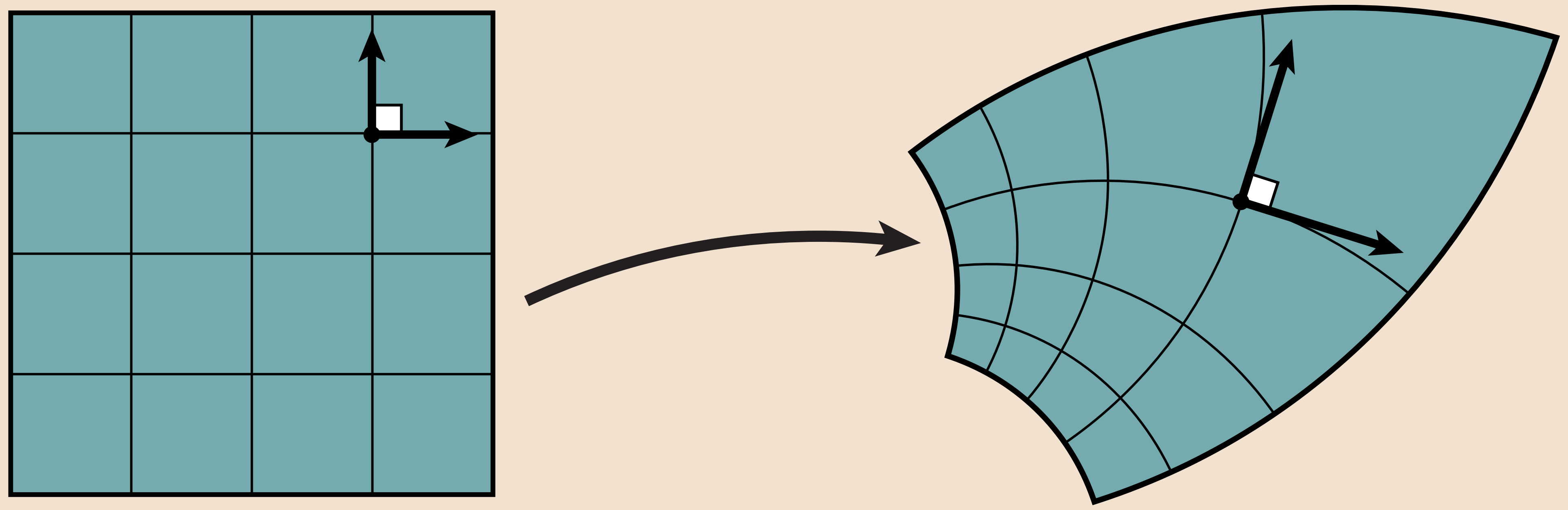
Discrete Conformal Parameterization

with Ptolemy flips

What is a discrete conformal map?

Put the presenter's video feed here!

