Astroparticle physics



CERN Summer Student Lecture Programme 2010

What is particle astrophysics?

What is physics?

What is physics?

Oxford dictionary :

Physics, *plural noun* [treated as singular] : the branch of science concerned with the nature and properties of matter and energy.

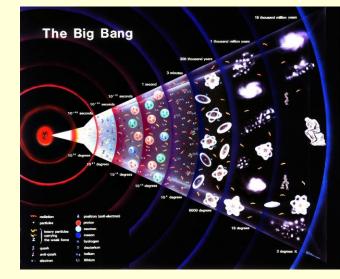
What is particle astrophysics?

Particle astrophysics :

the branch of science concerned with the nature and fundamental properties of matter and energy in the Universe. Many different aspects:

• The early Universe a a particle physics laboratory

In the big bang theory, the early Universe is hot and dense



• The matter and energy content of the present Universe: dark and luminous matter, neutrinos, radiation, dark energy...

• Study of violent phenomena in the Universe : particles ejected provide a complementary signal to visible (or electromagnetic) observations

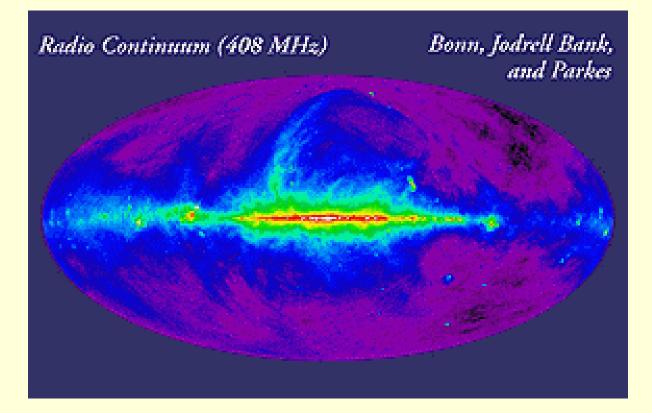
Astrophysics at the end of XXst century :



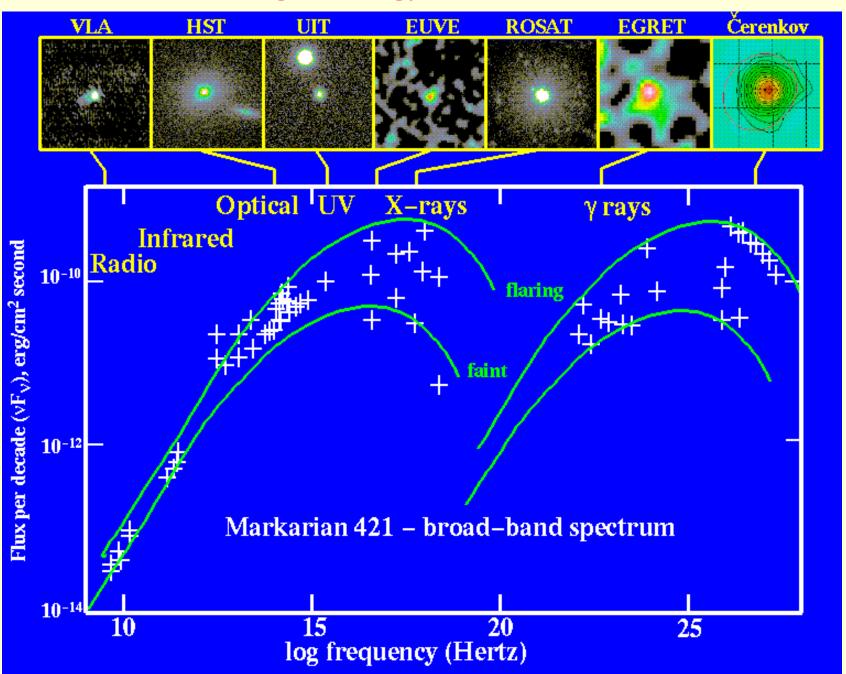




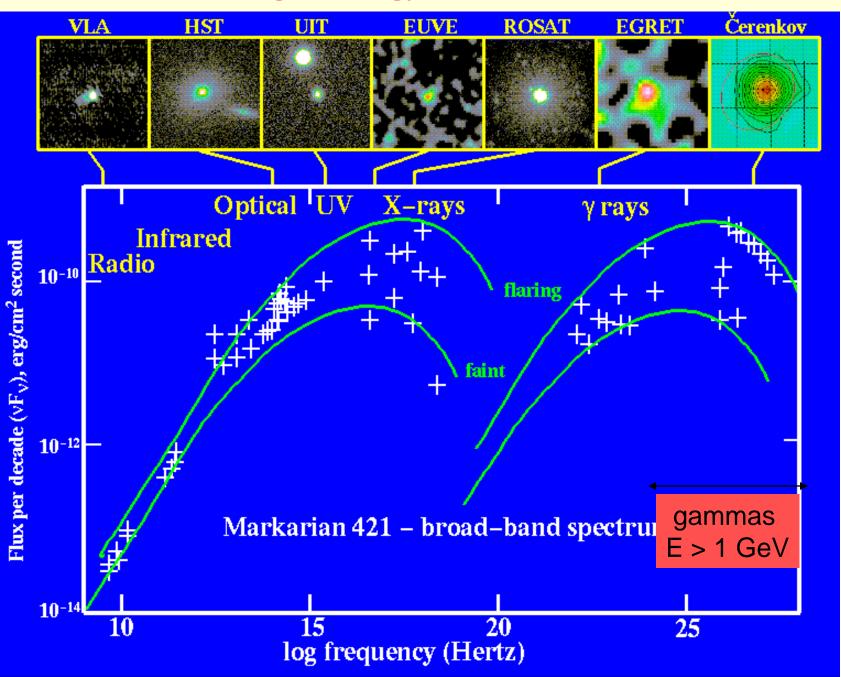
Astrophysics: a multi-wavelength strategy



A multi-wavelength strategy : the Ma 421 source



A multi-wavelength strategy : the Ma 421 source



A multi-messenger strategy

Ideally, one would like to study the same source by detecting the photons, protons, neutrinos and gravitational waves emitted :

- high energy photons trace populations of accelerated particules, as well as dark matter annihilation
- protons provide information on the cosmic accelerators that have produced them
- neutrinos give information on the deepest zones, opaque to photons (e.g. on the origin --hadronic or electromagnetic-- of γ).
 - gravitational waves give information on the bulk motion of matter in energetic processes

Chapter I The tools of the trade

Chapter II The violent universe

Chapter III The Universe at large

Astroparticle physics

I - The tools of the trade



CERN Summer Student Lecture Programme 2010

Outline

- 1. An extraterrestrial radiation
- 2. The light of particles
- 3. Detector Earth
- 4. Particle detectors in space
- 5. Underground: neutrinos and dark matter
- 6. Ripples of spacetime

1. An extraterrestrial radiation

QuickTime™ et un dé compresseur sont requis pour visionner cette im age.

