Fine-Scale Structure Design

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SUMMARY

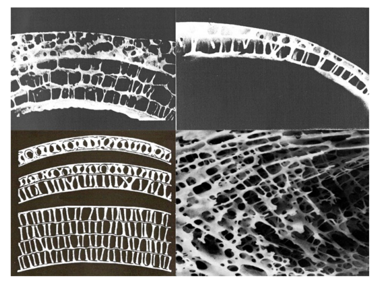
Motivation

- Additive fabrication technologies enable fabrication of customized, spatially varying microstructures.
- Microstructures allow continuous variation of *homogenized* material properties, in departure from standard structural design approaches.
- Exciting new possibilities can be achieved by combining macro- and micro-scale design: e.g., manufacturing negative Poisson's ratio and pentamode (fluid-like) materials adapting to complex surface shapes.

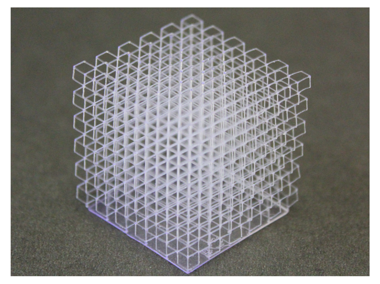
Goals

- Bridge between advanced mathematical theory of microstructures, practical computational techniques, and experiment by designing and fabricating parametric families of microstructures.
- Use these structures to solve specific shape optimization problems.

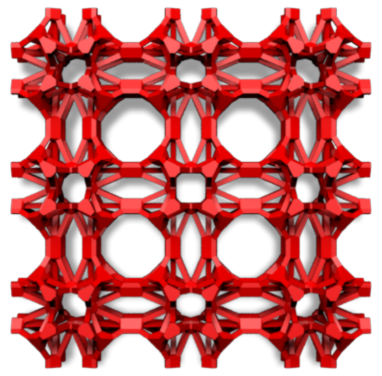
COMPLEX STRUCTURES


Bone structures

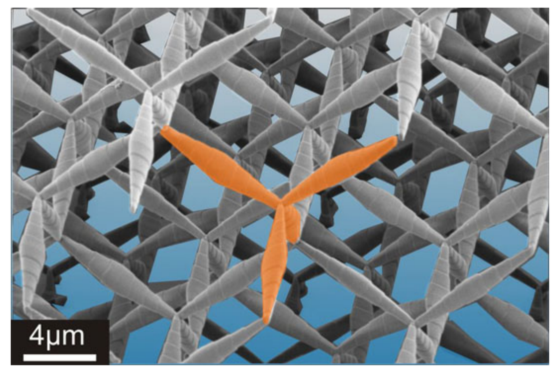

Bridge truss structures


3D printed fine-scale structure

Exotic materials

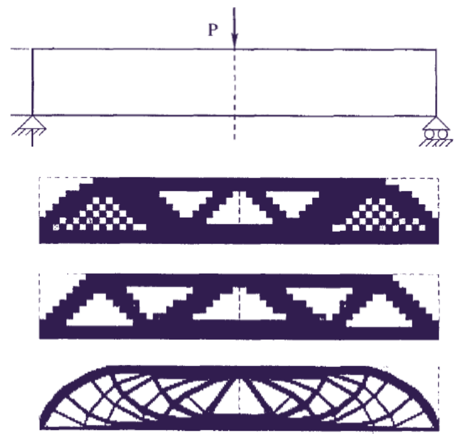


Auxetic (negative Poisson's ratio)
- Friis, Lakes and Park (1988)
- Image: a negative Poisson's ratio pattern from our database



Pentamode (shear moduli ≈ 0)
- Proposed: Milton and Cherkaev (1995)
- Fabricated: Kadic et al.(2012) using dip-in direct-laser-writing optical lithography

TOPOLOGICAL STRUCTURE OPTIMIZATION



Sigmund and Petersson, 1998

Resolution- and regularization-dependent:

Spurious checkerboard solution

Coarse discretization

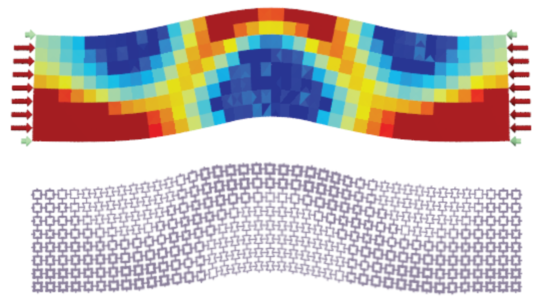
Fine discretization

- Topological optimization removes and adds material based on the topological derivative (effect on objective of introducing voids).
- Requires regularization to avoid spurious solutions.
- Difficult to design at very high resolutions.

OPTIMIZATION BY HOMOGENIZATION

Observation 1: when the structure becomes very fine, it is equivalent to a homogeneous material.

Observation 2: variable structure → variable material properties.



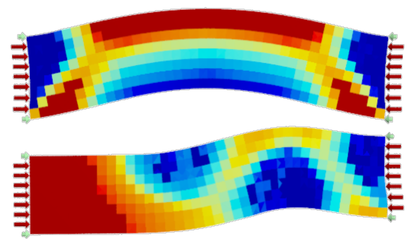
Approach

- Partition shape into small cells.
- Design with per-cell material properties as variables.
- For each cell, convert material properties into a printable structure.

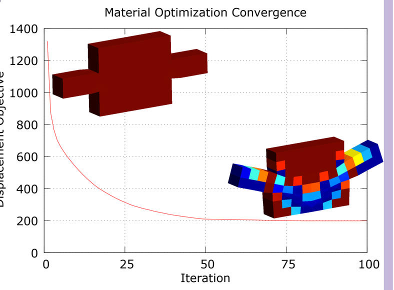
EXAMPLE PROBLEM: TARGET DEFORMATION

- Given loads, t, and target surface displacements, u^*, find material distribution, p, achieving these displacements:

$$\min_p \frac{1}{2}\int_{\partial\Omega} ||u(p) - u^*||^2 \,\mathrm{d}A$$
$$\text{s.t.} \; -\nabla\cdot C(p) : e(u) = 0 \quad \text{in } \Omega$$
$$C(p) : e(u)\hat{n} = t \quad \text{on } \partial\Omega$$


Young's modulus distributions for same loads, different target displacements.

- Standard gradient-based optimization approach exhibits poor convergence.
- Better convergence with a local-global iteration:
  1. With current material distribution, run two simulations:
     a) with specified loading scenario→ stress $\sigma(u_{\text{neumann}})$ estimates stress in optimal design
     b) with target displacement as Dirichlet condition → strain $e(u_{\text{dirichlet}})$ estimates strain in optimal design
  2. Update material parameters with a local nonlinear least squares fit and repeat:

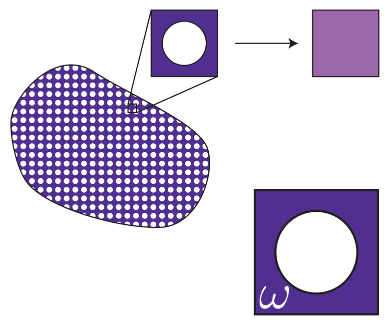


$$\min_p \int_{\Omega} ||e(u_{\text{dirichlet}}) - C^{-1}(p) : \sigma(u_{\text{neumann}})||_F^2 \,\mathrm{d}V$$

HOMOGENIZATION

- Goal: determine material properties from microstructure geometry (map from average strain to average stress in a periodic tiling)
- Stretch the cell in 6 different ways, applying the unit basis strains e^{ij}:

$$-\nabla\cdot(C^{\text{base}} : [e(\mathbf{w}^{ij}) + e^{ij}]) = \mathbf{0} \quad \text{in } \omega$$
$$\hat{\mathbf{n}}\cdot(C^{\text{base}} : [e(\mathbf{w}^{ij}) + e^{ij}]) = \mathbf{0} \quad \text{on } \partial\omega \setminus \partial Y$$
$$\mathbf{w}^{ij}(\mathbf{y}) \;\; Y\text{-periodic}$$
$$\int_\omega \mathbf{w}^{ij}(\mathbf{y})\,\mathrm{d}\mathbf{y} = \mathbf{0},$$