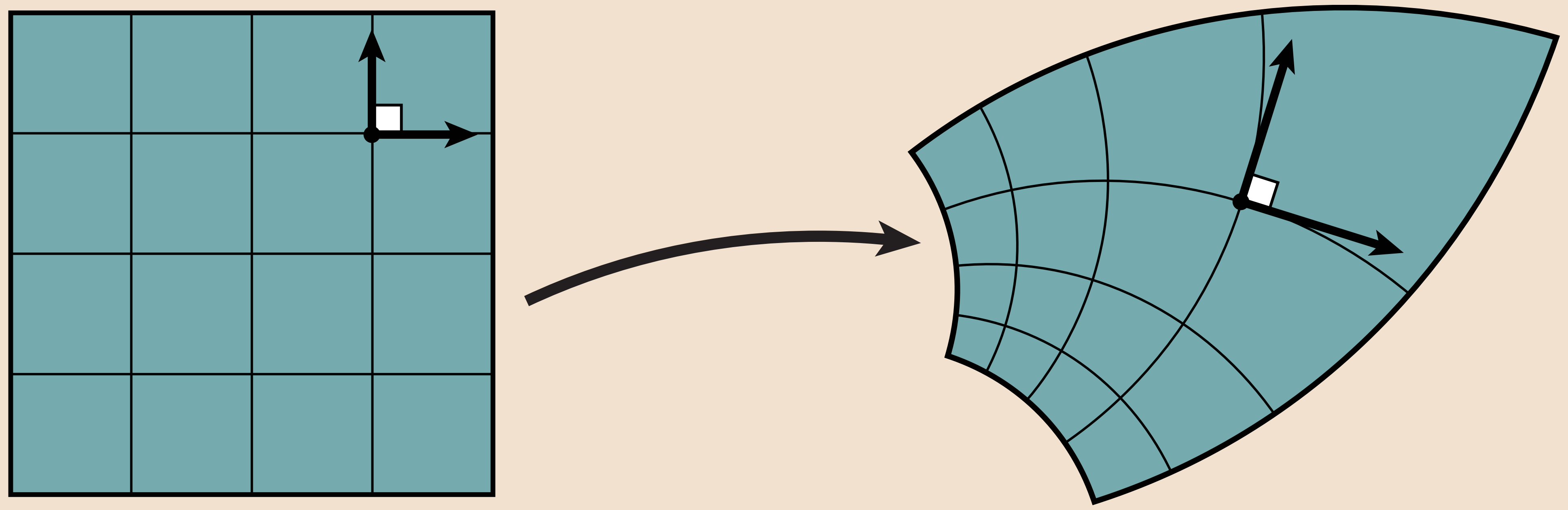
- *"Conformal maps preserve angles"*

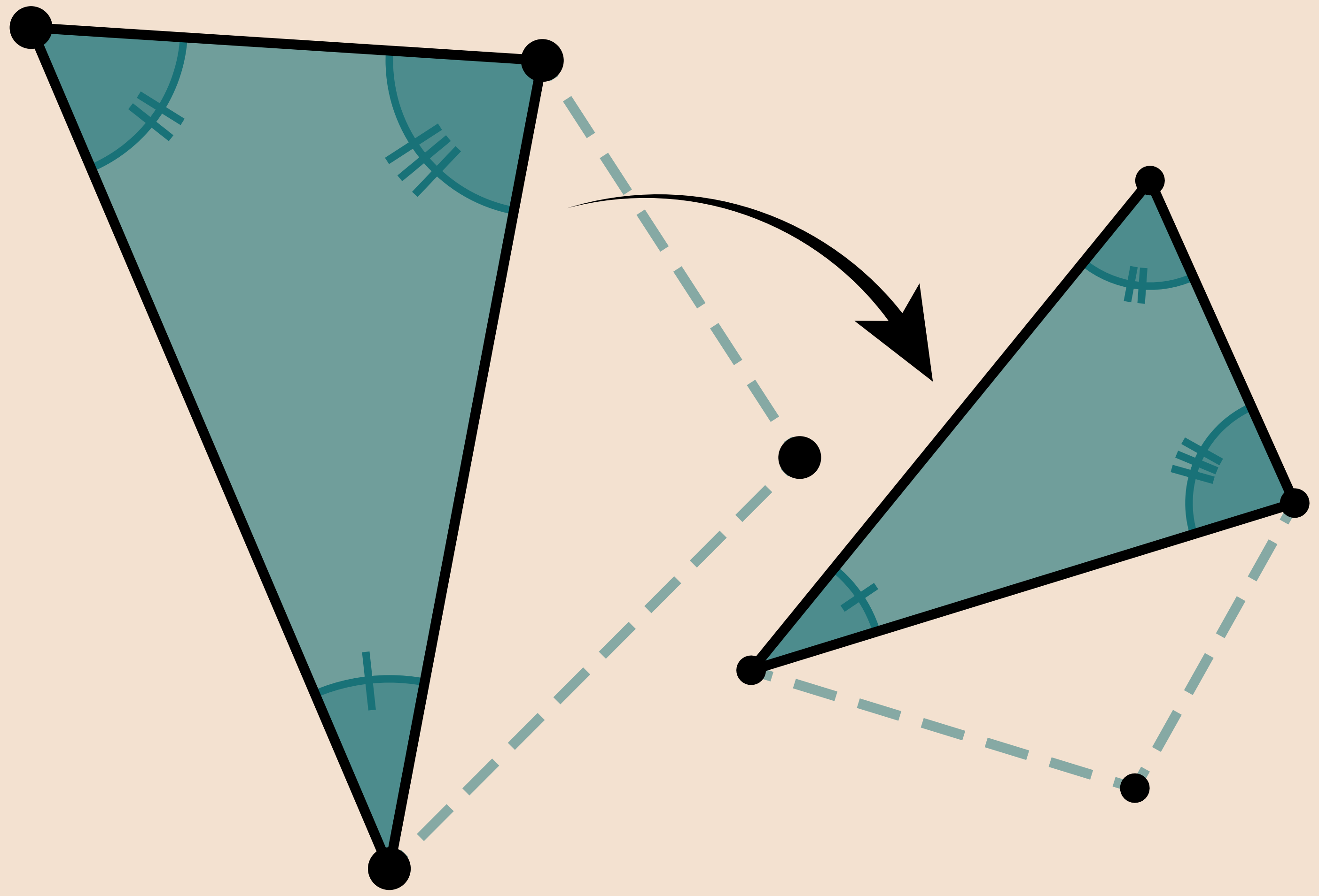


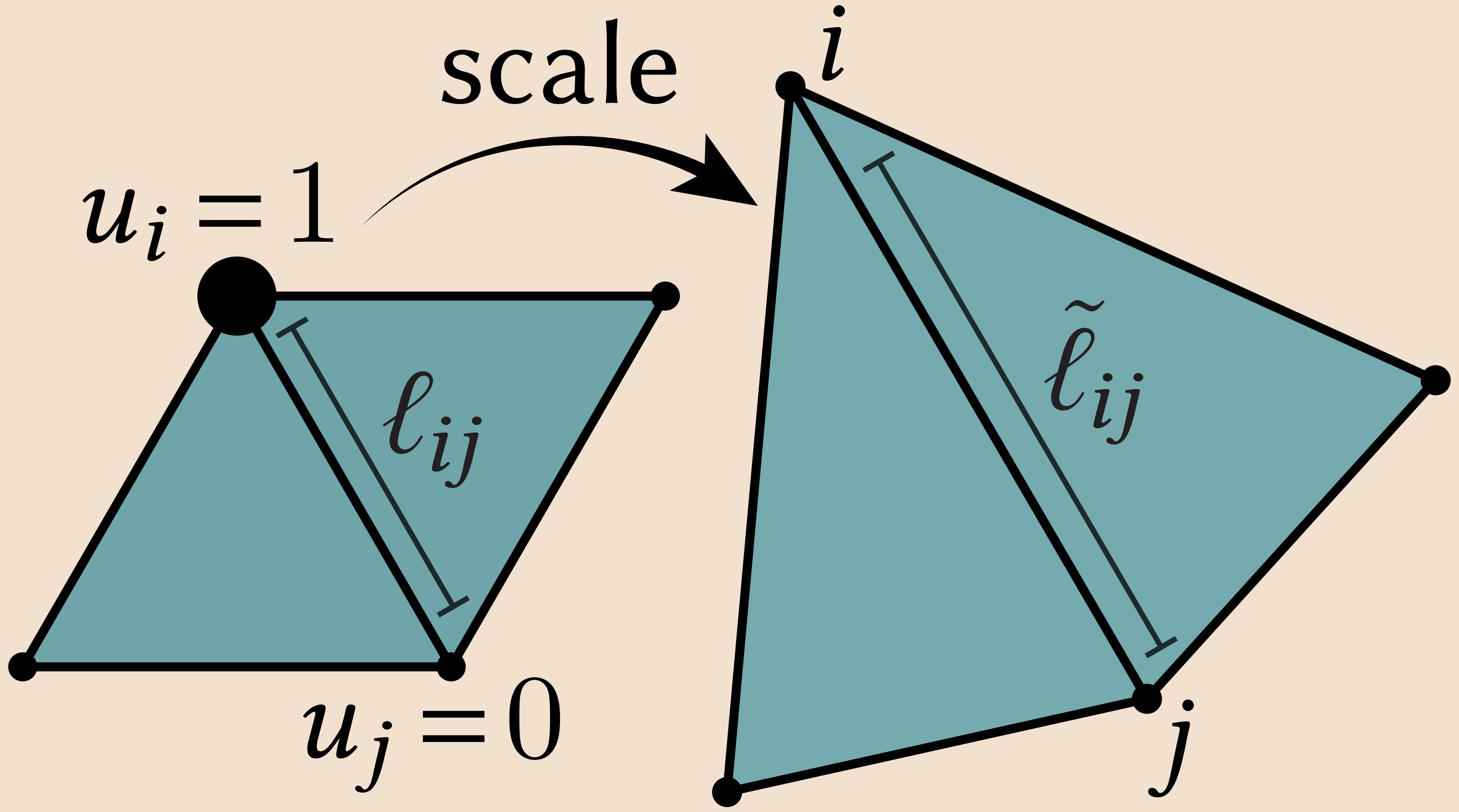
What is a discrete conformal map?

Put the presenter's video feed here!

- ~~“*Conformal maps preserve angles*”~~
  - Too strict
- Metric scaling
  - Locally, a conformal map just scales
- Discrete analogue: *vertex scaling*
  - $u : V \to \mathbb{R}$
  - $\tilde{\ell}_{ij} = e^{(u_i+u_j)/2}\ell_{ij}$
  - Just flexible enough [Bobenko+ 2011]







Uniformization