1785 Coulomb

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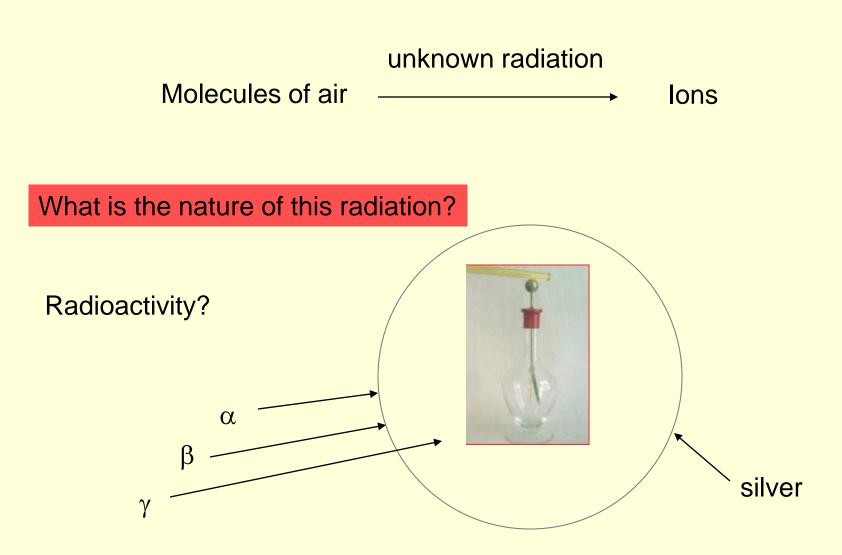
1785 Coulomb

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The electrometer discharges with time. Why?

Faraday : the air is a conductor because of ionisation



The cosmic adventure begins

In 1909, Theodore Wurtz who had developed an ultrasensitive electroscope sets it up at the top of the Eiffel Tower...

1

R ns les laboratoiz s el s phy iciens au ll u du sè le, les E ectroscopes se el hargeaient el manè ncompé ensible endant a uit...

Si nos appareils se déchargent pendant la nuit, c'est sans doute à cause d'un rayonnement venant du sol : en montant, par exemple en haut de la Tour Eiffel, le phénomène devrait alors s'atténuer...

A • , un ğ uite allemand, le P WUE (6 •), quiavait elop un Ecctroscope ultra stable, li nstalla en haut d la Tour Eiffel. Il constata alos que la diminution du taux d d harge d son Ecctroscope é ait moinde que pé u si tout l'effet ionisant é ait du à un re onneen nt uniquen nd 'originé ere ste.

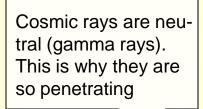
> Nous allons bien voir ce qui se passe en montant beaucoup plus haut !

10 00 00 00 00 00 a

In 1912, Victor Hess climbs to an altitude of 4200 m to prove that the ionising radiation decreases with altitude: it has a cosmic origin

Crédits Images : NASA, ESA, ESO, CNES, CEA, CNRS, OBSPM, APC, Droits réservés.

Millikan calls this radiation cosmic rays



QuickTime™ etun décompresseur QuickTime™ et un sontreguis pour visionner cete image. sont reguis pour visionner cete image. Cosmic rays are charged. This why they are so energetic. They can be accelerated by cosmic magnetic fields

1868-1953

Millikan - Compton controversy

Millikan calls this radiation cosmic rays



Cosmic rays are charged. This why they are so energetic. They can be accelerated by cosmic magnetic fields

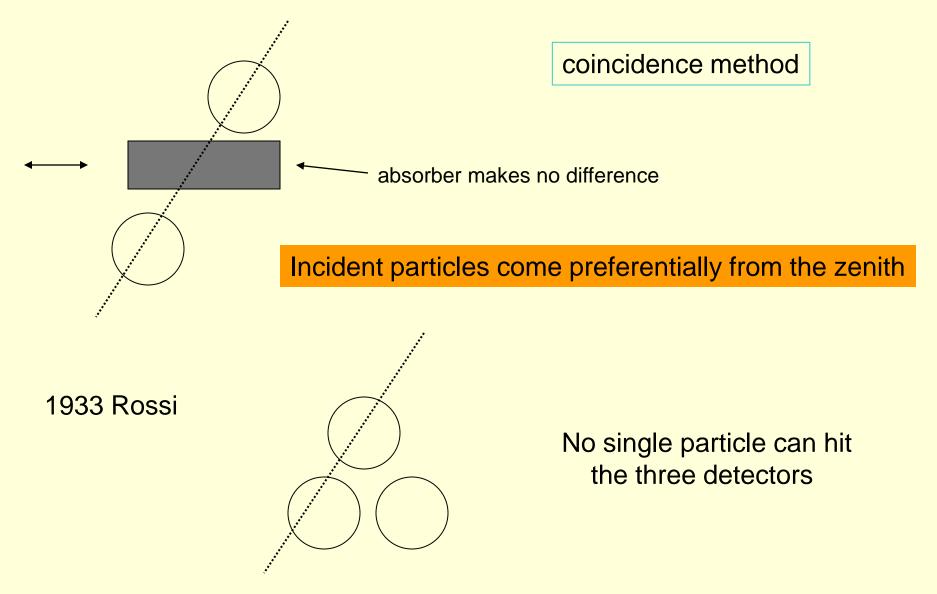
1868-1953

Millikan - Compton controversy

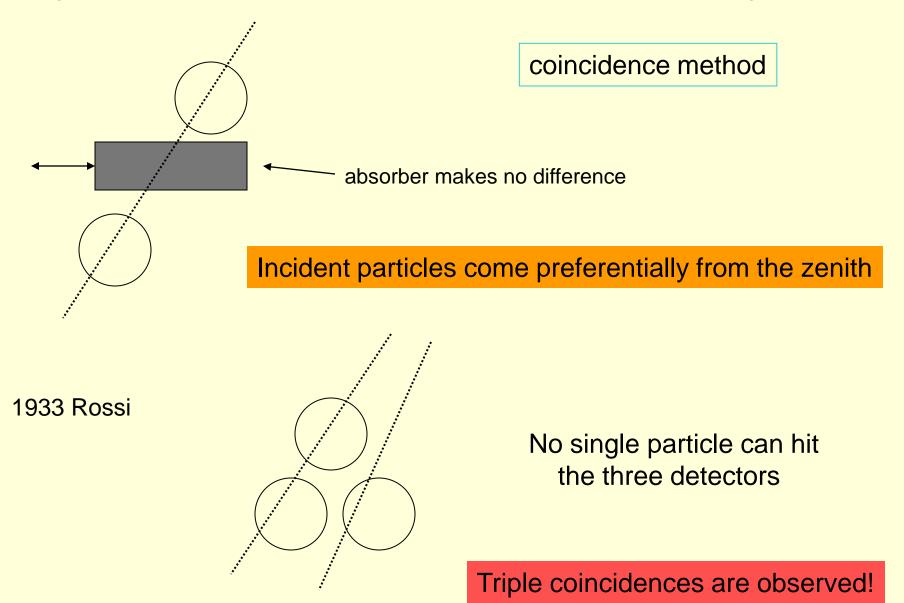
Solved by Jacob Clay on a trip from Genova to Java

Cosmic rays are charged!

