Origami and Geometry

Mark Gillespie

Graphics Lab Meeting 10/9/19

http://nicolaeene.blogspot.no

[Satoshi Kamiya]

atilla yurtkul

[Robert Lang]

[Joel Cooper]

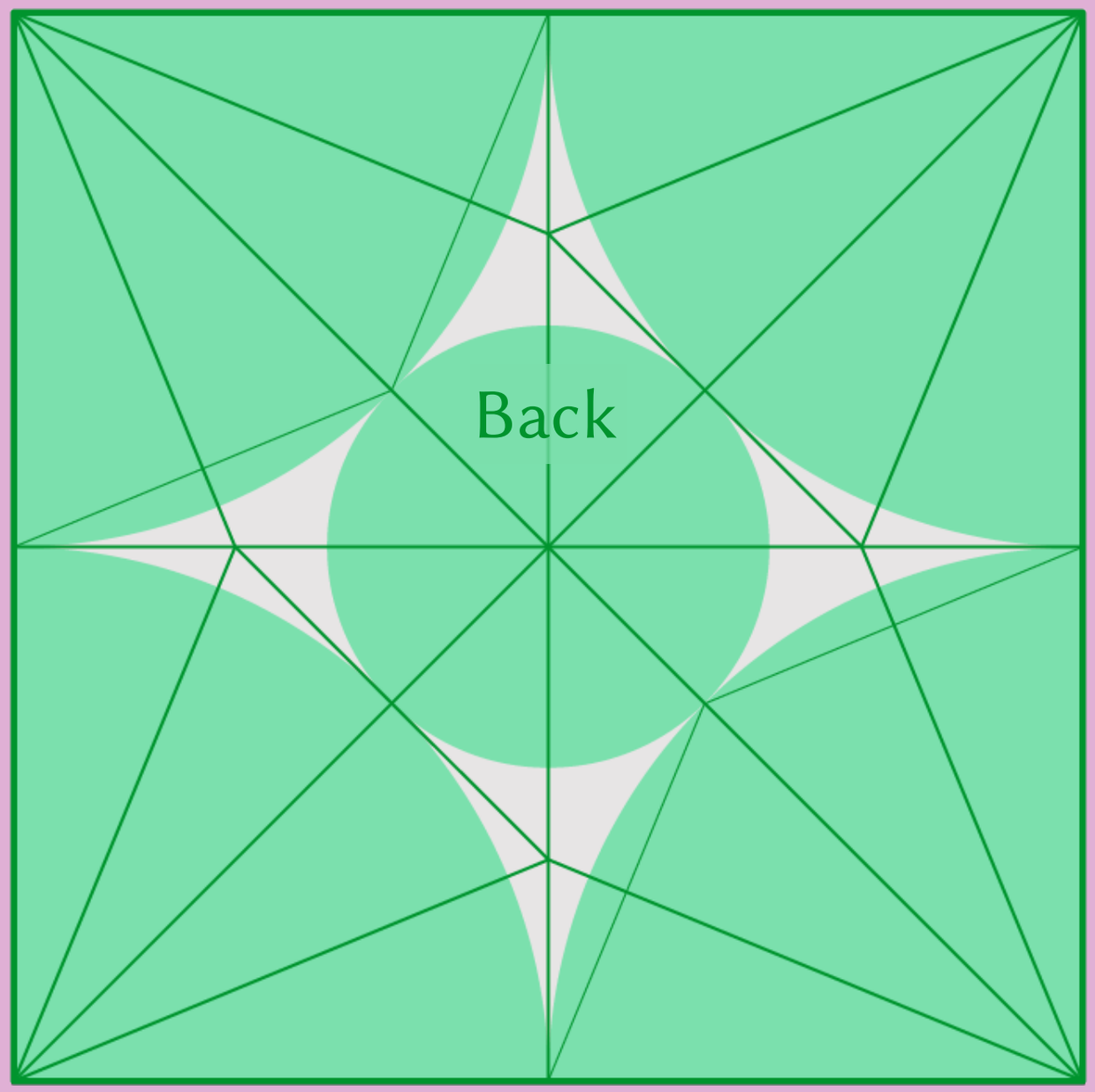
[Eric Joisel]

Circle Packing for Origami Design

Flaps from Circles

[Meguro 1992, Lang 1994]

Head

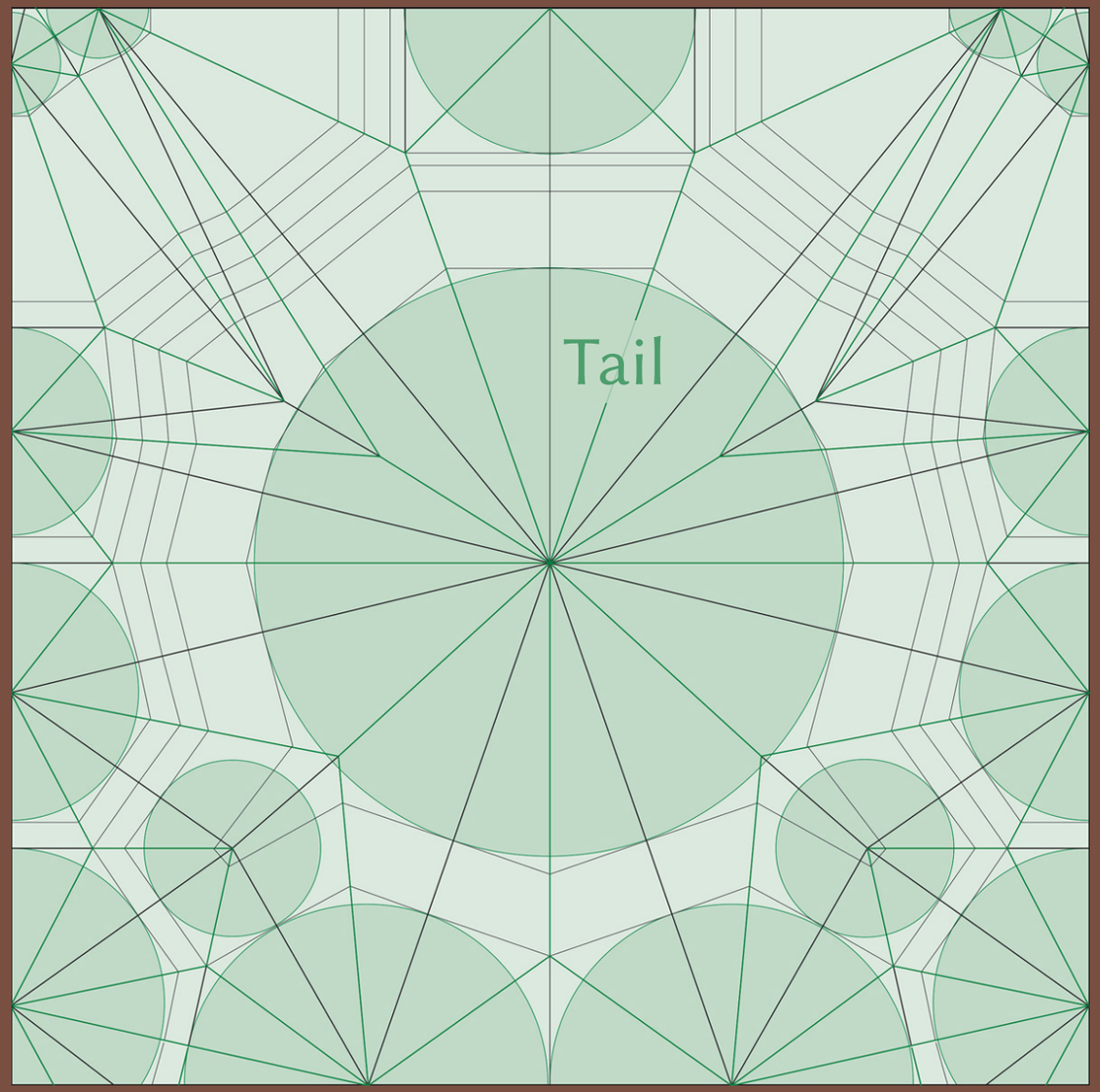


Wing

[Origami.guide]

