

## Subleading contributions to the decay width of the $T_{cc}^+$ tetraquark

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Recently, the LHCb Collaboration announced the discovery of the  $T_{cc}^+$  tetraquark. Being merely a few hundred keV below the  $D^{*+}D^0$  threshold, the  $T_{cc}^+$  is expected to have a molecular component, for which there is a good separation of scales that can be exploited to make reasonably accurate theoretical predictions about this tetraquark. Independently of its nature, the most important decay channels will be  $D^+D^0\pi^0$ ,  $D^0D^0\pi^+$ , and  $D^+D^0\gamma$ . Its closeness to threshold suggests that the mass and particularly the width of the  $T_{cc}^+$  tetraquark depend on the resonance profile. While the standard Breit-Wigner parametrization generates a  $T_{cc}^+$  that is too broad for current theoretical calculations to reproduce, a three-body unitarized Breit-Wigner shape reveals instead a decay width ( $\Gamma_{\text{pole}} = 48 \pm 2_{-12}^{+0}$  keV) consistent with theoretical expectations. Here, we consider subleading-order contributions to the decay amplitude, which, though having at most a moderate impact in the width, still indicate potentially significant differences with the experimental width that can be exploited to disentangle the nature of the  $T_{cc}^+$ . Concrete calculations yield  $\Gamma^{\text{LO}} = 49 \pm 16$  keV and  $\Gamma^{\text{NLO}} = 58_{-6}^{+7}$  keV, though we expect further corrections to the next-to-leading-order (NLO) decay widths from asymptotic normalization effects. We find that a detailed comparison of the NLO total and partial decay widths with experiment suggests the existence of a small (but distinguishable from zero) nonmolecular component of the  $T_{cc}^+$ .

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### I. INTRODUCTION

The LHCb Collaboration has recently observed [1] a tetraquark in the  $D^0D^0\pi^+$  mass spectrum. The Breit-Wigner parameters of this tetraquark, the  $T_{cc}^+$ , are

$$\delta m_{\text{BW}} = -273 \pm 61 \pm 5_{-14}^{+11} \text{ keV}, \quad (1)$$

$$\Gamma_{\text{BW}} = 410 \pm 165 \pm 43_{-38}^{+18} \text{ keV}, \quad (2)$$

where the mass difference is with respect to the  $D^{*+}D^0$  threshold. Alternatively, if the data are analyzed with a resonance profile more suitable to the closeness of the  $T_{cc}^+$  to the  $D^{*+}D^0$  threshold [2], the parameters of the  $T_{cc}^+$  pole turn out to be

$$\delta m_{\text{pole}} = -360 \pm 40_{-0}^{+4} \text{ keV}, \quad (3)$$

$$\Gamma_{\text{pole}} = 48 \pm 2_{-12}^{+0} \text{ keV}. \quad (4)$$

Of course, the question is what the nature of this state is, where the two contending explanations are a compact tetraquark or a loosely bound  $D^{*+}D^0$ - $D^*D^+$  system.

Actually, there is a long list of predictions of a  $cc\bar{u}\bar{d}$  state with  $J = 1^+$  and  $I = 0$ , beginning with the pioneering realization by Zouzou *et al.* [3] that this tetraquark could be below the  $D^*D$  threshold, followed by a large series of works till nowadays [4–12]. The predictions of  $I = 0$   $D^*D$  and  $D^*D^*$  bound states with  $J^P = 1^+$  (for which heavy quark-spin symmetry predicts identical potentials<sup>1</sup>) are in contrast somewhat more recent, with Manohar and Wise [16] and Törnqvist [17] considering it unlikely (from pions alone), but then Ericson and Karl [18] realizing that this conclusion might change if other meson exchanges are considered, an observation later confirmed in Ref. [19] for  $D^*D^*$ , in Ref. [20] for  $D^*D$  (corresponding to the  $T_{cc}^+$ ), in Ref. [21] for  $D^*D$  and  $D^*D^*$ , in Refs. [22,23] for  $D^*D$ , etc. (plus the attention this hypothesis has received [24–28] after the observation of the  $T_{cc}^+$ ). Here, it is worth noticing

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<sup>1</sup>The situation is completely analogous to the  $Z_b(10610)$  and  $Z_b(10650)$  [13] or to the  $X(3872)$  and its hypothetical  $J^{PC} = 2^{++} X(4012)$  partner [14]. However, for the  $D^*D^*$ , a similar caveat applies as for the  $X(4012)$  [15]: the actual location of the compact  $cc\bar{u}\bar{d}/c\bar{c}$  states might make the higher mass partner disappear.

that there might be up to three states with the quantum numbers of the  $T_{cc}^+$  (with the molecular ones usually close to threshold).

In view of the aforementioned theoretical landscape, there are reasons to believe that the two hypotheses (i.e., the molecular and tetraquark explanations) are not mutually exclusive and the  $T_{cc}^+$  could be a superposition of both. The molecular component of the  $T_{cc}^+$  has the theoretical advantage of being a shallow bound state, presumably with a good separation of scales between its long-range and short-range components. This in turn allows us to use the existent theoretical toolbox for shallow bound states [29,30], from which in principle it would be possible to make predictions accurate enough as to analyze its structure.

In this regard, the decay width of the  $T_{cc}^+$  is particularly important (and indeed it has already received due attention [31–33]); if the experimental measurements and theoretical predictions are on par with each other in terms of accuracy, we will be able to determine whether the  $T_{cc}^+$  is purely molecular or compact, or what the degree of admixture is between these two explanations. Given small enough uncertainties, a calculated decay width that is too small or too large in comparison with the experiment might point to (or even determine) the existence of physics beyond the naive molecular explanation, like a tetraquark component or unobserved states. However, this might prove difficult; the wave function of a tetraquark close to the  $D^*D$  threshold might be indistinguishable from that of two separate  $D^*$  and  $D$  mesons, as noted in Ref. [8], which already considered the possibility of a tetraquark lying between the  $D^*D$  and  $DD\pi$  thresholds (see also the discussion in Ref. [34]).

## II. DECAY CHANNELS

The  $T_{cc}^+$  decay width is expected to be saturated by its strong and electromagnetic decays, which are in principle limited to three possibilities:  $T_{cc}^+ \rightarrow D^+D^0\pi^0$ ,  $T_{cc}^+ \rightarrow D^0D^0\pi^+$ , and  $T_{cc}^+ \rightarrow D^+D^0\gamma$ .

However, this is not necessarily the whole story; if the predicted  $cc\bar{u}\bar{d}$  tetraquark happens to be a different state than the  $T_{cc}^+$  but with a lower mass, which we might call  $T_{cc}^{\prime+}$  for concreteness, we will have to add up to two new decay channels,  $T_{cc}^+ \rightarrow T_{cc}^{\prime+}\gamma$  (M1 magnetic and E2 quadrupole transitions) and  $T_{cc}^+ \rightarrow T_{cc}^{\prime+}\pi^0$ , of which the second requires isospin breaking (e.g., stemming from the isospin breaking in the mass of the  $D^{*+}D^0$  and  $D^+D^{*0}$  channels) and a compact  $T_{cc}^{\prime+}$  located close or below the  $D^+D^0$  threshold. This last condition is more difficult to meet as there are fewer predictions of a  $cc\bar{u}\bar{d}$  state close to or below the  $DD$  threshold [35–38] (as to allow some phase space for  $T_{cc}^+ \rightarrow T_{cc}^{\prime+}\pi^0$ ) than between the  $D^*D$  and  $DD$  thresholds [5,39–43]. A different variation over this idea—the possibility of a  $DD$  bound state,  $\tilde{T}_{cc}^+$ , and its potential effect on the  $T_{cc}^+$  decay width—has been recently explored in Ref. [44].

The most straightforward calculation of the  $T_{cc}^+$  decay width into pions involves sandwiching the  $D^* \rightarrow D\pi$  one-body decay operators between the initial and final wave functions [31–33], in which case the total decay width of a molecular  $T_{cc}^+$  falls short of the Breit-Wigner width [1], but agrees well with the width from the improved resonance profile introduced in Ref. [2] (which is in turn consistent with the well-known fact that the Breit-Wigner parametrization will lead to distortions for two-body states close to threshold [45,46]). Here, we include a series of subleading-order effects, including two-body decay operators and rescattering effects in the final  $DD$  pair, which refine the aforementioned theoretical estimations and might allow us to eventually disentangle the molecular and nonmolecular components of the  $T_{cc}^+$ .

## III. POWER COUNTING

Effective field theories (EFTs) are expansions in terms of the ratio  $Q/M$ , with  $Q$  and  $M$  characteristic soft and hard scales of the system at hand. If the  $T_{cc}^+$  is molecular, its natural momentum scale  $Q$  is given by the wave number of its  $D^{*+}D^0$ - $D^{*0}D^+$  components, i.e., 23–26 and 57–59 MeV, respectively, depending on whether we use  $\delta m_{\text{BW}}$  or  $\delta m_{\text{pole}}$ ; see Eqs. (1) and (3). The ratio of these two scales with respect to the pion mass is about 0.18 and 0.42, from which it would be perfectly possible to consider the pion mass as a heavy scale  $M \sim m_\pi$  in a first approximation. If we consider the strong decay products of the  $T_{cc}^+$ , the maximum momentum and energy of the final pion are about 40 and 6 MeV, which are again small in comparison with the pion mass. The situation is less clear with the momentum of the final  $DD$  pair, which can reach 100 MeV; however, one pion exchange does not happen in this system, with the longest range piece of the  $DD$  potential being the two-pion exchange football diagram, with a range of  $2m_\pi$ . From this, the ratio of scales for the final  $DD$  system is 0.37. At this point, it is worth noticing that a pion exchanged between a  $D^*D$  and a  $DD^*$  initial and final state is almost on mass shell and will in principle follow naive dimensional analysis (NDA) as its power counting (also referred to as Weinberg’s counting [47,48], which was originally formulated for the two-nucleon system but can be applied to other non-relativistic two-hadron systems as well). This conclusion changes, though, once we consider the relatively large momentum scale at which pions become nonperturbative in the two-charmed meson system [49]. In summary, an effective field theory description in which the pion mass is considered a hard scale is expected to have a convergence parameter in the range  $Q/M \sim 0.2$ –0.4.