QuickTime™ et un décompresseur sont requis pour visionner cette image. Geiger counter developped in 1912-1928 proved to be very useful for studying cosmic rays



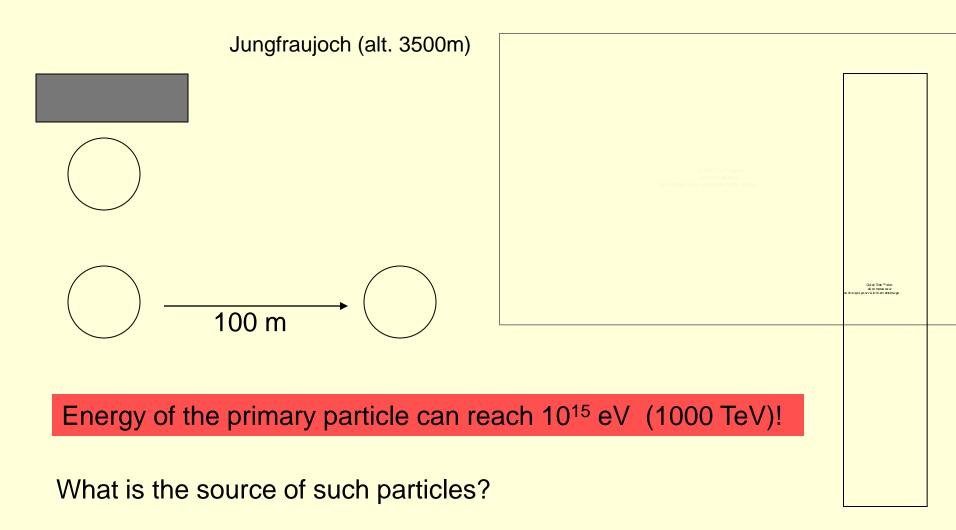
Geiger counter developped in 1912-1928 proved to be very useful for studying cosmic rays



Several particles arrive in coincidence: cosmic ray showers

QuickTime™ et un décompresseur sont requis pour visionner cette image.

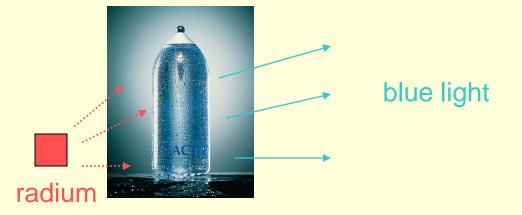
In lates '30s, Pierre Auger and collaborators study these showers



2. The light of particles



Early '30s, young Ph.D. student Pavel Cherenkov is asked to study this blue light



Is this fluorescence?

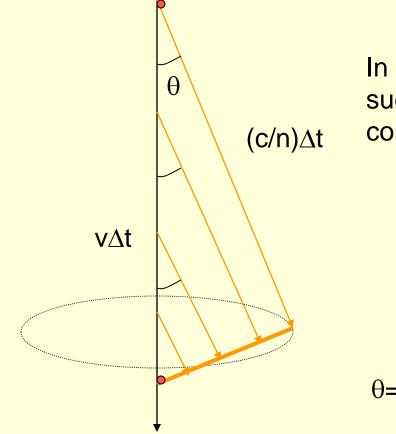
Fluorescence is associated with the emission of a photon by an excited molecule returning to its fundamental state.

Cherenkov: no, because the phenomenon does not depend on the type of liquid

1937, Frank and Tamm: a (light) shock wave due to the fact that the energetic particles travel at a velocity larger than the speed of light.

In a medium of index n, light travels at a speed c/n

Hence, particles can travel at velocity v > c/n

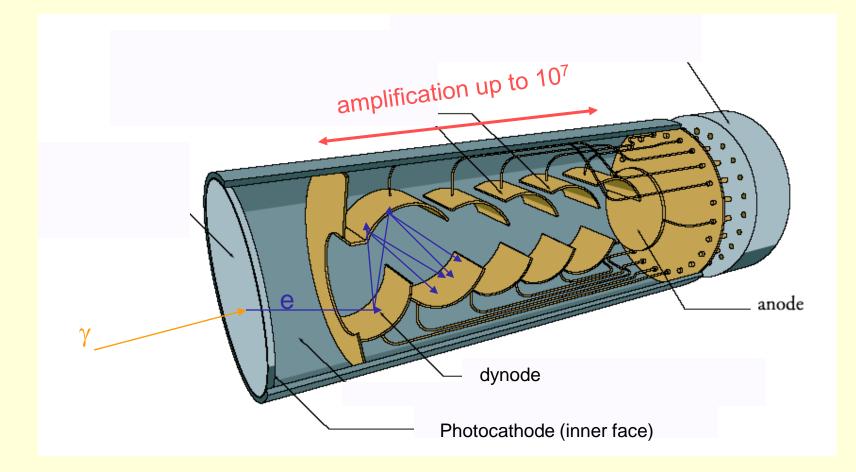


In direction θ , all light waves emitted successively by the particle interfere constructively :

$$\cos \theta = \frac{(c/n)}{v}$$

 θ = 1° in air 42° in water

How to collect this light? Photomultipliers



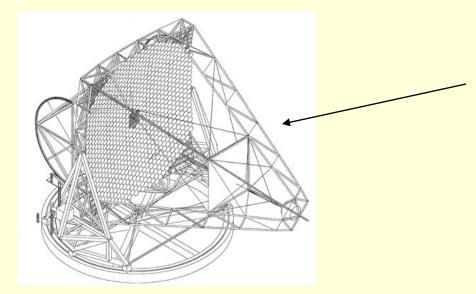


Examples of photomultipliers

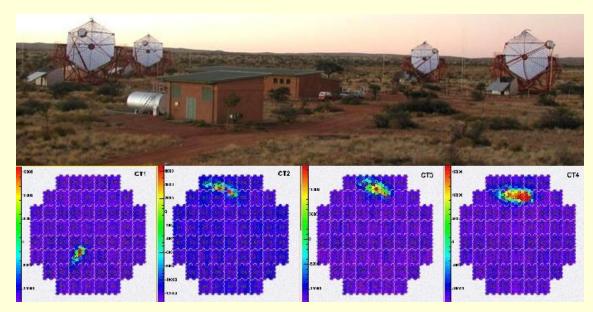
Detecting high energy gammas through their Cherenkov light: the Cherenkov telescopes of the HESS experiment in Namibia