Wing

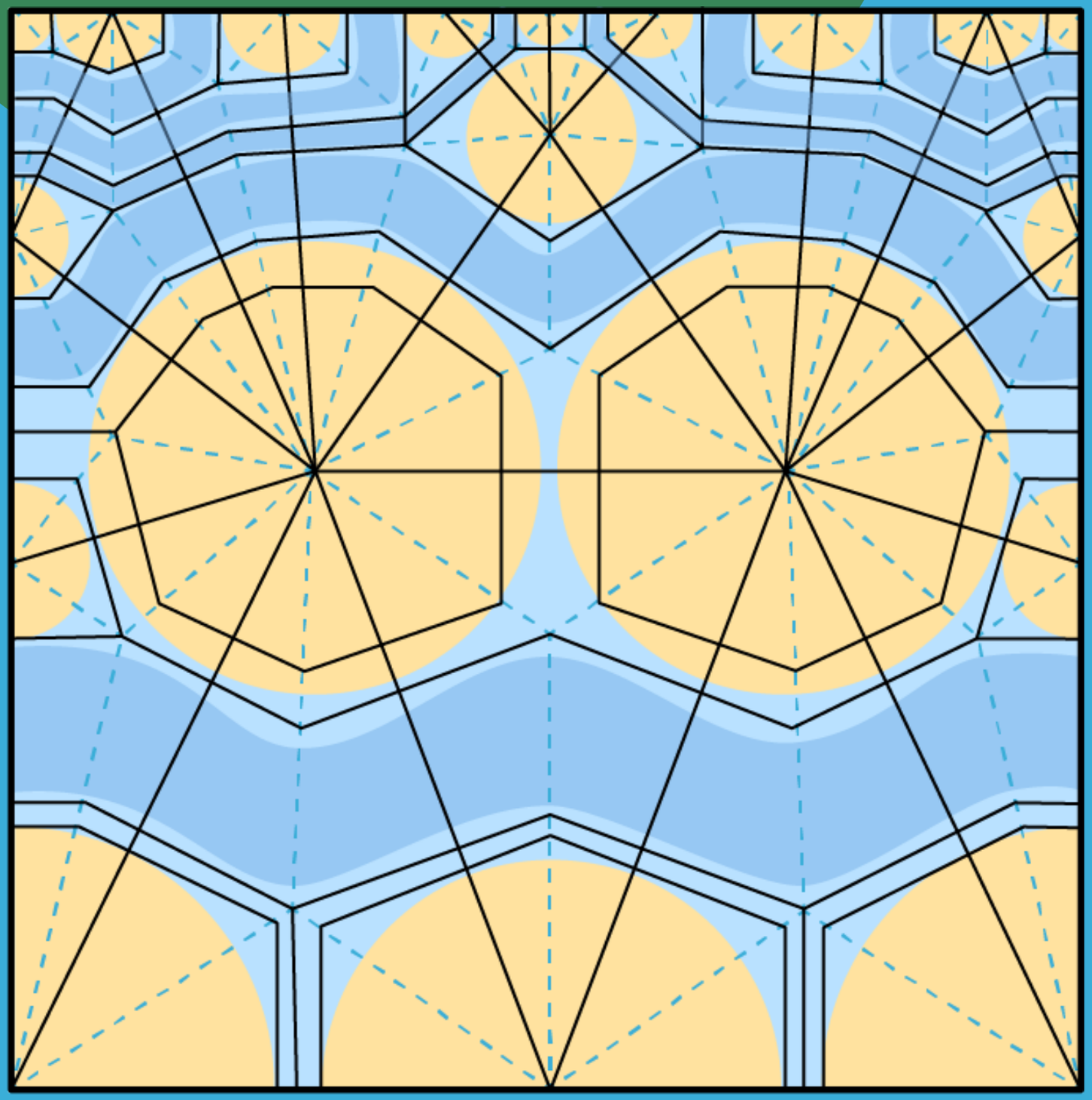
Tail

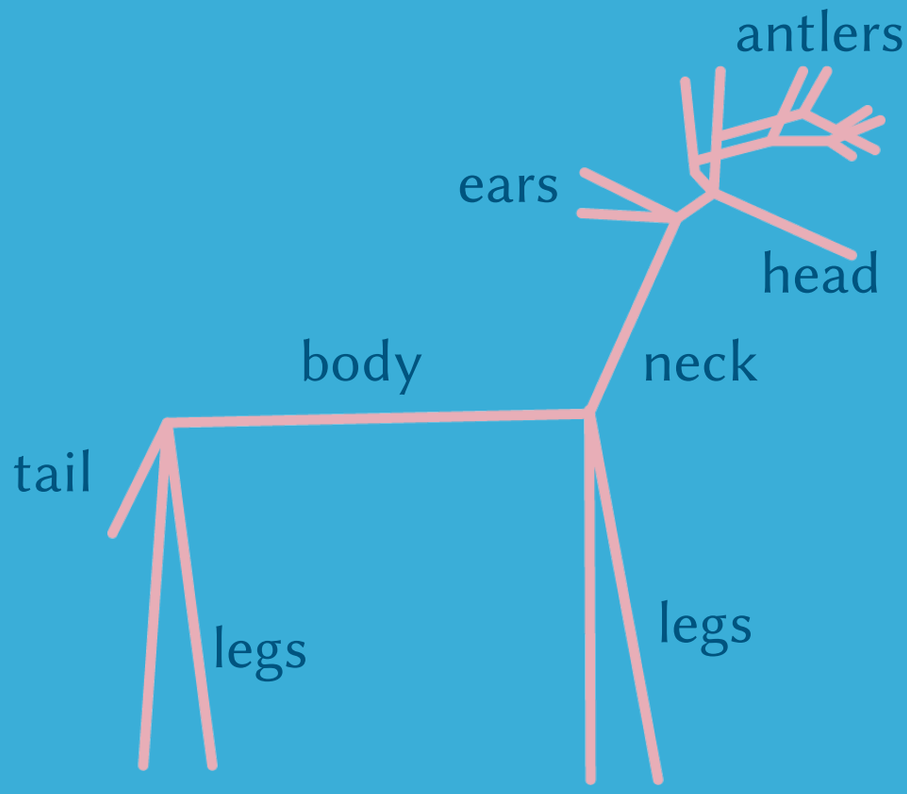


[Scorpion varileg, Opus 379, by Robert Lang]

Circle-River Origami Design

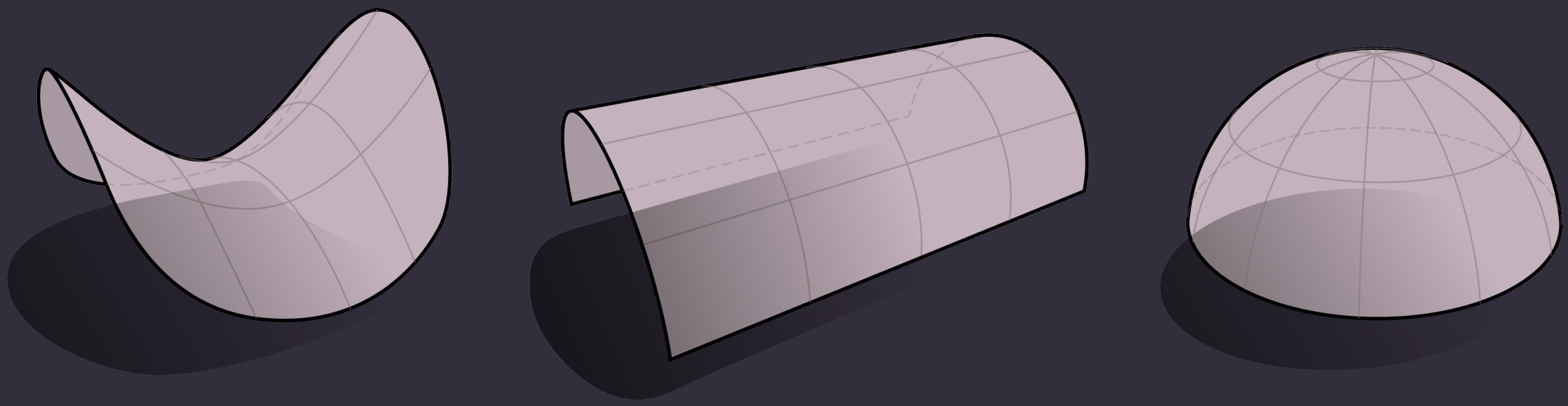
[Robert Lang - White-tailed Deer, Opus 550]





Curvature

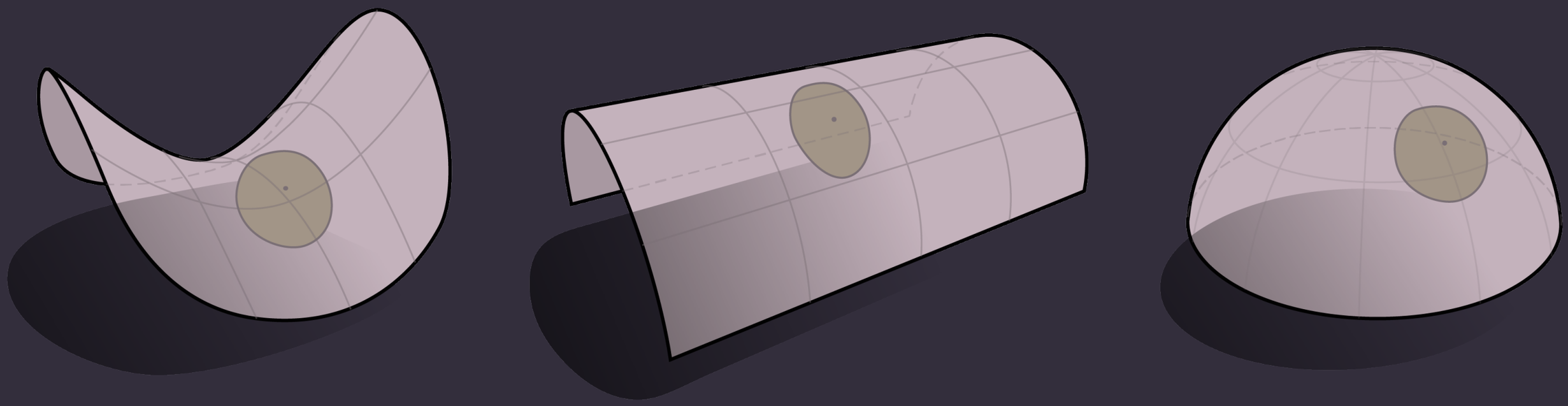
Gaussian Curvature



$K < 0$ $K = 0$ $K > 0$

intrinsic!