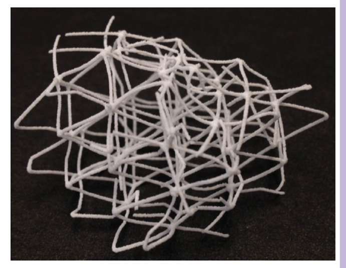


and average the resulting stresses:

$$C^H_{ijkl} = \frac{1}{|Y|}\int_Y C^{\text{base}}_{ijpq}[e(\mathbf{w}^{kl}) + e^{kl}]_{pq}\,\mathrm{d}\mathbf{y}$$

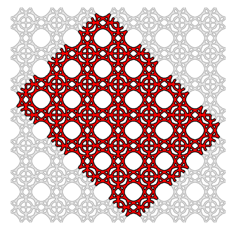
PRINTABILITY AND TILEABILITY CONSTRAINTS

- We design **printable** patterns: **Connected**, **no enclosed voids, above minimum thickness, self-supporting** (for stereolithography)
- Each pattern must also **tile** with every other pattern to enable spatially varying properties.



PATTERN ISOTROPY

- Our main goal is to design structures with **isotropic** properties.
- Cubic symmetry (square in 2D) ensures an orthotropic homogenized material, but isotropy is challenging.
- We validated our structures' isotropy both numerically and experimentally (compressing a fabricated structure that was rotated by 45° and clipped to a box).



PATTERN FAMILIES

- We consider truss-like structures defined by bar connectivity (topology) and bar thickness/node offset parameters.
- Easy to formulate **printability** and **tileability** constraints.
- One single topology is not enough to cover the full range of material properties; we run a combinatorial exploration of topologies:

- Symmetry and simplicity constraints reduce space to **1205 topologies**.
- We divide these topologies into **138 families** that can be tiled together in a single design (matching interface topology).

SHAPE OPTIMIZATION

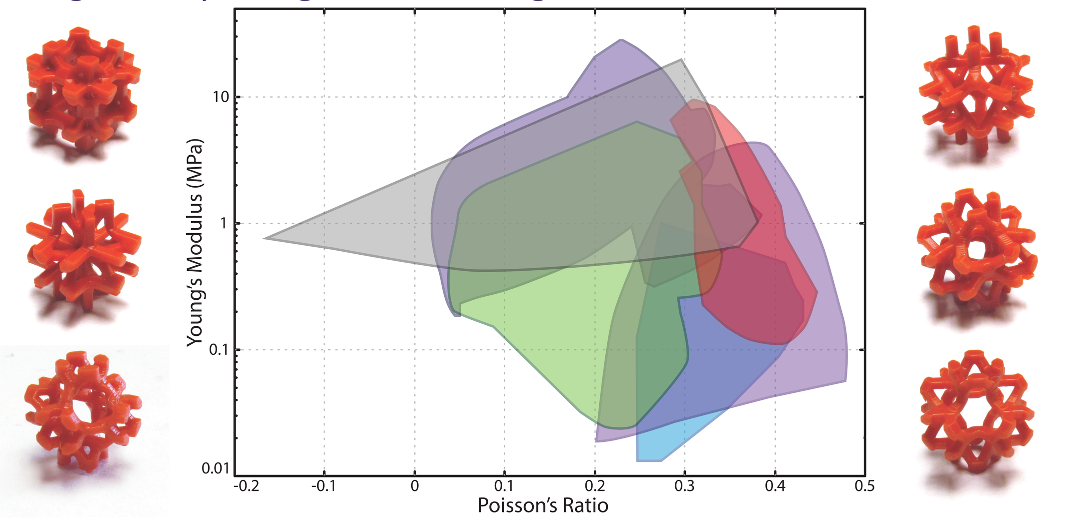
- For each pattern topology, we optimize the shape parameters (offsets, thicknesses) to achieve a wide range of isotropic material properties.
- We run a nonlinear least squares optimization to fit a pattern's homogenized tensor, C^H, to each isotropic target tensor, C^*:

$$\min_\omega ||C^H(\omega) - C^*||_F^2$$

- We use a **shape derivative** to determine how perturbations of the shape, ω, change C^H, allowing us to differentiate the fitting energy with respect to the offsets and thicknesses.