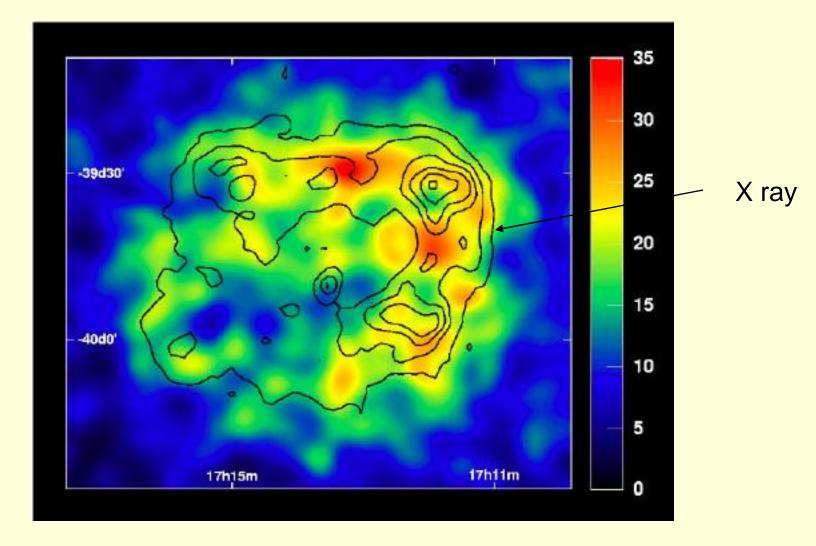
Detection of gamma rays in the 100 GeV to 10 TeV range



one HESS telescope



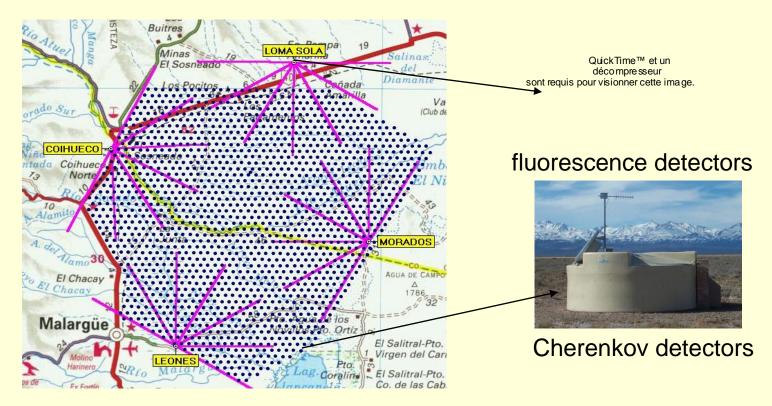
stereoscopic view



Detection of the shell structure of a supernova remnant

The Pierre Auger observatory

To study the cosmic ray showers of the highest energy, one needs to instrument a very large area

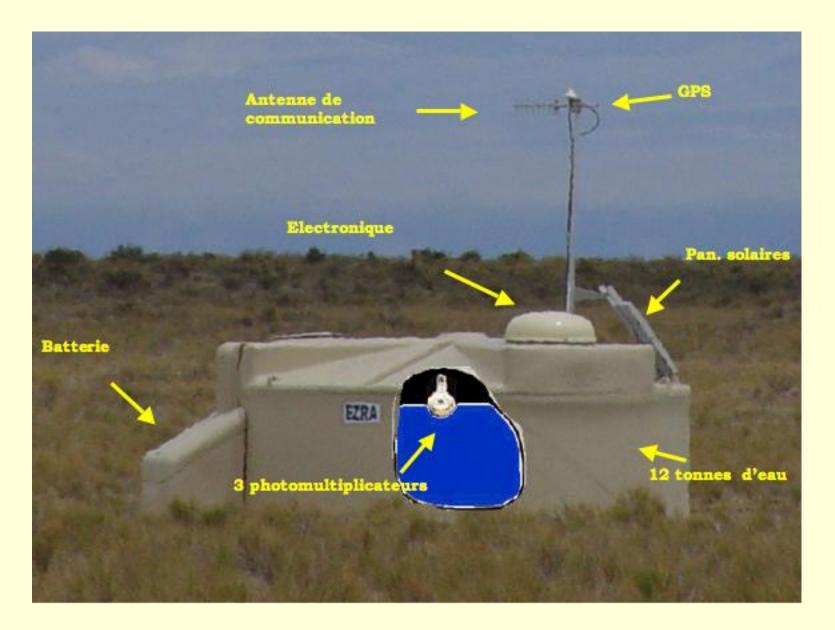


In Argentina, the Pierre Auger Observatory uses an array of 1600 detectors to cover an area of 3000 km²

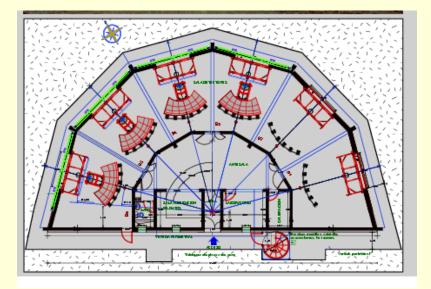
QuickTime™ et un décompresseur sont requis pour visionner cette image.

tanks filled with water fpr Cherenkov light detection telescopes for detecting fluorescence light produced by shower

One tank of Auger (Cherenkov light)

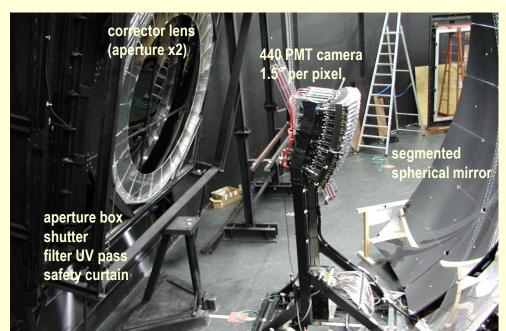


Fluorescence detectors

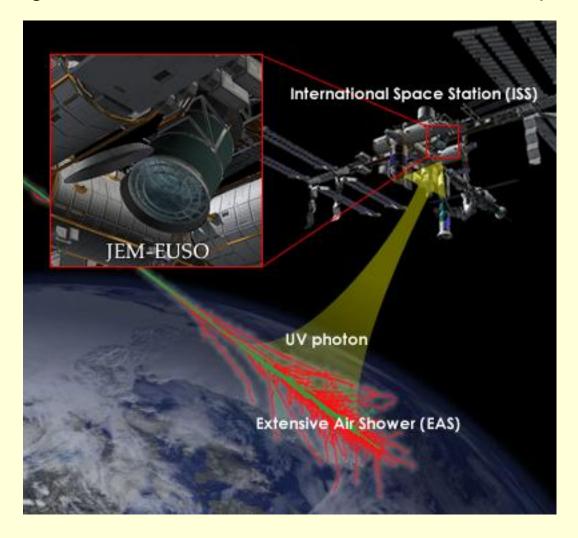


optical system

Internal lay out

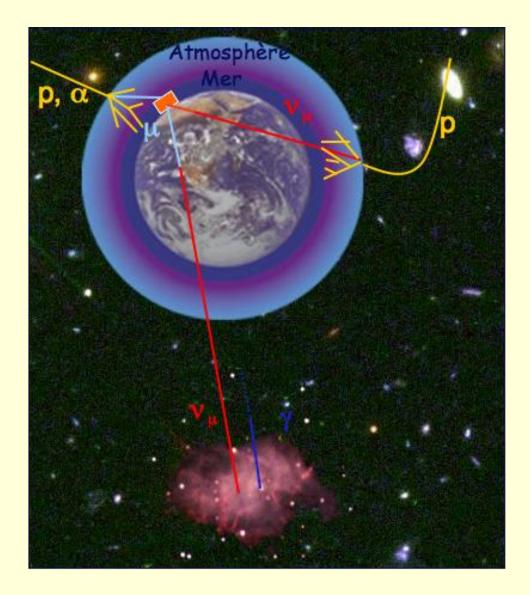


Using fluorescence to observe these showers from space: JEM-EUSO on the ISS



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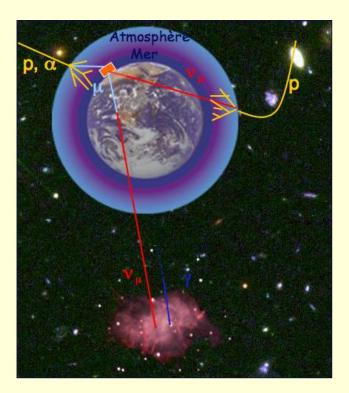
3. Detector Earth