Gaussian Curvature



$K < 0$

$C > 2\pi R$

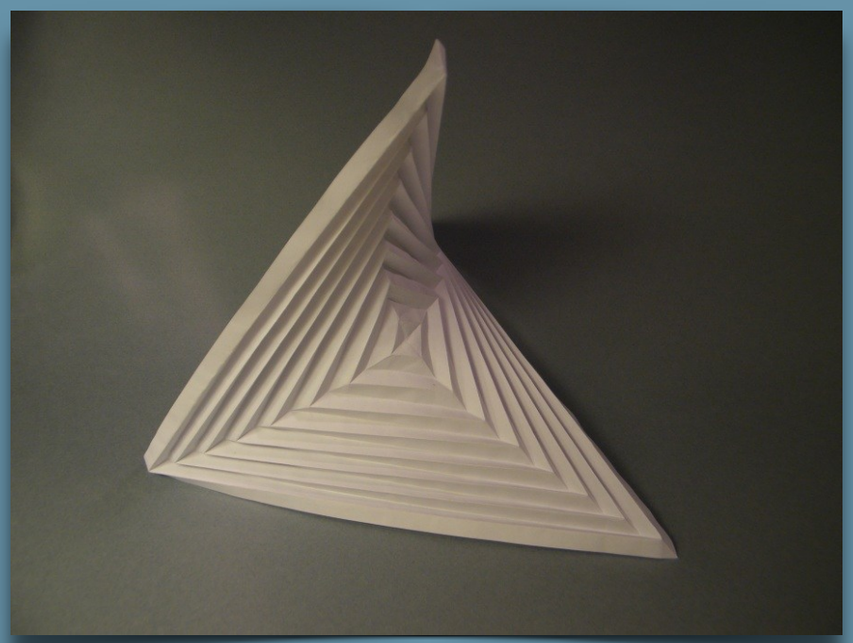
$K \equiv 0$

$C \equiv 2\pi R$

$K > 0$

$C < 2\pi R$

Curvature in Origami



[Eric Gjerde]

[FearlessFlourish]

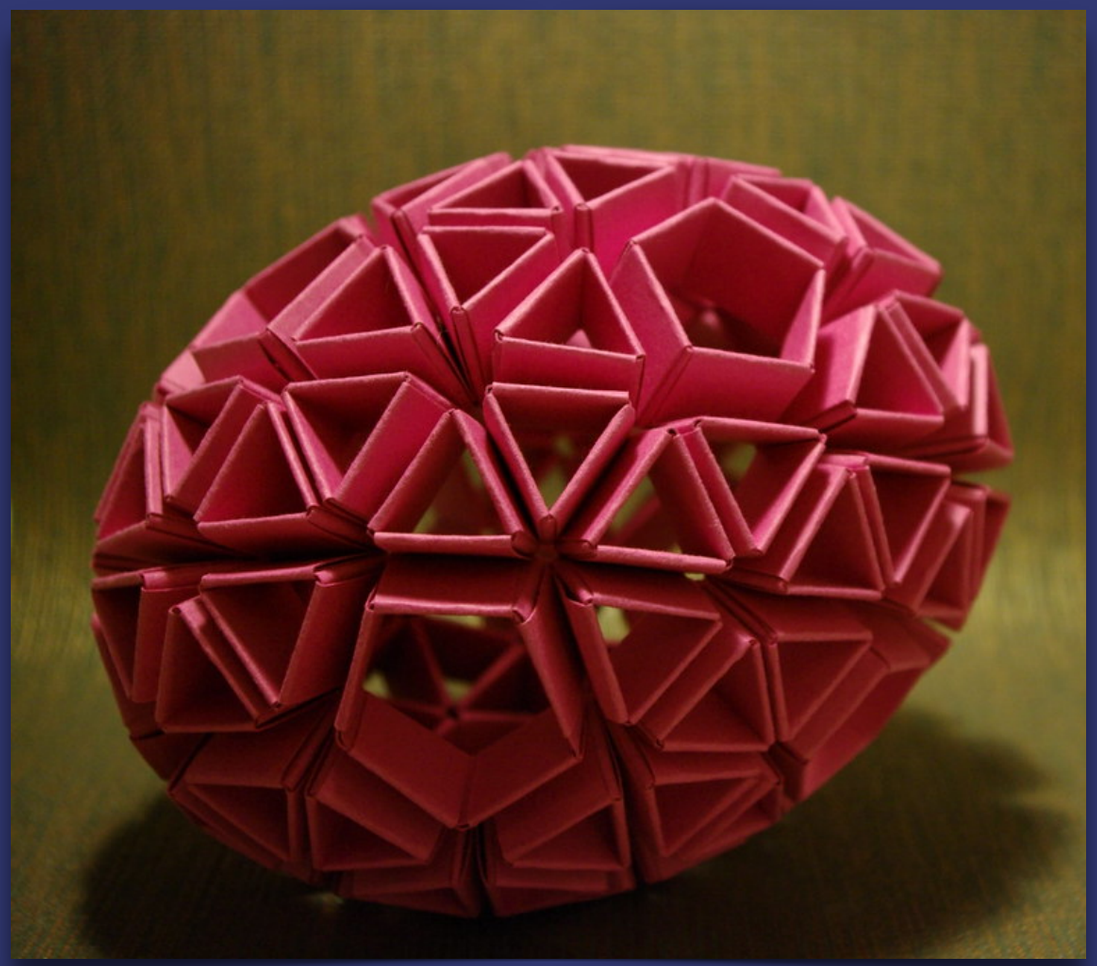
More Curvature

Heptagons give negative curvature

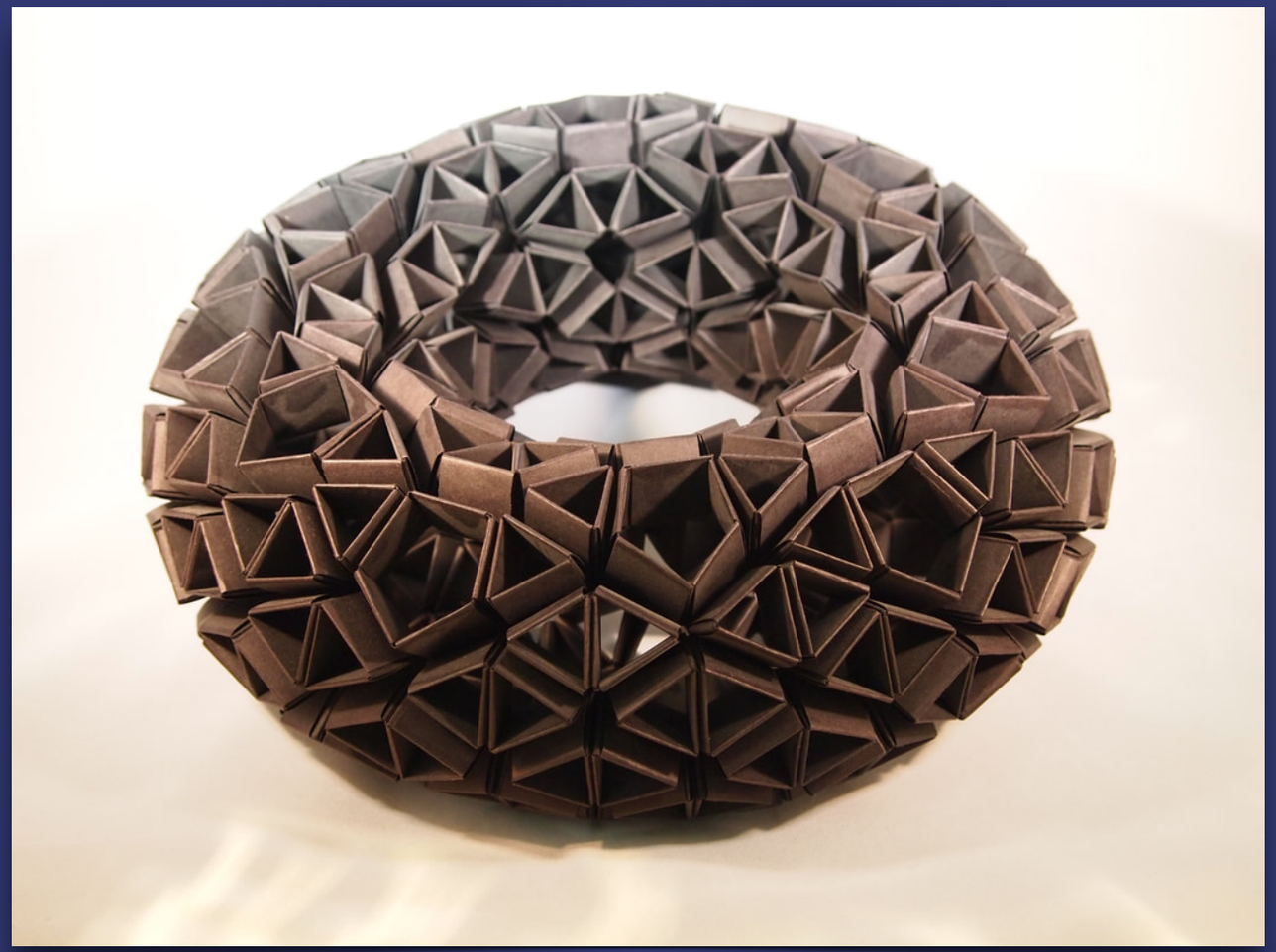
Pentagons give positive curvature

[ServeSmasher]

Even More Curvature



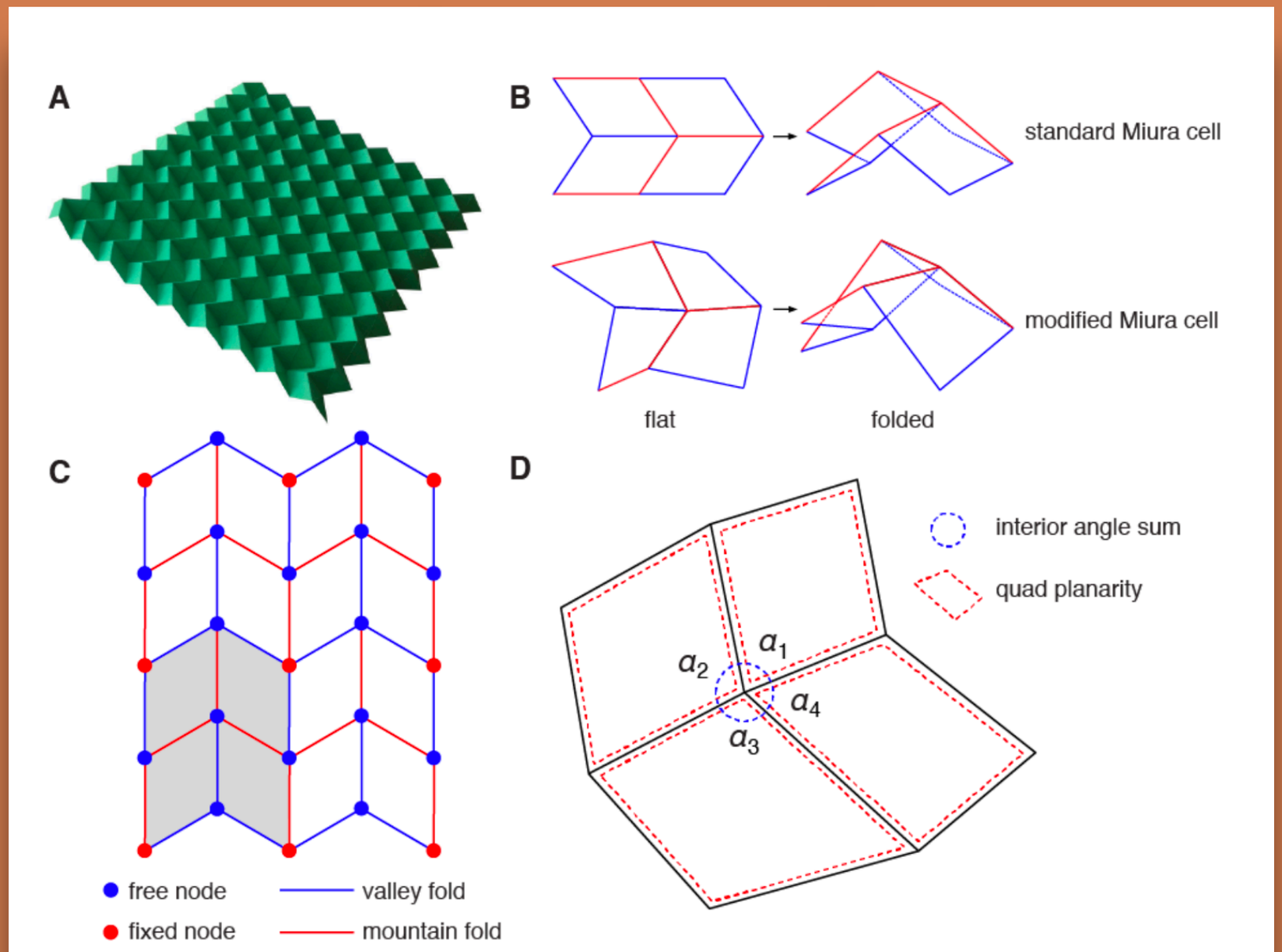
[Natalia Guzowska]

