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THERE ARE NO ITERATED MORPHISMS THAT DEFINE THE ARSHON SEQUENCE AND THE σ-SEQUENCE

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ABSTRACT

In [3], Berstel proved that the Arshon sequence cannot be obtained by iteration of a morphism. An alternative proof of this fact is given here. The σ-sequence was constructed by Evdokimov in order to construct chains of maximal length in the n-dimensional unit cube. It turns out that the σ-sequence has a close connection to the Dragon curve [10]. We prove that the σ-sequence cannot be defined by iteration of a morphism.

Keywords: Symbolic sequence, iterated morphism, Arshon sequence

1. Introduction and Background

In 1937, Arshon gave a construction of a sequence of symbols w over the alphabet $\{1,2,3\}$, constructed as follows: Let $w_1 = 1$. For $k \geq 1$, w_{k+1} is obtained from w_k by replacing the letters of w_k in odd positions as follows:

$$1 \to 123,\ 2 \to 231,\ 3 \to 312$$

and in even positions as follows:

$$1 \to 321,\ 2 \to 132,\ 3 \to 213.$$

Then

$$w_2 = 123, \quad w_3 = 123132312,$$

and each w_i is a prefix of w_{i+1}. Thus the infinite symbolic sequence $w = \lim_{n\to\infty} w_n$ is well defined. It is called the *Arshon sequence.*

This method of constructing w is called the *Arshon Method* (**AM**), and ψ will denote the indicated map of the letters 1, 2, 3 according to position as described above.

We will denote the natural decomposition of w in 3-blocks by lower braces:

$$w = \underbrace{123}\underbrace{132}\underbrace{312}\ldots$$