Neutrinos are very difficult to detect because of their small cross section

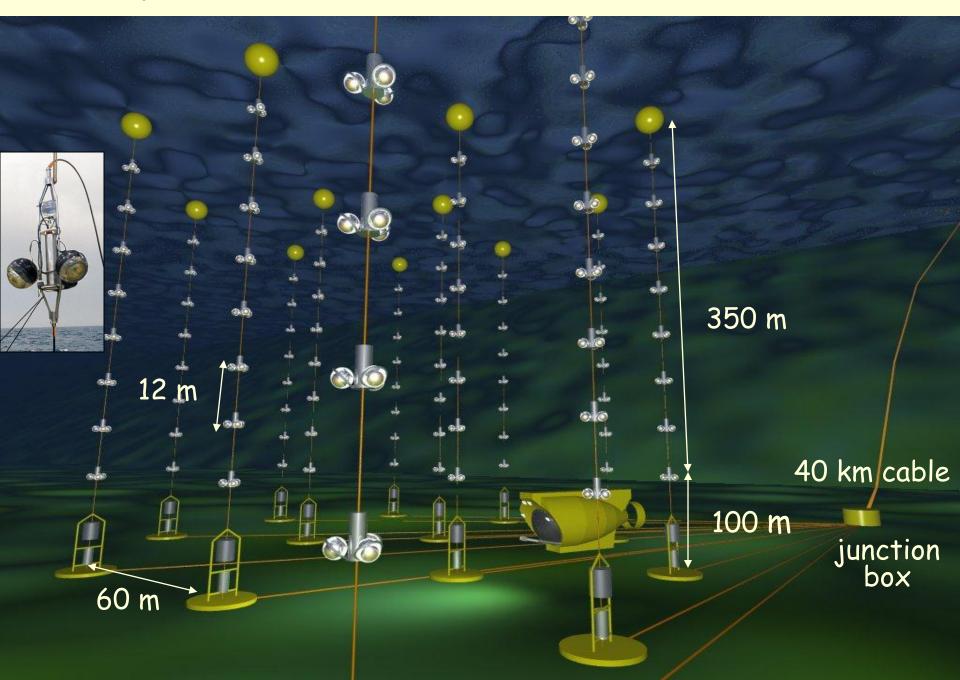
e.g. neutrinos produced in the Sun reach the Earth with a flux of 60 billion/cm², only 1 in 10¹² are stopped

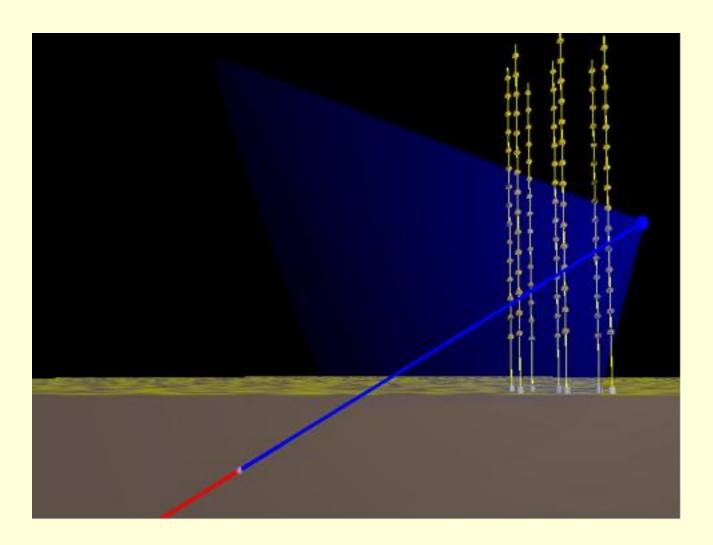
Positive side: cosmic neutrinos arrive on Earth unperturbed they trace the sources



To track the elusive neutrinos, one uses the Earth as detector.

Example of ANTARES in the Mediterranean

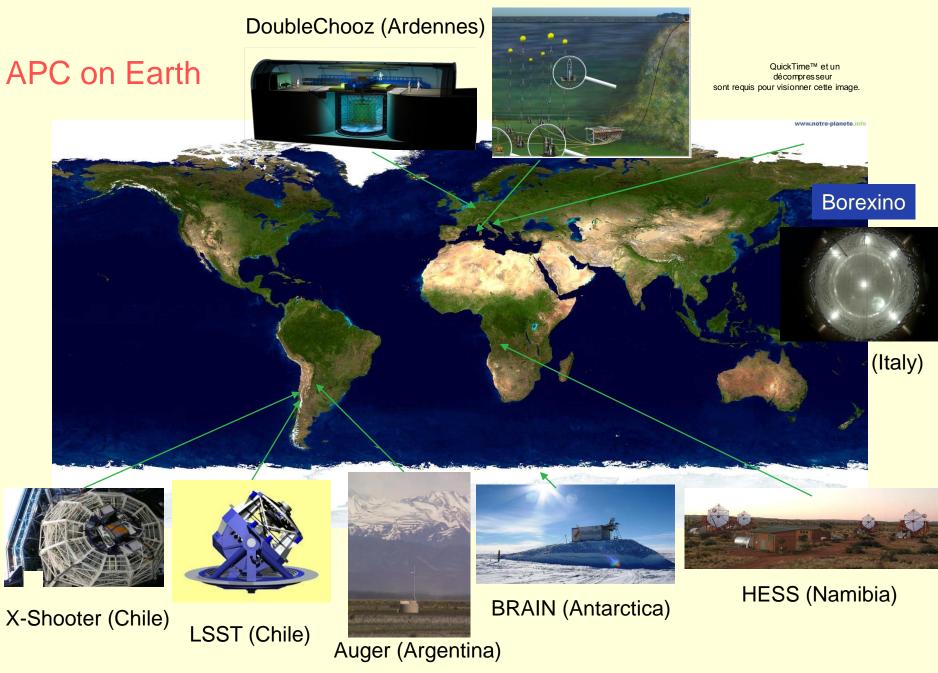




QuickTime™ et un décompresseur sont requis pour visionner cette image.

QuickTime™ et un décompresseur codec YUV420 sont requis pour visionner cette image.

ANTARES (Mediterranean) VIRGO (Italy)



Outline

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4. Particle detectors in space

Why go into space?

Catch the primary cosmic particles before they hit the atmosphere and form cosmic ray showers.