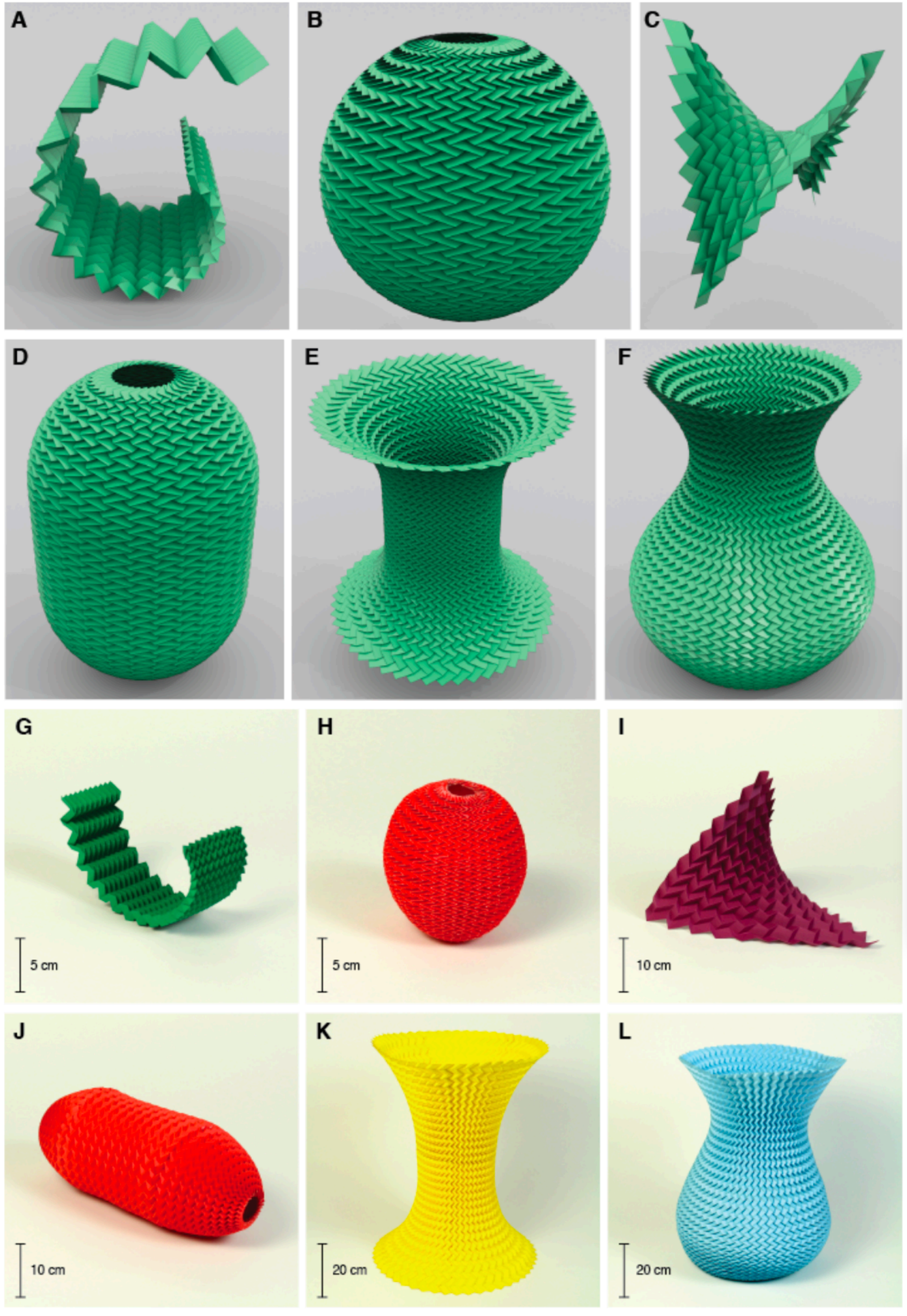


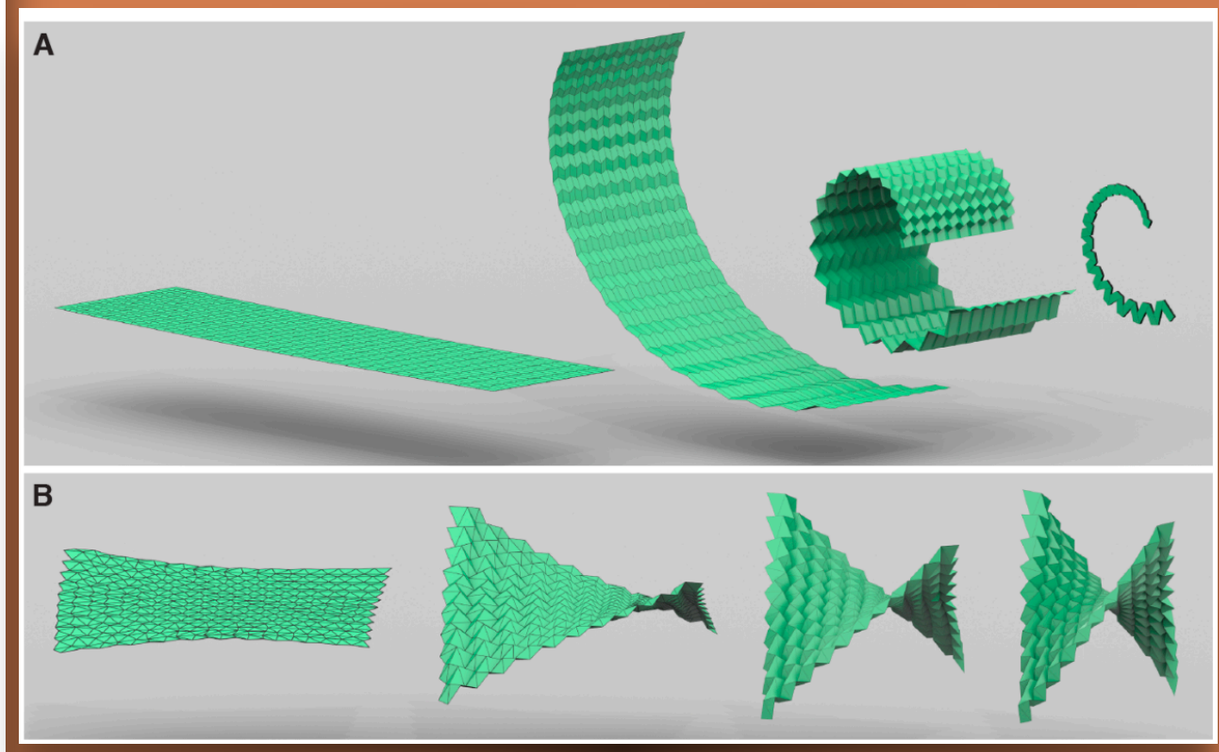
[Dave Honda]

Programming Curvature using Origami Tessellations

[Dudte, Vouga, Tachi, Mahadevan 2018]