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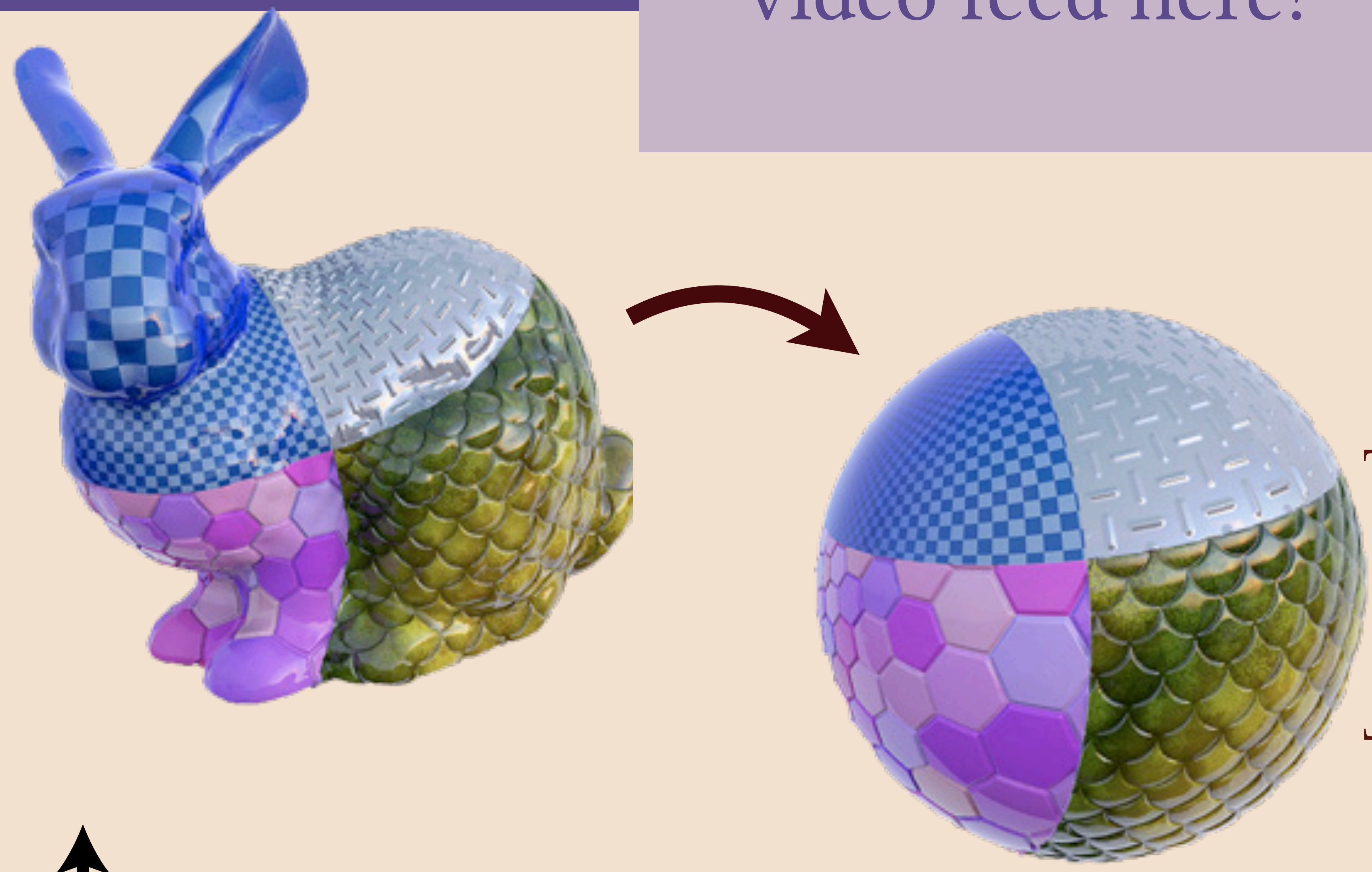
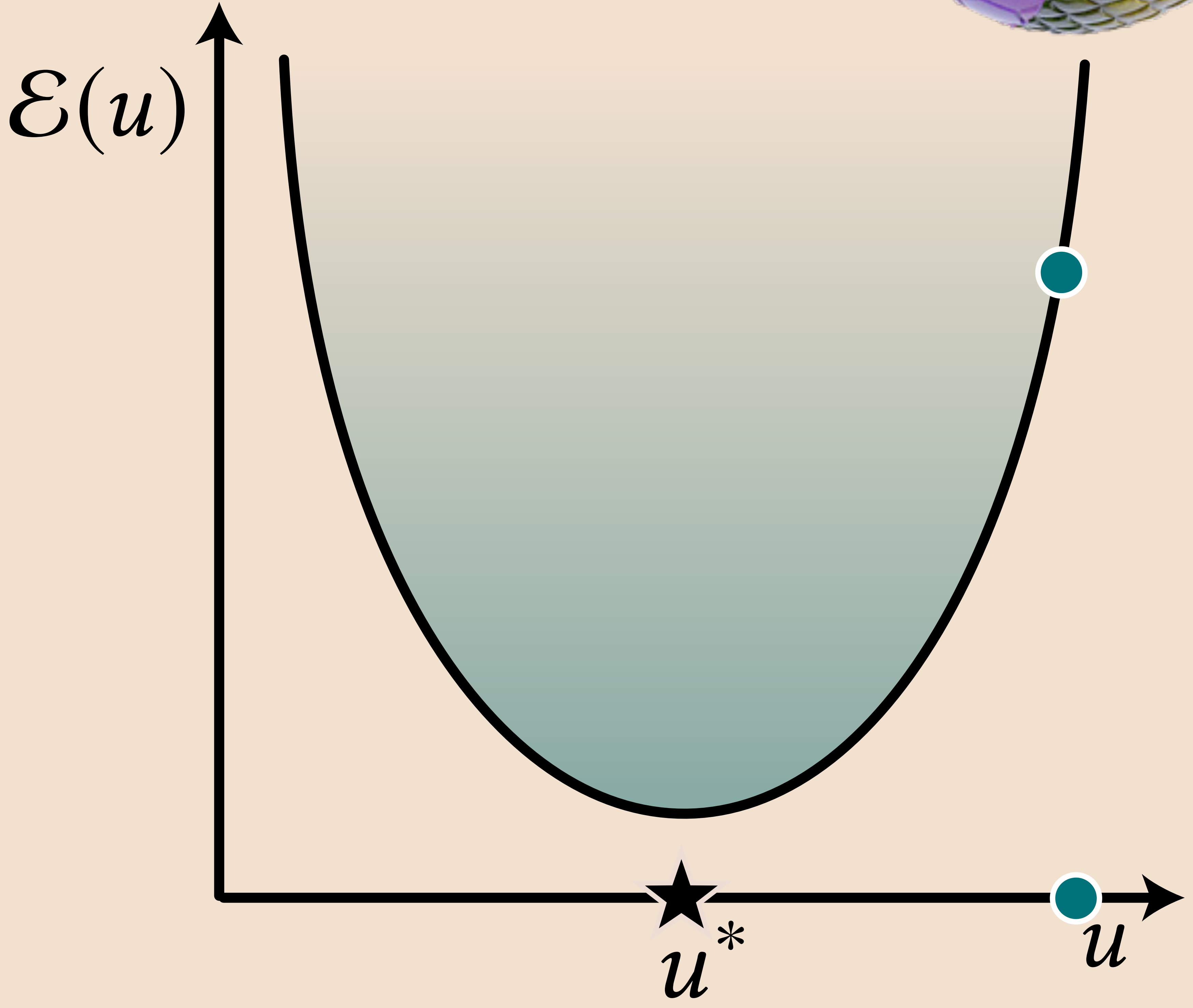
- Smooth uniformization [Poincaré 1907; Abikoff 1981]
  - *Any surface can be conformally mapped to one of constant curvature*
- Discrete uniformization [Gu+ 2018ab; Springborn 2019]
  - *Any valid curvatures can be realized by some vertex scaling*
  - [Luo 2004]: follow flow
  - [Springborn+ 2008]: minimize energy
  - **Main idea:** find discrete conformal maps by minimizing a convex energy