With this, if we consider the strong decays of the  $T_{cc}^+$  and the diagrams in Fig. 1, their counting will be as follows:

- (a) The one-body decay diagram is order  $Q^{-2}$ , and being the lowest-order one, it is leading order (LO),
- (b) In the Weinberg counting, the seagull diagram is  $Q^0$ , but in the decay of the  $T_{cc}^+$ , the Weinberg-Tomozawa

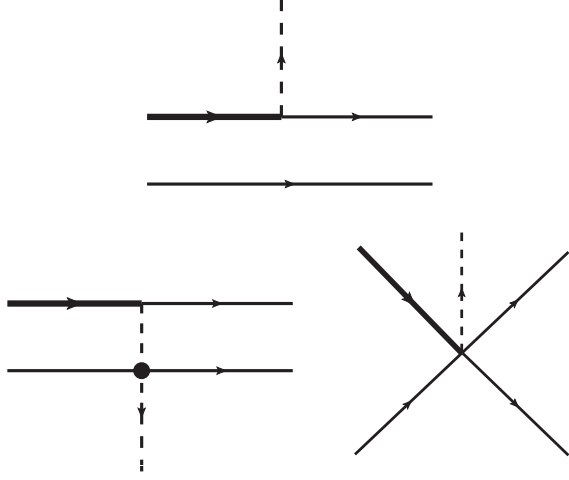


FIG. 1. Lowest-order operators involved in the  $T_{cc}^+ \rightarrow DD\pi$  decay: the one-body decay operator is of order  $Q^{-2}$ , while the seagull and contact two-body currents are naively of order  $Q^0$  and  $Q^1$  but get promoted to  $Q^{-1}$  and  $Q^0$ , respectively.

term is proportional to  $m_\pi$ , which we count as a hard scale. Thus, this diagram is promoted to  $Q^{-1}$  and is next-to-leading order (NLO),

- (c) The contact-range two-body operator is naively  $Q^1$ , but if the  $T_{cc}^+$  is a bound state and if we apply the arguments of Ref. [50] for the counting of two-body operators, it will be promoted to  $Q^0$  and will be next-to-next-to-leading order ( $N^2$ LO).

Indeed, following the logic in Ref. [50], if we apply renormalization group invariance to the contact-range two-body operator, we obtain

$$\begin{aligned} \frac{d}{dR_c} \langle \Psi(DD) | \hat{\mathcal{O}}_C^{2B} | \Psi(T_{cc}^+) \rangle &\propto \\ \frac{d}{dR_c} \left[ \vec{\epsilon}_1 \cdot \vec{q} \frac{C_{2B}(R_c)}{R_c} \right] &= 0, \end{aligned} \quad (5)$$

where  $|\Psi(T_{cc}^+)\rangle$  and  $|\Psi(DD)\rangle$  are the initial and final two-meson wave functions,  $R_c$  is a cutoff radius,  $C_{2B}$  is the contact-range coupling,  $\vec{\epsilon}_1$  is the polarization of the  $T_{cc}^+$  tetraquark, and  $\vec{q}$  is the momentum of the pion. The  $R_c^{-1}$  factor in front of  $C_{2B}(R_c)$  comes from the tetraquark wave function, which scales as  $\langle r | \Psi(T_{cc}^+) \rangle \propto 1/r$  at short distances. This factor also implies that in the infrared limit ( $R_c^{-1} \sim Q$ ), the  $C_{2B}$  coupling is proportional to  $1/Q$  and thus enhanced by one order with respect to NDA.

It is worth noticing that the appearance of the contact-range two-body operator sets the limit of predictability of the EFT: at  $N^2$ LO, this contact can be calibrated to reproduce the  $T_{cc}^+$  decay width into pions, which means that the decay width becomes the input of the theory (instead of its output, which is what we want). For comparison purposes, this counting has a few similarities with X-EFT [an effective

field theory tailor-made for the  $X(3872)$ ][51] and a very significant difference in what regards the counting of pion exchanges [which we count at least as  $N^2$ LO, as will be explained later; see the discussion below Eq. (32)]. We find it also interesting to comment on Ref. [44], which proposes an EFT description of the  $T_{cc}^+$  decays when there is a bound state in the final  $DD$  state; if this were to be the case, the final  $DD$  wave function will behave in exactly the same way as the initial  $T_{cc}^+$  one, i.e.,  $\langle r | \Psi(DD) \rangle \propto 1/r$ , which will result in a  $1/Q^2$  enhancement of the contact-range two-body operator, which will then enter at NLO. In this case, EFT will only be able to predict the  $T_{cc}^+$  decay at LO (instead of NLO as we propose here).

Even if the NLO limitation and the potentially slow convergence parameter look disappointing, they are indeed more than enough for the current situation; the relative uncertainty in the experimental decay width is 0.43 for the standard Breit-Wigner parametrization and 0.25 for the unitarized Breit-Wigner. This naively indicates that either a LO or NLO calculation will be enough to match it, but at which order this exactly happens is not completely obvious *a priori*; EFT arguments allow for the existence of numerical factors of  $\mathcal{O}(1)$ , which might subvert the original power counting expectations. If we ignore these numerical factors and consider the expansion parameter to lie between 0.18–0.42, we find that the uncertainty in the LO and NLO decay widths will be

$$\frac{\Delta\Gamma^{\text{LO}}}{\Gamma^{\text{LO}}} \sim 0.18\text{--}0.42 \quad \text{and} \quad \frac{\Delta\Gamma^{\text{NLO}}}{\Gamma^{\text{NLO}}} \sim 0.03\text{--}0.18, \quad (6)$$

which indicates that a NLO calculation is necessary to be fully competitive with experiment, particularly if we want to match the accuracy of Ref. [2]. As we will see, calculations of the decay width will turn out to be compatible with the average estimations of the EFT convergence. Thus, it happens that all the pieces fit together to put the current limit at NLO, as it is simply not possible to achieve a better accuracy at  $N^2$ LO where the decay width is no longer a prediction.

#### IV. DECAY AMPLITUDES

For the decay of the  $T_{cc}^+$  into  $DD\pi$ , we will consider a decay amplitude in the form

$$\langle DD(\vec{p}') \pi^c | H | D^* D(\vec{p}) \rangle = A^c(\vec{p}', \vec{p}, \vec{q}), \quad (7)$$

where  $c$  is the isospin index of the outgoing pion,  $\vec{q}$  is its momentum, and  $\vec{p}$  ( $\vec{p}'$ ) is the center-of-mass relative momentum of the incoming (outgoing)  $D^* D$  ( $DD$ ) system. This amplitude will be sandwiched between the initial and final state wave functions

$$\langle A^c \rangle = \langle DD(\vec{k}) | A^c | T_{cc}^+ \rangle, \quad (8)$$

and then inserted into Fermi's golden rule to obtain the decay width

$$\Gamma(T_{cc}^+ \rightarrow DD\pi) = 2\pi \int \frac{d^3\vec{k}}{(2\pi)^3} \frac{d^3\vec{q}}{(2\omega)(2\pi)^3} \times \delta\left(\omega + \frac{k^2}{2\mu_{DD}} + \frac{q^2}{2m_{DD}} - \Delta\right) |\langle A^c \rangle|^2, \quad (9)$$

where  $\vec{k}$  is the center-of-mass momentum of the  $DD$  pair,  $\vec{q}$  refers to the momentum of the outgoing pion and  $\omega = \sqrt{m_\pi^2 + q^2}$  refers to its energy (with  $m_\pi$  the pion mass),  $\mu_{DD}$  and  $m_{DD}$  are the reduced and total mass of the final  $DD$  pair, and  $|\langle A^c \rangle|^2$  represents the sum over final states and average over initial states. The amplitude  $A^c$  is represented by the diagrams in Fig. 1, the evaluation of which yields

$$A_c = \frac{g_1}{\sqrt{2}f_\pi} \left[ \vec{\epsilon}_1 \cdot \vec{q} \tau_{1c} (2\pi)^3 \delta^{(3)}\left(\vec{p}' - \vec{p} + \frac{\vec{q}}{2}\right) + i(\vec{\tau}_1 \times \vec{\tau}_2)_c \frac{m_\pi}{f_\pi^2} \frac{\vec{\epsilon}_1 \cdot (\vec{p}' - \vec{p} - \frac{\vec{q}}{2})}{\mu_\pi^2 + (\vec{p}' - \vec{p} - \frac{\vec{q}}{2})^2} \right], \quad (10)$$

where  $\vec{p}$ ,  $\vec{p}'$  are the relative momenta of the incoming and outgoing  $D^*D$  and  $DD$  systems,  $\vec{q}$  and  $c$  are the momentum and the isospin index (in the Cartesian basis) for the outgoing pion,  $\tau_i^c$  is the isospin operator (a Pauli matrix) for the pion as applied to vertex  $i = 1, 2$ ,  $f_\pi \simeq 130$  MeV is the pion weak decay constant,  $g_1$  is the axial coupling for the charmed mesons,  $m_\pi$  is the pion mass, and  $\mu_\pi^2 = m_\pi^2 - (m(D^*) - m(D))^2$  is the effective pion mass for the in-flight pion, which can be on shell and which we simplify to  $\mu_\pi = 0$  from now on.<sup>2</sup> Besides, in the Weinberg-Tomozawa vertex, we have made the simplification that the energy of the incoming and outgoing pion is  $m_\pi$  (we notice that changing it to  $\omega_\pi = \sqrt{m_\pi^2 + \vec{q}^2}$  has a negligibly small effect). For the initial center-of-mass momentum coordinates, we have also ignored the mass difference between the  $D$  and  $D^*$  mesons.