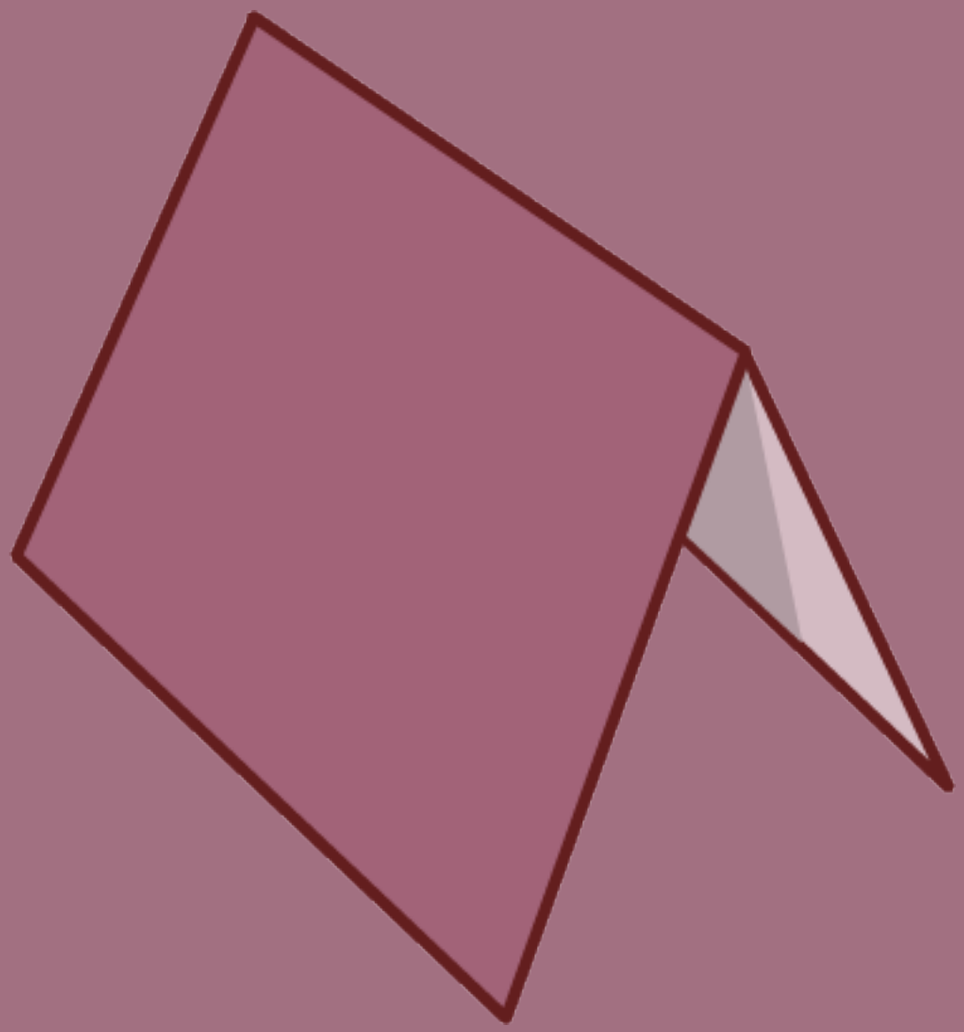

A
B
C
D
E
F
G
5 cm
H
5 cm
I
10 cm
J
10 cm
K
20 cm
L
20 cm

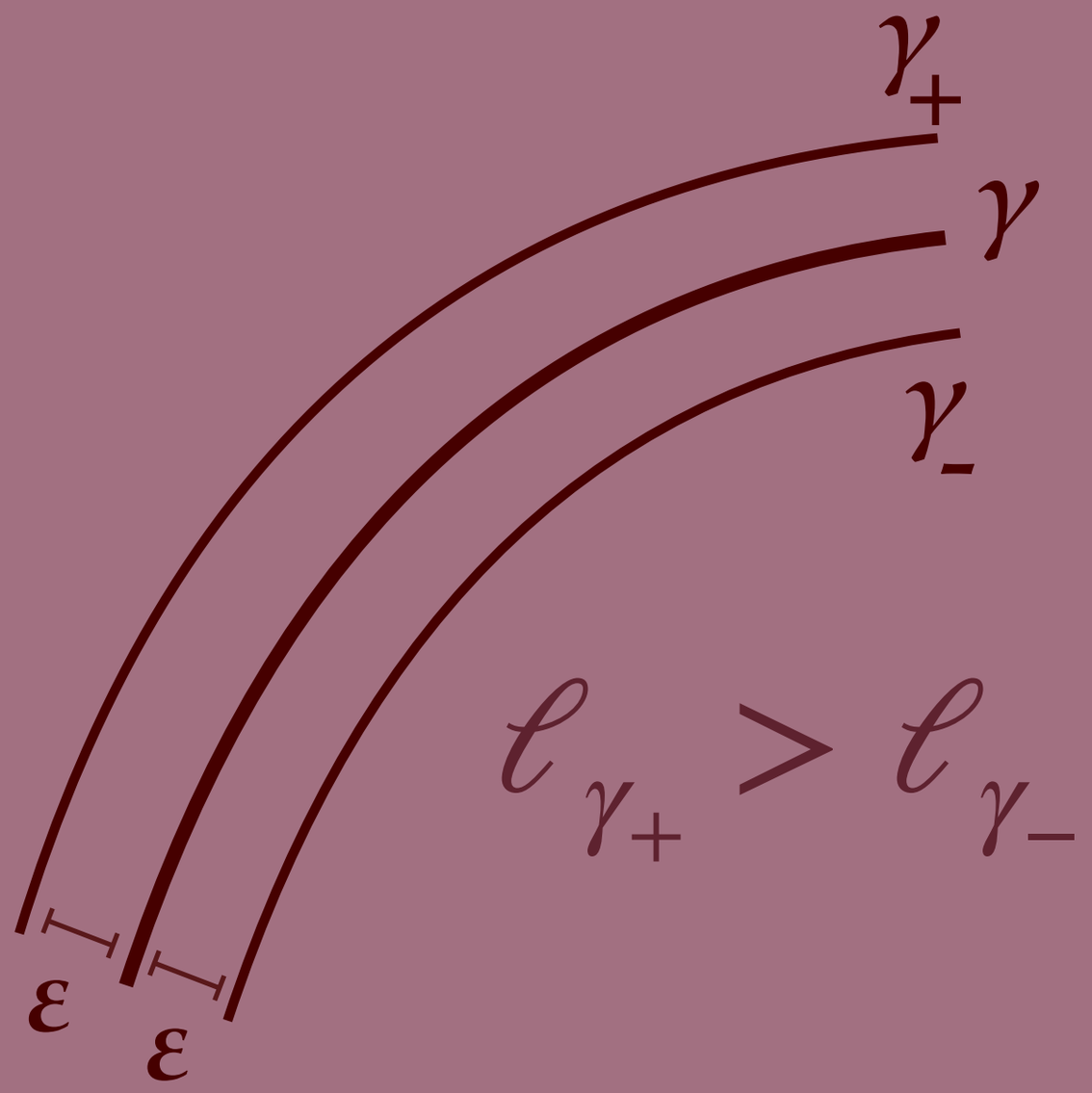

A
B

More on Paperfolding

[Fuchs and Tabachnikov 1999]

Folds are Straight Lines





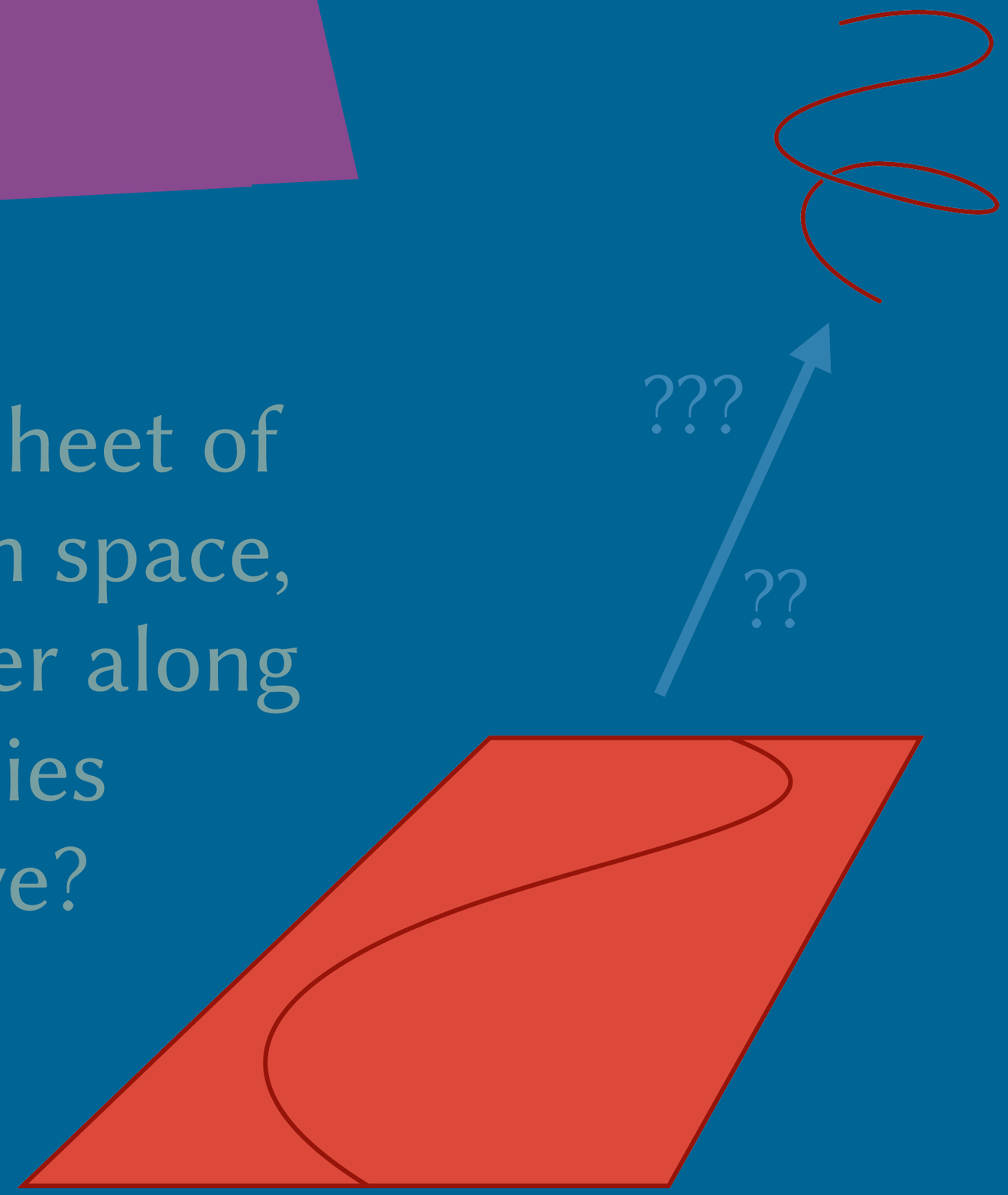
$$\ell_{\gamma_+} > \ell_{\gamma_-}$$

$$\ell_{\gamma_+} \approx \ell_{\gamma_-} + \epsilon \cdot \ell_{\gamma} \cdot curvature$$

[Eric and Martin Demaine]