[Crane+ 2013]



Challenges with discrete uniformization

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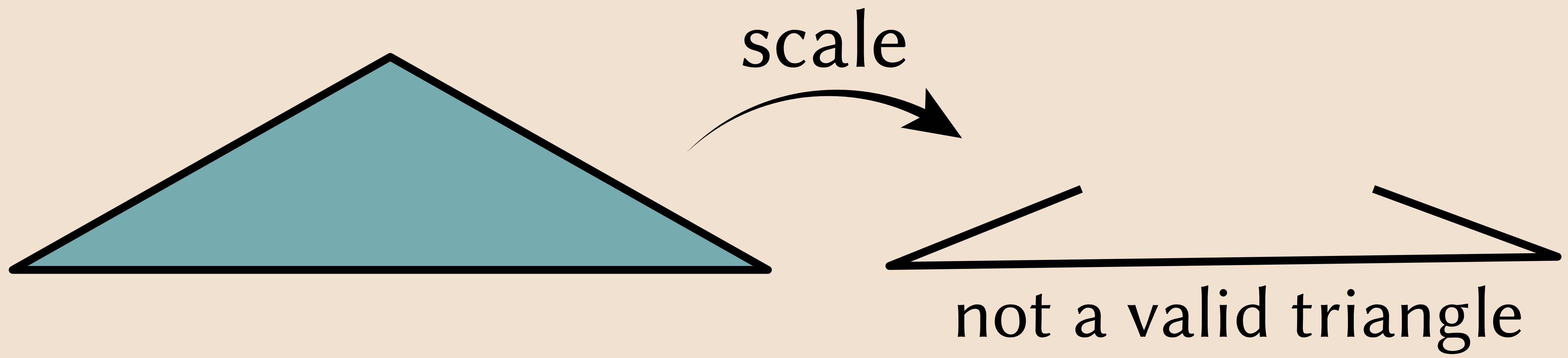
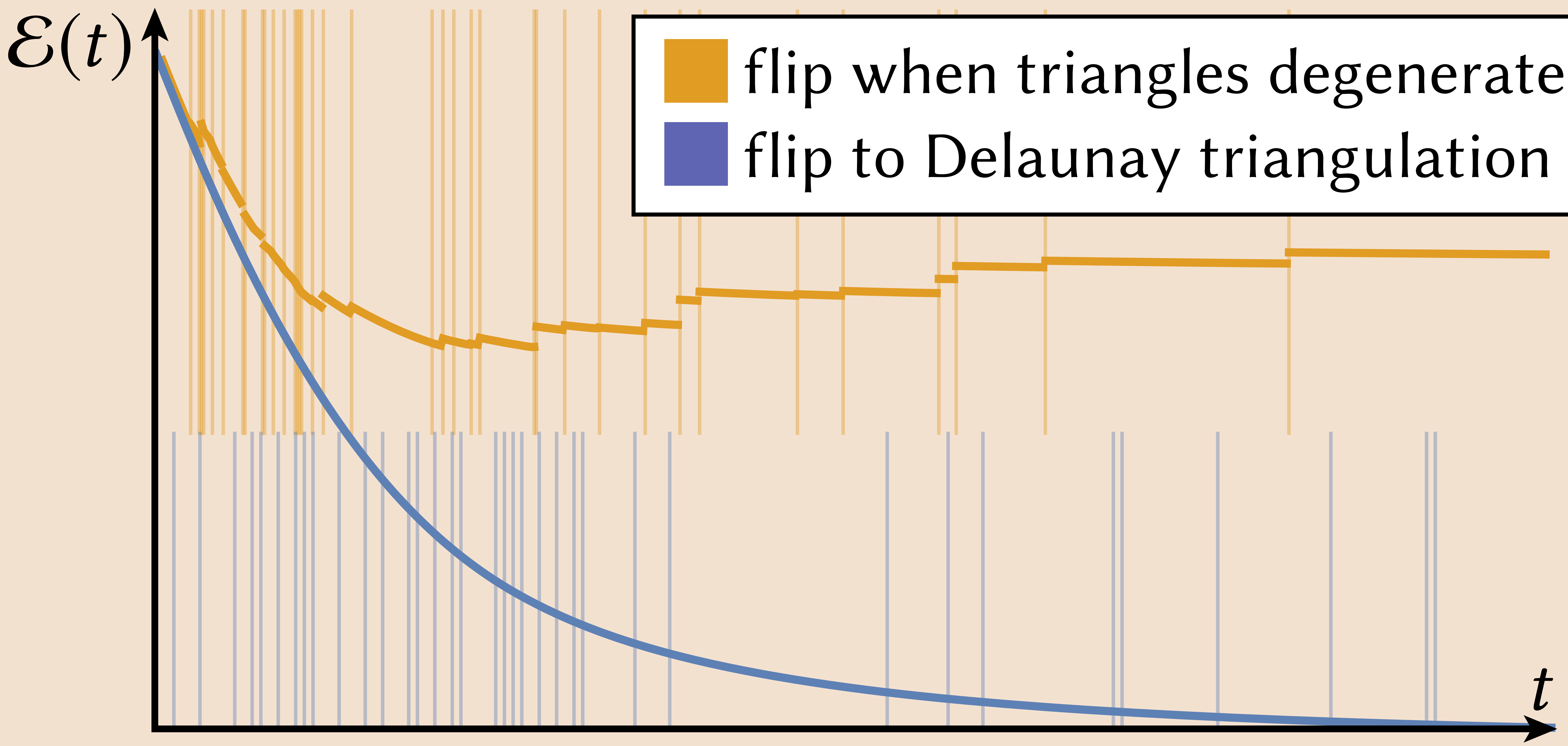
- **Big Problem:** Discrete uniformization doesn't always work on a fixed mesh because triangles can degenerate



- **Idea**: flip edges when triangles break
  - **Problem**: energy discontinuous at flips (vertical lines)
- [Gu+ 2018a]: maintain Delaunay
  - **Problem**: stop to flip



Hyperbolic geometry to the rescue

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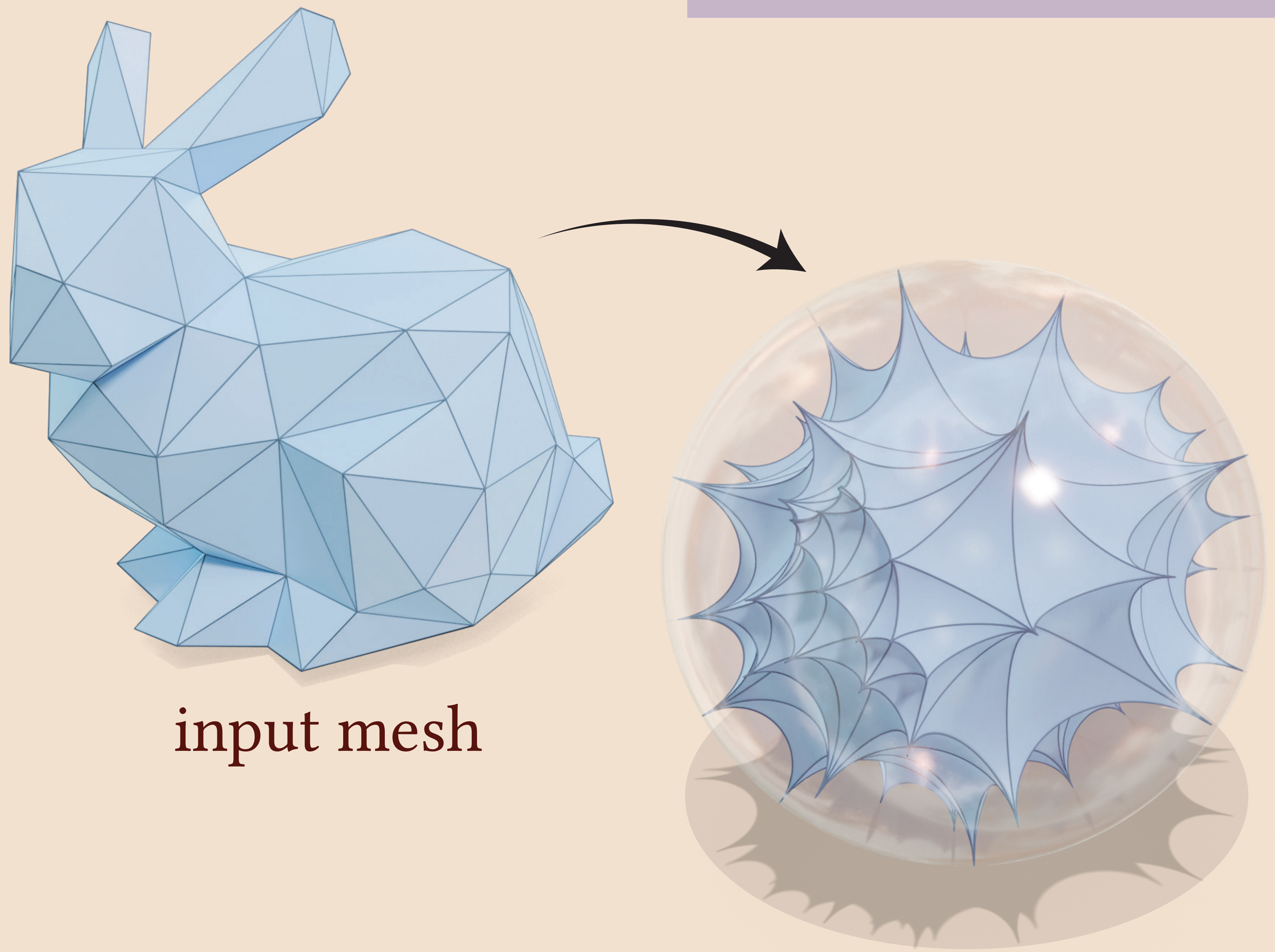
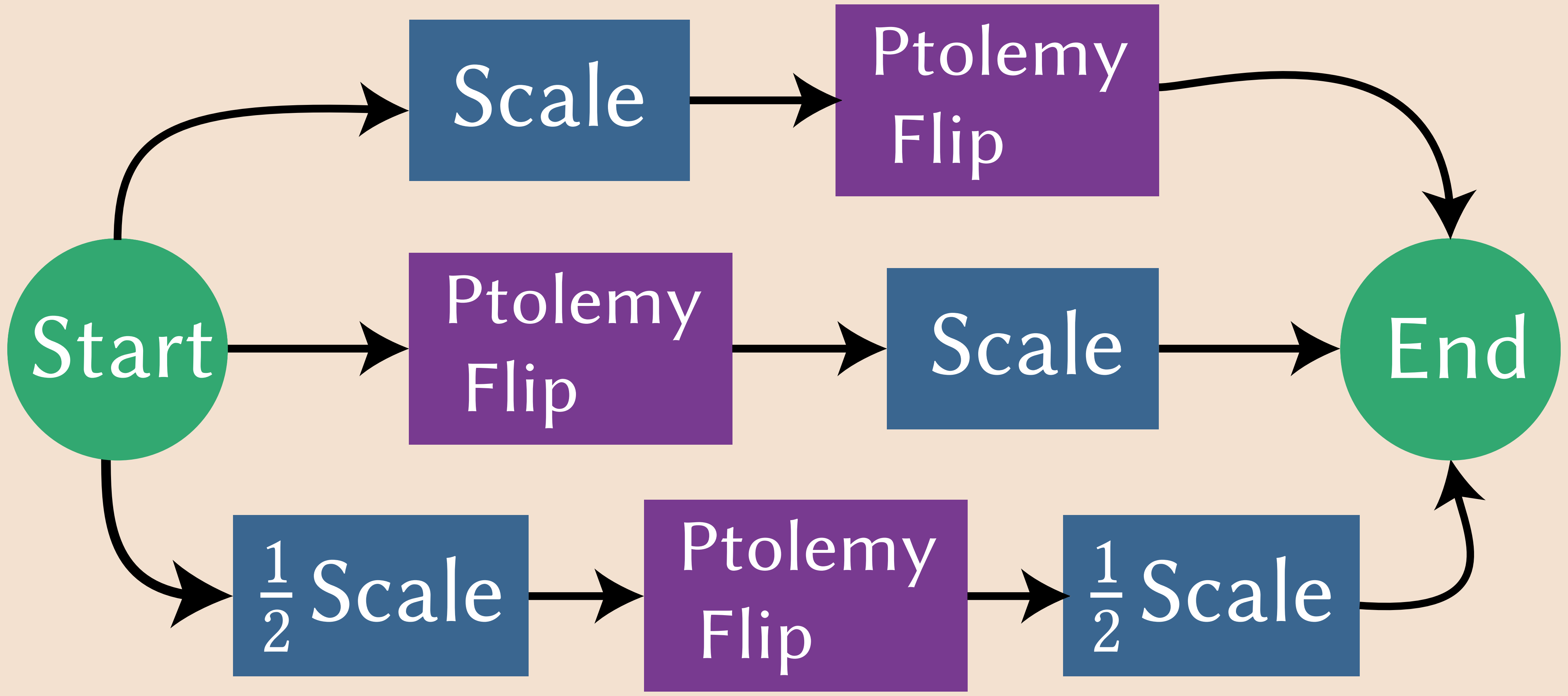
- Reinterpret mesh as ideal polyhedron [Bobenko+ 2010]
- Compute flipped edge lengths via *Ptolemy's formula*
  - $\ell_{ij} := (\ell_{lj}\ell_{ki} + \ell_{il}\ell_{jk})/\ell_{lk}$
  - Well-defined for any positive edge lengths
- Decouples scaling and flipping [Springborn 2019]