We now evaluate the decay operator between the initial and final states. If we assume wave functions of the type

$$\langle \vec{x} | DD(\vec{k}) \rangle = e^{i\vec{k}\cdot\vec{x}} |DD\rangle \quad \text{and} \quad \langle \vec{x} | T_{cc} \rangle = \psi(\vec{x}) |D^*D\rangle, \quad (11)$$

<sup>2</sup>Actually,  $\mu_\pi^2 \leq 0$ , which means that we can interpret the two-body operator as the rescattering of the outgoing pion with the second charmed meson (indeed, this is how this operator is interpreted in X-EFT [52]). By taking the  $\mu_\pi = 0$  limit, we are effectively considering that this rescattering happens at zero energy, which is a good approximation taking into account that the maximum momentum of the pion is about 35–40 MeV.

the matrix element of the decay amplitude can be explicitly evaluated as follows,

$$\langle A_c \rangle = \frac{g_1}{\sqrt{2}f_\pi} \left[ \vec{\epsilon}_1 \cdot \vec{q} \langle \tau_{1c} \rangle I_{1B}(\vec{k}, \vec{q}) + i(\vec{\tau}_1 \times \vec{\tau}_2)_c \frac{m_\pi}{f_\pi^2} \vec{\epsilon}_1 \cdot \vec{I}_{2B}(\vec{k}, \vec{q}) \right], \quad (12)$$

where the one- and two-body integrals take the form

$$I_{1B}(\vec{k}, \vec{q}) = \int d^3\vec{x} \psi(\vec{x}) e^{-i(\vec{k} - \frac{\vec{q}}{2})\cdot\vec{x}}, \quad (13)$$

$$\vec{I}_{2B}(\vec{k}, \vec{q}) = \int d^3\vec{x} \psi(\vec{x}) \vec{\nabla}_x \left[ \frac{1}{4\pi|\vec{x}|} \right] e^{-i(\vec{k} + \frac{\vec{q}}{2})\cdot\vec{x}}. \quad (14)$$

These expressions can be further simplified by assuming the  $T_{cc}$  to be an S-wave bound state

$$\langle \vec{r} | T_{cc} \rangle = \frac{1}{\sqrt{4\pi}} \frac{u(r)}{r} |D^*D\rangle, \quad (15)$$

and by expanding the decay amplitude in partial waves

$$\langle A(\vec{k}, \vec{q}) \rangle = \vec{\epsilon}_1 \cdot \vec{q} A_{SP}(k, q) + \vec{\epsilon}_1 \cdot \vec{k} A_{PS}(k, q) + (\text{D-waves and higher}), \quad (16)$$

where we will ignore contributions in which the final  $DD$  pair has orbital angular momentum  $L \geq 2$ . After a few manipulations, we arrive at

$$\langle A_c \rangle = \frac{g_1}{\sqrt{2}f_\pi} \left[ \vec{\epsilon}_1 \cdot \vec{q} \langle \tau_{1c} \rangle I_{00}(k, q) - \frac{m_\pi}{f_\pi^2} \langle i(\vec{\tau}_1 \times \vec{\tau}_2)_c \rangle \left( \vec{\epsilon}_1 \cdot \frac{\vec{q}}{2} I_{01}(k, q) + \vec{\epsilon}_1 \cdot \vec{k} I_{10}(k, q) \right) \right], \quad (17)$$

where the integrals  $I_{00}$ ,  $I_{01}$ , and  $I_{11}$  are given by

$$I_{00}(k, q) = \sqrt{4\pi} \int_0^{+\infty} dr r u(r) j_0(kr) j_0\left(\frac{q}{2}r\right), \quad (18)$$

$$I_{01}(k, q) = \frac{1}{\sqrt{4\pi}} \int_0^{+\infty} dr u(r) j_0(kr) \frac{j_1\left(\frac{q}{2}r\right)}{\left(\frac{q}{2}r\right)}, \quad (19)$$

$$I_{10}(k, q) = \frac{1}{\sqrt{4\pi}} \int_0^{+\infty} dr u(r) \frac{j_1(kr)}{kr} j_0\left(\frac{q}{2}r\right), \quad (20)$$

with  $j_n(x)$  the spherical Bessel functions. We notice that in the theory we are using, the reduced wave function takes the form  $u(r) = A_S e^{-\gamma r}$ , for which the  $I_{00}$ ,  $I_{01}$ , and  $I_{10}$  integrals can be evaluated analytically.

The scattering of the  $DD$  in the final state can be taken into account by changing the  $j_0(kr)$  (which comes from assuming the final  $DD$  state is a plane wave) in the integral  $I_{00}$  to

$$j_0(kr) \rightarrow \cos \delta(k)j_0(kr) - \sin \delta(k)y_0(kr), \quad (21)$$

where  $\delta(k)$  is the  $S$ -wave  $DD$  phase shift and  $y_n(x)$  the spherical Neumann functions. If we assume that scattering in the final state is weak (as dictated by our counting), we can simply approximate

$$\cos \delta \approx 1 \quad \text{and} \quad \sin \delta \approx -a_0k, \quad (22)$$

with  $a_0$  the  $DD$  scattering length.<sup>3</sup> Thus, the only change we have to do is the substitution

$$I_{00}(k, q) \rightarrow I_{00}(k, q) + (a_0k)Y_{00}(k, q), \quad (23)$$

with  $Y_{00}$  defined as

$$Y_{00}(k, q) = \sqrt{4\pi} \int_0^\infty dr r u(r) y_0(kr) j_0\left(\frac{q}{2}r\right). \quad (24)$$

We notice that the combination of the  $DD$  rescattering with the seagull diagram, which is N<sup>2</sup>LO, is logarithmically divergent and requires the inclusion of a contact-range two-body operator. This represents a nontrivial check of our initial power counting estimation for this operator.

For the decay of the  $T_{cc}^+$  into  $DD\gamma$ , we use basically the same formalism, though in this case there is no two-body operator; the lowest-order one enters at N<sup>2</sup>LO. The decay amplitude takes the form

$$\langle DD(\vec{p}')\gamma|H|D^*D(\vec{p})\rangle = A_{M1}(\vec{p}', \vec{p}, \vec{q}), \quad (25)$$

with  $A_{M1}$  given by

$$A_{M1} = \mu(D^*)\vec{\alpha} \cdot (\vec{\epsilon}_1 \times \vec{q})(2\pi)^3\delta^{(3)}\left(\vec{p}' - \vec{p} + \frac{\vec{q}}{2}\right), \quad (26)$$

where  $\vec{\alpha}$  is the polarization vector of the photon and  $\mu(D^*)$  is the magnetic moment of the relevant  $D^* \rightarrow D\gamma$  transition, which if written in the isospin basis reads

$$\mu(D^*) = \mu_+ \left(\frac{1 + \tau_{1z}}{2}\right) + \mu_0 \left(\frac{1 - \tau_{1z}}{2}\right), \quad (27)$$

with  $\mu_+$  and  $\mu_0$  the magnetic moments for  $D^{*+} \rightarrow D^+\gamma$  and  $D^{*0} \rightarrow D^0\gamma$ , though for this decay, one might as well simply use the particle basis. The calculation of the decay

width uses Eq. (9) but with the substitution  $\omega \rightarrow q$  for adapting it to the photon case. The matrix elements of  $A_{M1}$  are obtained as before, leading to

$$\langle A_{M1} \rangle = \langle \mu(D^*) \rangle \vec{\alpha} \cdot (\vec{\epsilon}_1 \times \vec{q}) I_{00}(k, q), \quad (28)$$

which is completely analogous to Eq. (17). The inclusion of rescattering effects in the final  $DD$  state is done again with Eq. (23).