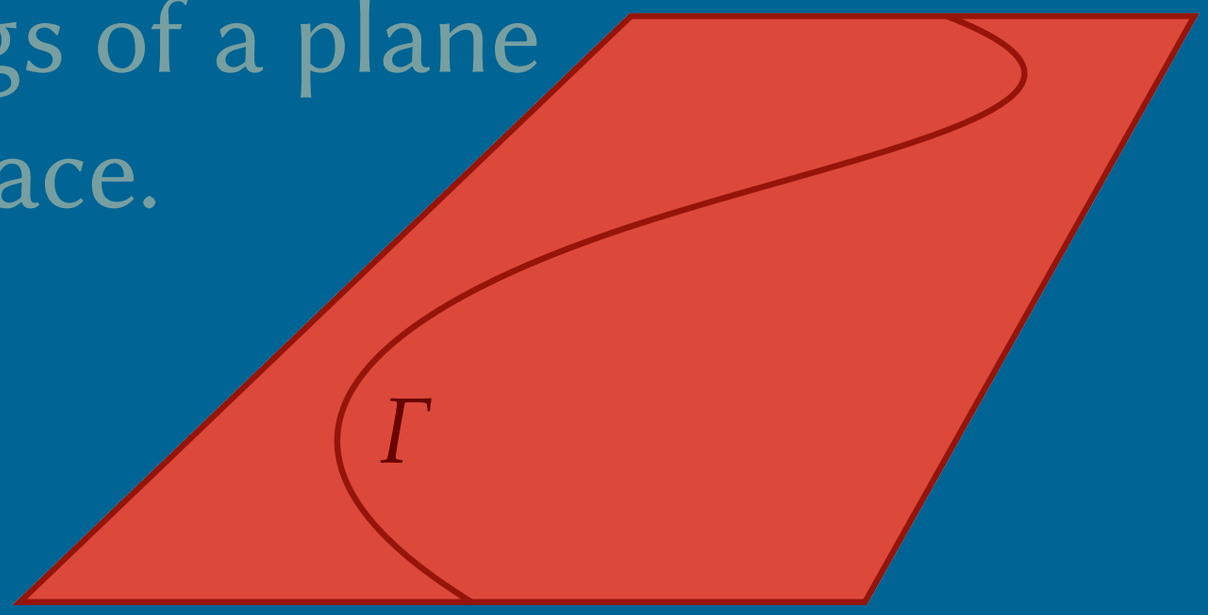
Question:

Given a curve on a sheet of paper, and a curve in space, can we fold the paper along the curve so that it lies along the other curve?



Answer:

“Assume that for every $x \in \Gamma$ the absolute value of the curvature of γ at point $f(x)$ is greater than that of Γ at x. Then there exist exactly two extensions of f to isometric embeddings of a plane neighborhood of Γ to space.



Two Isometric Embeddings

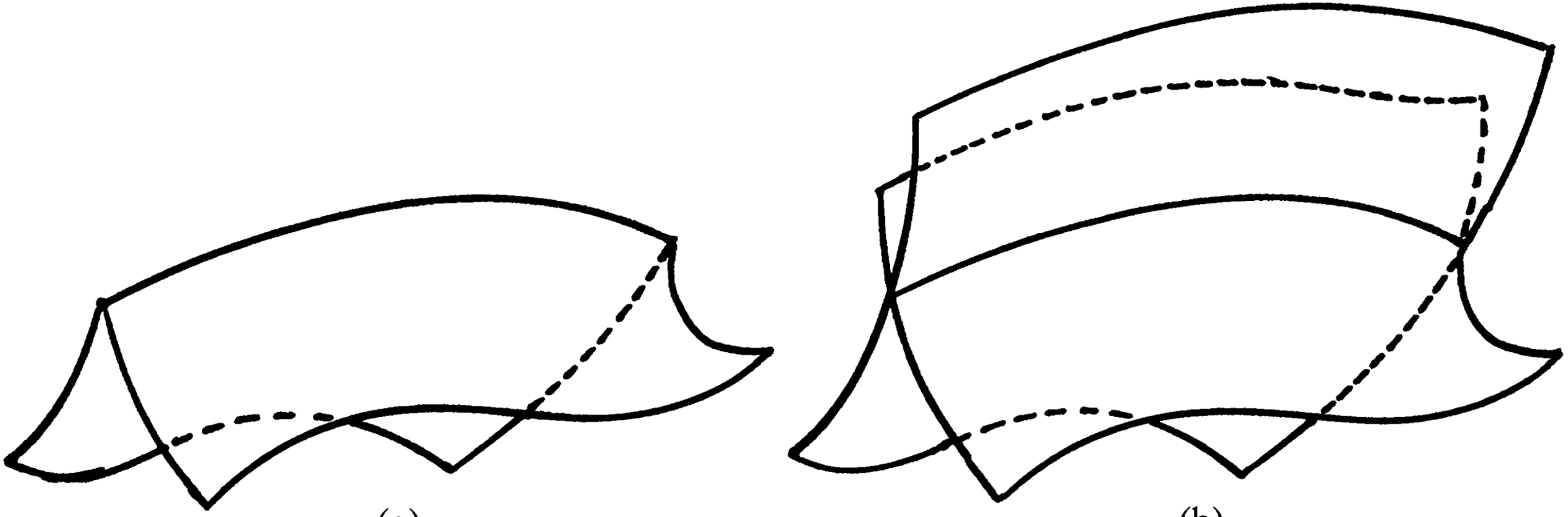
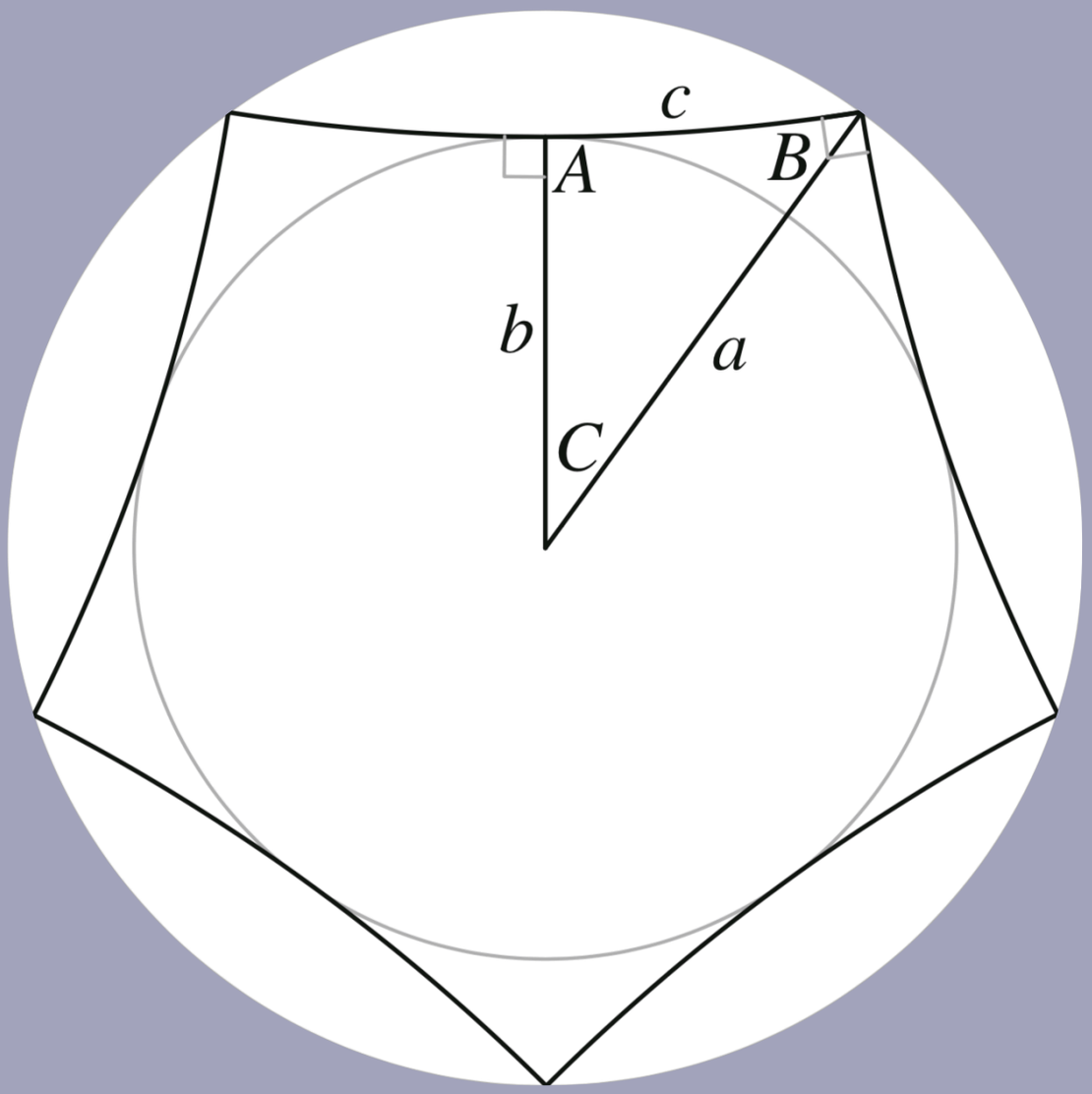