RESULTS: 3D ISOTROPIC PATTERNS

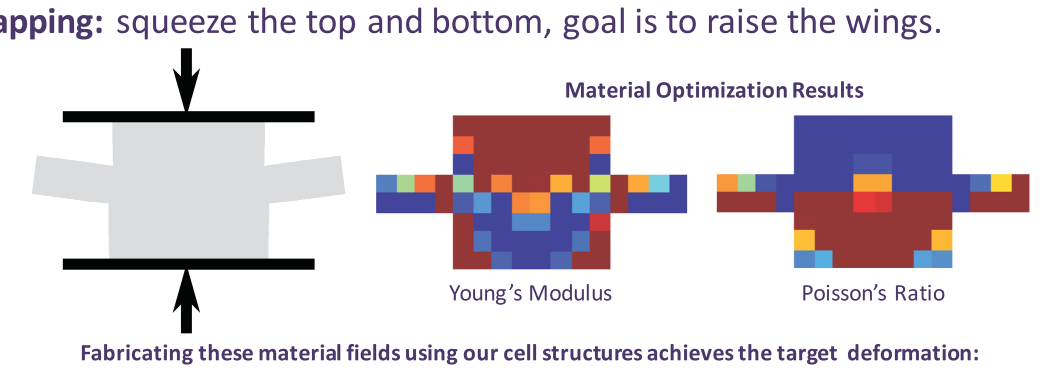
We show the space of isotropic materials reached by the six topologies in the single family with greatest coverage:



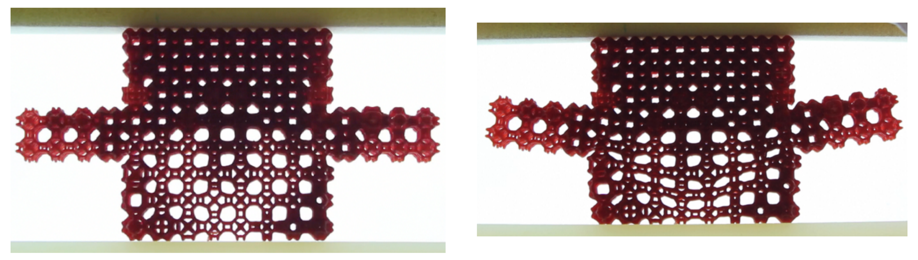
- Young's moduli range from under 1/14,000 up to over 1/10 of the printer material's Young's modulus.
- Poisson's ratios span from -0.16 to nearly 0.5 (theoretical limit).

OPTIMIZED EXAMPLES

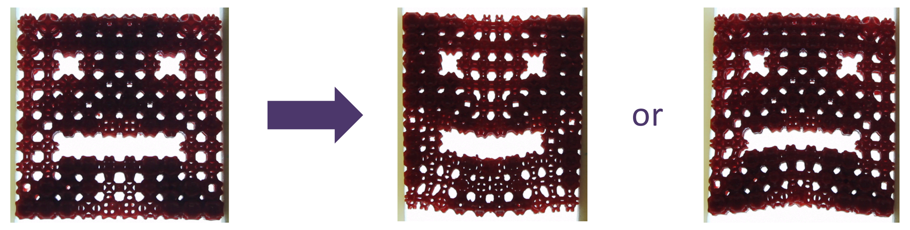
Flapping: squeeze the top and bottom, goal is to raise the wings.