## V. MOLECULAR $T_{cc}^+$ WAVE FUNCTION

If isospin symmetry were to be conserved in the masses, the wave function of the  $T_{cc}$  would be written as

$$\langle \vec{x} | T_{cc}^{(I)} \rangle = \psi(\vec{x}) | D^* D(I) \rangle, \quad (29)$$

depending on whether its isospin is  $I = 0$  or  $1$  (where molecular models show a clear preference for  $I = 0$ ). However, the  $T_{cc}^+$  is located merely a few hundred of keV below the  $D^{*+}D^0$ , which is small in comparison with the mass difference between the  $D^{*+}D^0$  and  $D^{*0}D^+$  thresholds (about 1.4 MeV). For this reason, we instead consider the  $T_{cc}^+$  wave function to be a linear combination of a *low* and a *high mass* channel contributions,

$$\langle \vec{x} | T_{cc} \rangle = \psi_L(\vec{x}) | L \rangle + \psi_H(\vec{x}) | H \rangle, \quad (30)$$

with  $|L\rangle$  and  $|H\rangle$  given by

$$|L\rangle = |D^{*+}D^0\rangle \quad \text{and} \quad |H\rangle = |D^{*0}D^+\rangle. \quad (31)$$

For determining the wave function, we first consider the EFT expansion of the  $D^*D$  interaction

$$V_{\text{EFT}}(\vec{q}) = C_I + D_I \vec{q}^2 + E_I \left( \vec{\epsilon}_1 \cdot \vec{q} \vec{\epsilon}_1^* \cdot \vec{q} - \frac{1}{3} \vec{q}^2 \right) + V_{\text{OPE}}(\vec{q}), \quad (32)$$

where  $\vec{q}$  is the exchanged momentum between the mesons,  $I = 0, 1$  indicates isospin,  $C_I$  and  $D_I$  represent momentum-independent and momentum-dependent  $S$ -wave interactions,  $E_I$  is an  $S$ -to- $D$ -wave contact interaction, and  $V_{\text{OPE}}$  is the one pion exchange (OPE) potential. We will count  $C_0$  as LO,  $C_1$  and  $D_0$  as NLO, and  $E_0$  as N<sup>2</sup>LO. As for the OPE potential, it is nominally NLO, but the actual momentum scale at which central and tensor pion exchanges become nonperturbative in the  $D^*D/DD^*$  systems has been estimated to be  $\Lambda_C > 1$  GeV and  $\Lambda_T = 290$  MeV in Ref. [49], but this corresponds to  $g_1 = 0.6$ . If we use the updated value of the axial coupling  $g_1 = 0.56$ , the tensor scale will become  $\Lambda_T = 330$  MeV. The size of the tensor corrections is thus expected to be  $\gamma_L/\Lambda_T \sim 0.08$ , which is in between N<sup>1.7</sup>LO and N<sup>2.9</sup>LO for our estimation of the expansion parameter [(0.2)<sup>1.7</sup> and (0.4)<sup>2.9</sup> are

<sup>3</sup>Notice that in the sign convention we are using  $k \cot \delta(k) \rightarrow -1/a_0$  for  $k \rightarrow 0$ , and a bound (virtual) state entails a positive (negative) scattering length.

approximately 0.08]. Yet, we warn that this estimation will require further attention; owing to the effective mass of the pion being relatively small ( $\mu_\pi \rightarrow 0$ ), the  $S$ -to- $D$  wave tensor force effectively becomes a  $1/r^3$  potential, which has a really long range and might generate a  $D$ -wave component of the wave function that is larger than expected.

As long as pions remain subleading, we have a contact theory with a wave function of the type

$$\psi_L(\vec{x}) = \frac{A_S}{\sqrt{4\pi}} \phi_L \frac{e^{-\gamma_L r}}{r}, \quad (33)$$

$$\psi_H(\vec{x}) = \frac{A_S}{\sqrt{4\pi}} \phi_H \frac{e^{-\gamma_H r}}{r}, \quad (34)$$

with  $A_S$  the asymptotic normalization of the wave function,  $\phi_L$  and  $\phi_H$  (such that  $|\phi_L|^2 + |\phi_H|^2 = 1$ ) the amplitudes of the  $L$  and  $H$  channels, and  $\gamma_L = 26.4$  MeV and  $\gamma_H = 58.5$  MeV the wave numbers for the central value of  $\delta m_{\text{pole}}$ . It will prove useful to also define  $\phi_L$  and  $\phi_H$  in terms of the isospin angle  $\theta_I$ :

$$\phi_L = \cos \theta_I \quad \text{and} \quad \phi_H = \sin \theta_I. \quad (35)$$

At LO, if we assume that the  $T_{cc}^+$  is predominantly an  $I = 0$  state (at least at short distances), we will only have an isoscalar contact-range interaction, which basically fixes  $\phi_L^{\text{LO}} = -\phi_H^{\text{LO}} = 1/\sqrt{2}$  (modulo corrections from the difference in the reduced masses of the  $L$  and  $H$  channels). The LO asymptotic normalization will be determined by the normalization of the wave function, i.e.,

$$|A_S^{\text{LO}}|^2 \int_0^\infty dr [|\phi_L^{\text{LO}}|^2 u_L^2(r) + |\phi_H^{\text{LO}}|^2 u_H^2(r)] = 1, \quad (36)$$

with  $u_L(r) = e^{-\gamma_L r}$  and  $u_H(r) = e^{-\gamma_H r}$ , from which

$$\frac{1}{A_S^{\text{LO}}} = \sqrt{\frac{\phi_L^{\text{LO}2}}{2\gamma_L} + \frac{\phi_H^{\text{LO}2}}{2\gamma_H}}. \quad (37)$$

At NLO, we will have corrections from i) the potential in the  $I = 1$  channel, i.e.,  $C_1$ , which actually does not change the form of the wave function at all, and ii) the momentum-dependent contact-range interaction in the  $I = 0$  channel, i.e.,  $D_0$ , which breaks the relation between  $A_S$  and the normalization of the wave function. Basically, this implies the correction:

$$A_S^{\text{NLO}} = A_S^{\text{LO}} + \delta A_S. \quad (38)$$

Now, two strategies are possible here: i) to determine  $\delta A_S$  from the isoscalar effective range or ii) to determine them from external information. The first strategy is the more usual one in pionless EFT, though besides expanding in

terms of range corrections [53], it is also possible to expand in terms of the wave function renormalization [54].

The second strategy is equivalent (up to higher-order corrections) to the first one and might be easier to pull off simply because  $A_S$  (and also  $\theta_I$ ) can be determined from potential models. In fact, provided that the interaction binding the  $T_{cc}^+$  is attractive at most distance scales, we will have  $A_S > A_S^{\text{LO}}$  (the only way in which to obtain  $A_S < A_S^{\text{LO}}$  is with an attractive short-range potential surrounded by a repulsive barrier), which works in the direction of increasing the decay widths.

Finally, for completeness and as a nontrivial crosscheck of isospin symmetry, we will explicitly consider changes of the isospin angle at NLO,

$$\theta_I^{\text{NLO}} = \theta_I^{\text{LO}} + \delta\theta_I. \quad (39)$$

From the expansion of the potential in Eq. (32), we expect  $\theta_I^{\text{LO}} \approx \theta_I^{\text{NLO}} \approx -45^\circ$ , modulo negligible corrections from the difference in the reduced masses of the  $L$  and  $H$  channel and the relative effect of the range corrections (the  $D_0$  coupling) in these two channels. Yet, considering a nontrivial  $\delta\theta_I$  might reveal the existence of isospin breaking contact terms at short distances (though here short distances actually include two-pion exchange diagrams that might generate a larger than expected  $\delta\theta_I$ ). We will see that the NLO calculation yields a  $\theta_I^{\text{NLO}}$  compatible with  $-45^\circ$  when compared with the relevant experimental data.

## VI. NONMOLECULAR COMPONENT OF THE $T_{cc}^+$

Predictions for the  $T_{cc}^+$  tetraquark fall into two categories depending on whether they are based on its quark or charmed meson degrees of freedom. We might loosely refer to them as compact and molecular. This is not necessarily a clear-cut distinction though, as four-quark explanations can perfectly generate a noncompact, two-charmed meson component of the wave function if the mass of the tetraquark happens to be close to the  $D^*D$  threshold [8]. For the sake of simplicity, we might consider that the  $T_{cc}^+$  wave function can be subdivided into a noncompact and compact component,

$$|T_{cc}^+\rangle = \cos \theta_C |D^*D\rangle + \sin \theta_C |cc\bar{q}\bar{q}\rangle, \quad (40)$$

with the noncompact piece corresponding to the  $D^*D$  molecular explanation we have referred to previously, while the compact piece represents the nonmolecular components and  $\theta_C$  represents the mixing angle between these two pieces of the wave function, where  $\theta_C = 0$  corresponds to the usual molecular interpretation of the  $T_{cc}^+$ .

The interesting point is how a compact component will enter the description of the decay widths. The contribution of a wave function component to the decay amplitude

depends on the momentum scales involved. For the molecular component, we have

$$\begin{aligned} \langle T_{cc}(D^*D) | A_c^{1B} | DD\pi \rangle & \\ &= \frac{g_1}{\sqrt{2}f_\pi} \vec{\epsilon}_1 \cdot \vec{q} \int d^3\vec{x} \langle \psi | \vec{x} \rangle e^{-i(\vec{k}-\vec{q}/2)\cdot\vec{x}} \\ &= \frac{g_1}{\sqrt{2}f_\pi} \vec{\epsilon}_1 \cdot \vec{q} \langle \psi | \vec{k} - \vec{q}/2 \rangle, \end{aligned} \quad (41)$$

where  $\langle \psi | \vec{x} \rangle$  and  $\langle \psi | \vec{p} \rangle$  are the r- and p-space molecular wave functions. By taking into account that in a contact-range theory  $\langle \psi | \vec{p} \rangle = \sqrt{8\pi\gamma}/(p^2 + \gamma^2)$ , we expect that this matrix element scales as

$$\langle T_{cc}(D^*D) | A_c^{1B} | DD\pi \rangle \propto \frac{g_1}{\sqrt{2}f_\pi} \vec{\epsilon}_1 \cdot \vec{q} \frac{\sqrt{2\pi}}{Q^{3/2}}, \quad (42)$$

with  $Q \sim \gamma_L, \gamma_H, k$ , or  $q$  the characteristic low-energy momentum scale for a molecular  $T_{cc}^+$ . From this, the natural expectation for the scaling of the decay amplitude of a compact component would be

$$\langle T_{cc}(cc\bar{q}\bar{q}) | A_c | DD\pi \rangle \propto \frac{g'_1}{\sqrt{2}f_\pi} \vec{\epsilon}_1 \cdot \vec{q} \frac{\sqrt{2\pi}}{M_C^{3/2}}, \quad (43)$$

with  $M_C$  the characteristic momentum scale for a compact tetraquark, which we expect to be of the order of the natural hadronic scale  $M_C \sim (0.5-1.0)$  GeV (and  $g'_1$  the coupling of the compact component to  $DD\pi$ , which we have assumed to be of roughly the same size as  $g_1$ ). This scaling argument also applies to the  $DD\gamma$  decays.