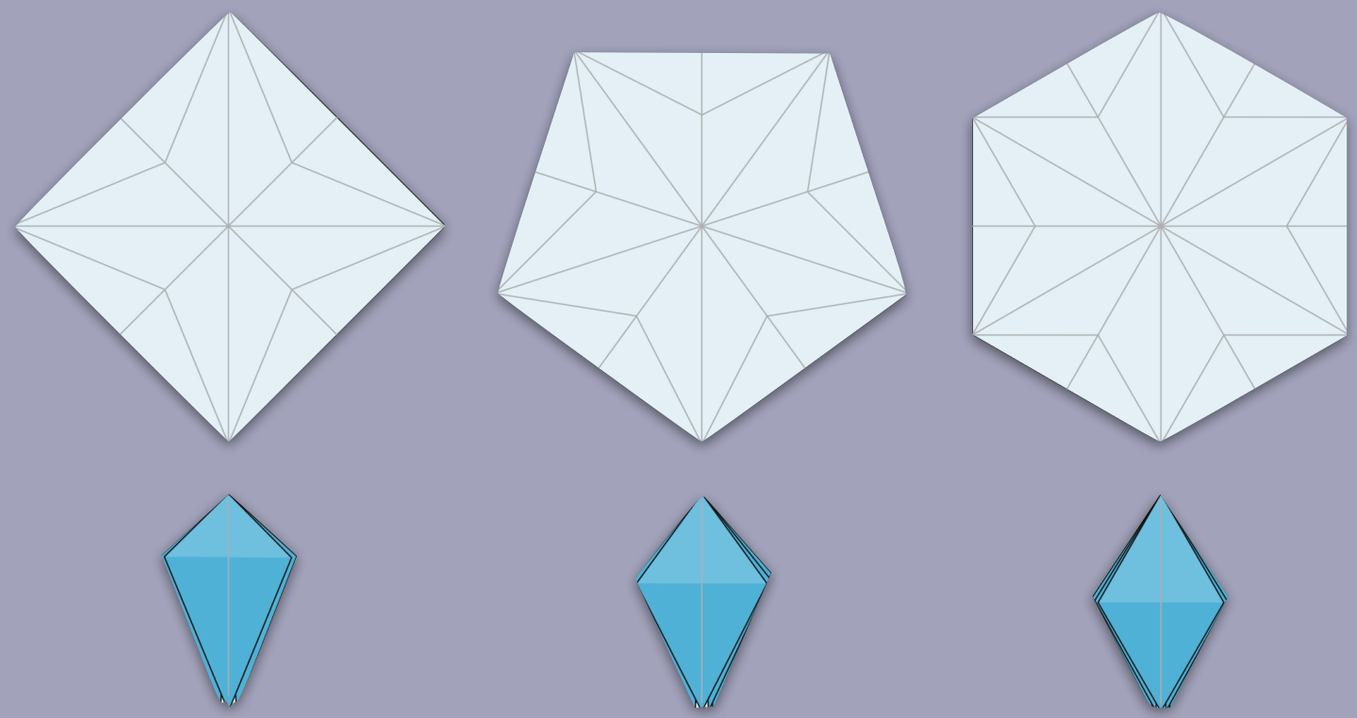
Figure 3

Folding the Hyperbolic Crane

[Alperin, Hayes, Lang 2012]

Idea: Right Angle Pentagon



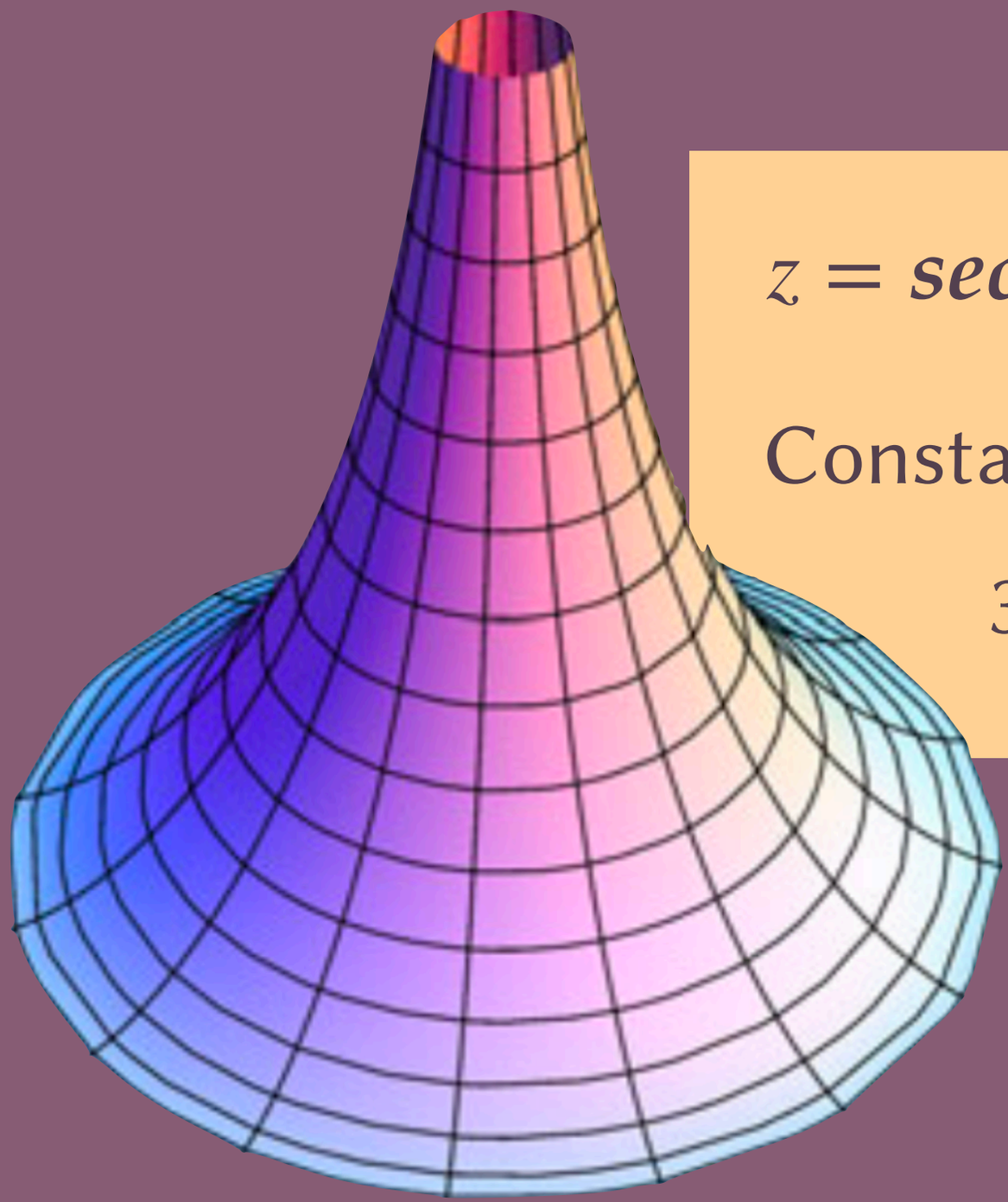


“We can, actually make hyperbolic paper and carry out folding experiments with it. To do so, we need two things:

(a) a hyperbolic sheet of paper,

(b) a hyperbolic desk to fold on.

A Hyperbolic Desk



$$z = \mathit{sech}^{-1}\sqrt{x^2+y^2} - \sqrt{1-(x^2+y^2)}$$

Constant curvature -1. Lets us fold

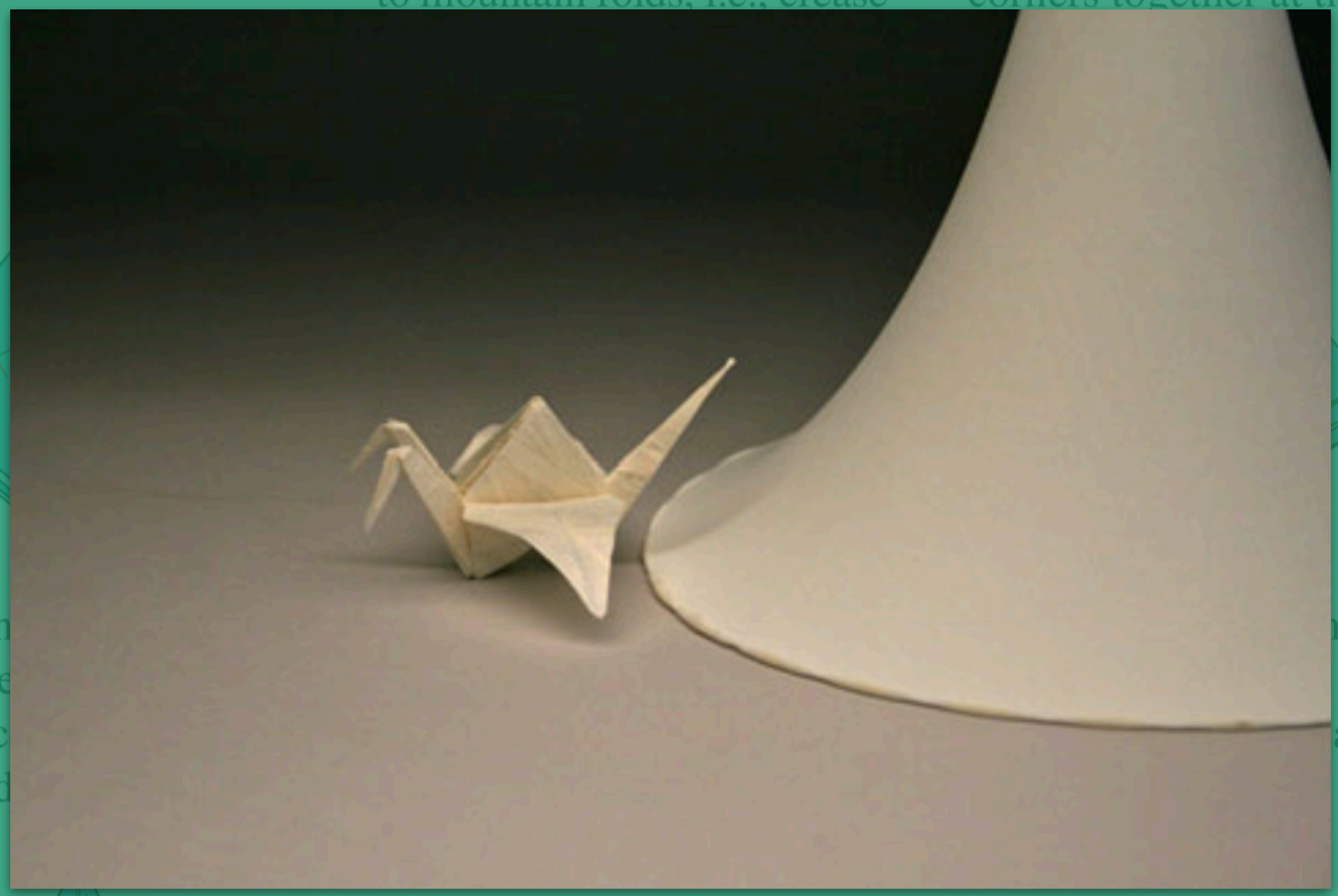
3D print it!