Fabricating these material fields using our cell structures achieves the target deformation:

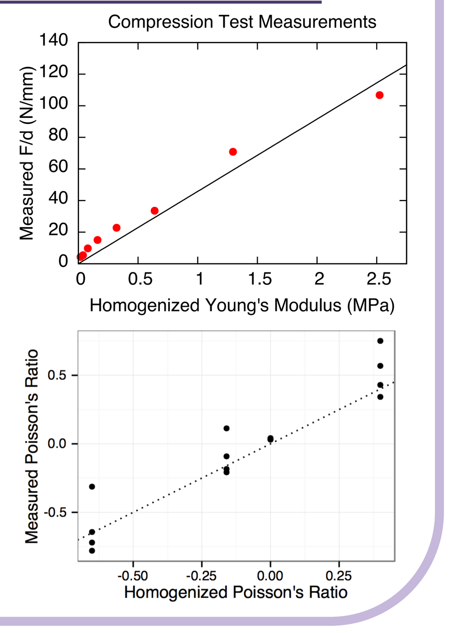


Faces: different microstructures cause the face to smile or frown



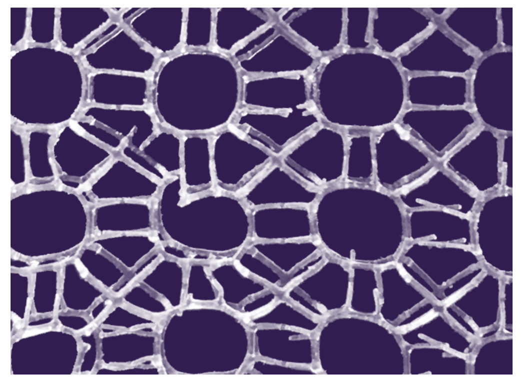
FABRICATION AND PHYSICAL VALIDATION

- We fabricated tilings of 5mm cells on a stereolithography printer (B9Creator) with 30 micron resolution and 200 micron min feature size.
- Compression tests showed patterns behaved nearly linearly for small deformations and had stiffness consistent with their target Young's moduli.
- Our lower-accuracy Poisson's ratio testing setup also gave consistent measurements.



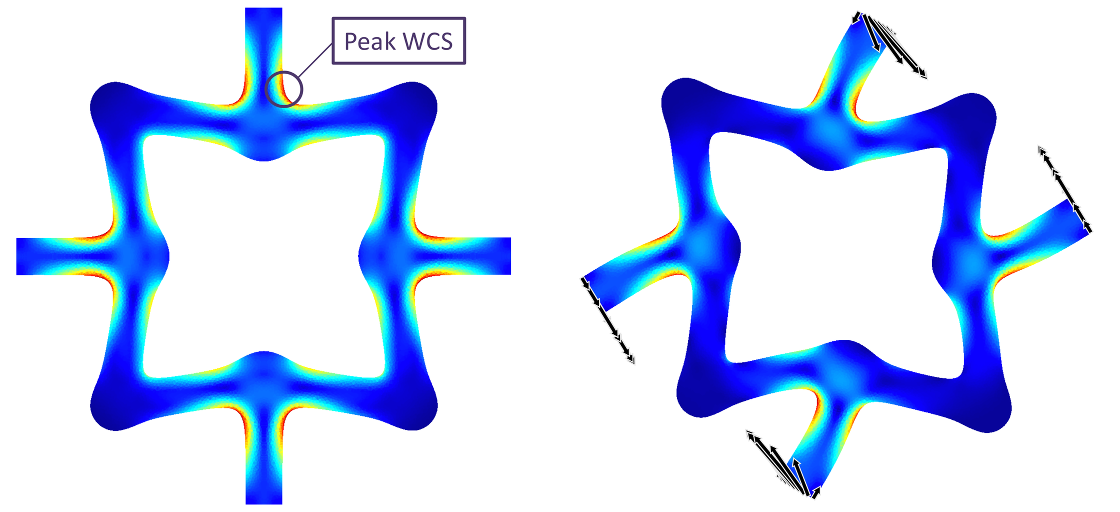
WORST-CASE MICRO-STRESS ANALYSIS

- Microstructures tend to be **fragile**, developing stress concentrations that cause brittle or ductile fracture.
- We want to design general-purpose structures that are structurally robust under **arbitrary use cases**.
- To make robustness guarantees, we must analyze the **worst-case stress** occurring at points in the structure under any unit magnitude loading:



$$\mathrm{WCS}(x) = \max_{\text{unit load}} ||\sigma(x)|| \qquad (x\text{: Point in microstructure})$$