From the previous estimation in Eq. (69) the decay amplitude of a compact component of the  $T_{cc}$  wave function is expected to be suppressed by a factor of  $(Q/M_C)^{3/2}$  with respect to the LO contribution. Were  $M_C$  to be of the order of the hard scale in the EFT we are using here, i.e.,  $M \sim (1-2)m_\pi$ , the contribution from a compact component to the decay would enter at  $N^3/2$ LO. It turns out that  $M_C > M$ , which means that this contribution enters at a considerably higher order. Thus, at lower orders in the EFT expansion, the effect of a compact tetraquark component is simply to reduce the total decay width:

$$\Gamma(T_{cc}^+) = \cos^2\theta_C \Gamma(T_{cc}^+(D^*D)). \quad (44)$$

That is, if a molecular prediction overshoots the experimental decay width by a noticeable amount, this might indicate the existence of a nonmolecular component for the  $T_{cc}^+$  tetraquark.

It is, however, worth noticing that the explicit separation of the  $T_{cc}^+$  wave function into molecular and nonmolecular components generates a parameter redundancy problem, as the observable effects of the compact mixing angle  $\theta_C$  can be reabsorbed into the EFT's subleading range corrections,

i.e., into  $A_S$ . Indeed, at NLO, the decay amplitude is proportional to these two factors,

$$\Gamma(T_{cc}^+) \propto \cos^2\theta_C A_S^2, \quad (45)$$

which means that *compactness* can be recast into a negative contribution to the effective range (as this reduces  $A_S^2$ ; see Appendix). Thus, the angle  $\theta_C$  should be considered as a model-dependent quantity, at least in the absence of a model-independent disentanglement of the dynamics between the molecular and nonmolecular degrees of freedom. Unfortunately, though the inclusion of a compact  $T_{cc}^+$  field is straightforward, this still does not resolve the parameter redundancy problem (which probably requires invoking phenomenological models).

## VII. COUPLINGS

The width of a molecular  $T_{cc}$  depends on the axial coupling  $g_1$  and the magnetic moments  $\mu_+$  and  $\mu_0$  for the  $D^*$  to  $D$  transitions, which can be extracted from the decay widths of the charmed mesons. We begin with  $g_1$ , for which we use the decays of  $D^{*+}$  into  $D\pi$ ,

$$\Gamma(D^{*+} \rightarrow D^0\pi^+) = \frac{g_1^2}{6\pi f_\pi^2} \frac{m_{D^0}}{m_{D^{*+}}} q_{\pi^+}^3, \quad (46)$$

$$\Gamma(D^{*+} \rightarrow D^+\pi^0) = \frac{g_1^2}{12\pi f_\pi^2} \frac{m_{D^+}}{m_{D^{*+}}} q_{\pi^0}^3, \quad (47)$$

where  $f_\pi = 130$  MeV and  $q_\pi$  the momentum of the emitted pion. From the  $D^{*+}$  decay width and branching ratios provided in the Review of Particle Physics [55], i.e.,  $\Gamma(D^{*+}) = 83.4 \pm 1.8$  keV,  $\Gamma(D^0\pi^+)/\Gamma = (67.7 \pm 0.5)\%$ , and  $\Gamma(D^+\pi^0)/\Gamma = (30.7 \pm 0.5)\%$ , we obtain  $g_1 = 0.56 \pm 0.01$ .

For the magnetic moments  $\mu_+$  and  $\mu_0$ , we use the  $D^*$  decays into  $D\gamma$ ,

$$\Gamma(D^* \rightarrow D\gamma) = \frac{|\mu|^2}{3\pi} \frac{m_D}{m_{D^*}} q_\gamma^3, \quad (48)$$

with  $q_\gamma$  the momentum of the outgoing photon. For  $\mu_+$ , we use again the  $D^{*+}$  decay width and its branching ratio into  $D^+\gamma$  (i.e.,  $1.6 \pm 0.4\%$ ), yielding  $\mu_+ = 0.46 \pm 0.06\mu_{\text{n.m.}}$  where the sign is chosen as to coincide with that of the magnetic moment of the  $\bar{d}$  antiquark within the  $D^{*+}$  and with  $\mu_{\text{n.m.}} = |e|/2m_N$  the nuclear magneton. The determination of  $\mu_0$  is more indirect as the  $D^{*0}$  decay width is not experimentally known (beyond an upper bound). However, its branching ratios into  $D^0\pi^0$  and  $D^0\gamma$  are well determined [55], and the partial decay width into  $D^0\pi^0$  can be calculated from  $g_1$  [resulting in  $\Gamma(D^{*0} \rightarrow D^0\pi^0) = 35.9 \pm 1.3$  keV], which all together yields  $\Gamma(D^{*0} \rightarrow D^0\gamma) = 19.6 \pm 1.0$  keV. From this, we obtain  $\mu_0 = -(1.72 \pm 0.05)\mu_{\text{n.m.}}$

### VIII. PARTIAL DECAY WIDTHS

With the previous ingredients, we are ready to calculate the  $T_{cc} \rightarrow DD\pi$  and  $T_{cc} \rightarrow DD\gamma$  decay widths. For this, we have to sandwich the decay operator between the initial and final states, which, though laborious (we have to take into account isospin breaking in the  $L$  and  $H$  channels), is nonetheless straightforward. We will use the  $\delta m_{\text{pole}}$  solution, from which the binding energy of the  $L$  ( $H$ ) components of a molecular  $T_{cc}^+$  is  $B_L = 0.36 \pm 0.04$  MeV ( $B_H = 1.77 \pm 0.04$  MeV). In addition, we will assume a purely molecular  $T_{cc}^+$  (i.e.,  $\theta_C = 0$ ) unless stated otherwise.

We begin with the tree level amplitudes. For the  $T_{cc}^+ \rightarrow D^0 D^0 \pi^+$  decay, the final state contains two identical bosons and requires symmetrization, which is done by adding the  $A(\vec{k}, \vec{q})$  and  $A(-\vec{k}, \vec{q})$  amplitudes and then changing the phase space factor for the final  $D^0 D^0$  pair from  $d^3\vec{k} \rightarrow d^3\vec{k}/2$  to avoid counting the final  $D^0 D^0$  states twice. We obtain

$$\Gamma^{\text{LO(1B)}}(T_{cc}^+ \rightarrow D^0 D^0 \pi^+) = 29.6_{-1.0}^{+1.1} {}_{-1.8}^{+1.8} \text{ keV}, \quad (49)$$

$$\Gamma^{\text{LO(1B)}}(T_{cc}^+ \rightarrow D^+ D^0 \pi^0) = 13.7_{-0.5}^{+0.5} {}_{-0.6}^{+0.5} \text{ keV}, \quad (50)$$

$$\Gamma^{\text{LO(1B)}}(T_{cc}^+ \rightarrow D^+ D^0 \gamma) = 5.8 \pm 0.4 \pm 0.2 \text{ keV}, \quad (51)$$

which basically agrees with Ref. [31] and where we have taken  $\phi_L = -\phi_H = 1/\sqrt{2}$  and  $A_S = 8.5 \text{ MeV}^{1/2}$  (obtained from the normalization of the wave function). The first uncertainty corresponds to varying  $g_1$  for the strong decays and  $\mu_+$  and  $\mu_0$  for the electromagnetic one, while the second comes from the binding energy. We notice that the previous amplitudes only take into account a final  $D^0 D^0$  state in the  $S$ -wave or a final  $D^+ D^0$  state in the  $S$ - or  $P$ -wave; adding the contributions from higher  $L = 2, 4, 6, \dots$  ( $L = 2, 3, 4, \dots$ ), partial waves of the  $D^0 D^0$  ( $D^+ D^0$ ) final state will change the partial decay widths to 29.9, 14.1, and 6.4 keV, respectively, i.e., a small 1.3 keV increase in the total decay width. With the exception of the electromagnetic decay, the decay widths increase if the binding energy is reduced (e.g., if we were to use  $\delta m_{\text{BW}}$  instead of  $\delta m_{\text{pole}}$ , we would obtain 33.7, 14.9, and 5.5 keV for the partial decay widths). The combined LO decay width is

$$\begin{aligned} \Gamma^{\text{LO}}(T_{cc}^+) &= 49.1_{-1.5}^{+1.6} {}_{-0.4}^{+0.4} {}_{-2.1}^{+2.2} \text{ keV}, \\ &= 49.1_{-2.6}^{+2.7} \text{ keV}, \end{aligned} \quad (52)$$

where the uncertainties refer only to the input parameters ( $g_1$ ,  $\mu_+/\mu_0$ , and  $\delta m$ ), not to the EFT convergence rate (which we have not discussed yet). It is interesting to notice that this width is in line with most of the other LO calculations available: 47 keV in Ref. [31] (which uses  $\delta m_{\text{BW}}$  instead of  $\delta m_{\text{pole}}$ ), 53 keV in Ref. [32] (which calculates the decay width in the isospin limit), 43 keV

(80 keV) for  $\delta m_{\text{pole}}$  ( $\delta m_{\text{BW}}$ ) in Ref. [33] (which directly convolutes the width of the charmed mesons to obtain the  $T_{cc}^+$  width), and 52 keV in Ref. [44] (which also uses  $\delta m_{\text{BW}}$ ). A cursory comparison of the previous predictions suggest a LO error of the order of 10 keV, a figure compatible with the EFT uncertainties we will later obtain in Eqs. (63) and (64).