Next, Obtain a Hyperbolic Sheet of Paper

1. Smoosh flat paper onto pseudosphere

2. Adhere with methylcellulose

Folding the Hyperbolic Crane



Repeat with each of other 4 corners.

3. Change each of the creases that run from corner to center to mountain folds, i.e., crease

4. Using the existing creases, gather all 5 corners together at the

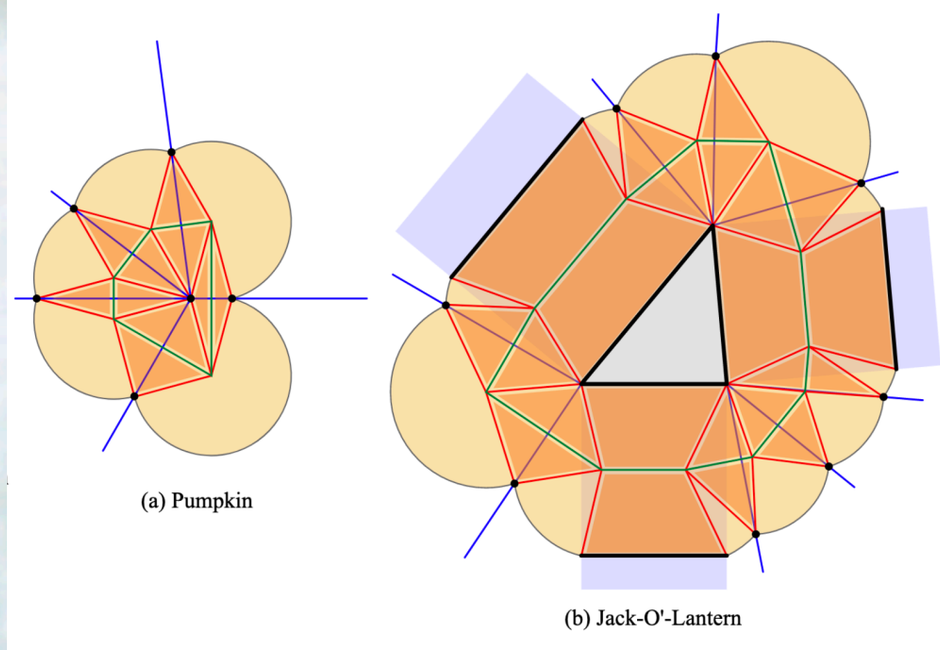
5. There is one extra flap in the middle. Fold it over to the right.

6. Fold on
so that the
with the c
firmly and
the right.

n
sides
along

Origamizer

[Demaine and Tachi 2017]

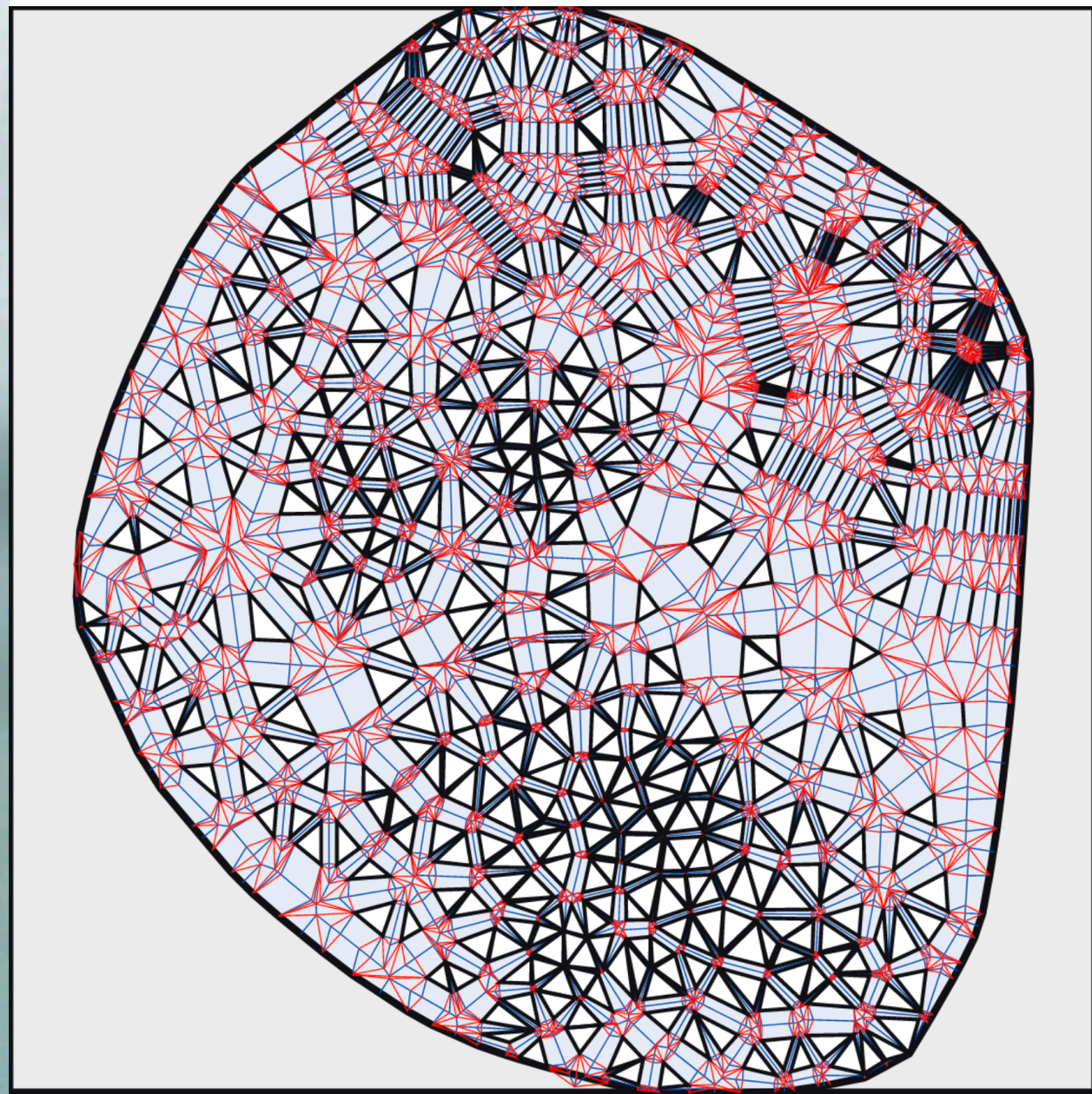


(a) Pumpkin

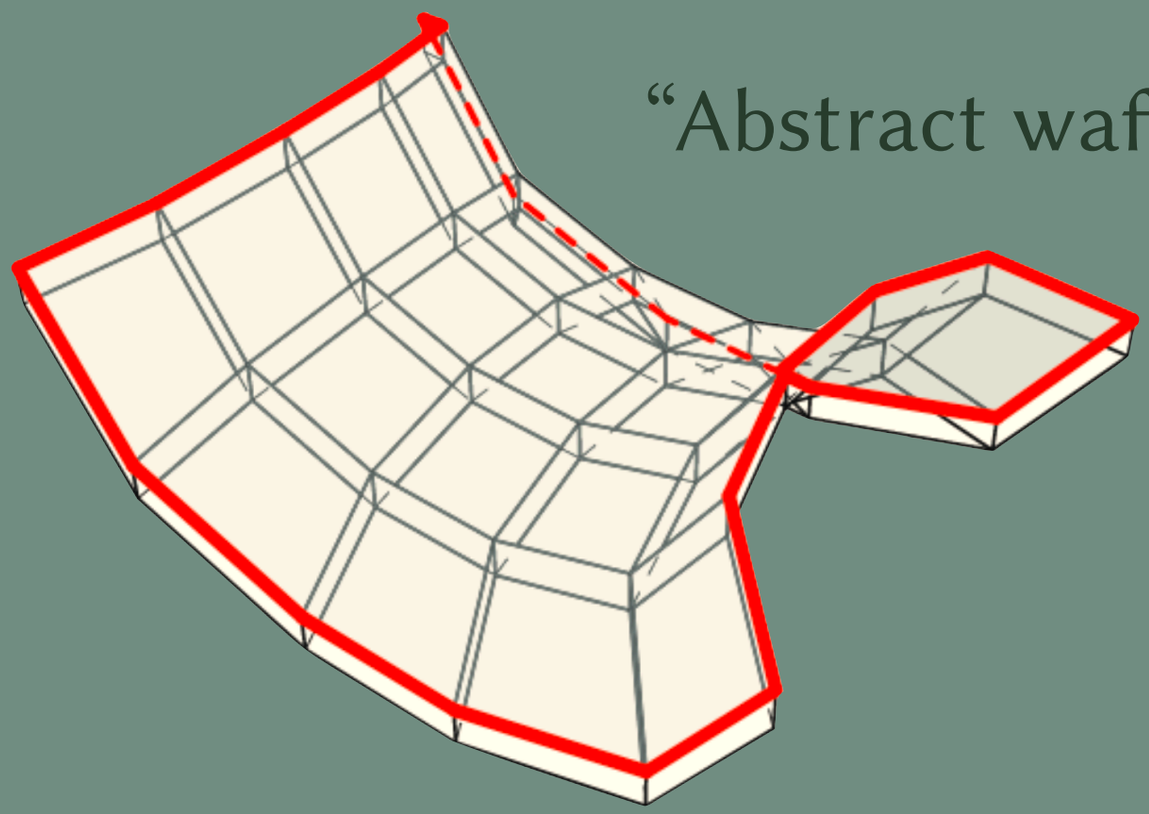
(b) Jack-O'-Lantern

(c) Calabash

(d) Cucumber



Origamizer



"Abstract waffle"

https://www.youtube.com/watch?v=GAnW-KU2yn4

> "In this design, 22.3% of the paper area makes up the actual surface of the target shape—about a 2:1 scale factor in each dimension—which matches the material usage ratio in most practical origami design