- Stress norm can be chosen based on the printing material:
  - Max norm (principal stress) for brittle materials
  - Frobenius norm of deviatoric stress (von Mises) for ductile materials
- With either norm, the worst-case load and stress can be computed efficiently at every point in the structure:

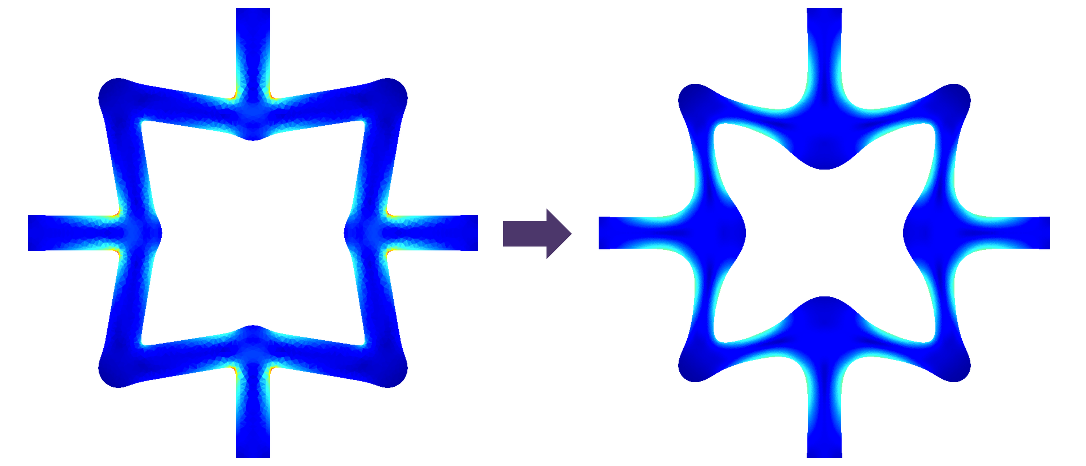


Worst-case stress at every point in the microstructure; one of the eight peak stress locations is circled.

The unit load inducing the worst-case stress at the circled peak stress location on the left.

MINIMIZING WORST-CASE STRESS

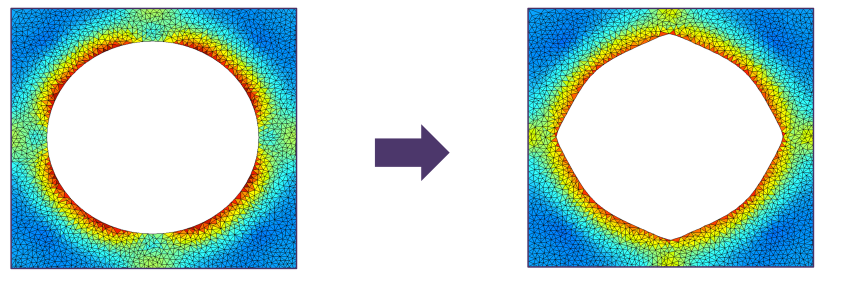
- The worst-case stress can also be **shape-differentiated,** enabling us to design patterns with reduced peak stress while still achieving the desired effective material properties.



Worst-case stress at each point of a microstructure achieving particular effective material properties.

This optimized design achieves **identical material properties** but experiences only **50% of the old peak worst-case stress.**

- Rounder designs usually experience lower stress, but the global geometry can cause different preferred curvature distributions:



Circular hole worst-case micro stress

Minimized worst-case micro stress

REFERENCES

The inverse homogenization design approach and the target-deformation material property optimization were presented at SIGGRAPH 2015:

Panetta J*., **Zhou Q***., Malomo L., Pietroni, N., Cignoni, P., and Zorin, D. 2015. Elastic Textures for Additive Fabrication. ACM Trans. Graph. 34, 4, 135:1-135:122.

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