Next, we consider the rescattering of the  $DD$  pair in the final state, which requires the scattering length of this system as input. The only phenomenological calculation of this quantity we are aware of is Ref. [21], which estimates  $a_0(DD) = -0.4_{-0.2}^{+0.1}$  fm (and also predicts  $\delta m = -3_{-15}^{+4}$  MeV for the  $T_{cc}$  in the isospin symmetric limit, from which we may assume that the actual  $DD$  scattering length will also fall within the error bars<sup>4</sup>). If we use the LO values of  $A_S$ ,  $\phi_L$ , and  $\phi_H$  (which we will from now on, unless stated otherwise), we obtain

$$\Gamma^{(\text{1B+DD})}(T_{cc}^+ \rightarrow D^0 D^0 \pi^+) = 33.3_{-1.2}^{+1.2} {}_{-1.7}^{+1.9} {}_{-0.9}^{+2.0} \text{ keV}, \quad (53)$$

$$\Gamma^{(\text{1B+DD})}(T_{cc}^+ \rightarrow D^+ D^0 \pi^0) = 15.9_{-0.5}^{+0.6} {}_{-0.5}^{+0.5} {}_{-0.6}^{+1.2} \text{ keV}, \quad (54)$$

$$\Gamma^{(\text{1B+DD})}(T_{cc}^+ \rightarrow D^+ D^0 \gamma) = 7.5 \pm 0.6 \pm 0.2 {}_{-0.4}^{+0.9} \text{ keV}, \quad (55)$$

where the source of the first two errors is as in the LO calculation and the third error comes from the propagation of the uncertainty in  $a_0$ . We stress that in the counting used here, the  $DD$  interaction is perturbative. Previously, a nonperturbative final state interaction was considered for instance in case of the  $X(3872)$  as a  $D^* \bar{D}$  system and its decays into  $D \bar{D} \pi$  [52,58]. For the  $T_{cc}^+$ , Ref. [44] has recently considered the case in which the final  $DD$  interaction is able to form a bound state.

Then, we consider the inclusion of the seagull diagram (but without including the  $DD$  rescattering or the changes in asymptotic normalization), which only affects the  $DD\pi$  decays, arriving at

$$\Gamma^{(\text{1B+2B})}(T_{cc}^+ \rightarrow D^0 D^0 \pi^+) = 30.2_{-1.0}^{+1.1} {}_{-1.6}^{+1.8} \text{ keV}, \quad (56)$$

$$\Gamma^{(\text{1B+2B})}(T_{cc}^+ \rightarrow D^+ D^0 \pi^0) = 14.1_{-0.4}^{+0.6} {}_{-0.5}^{+0.7} \text{ keV}, \quad (57)$$

which implies that the two-body corrections are actually smaller than the rescattering of the final  $DD$  mesons and where the uncertainties are the same as in the LO calculation ( $g_1$  and  $\delta m_{\text{pole}}$ ). Here, a comparison with the

<sup>4</sup>We mention, though, that calculations of the two-bottom-meson potential in the lattice indicate that the  $I = 1$   $BB$  configuration is attractive overall [56] (which in our sign convention will generate  $a_0 < 0$  if the attraction is not strong enough to generate a bound state), particularly at short distances. Chiral EFT also predicts an attractive two-pion exchange potential for  $I = 1$   $BB$  [57]. Finally, from heavy flavor symmetry, we expect the  $DD$  and  $BB$  potentials to be similar.



$D^*\bar{D}$  system—the  $X(3872)$ —is in order: Ref. [52], which previously considered the rescattering and two-body corrections for the  $X(3872)$  decays, also arrived to the conclusion that rescattering effects are much larger than the contribution from the seagull diagram, where the later contribution happens to be fairly small.

Finally, including these two subleading-order corrections together (but using the LO values of the wave function parameters), we obtain an *abridged* NLO result for the decay widths,

$$\Gamma^{\text{NLO}(*)}(T_{cc}^+ \rightarrow D^0 D^0 \pi^+) = 34.0 \pm 1.2_{-1.7}^{+1.8} \pm 1.9_{-1.0}^{+1.9} \text{ keV}, \quad (58)$$

$$\Gamma^{\text{NLO}(*)}(T_{cc}^+ \rightarrow D^+ D^0 \pi^0) = 16.4_{-0.5}^{+0.6} \pm 0.6_{-0.5}^{+0.6} \pm 1.2_{-0.5}^{+1.2} \text{ keV}, \quad (59)$$

$$\Gamma^{\text{NLO}(*)}(T_{cc}^+ \rightarrow D^+ D^0 \gamma) = 7.5 \pm 0.6 \pm 0.2_{-0.4}^{+0.9} \text{ keV}, \quad (60)$$

where the uncertainties are as in the previous  $DD$  rescattering partial widths ( $g_1$  or  $\mu_+$  and  $\mu_0$ ,  $\delta m_{\text{pole}}$ , and  $a_0$ ). The combined decay width will then be

$$\begin{aligned} \Gamma^{\text{NLO}(*)}(T_{cc}^+) &= 57.9_{-1.8}^{+1.8} \pm 0.6_{-2.0}^{+4.1} \pm 2.2_{-2.1}^{+2.2} \text{ keV} \\ &= 57.9_{-3.4}^{+5.0} \text{ keV}, \end{aligned} \quad (61)$$

where for the moment the uncertainty only refers to the one coming from the input parameters of the calculation. If we are interested in the EFT uncertainty, we can compare the abridged NLO(\*) decay width with the LO one, suggesting

$$\left| \frac{\Gamma^{\text{NLO}(*)} - \Gamma^{\text{LO}}}{\Gamma^{\text{LO}}} \right| \approx 0.18, \quad (62)$$

in line with the naive estimations in Eq. (6) of a convergence rate within 0.2–0.4. However, this does not take into account the possible corrections to the asymptotic normalization and isospin angle, which might worsen the convergence of the EFT. For exploring what to expect from the corrections to the asymptotic normalization, we might look at the two nucleon system. There, the LO wave function in a pionless theory would yield  $A_S^{\text{LO}} = \sqrt{2\gamma_d} = 0.6806 \text{ fm}^{-1/2}$ , which is to be compared with  $A_S = 0.8846(9) \text{ fm}^{-1/2}$  [59], yielding  $A_S^2/(A_S^{\text{LO}})^2 = 1.69$ . Were this ratio to hold for the  $T_{cc}^+$  case, we would have a 70% increase in the NLO decay width if we

were to expand directly in terms of  $A_S$ . This is probably not the case, though; the  $A_S^2/(A_S^{\text{LO}})^2$  ratio scales as  $1/(1 - \gamma r_e)$  with  $\gamma$  the binding momentum and  $r_e$  the effective range (with this simple approximation resulting in 1.68 for the deuteron). The  $T_{cc}^+$  is less bound than the deuteron, though, and naively we expect range corrections in the  $T_{cc}^+$  to be of the same order of magnitude as those of the  $X(3872)$ , which are considerably smaller than in the deuteron case [51,52]. If we notice that nontensor OPE almost cancels in the  $D^*D$  and  $D^*\bar{D}$  systems, the scale of range corrections is probably set by the tensor scale  $\Lambda_T = 330 \text{ MeV}$  [check the previous discussion below Eq. (32)], while in the deuteron, the range will be given by the pion mass, indicating that the expected range of the  $D^*D$  potential is about 0.42 times that of the two-nucleon case. From this, we could expect  $A_S^2/(A_S^{\text{LO}})^2 \sim 1.1$ , which will suggest a convergence parameter of

$$\left| \frac{\Gamma^{\text{NLO}} - \Gamma^{\text{LO}}}{\Gamma^{\text{LO}}} \right| \approx 0.3. \quad (63)$$

If this estimation were to hold, the full uncertainties in the LO and NLO calculation would be

$$\Gamma^{\text{LO}}(T_{cc}^+) = 49 \pm 3 \pm 16 \text{ keV}, \quad (64)$$

$$\Gamma^{\text{NLO}(*)}(T_{cc}^+) = 58_{-3}^{+5} \pm 5 \text{ keV}, \quad (65)$$

where the first and second errors refer to the input parameters and the intrinsic EFT uncertainty, respectively. But again, there might be factors which we have not properly considered and which might alter the current conclusions, which should be taken as temporary. As a cross-check, we notice that most LO calculations available [31–33,44] lie within the (43–53) keV window and are thus compatible with our LO result within EFT uncertainties.