input mesh

ideal hyperbolic polyhedron



Uniformization with Ptolemy Flips

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Now, all scale factors are valid

Energy remains convex and C^2

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Spherical Uniformization

a very brief overview

Discrete spherical uniformization

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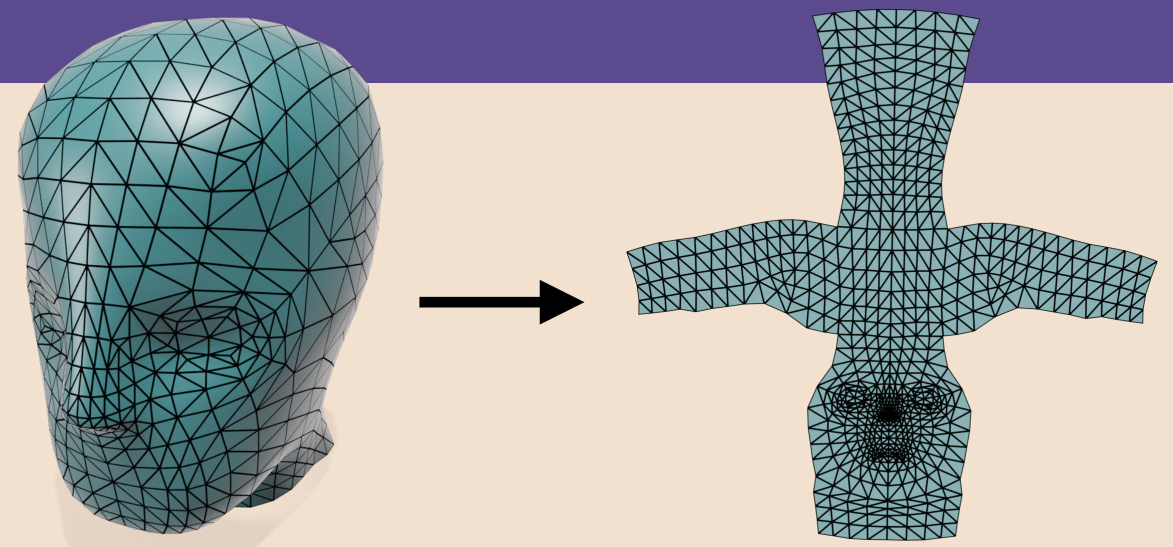
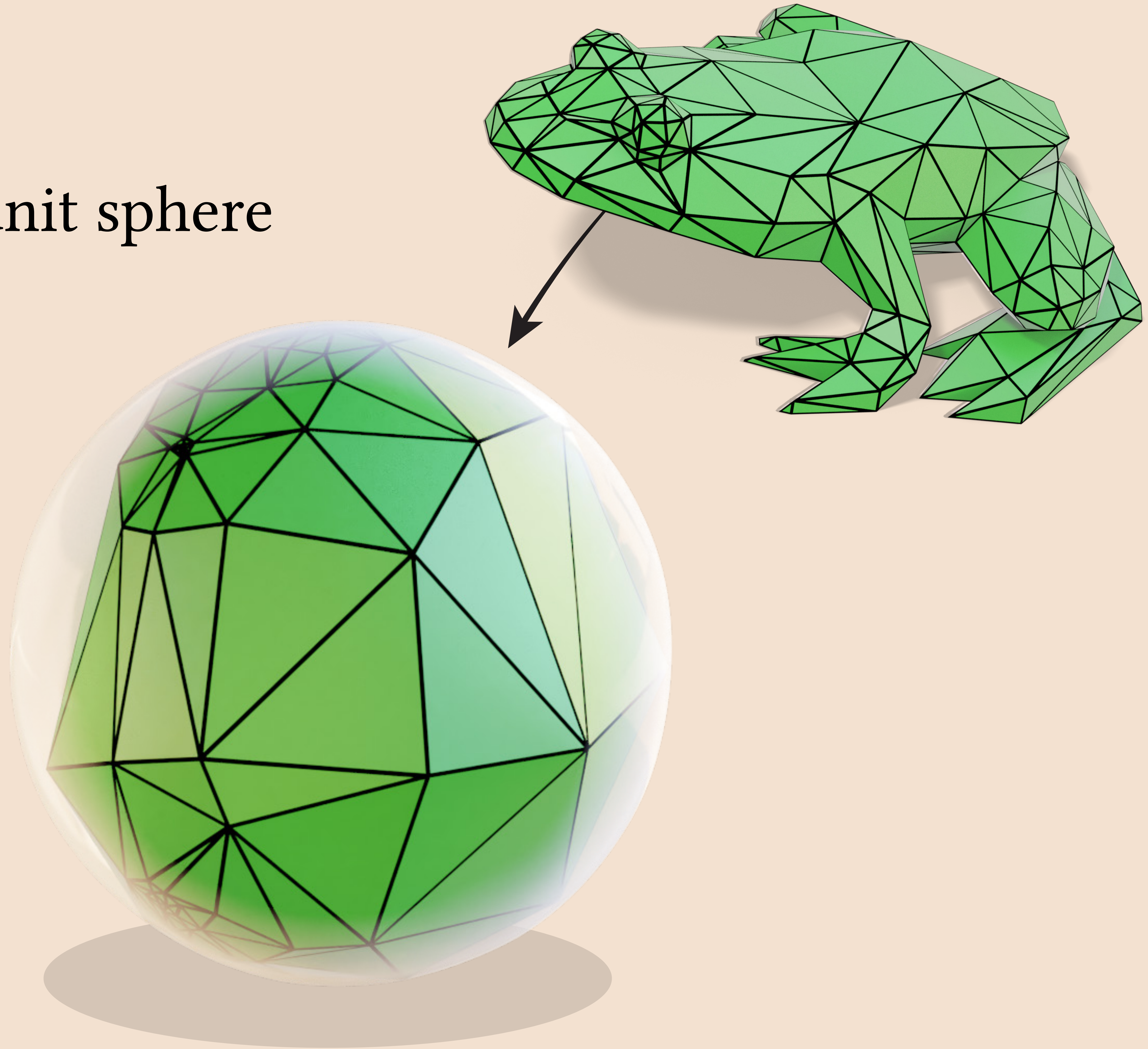
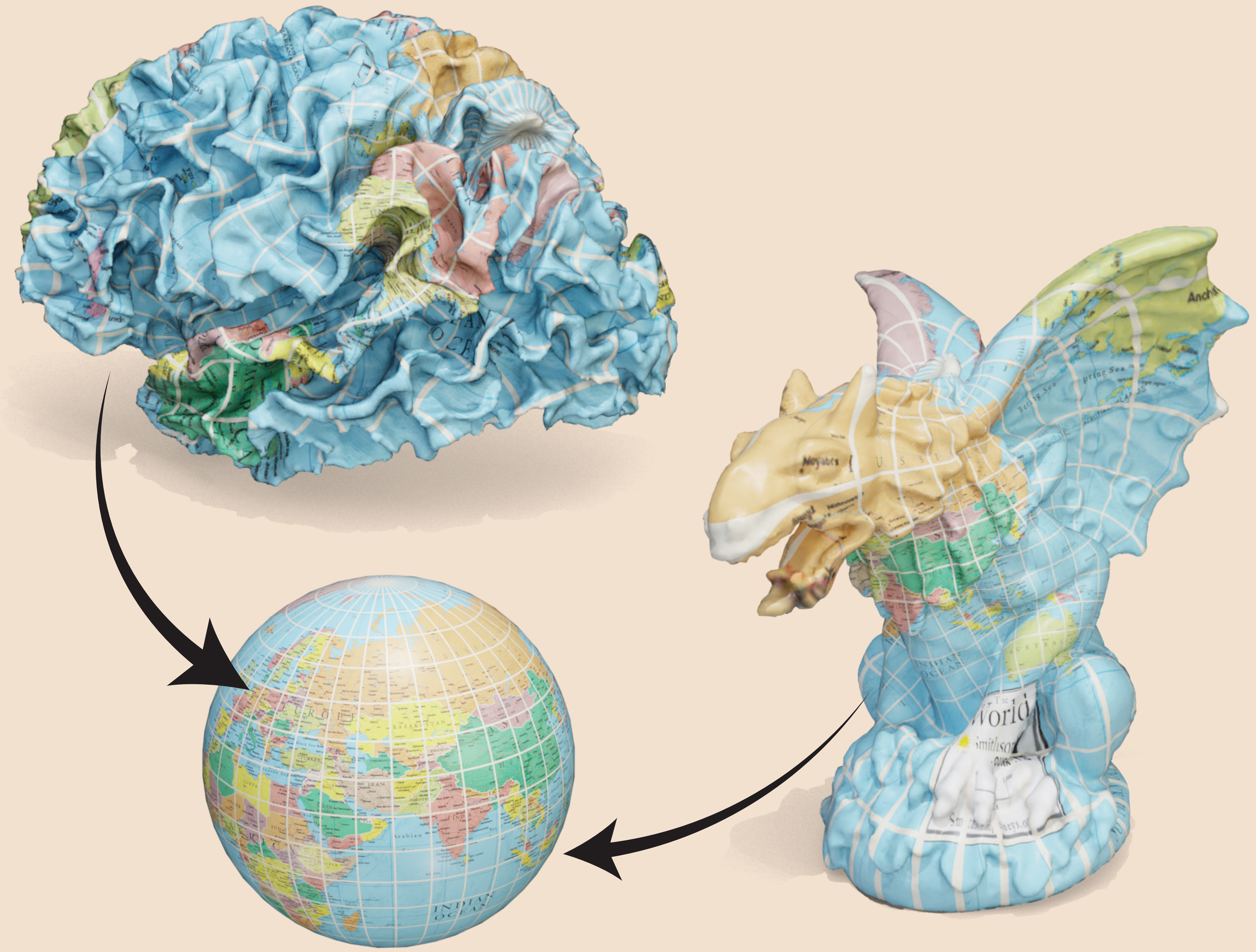
- So far: cone flattenings