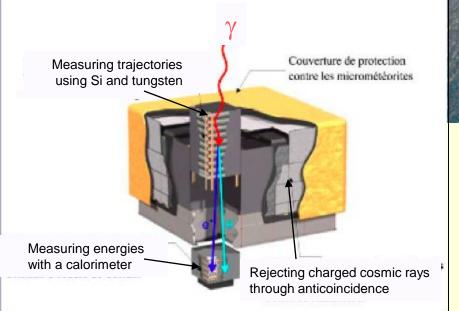


Launch in 2008

γ from 10 keV to 300 GeV

GAMMA-RAY LARGE AREA SPACE TELESCOPE

Large Area Telescope (20 to 300 MeV)



Exploded View: One of Forty-nine Towers

10 Layers of 0.5 rad Length Converter (pb)
 12 Layers of XY Silicon Strips
 Gamma Rays
 Positrons/Electrons

Detecting the primary particles responsible for cosmic rays: AMS on the ISS

(launch planned in 2010)

permanent magnet (charged particles) QuickTime[™] et un décompresseur

sont requis pour visionner cette image.

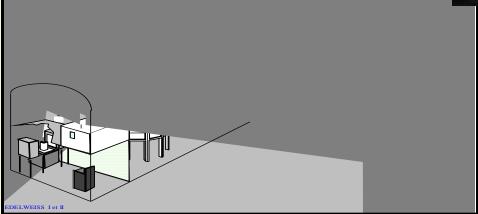
launched in 2006

QuickTime™ et un décompresseur sont requis pour visionner cette image.

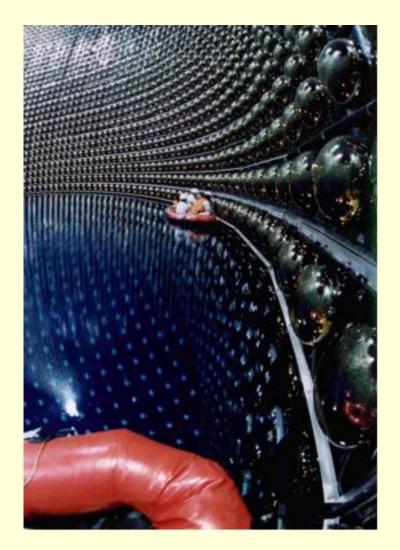


The detection of elusive particles such as neutrinos or dark matter particles Require very low background environments \rightarrow underground laboratories





Neutrino (solar and atmospheric) detectors



SuperKamiokande (in Japan) during the filling of the tank

Edelweiss-II experiment to detect dark matter

QuickTime[™] et un décompresseur sont requis pour visionner cette image. Astroparticle physics :

II - The violent Universe



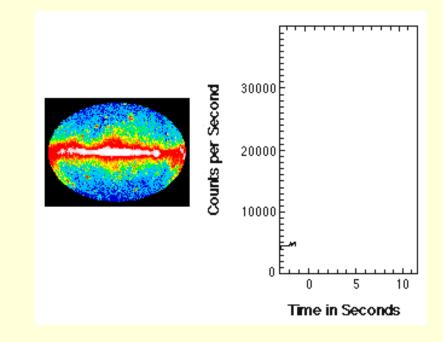
CERN Summer Student Lecture Programme 2010

<u>Outline</u>

- The example of gamma ray bursts 1.
- 2. The end of a star
- The story of black holes _____ Ripples in spacetime: gravitational waves 3.
- 4. Supernovae explosions
- 5. Cosmic rays and cosmic accelerators

1. Some very energetic events in the Universe: the example of Gamma Ray Bursts (GRB)





Vela, US military satellite looking for gamma emission from Soviet nuclear explosions

Some orders of magnitude

Energy released by the GRB : approximately 10^{44} to 10^{47} J i.e. $M_{\odot}c^2$

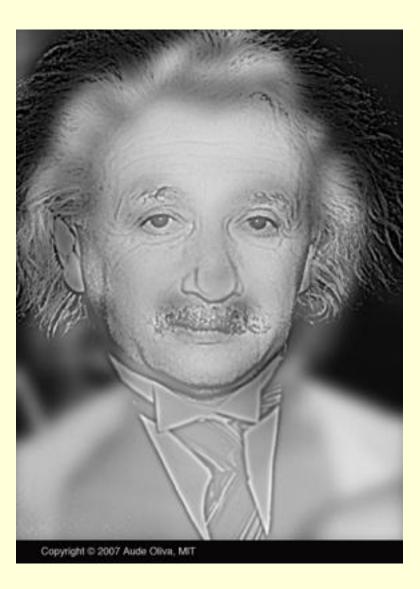
Distance that light travels in 5 seconds: 1 500 000 km i.e. 0.01 au

Hence the energy released occupies a very small volume on the scale of the Universe

 \rightarrow compact objects

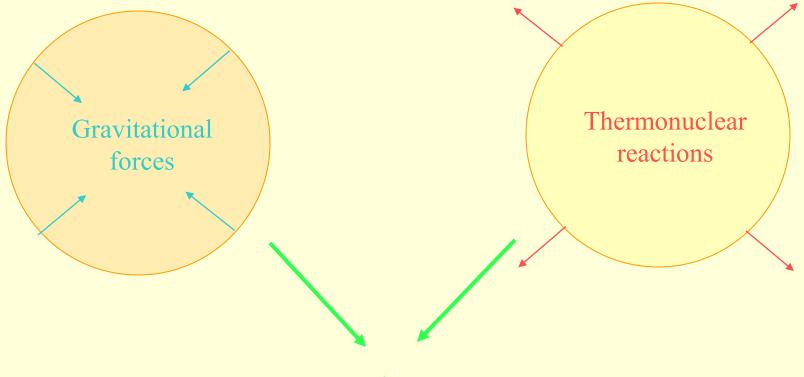
e.g. black holes, neutron stars, white dwarfs

2. The end of a star





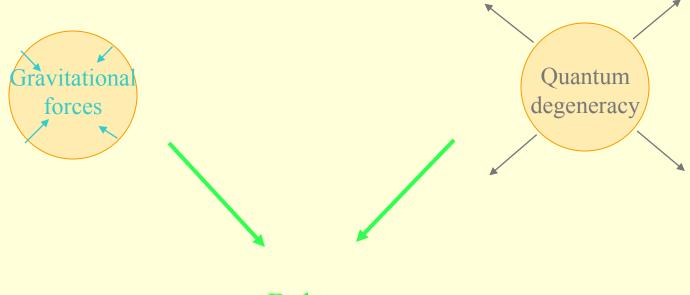
Some notions about star evolution (such as our Sun)



Balance

But when the nuclear fuel is exhausted, there is collapse under the effect of gravity \rightarrow FORMATION OF A COMPACT OBJECT

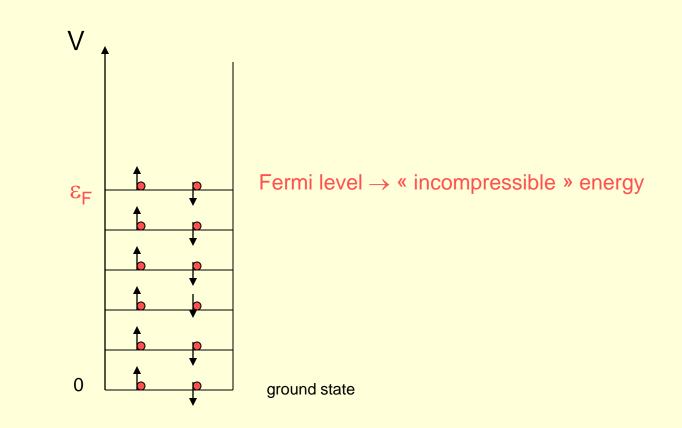
But when nuclear fuel becomes exhausted, quantum degeneracy comes to the rescue...