Of course, the previous NLO partial decay widths are incomplete; the full NLO calculation requires a modification in the asymptotic normalization  $A_S$  and the isospin angle  $\theta_I$ , and in principle, it is also possible to consider the mixing angle between a noncompact and compact  $T_{cc}^+$  component,  $\theta_C$ . Luckily, these contributions can be factored out easily, leading to the following expressions,

$$\begin{aligned} \Gamma^{\text{NLO}}(T_{cc}^+ \rightarrow D^0 D^0 \pi^+) &= \cos^2 \theta_C A_S^2 \times (\phi_L^2 [0.825(94) - 0.248(17)a_0 + 0.0188(7)a_0^2] \\ &\quad + \phi_L \phi_H [-0.00648(47) + 0.000947(35)a_0]), \end{aligned} \quad (66)$$

$$\begin{aligned} \Gamma^{\text{NLO}}(T_{cc}^+ \rightarrow D^+ D^0 \pi^0) &= \cos^2 \theta_C A_S^2 \times (\phi_L^2 [0.187(22) - 0.0552(39)a_0 + 0.00412(15)a_0^2] \\ &\quad + \phi_L \phi_H [-0.164(13) + 0.0729(48)a_0 - 0.00719(27)a_0^2] \\ &\quad + \phi_H^2 [0.0386(23) - 0.00230(11)a_0 + 0.00347(14)a_0^2]), \end{aligned} \quad (67)$$

$$\begin{aligned} \Gamma^{\text{NLO}}(T_{cc}^+ \rightarrow D^+ D^0 \gamma) &= \cos^2 \theta_C A_S^2 \times (\phi_L^2 [0.0119(30) - 0.0057(14) a_0 + 0.00078(19) a_0^2] \\ &\quad + \phi_L \phi_H [-0.0624(82) + 0.0381(49) a_0 - 0.000573(73) a_0^2] \\ &\quad + \phi_H^2 [0.0851(45) - 0.0638(34) a_0 + 0.0122(6) a_0^2]), \end{aligned} \quad (68)$$

which return the partial decay widths in keV and require as input  $A_S$  in units of  $\text{MeV}^{-1/2}$  and  $a_0$  in units of fm. The uncertainties are shown in parentheses and correspond to  $g_1$ ,  $\mu_+$ ,  $\mu_0$ , and  $\delta m_{\text{pole}}$  summed in quadrature and symmetrized (as these errors are almost symmetrical). The term proportional to  $\phi_H^2$  in  $\Gamma(T_c^+ \rightarrow D^0 D^0 \pi^+)$  is actually negligible (a small contribution coming from the seagull diagram), and we have not written it down. The same could be argued of a few of the terms we have kept, but in these cases, the difference is at most of 2 orders of magnitude, potentially up to about a 1% difference for  $|a_0| = 1$  fm. Here, it is worth remembering that these formulas are only expected to be valid for  $m_\pi a_0 < 1$ , i.e.,  $a_0 < 1.4$  fm (otherwise, a different power counting in which the interaction of the final  $DD$  pair is non-perturbative should be used). The actual accuracy of these formulas is limited by the EFT convergence, where for our estimation of 0.3 for the expansion parameter we should expect the previous expressions to have a 9% uncertainty.

## IX. COMPARISON WITH EXPERIMENT

The theoretical decay widths can be compared with the experimental analysis to obtain information about the  $T_{cc}^+$ . If we begin with the LO calculation, we quickly realize that the EFT calculation overshoots the experimental decay width as extracted from the unitarized Breit-Wigner profile [2]

$$\frac{\Gamma_{\text{pole}}}{\Gamma_{\text{LO}}} = 0.98_{-0.22}^{+0.04} {}_{-0.06}^{+0.06} {}_{-0.23}^{+0.42}, \quad (69)$$

where the first uncertainty comes from  $\Gamma_{\text{pole}}$  while the second and third are derived from the input parameters and the convergence rate of  $\Gamma_{\text{LO}}$ , respectively. The previous result is compatible with 1. If we allow for a nontrivial  $\theta_C$ , this ratio could be related to the molecular content of the  $T_{cc}^+$ ,

$$\frac{\Gamma_{\text{pole}}}{\Gamma_{\text{LO}}} = \cos^2 \theta_C^{\text{LO}}, \quad (70)$$

yielding

$$|\theta_C^{\text{LO}}| = (8.6_{-8.6}^{+7.7} {}_{-8.6}^{+21.3})^\circ, \quad (71)$$

which is compatible with the absence of a compact component, i.e.,  $\theta_C = 0$ , within errors.

Other interesting experimental information in Ref. [2] is the signal yields of the  $T_{cc}^+$  to the  $D^+ D^0$  and  $D^0 D^0$  channels:

$$N_S^+ \equiv N_S(D^+ D^0) = 171 \pm 26, \quad (72)$$

$$N_S^0 \equiv N_S(D^0 D^0) = 263 \pm 23. \quad (73)$$

As the number of events grows, the ratio of these two numbers is expected to approach the ratio of the decays to the  $D^+ D^0$  and  $D^0 D^0$  channels

$$\frac{\Gamma(T_{cc}^+ \rightarrow D^+ D^0 \pi^0 / \gamma)}{\Gamma(T_{cc}^+ \rightarrow D^0 D^0 \pi^+)} = \frac{N_S^+}{N_S^0} \left( 1 + \mathcal{O}\left(\frac{1}{\sqrt{N_S}}\right) \right), \quad (74)$$

with  $N_S = N_S^+ + N_S^0$ . Actually, the error from the finite number of signals can be easily estimated by assuming a binomial distribution for  $N_S^+$  and  $N_S^0$  and finding the expected 68% band for the  $N_S^+/N_S^0$  ratio. Putting the pieces together, we arrive at

$$\frac{\Gamma^+}{\Gamma^0} = 0.65 \pm 0.10_{-0.5}^{+0.6} {}_{-0.6}^{+0.7} = 0.65_{-0.13}^{+0.14}, \quad (75)$$

where the first two uncertainties come from  $N_S^+$  and  $N_S^0$  and the last one comes from the finite size of  $N_S^+ + N_S^0$  and then we add them in quadrature. The LO ratio is

$$\frac{\Gamma_{\text{LO}}^+}{\Gamma_{\text{LO}}^0} = 0.66_{-0.03}^{+0.03} \pm 0.20, \quad (76)$$

which is compatible with the experimental yields.

At NLO, the  $\Gamma^+/\Gamma^0$  ratio depends on the isospin angle, where we find that reproducing the experimental ratio requires

$$\theta_I^{\text{NLO}} = (-42.4_{-6.1}^{+8.1} {}_{-1.6}^{+1.7} \pm 3.8)^\circ, \quad (77)$$

where the first uncertainty is experimental, the second is the couplings/ $a_0/\delta m_{\text{pole}}$ , and the third is the NLO uncertainty. This in turn implies that

$$\frac{\Gamma_{\text{pole}}}{\Gamma_{\text{NLO}}} = \cos^2 \theta_C \frac{A_S^2}{A_{S(C)}^2} = 0.81_{-0.21}^{+0.03} {}_{-0.06}^{+0.04} \pm 0.07, \quad (78)$$

where the errors are as before and  $A_{S(C)}$  refers to the normalization for a contact-range theory with the isospin angle  $\theta_I^{\text{NLO}}$ , i.e., Eq. (37) but using  $\theta_I^{\text{NLO}}$  instead of  $\theta_I^{\text{LO}} = -45^\circ$ . If we assume that  $A_S = A_{S(C)}$  and the discrepancy comes exclusively from the probability of the compact component, we will obtain

$$|\theta_C^{\text{NLO}}| = (25.7_{-2.3}^{+13.0} {}_{-2.3}^{+4.2} \pm 2.3)^\circ, \quad (79)$$

with the errors as before and which would imply a  $\theta_C$  distinguishable from zero at NLO. However, this is contingent on two factors: what would be  $A_S$  for the molecular component (if considered as a separate degree of freedom from the compact one) and the fact that  $\theta_C$  is not model independent in the sense that its effects can be recast into a negative effective range instead (i.e., the compact component can be reabsorbed as energy dependence in the molecular one). In the first case, assuming  $A_S^2/A_{S(C)}^2 = 1.1(1.2)$  will entail a compact mixing angle of  $30.7(34.6)^\circ$ , larger than the one we have calculated. In the second case, we set  $\theta_C = 0$  and recast the effects of  $\theta_C \neq 0$  into a negative effective range by means of the formula (check Appendix)

$$\frac{A_S^2}{A_{S(C)}^2} = \frac{1}{1 - \frac{1}{2} r_{e0} \left( \frac{1 - \sin 2\theta_I}{2} \right) A_{S(C)}^2}, \quad (80)$$

where  $r_{e0}$  is the  $I = 0$  effective range, leading to

$$r_{e0}^{\text{NLO}} = -1.3_{-2.3}^{+0.3} {}_{-0.6}^{+0.3} \pm 0.1 \text{ fm}, \quad (81)$$

which is, as expected, negative and within the confidence limits (CL) of Ref. [2] [i.e.,  $0 \geq r_{eL} \geq -11.9(-16.9)$  fm within 90(95)% CL, where  $r_{eL}$  refers to the effective range in the  $L$  channel; if we assume no interaction in the isovector channel, we will have  $r_{eL} = 2r_{e0} - 1/\sqrt{\gamma_H^2 - \gamma_L^2}$ , or  $r_{eL} = -6.4$  fm for  $r_{e0} = -1.3$  fm].