- Also: map genus-0 surfaces to sphere
  - Explicitly, convex polyhedron w/ vertices on unit sphere

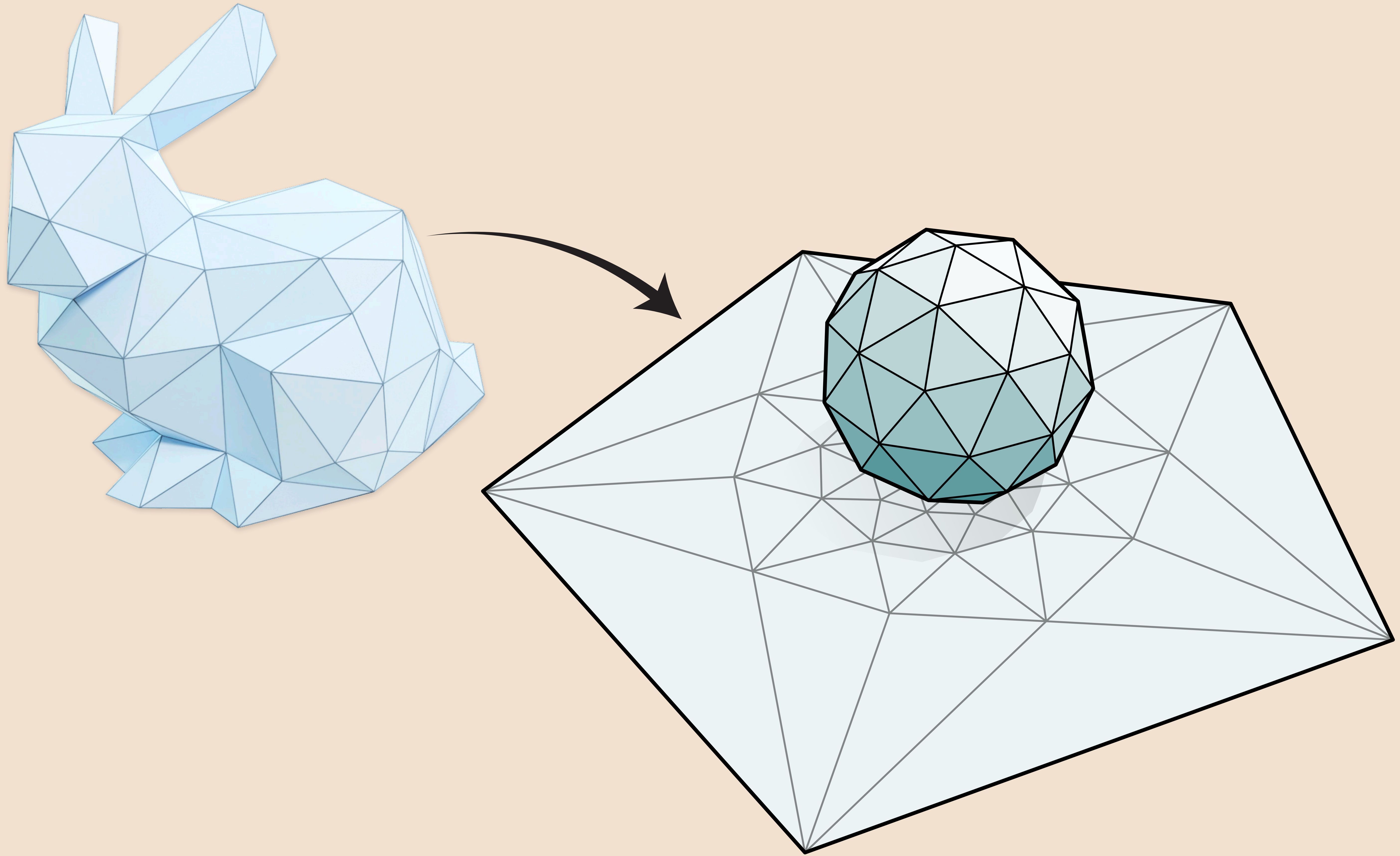


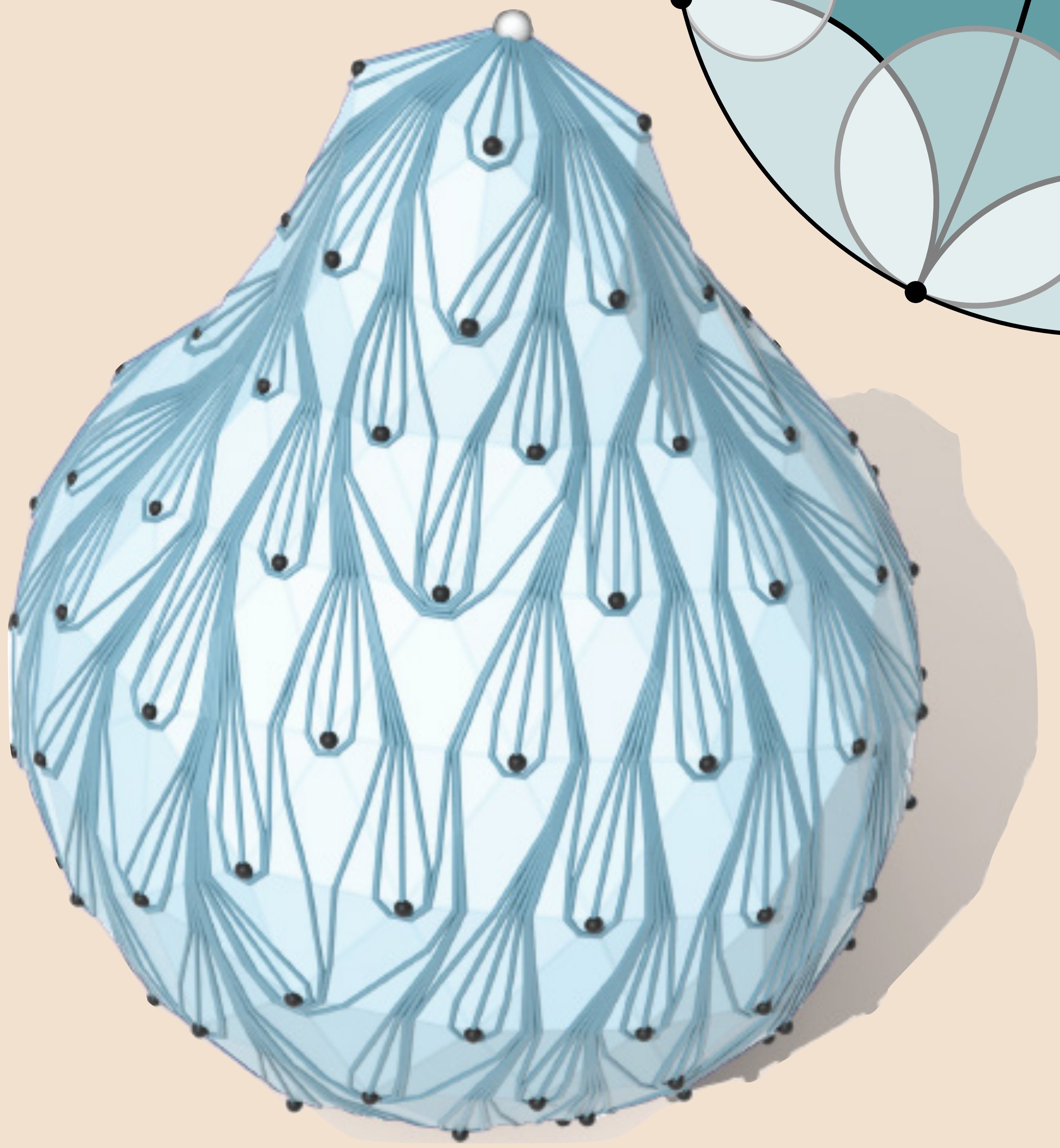


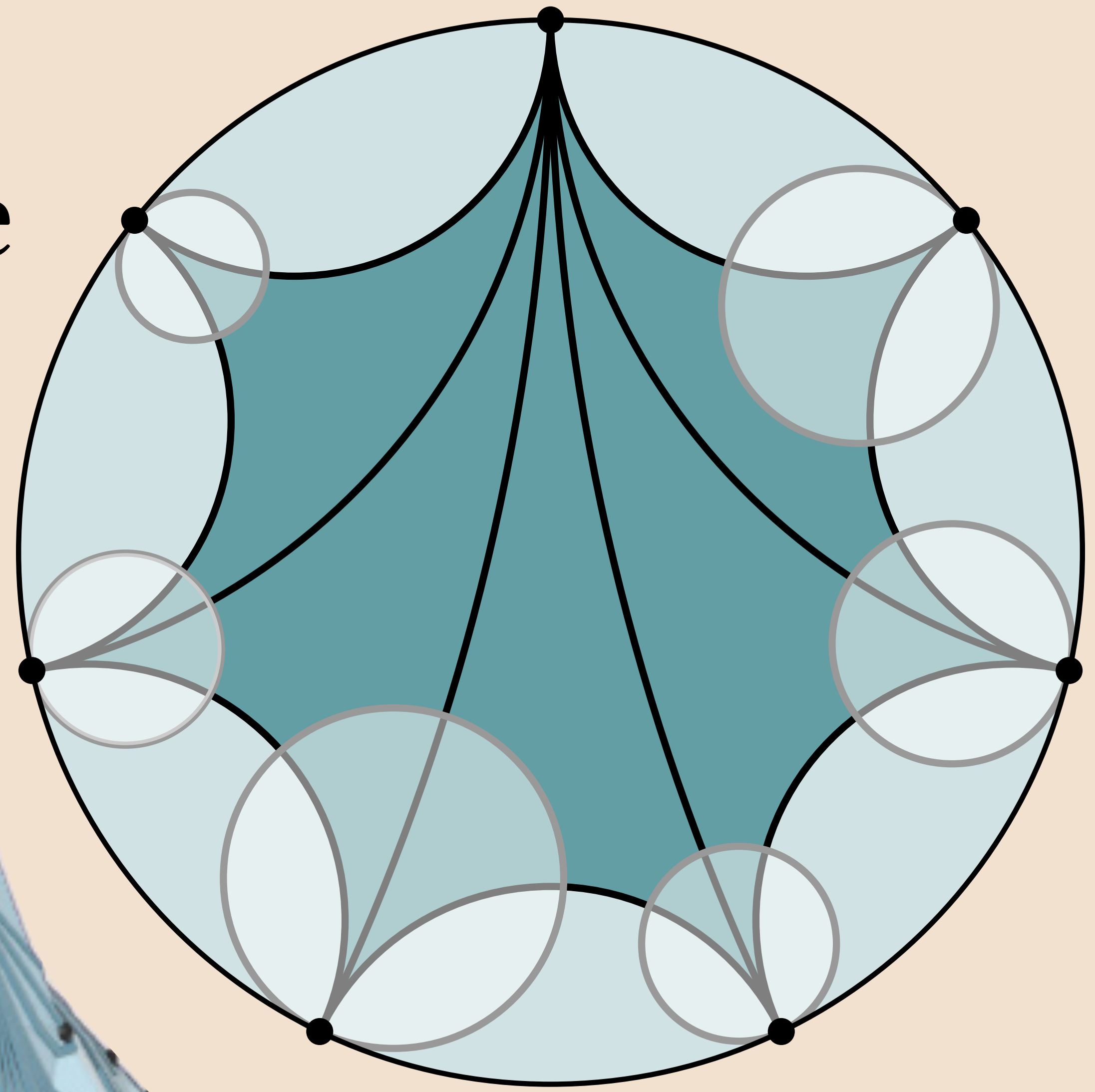
Discrete spherical uniformization

Put the presenter's video feed here!

- Similar optimization problem to cone flattening
- Algorithm complicated by the fact that mesh connectivity may change
  - Use even more hyperbolic geometry! [Springborn 2019]







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Results

Challenging datasets

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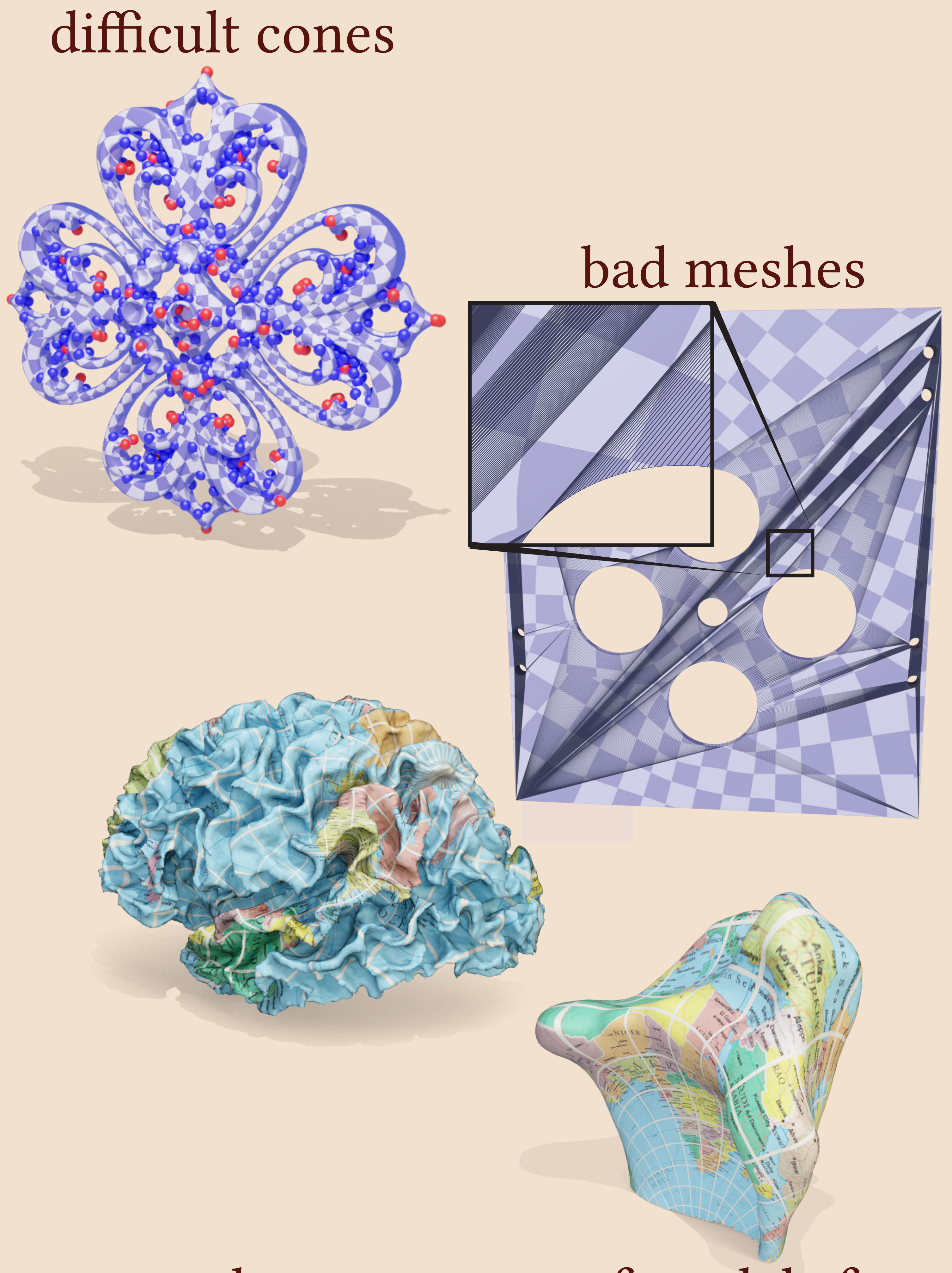

Dataset	# Models
MPZ [Myles+ 2014]	114
Thingi10k [Zhou+ 2016]	32,744*
brain scans [Yeo+ 2009]	78
anatomical surfaces [Boyer+ 2011]	187