Balance

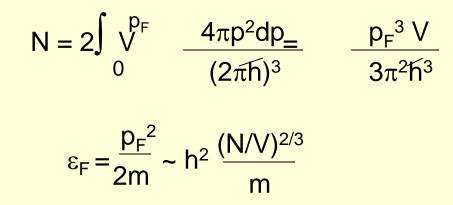
What is quantum degeneracy pressure?

Pauli principle: two fermions cannot be in the same state



A technical transparency





Hence the Fermi energy is larger for electrons than for neutrons.

When the nuclear fuel is exhausted, gravitational collape is first stopped by the quantum degeneracy of electrons :

Chandrasekhar limit

$$M_{WD} < 6 M_{\odot}/v^2$$

number of nucleons per electron, typ. 2

If density becomes larger, then

WHITE DWARFS

 $e + p \rightarrow n + v$ and gravitational collapse is stopped by the quantum degeneracy of neutrons

Oppenheimer-Volkoff bound

 $M_{\rm NS}$ < 0.7 M_{\odot}

If the mass is larger, then the gravitational collapse leads to

BLACK HOLES

NEUTRON STARS

3. The story of black holes



×

One month after the publication of Einstein's theory, Schwarzschild found an isotropic solution of Einstein's equations

$$ds^{2} = (1 - \frac{2G_{N}M}{r}) dt^{2} - (1 - \frac{2G_{N}M}{r}) dr^{2} - r^{2}(d\theta^{2} + \sin^{2}\theta d\phi^{2})$$

It describes the exterior of a static star of mass M and radius R if

$$R > 2G_N M/c^2 \equiv R_S$$
 Schwarzschild radius

For the Sun, $R_s = 2.9$ km

If $R < R_S$, the star undergoes gravitational collapse: it falls in a finite time Into a state of infinite energy density.

Oppenheimer and Snyder, 1939

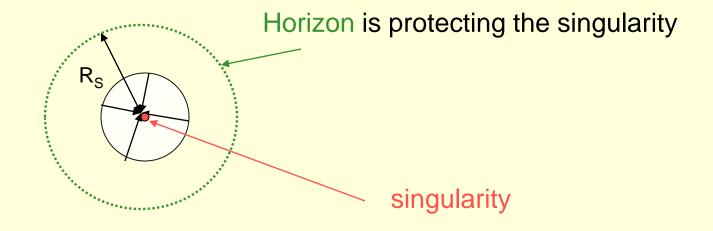
What is then the meaning of the Schwarschild radius?

Mitchell (1784) Laplace (1795)

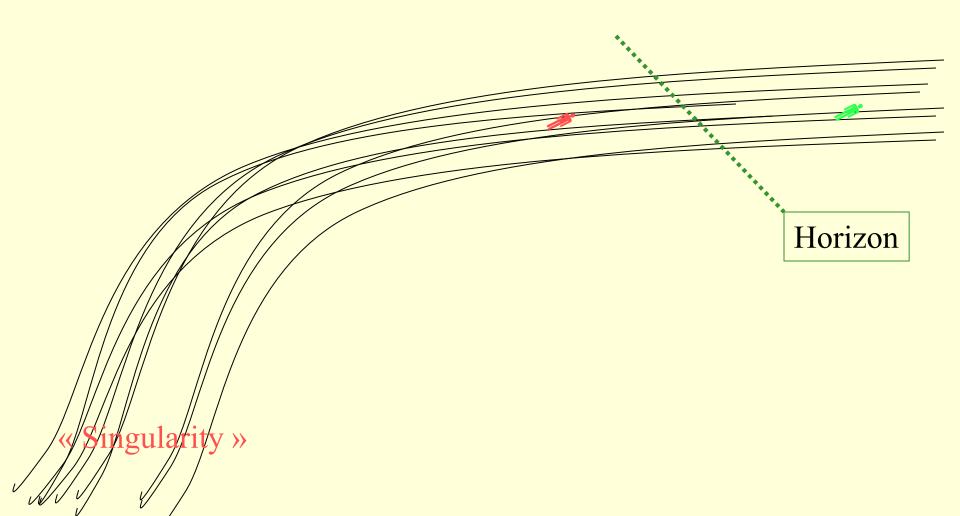
Classical condition for a body of mass m and velocity v to escape from a spherical star of mass M and radius R :

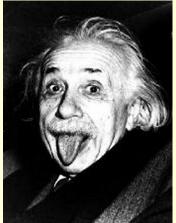
$$\frac{1}{2} \text{mv}^2 > \frac{\text{G}_{\text{N}}\text{Mm}}{\text{R}}$$

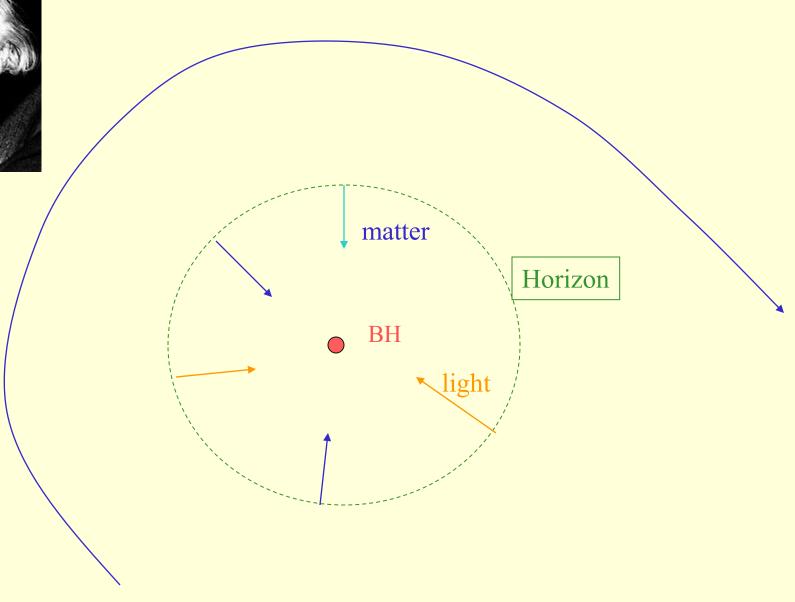
Hence even light (v=c) cannot escape if $R < 2 G_N M/c^2 = R_S$