Alternatively, had we simply assumed  $\theta_I = -45^\circ$ , the  $\Gamma^+/\Gamma_0$  ratio would have been

$$\left. \frac{\Gamma_{\text{NLO}}^+}{\Gamma_{\text{NLO}}^0} \right|_{\theta_I = -\frac{\pi}{4}} = 0.70 \pm 0.03 \pm 0.06, \quad (82)$$

which is still compatible with the experimental yields. Meanwhile, the nonmolecular ratio would have been

$$\left. \frac{\Gamma_{\text{pole}}}{\Gamma_{\text{NLO}}} \right|_{\theta_I = -\frac{\pi}{4}} = \cos^2 \theta_C \frac{A_S^2}{A_{S(LO)}^2} = 0.83_{-0.21}^{+0.03} {}_{-0.06}^{+0.05} \pm 0.07, \quad (83)$$

which also happens to be different from 1, again. Assuming  $A_S = A_{S(LO)}$ , this ratio would in turn imply a compact mixing angle of

$$|\theta_C^{\text{NLO}}|_{\theta_I = -\frac{\pi}{4}} = (24.1_{-2.3}^{+13.5} {}_{-2.3}^{+4.2} \pm 2.3)^\circ \quad (84)$$

or, alternatively, assuming that the ratio comes exclusively from range corrections, would imply an isoscalar effective range of

$$r_{e0}^{\text{NLO}}|_{\theta_I = -\frac{\pi}{4}} = -1.1_{-2.2}^{+0.3} {}_{-0.5}^{+0.4} \pm 0.1 \text{ fm}, \quad (85)$$

which is negative. As can be appreciated, all the numbers obtained for  $\theta_I = -45^\circ$  are indistinguishable within errors to the ones we obtain from fixing  $\theta_I$  to the  $\Gamma_+/\Gamma_0$  ratio derived from the experimental yields.

## X. CONCLUSIONS

The  $T_{cc}^+$  represents not only a fascinating discovery but also a wonderful opportunity for the study of hadron spectroscopy and decays.

While we do not know for sure its nature yet—the mass of the  $T_{cc}^+$  is in principle compatible with previous predictions of  $I = 0, J = 1$   $cc\bar{u}\bar{d}$  compact tetraquarks and  $D^*D$  shallow bound states—its closeness to the  $D^{*+}D^0$  threshold indicates that at least part of its wave function will be  $D^*D$ . This last component is amenable to relatively straightforward theoretical treatments, including the calculation of its expected width, which would be a crucial piece of information if we want to eventually know the structure of the  $T_{cc}^+$ .

The physical scales involved in the molecular components of the  $T_{cc}^+$  indicate a moderate convergence rate for the calculation of its decay width and a NLO calculation is required to achieve an accuracy comparable with the 25% to 40% relative uncertainty in the experimental result (depending on the resonance profile used). Our preliminary calculation shows that the inclusion of a seagull pion decay operator and the rescattering of the final  $DD$  pair increase the total decay width of the  $T_{cc}^+$  state from 50 keV in LO to 58 keV in our abridged NLO calculation (i.e., a NLO calculation with a LO wave function). This is still preliminary; there are corrections coming from the asymptotic normalization  $A_S$  of a molecular  $T_{cc}^+$  state, the particular isospin mixing  $\theta_I$  between its  $D^{*+}D^0$  and  $D^{*0}D^+$  components and the final physical pion rescattering with the charmed mesons at non-zero energy (not to mention that the  $DD$  system might interact more strongly than we expect).  $A_S$  and  $\theta_I$  could be easily estimated from phenomenological models and fed into the NLO calculation, where we expect a moderate increase from the 58 keV figure we obtain.

If we compare the NLO results with the total decay width extracted from the unitarized Breit-Wigner profile ( $\Gamma_{\text{pole}} = 48_{-12}^{+2}$  keV), the previously discussed factors (particularly  $A_S$ ) point toward an excess decay width for a purely molecular explanation of the  $T_{cc}^+$  at NLO. This excess can be interpreted as the existence of a compact component, where the ratio of  $\Gamma_{\text{pole}}$  and  $\Gamma_{\text{NLO}}$  suggest that the nonmolecular probability of the  $T_{cc}^+$  wave function is about 20%. This conclusion should be taken as temporary, though, as future experimental refinements regarding the  $T_{cc}^+$  mass and decay widths might alter the present picture. In addition, the interplay of the compact and molecular components of the  $T_{cc}^+$  could also be improved. Nonetheless, we find it worth mentioning that the picture that emerges from the current EFT description together

with the experimental analysis of Ref. [2] is compatible with that of Ref. [8], which underlined the importance of including both mesonic and quark degrees of freedom for the binding and description of a tetraquark below the  $D^*D$  threshold.

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### APPENDIX: RANGE CORRECTIONS TO THE ASYMPTOTIC NORMALIZATION

Here, we calculate the range corrections to  $A_S$  for the  $T_{cc}^+$ . Instead of the usual method of extracting  $A_S$  from the residue of the scattering amplitude, we will consider how the effective normalization of the wave function changes when range corrections are included. We will begin with a single channel system and assume that range corrections are generated by an energy-dependent contact interaction of the type

$$V_C^{(R)} = D k^2 \delta^{(3)}(\vec{r}), \quad (\text{A1})$$

where  $D$  is a coupling and  $k$  refers to the center-of-mass momentum of the two-body system, with  $k^2 = 2\mu E_{cm}$  and  $E_{cm}$  the center-of-mass energy. We regularize this potential with a delta-shell regulator of the type

$$V_C^{(R)} = \frac{D(R_c)}{4\pi R_c^2} k^2 \delta(r - R_c), \quad (\text{A2})$$

where  $D$  is a coupling and  $R_c$  is a cutoff. Fixing  $D(R_c)$  to the effective range  $r_e$  in two-body scattering gives [60]

$$D(R_c) = \frac{2\pi r_e}{\mu} R_c^2 + \mathcal{O}(R_c^3). \quad (\text{A3})$$

Energy-dependent potentials change the asymptotic normalization  $A_S$  in a way that is compatible with the following modified normalization condition [61],

$$1 = \int_0^\infty dr u^2(r) \left[ 1 - 2\mu \frac{d^2}{dk^2} V(r) \right], \quad (\text{A4})$$

with  $u(r)$  the reduced wave function of a two-body bound state. For a contact-range theory, we have  $u(r) = A_S e^{-\gamma r}$ , and after a few manipulations, we arrive at

$$\frac{A_{S(C)}^2}{A_S^2} = 1 - \frac{r_e}{2} A_{S(C)}^2, \quad (\text{A5})$$

for  $R_c \rightarrow 0$ , where  $A_{S(C)} = \sqrt{2\gamma}$  and  $A_S$  are the asymptotic normalizations in the absence and presence of range corrections. For a single channel problem, this is equivalent to the well-known result [54]

$$A_S = \sqrt{\frac{A_{S(C)}}{1 - \gamma r_e}} = \sqrt{\frac{2\gamma}{1 - \gamma r_e}}. \quad (\text{A6})$$

The advantage of the energy-dependent potential is that we can extend the previous result to the two-channel isospin-breaking case directly. In the  $\{|L\rangle, |H\rangle\}$  basis, the isospin effects can be included by considering that the potential (or for simplicity the coupling  $D$ ) is a matrix in said basis,

$$D \rightarrow \begin{pmatrix} \frac{1}{2}D_0 + \frac{1}{2}D_1 & -\frac{1}{2}D_0 + \frac{1}{2}D_1 \\ -\frac{1}{2}D_0 + \frac{1}{2}D_1 & \frac{1}{2}D_0 + \frac{1}{2}D_1 \end{pmatrix}, \quad (\text{A7})$$

where  $D_I$  is the coupling generating the  $I = 0, 1$  effective range. If we extend the modified normalization condition of Eq. (A4) to the two-channel case with the  $u_L(r) = A_S \cos \theta_I e^{-\gamma_L r}$  and  $u_H(r) = A_S \sin \theta_I e^{-\gamma_H r}$  wave functions, we arrive at

$$\frac{A_{S(C)}^2}{A_S^2} = 1 - \left[ \frac{r_{e0}(1 - \sin 2\theta_I)}{2} + \frac{r_{e1}(1 + \sin 2\theta_I)}{2} \right] A_{S(C)}^2, \quad (\text{A8})$$

where  $r_{eI}$  refers to the effective range in the isospin channel  $I = 0, 1$  and  $A_{S(C)}$  is given by Eq. (37). For a molecular  $T_{cc}^+$  that happens to be a pure  $I = 0, 1$  state at short distances (i.e.,  $\theta_I = \mp 45^\circ$ ), the range corrections will simplify to

$$\frac{A_{S(C)}^2}{A_S^2} = 1 - \frac{r_{eI}}{2} A_{S(C)}^2, \quad (\text{A9})$$

i.e., identical to the single channel case. Finally, it is interesting to connect the previous result with the following often-used definition of compositeness (check, e.g., Ref. [62]),

$$X_{\text{comp}} = - \sum_A |g_A|^2 \frac{dG_{0A}(E)}{dE} \Big|_{E=E_B}, \quad (\text{A10})$$

where  $A$  refers to the different two-body channels and  $g_A^2$  refers to the residue of the T-matrix at the bound state pole in the diagonal channels ( $g_A^2 = \lim_{E \rightarrow E_B} (E - E_B) T_{AA}(E)$ , with  $E$  the center-of-mass energy,  $E_B$  the binding energy, and  $T_{AA}$  the T-matrix in the diagonal channel  $AA$ , where the T-matrix is defined via  $T_{AB} = V_{AB} + \sum_C V_{AC} G_{0C}(E) T_{CB}$ ) and  $G_{0A} = 1/(E - H_{0A})$  is the resolvent operator for channel  $A$  (with  $H_{0A}$  the free Hamiltonian for that channel). From the previous definition, we obtain

$$X_{\text{comp}} = \frac{A_S^2}{A_{S(C)}^2}, \quad (\text{A11})$$

for the  $T_{cc}^+$ , which for a negative effective range gives  $X_{\text{comp}} < 1$ . For a positive effective range, the previous

result will probably have to be modified in the line of what is proposed in Ref. [63] for single channel scattering. Be that as it may, our calculations already suggest a negative effective range.

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