* connected components of models from Thingi10k

Challenging datasets

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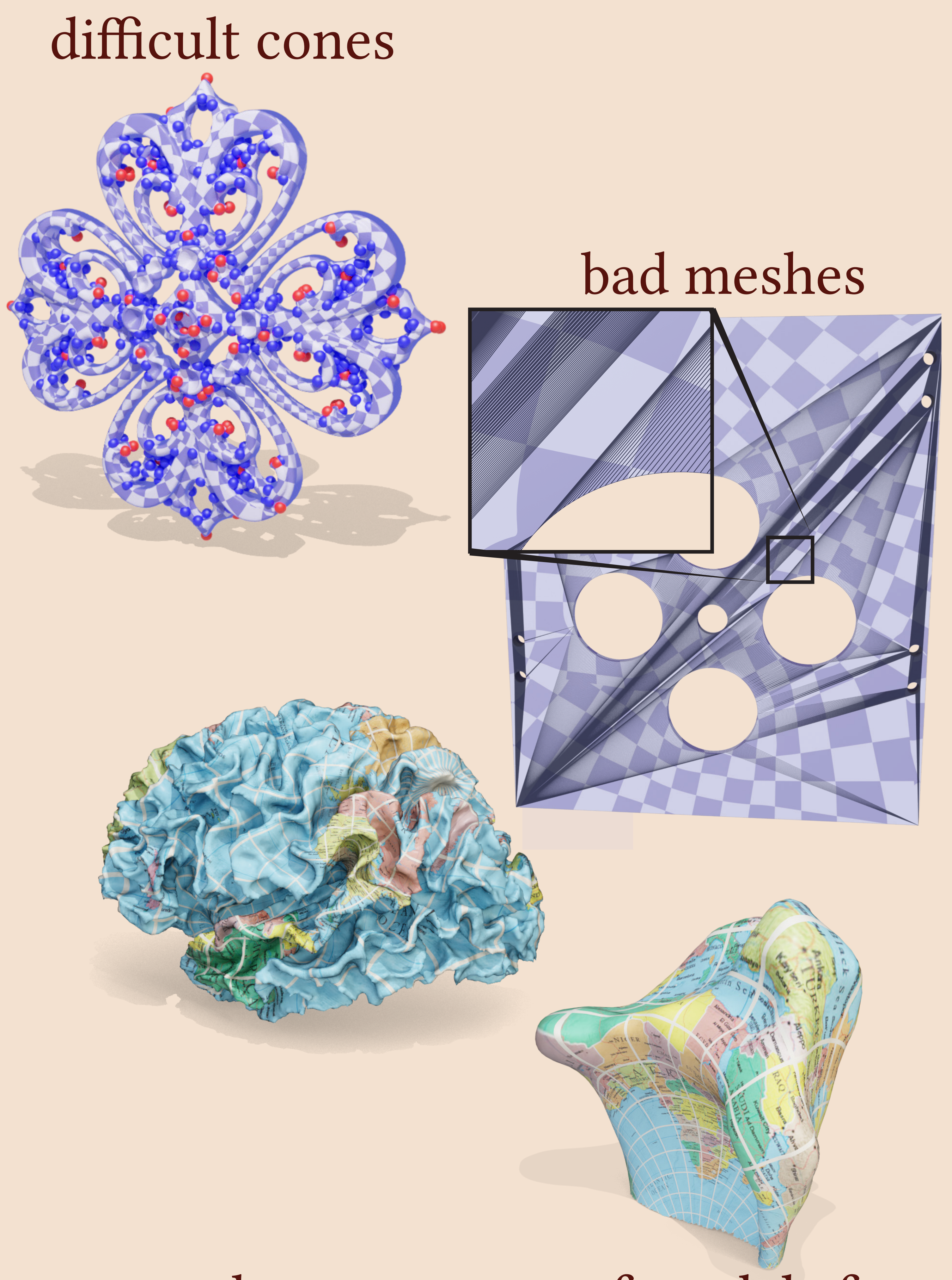

Dataset	# Models	Success rate
MPZ [Myles+ 2014]	114	100%
Thingi10k [Zhou+ 2016]	32,744*	97.7%
brain scans [Yeo+ 2009]	78	100%
anatomical surfaces [Boyer+ 2011]	187	100%

* connected components of models from Thingi10k

Challenging datasets

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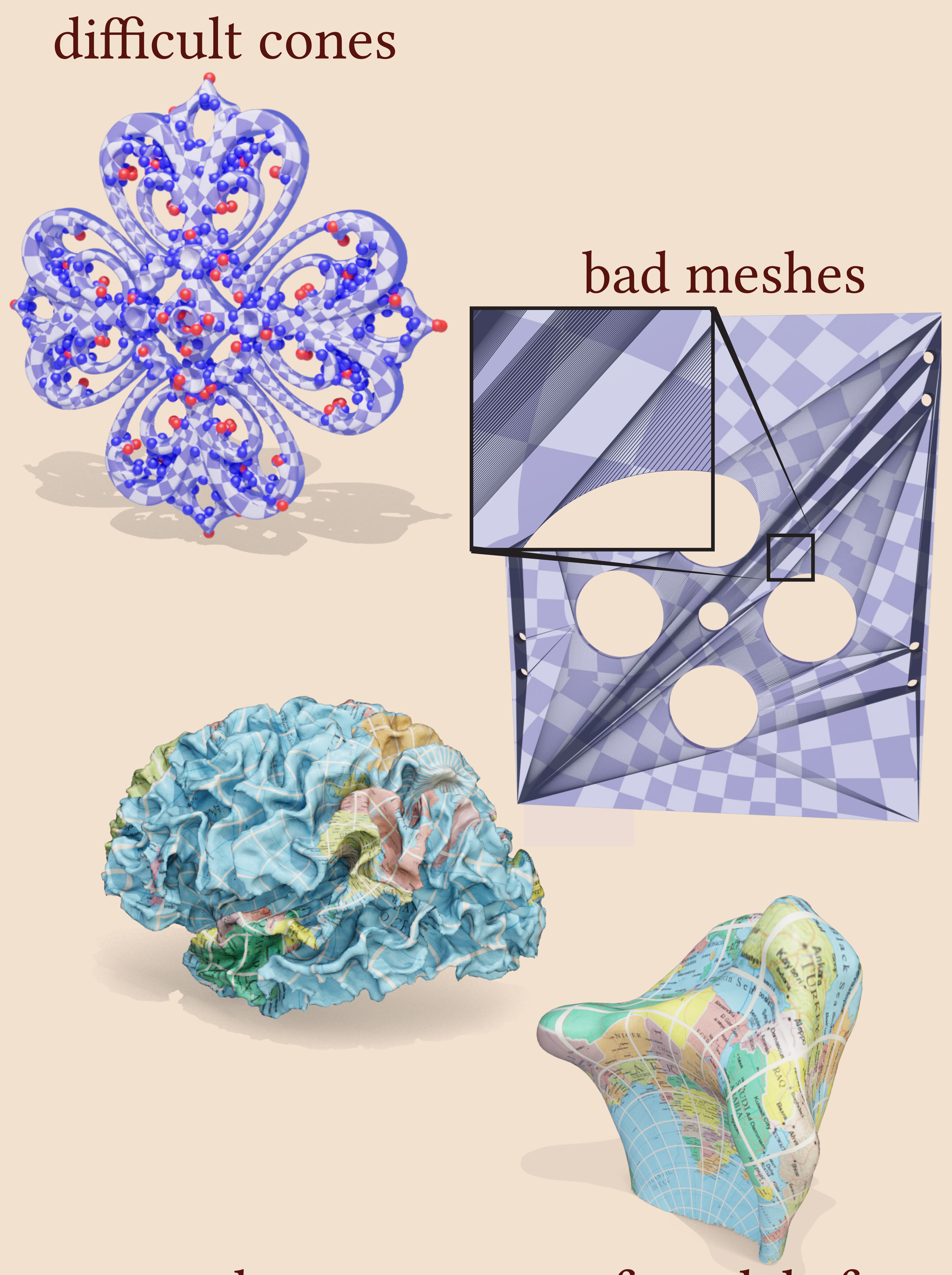

Dataset	# Models	Success rate	Average time
MPZ [Myles+ 2014]	114	100%	8s
Thingi10k [Zhou+ 2016]	32,744*	97.7%	57s†
brain scans [Yeo+ 2009]	78	100%	493s
anatomical surfaces [Boyer+ 2011]	187	100%	15s

* connected components of models from Thingi10k

* † average time on models with > 1000 vertices

Ptolemy flips improve performance

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Projective interpolation improves quality

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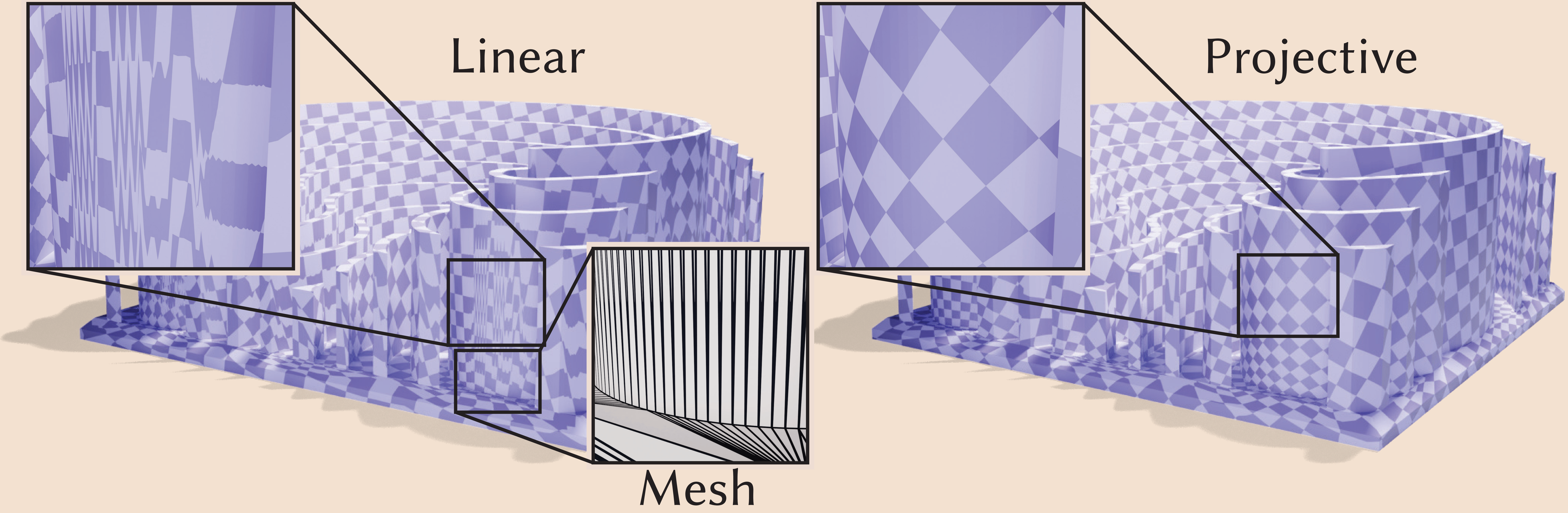

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Variable triangulation > fixed triangulation

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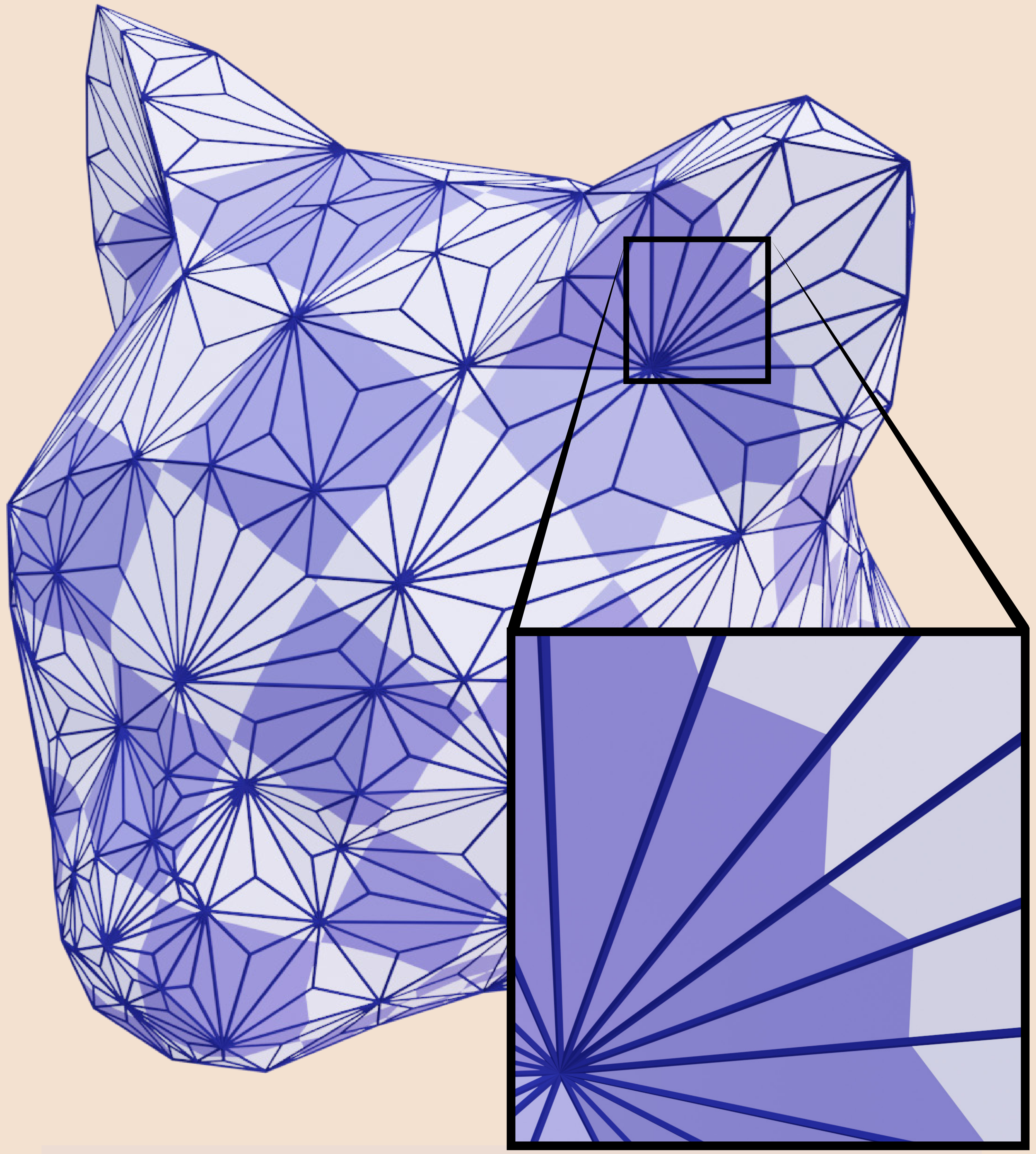
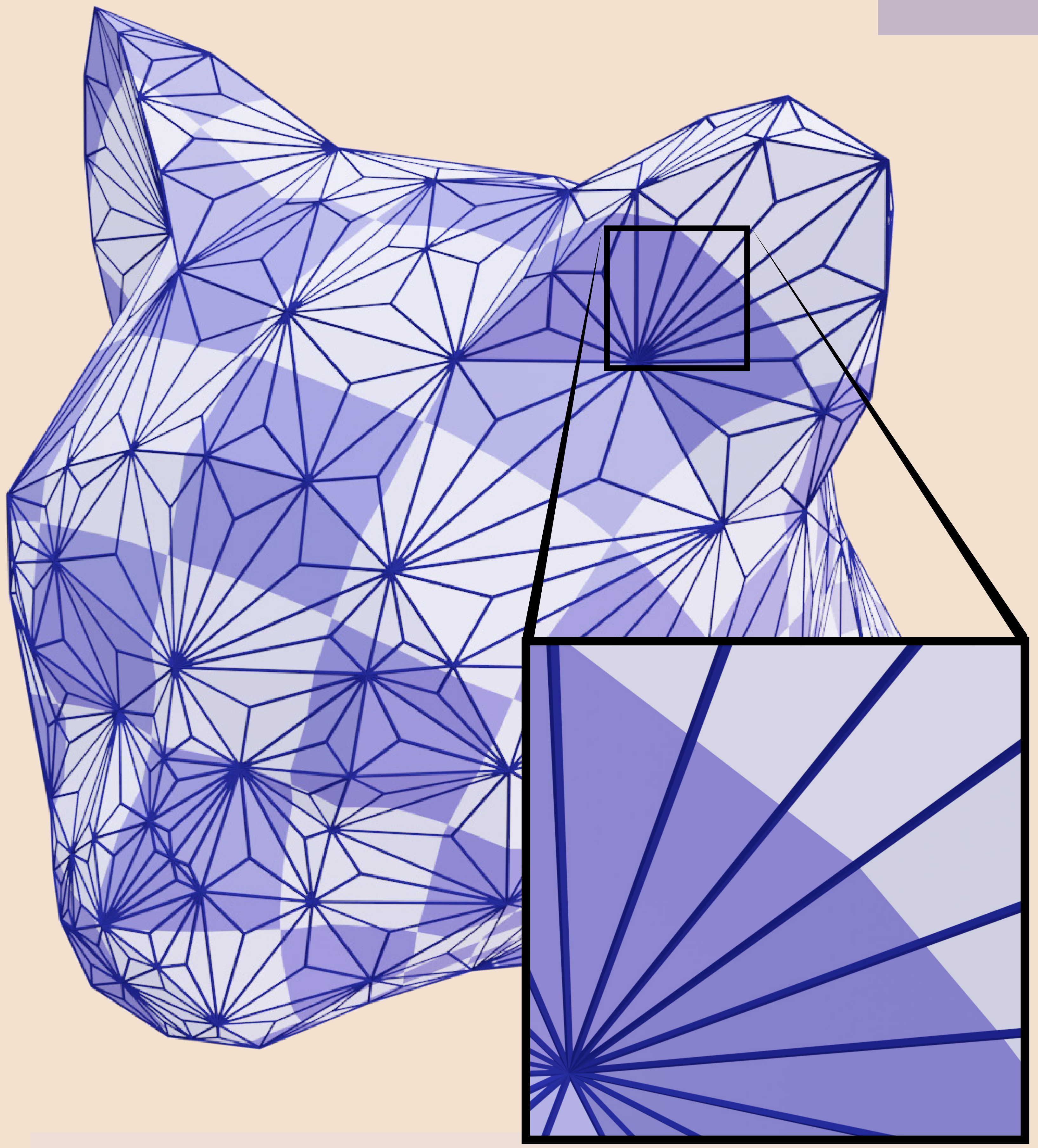


Fixed triangulation (CETM)



Variable triangulation (CEPS)

Even when fixed triangulation succeeds, variable triangulation projective interpolation is smoother

Limitations and future work

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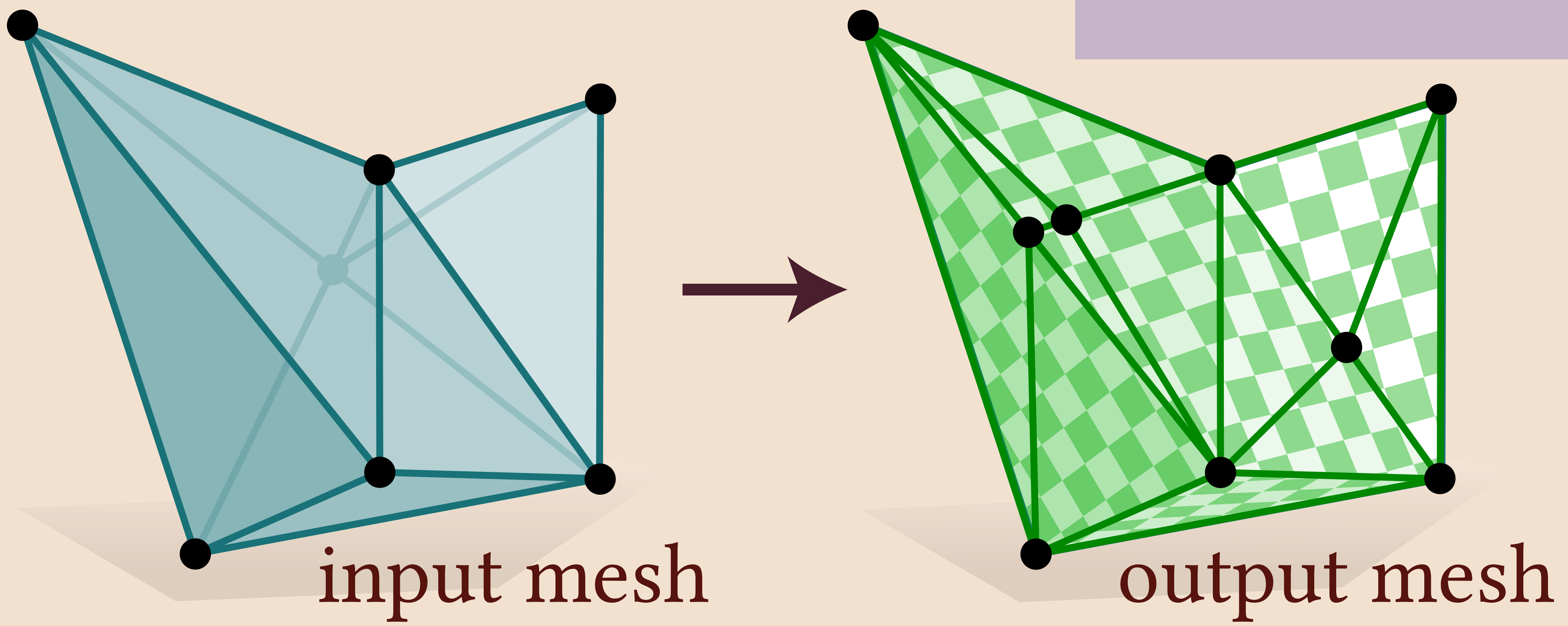
- Output is refined mesh
  - Could you unflip all flipped edges?
- If all you care about is injectivity, correspondence is simpler
- Going beyond 2D
  - 2D uniformization theorem ⟶ 3D geometrization theorem
  - 2D Delaunay triangulations ⟶ 3D Delaunay tetrahedralizations



Thanks!

Code is available at
github.com/MarkGillespie/CEPS

Put the presenter's video feed here!