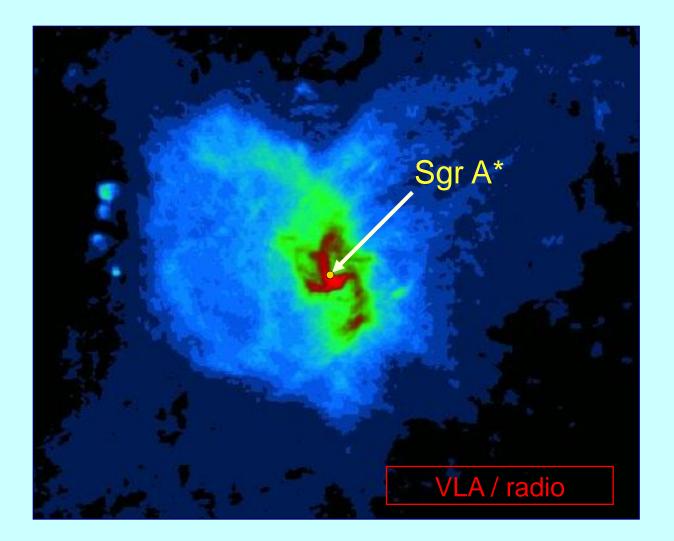
A comparison to understand the notion of (Schwarschild) horizon : the waterfall.







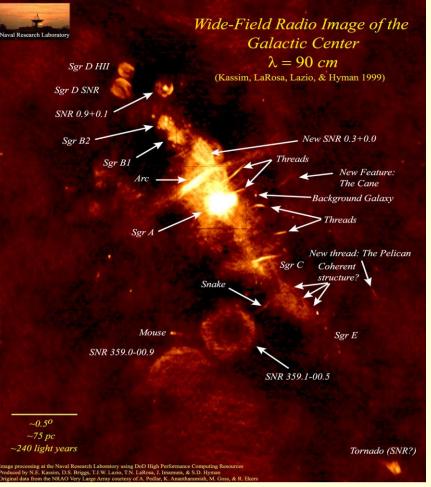
From « black holes » to black holes...



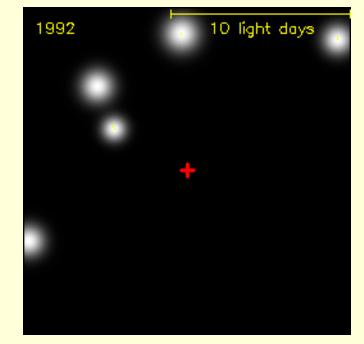
At the centre of our own galaxy, a source emits very energetic particles

Let us come closer!

Radio image (90 cm)

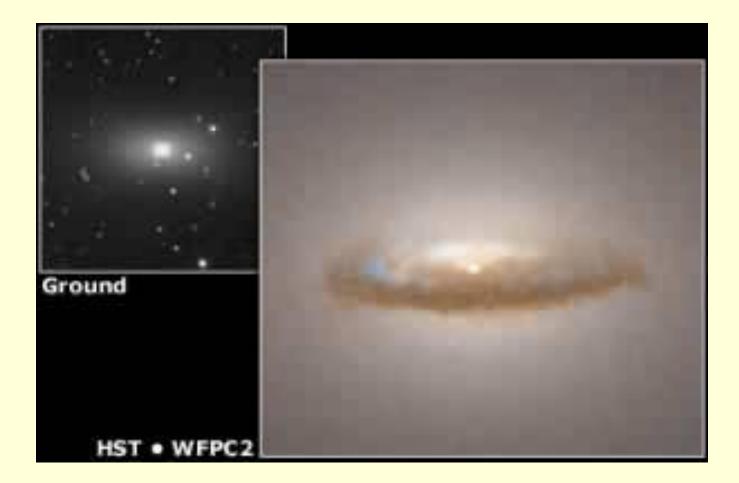


Infrared (1.6 μ m < λ <3.5 μ m) NAOS/CONICA

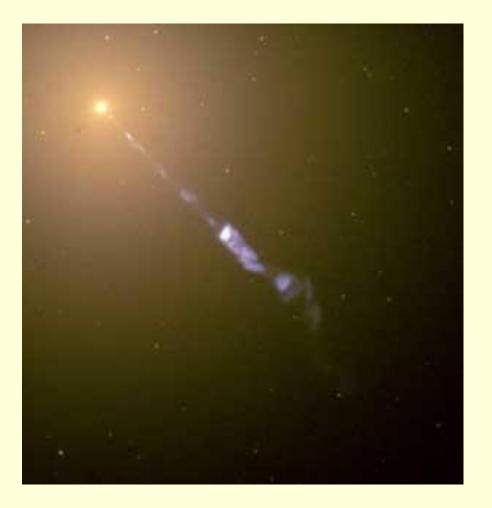


Black hole of mass of the order of 3 million solar masses Why is the central black hole associated with the emission of energetic particles?

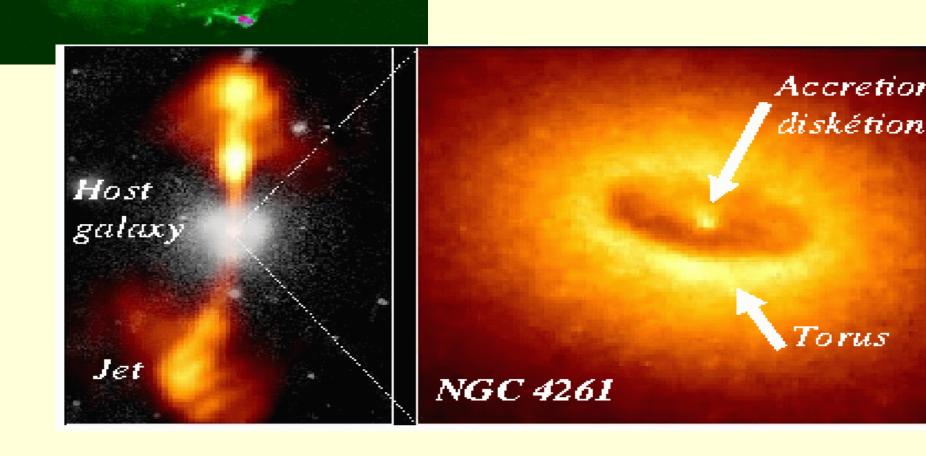
Because matter falling into the black hole is undergoing a very intense activity.



Torus of dust surrounding a black hole



A jet of particles associated with a black hole of M87 galaxy

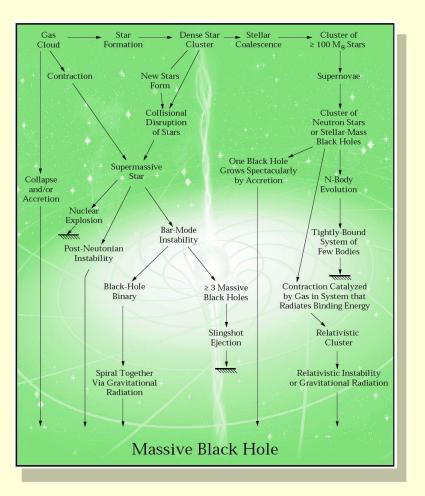


- Contraction

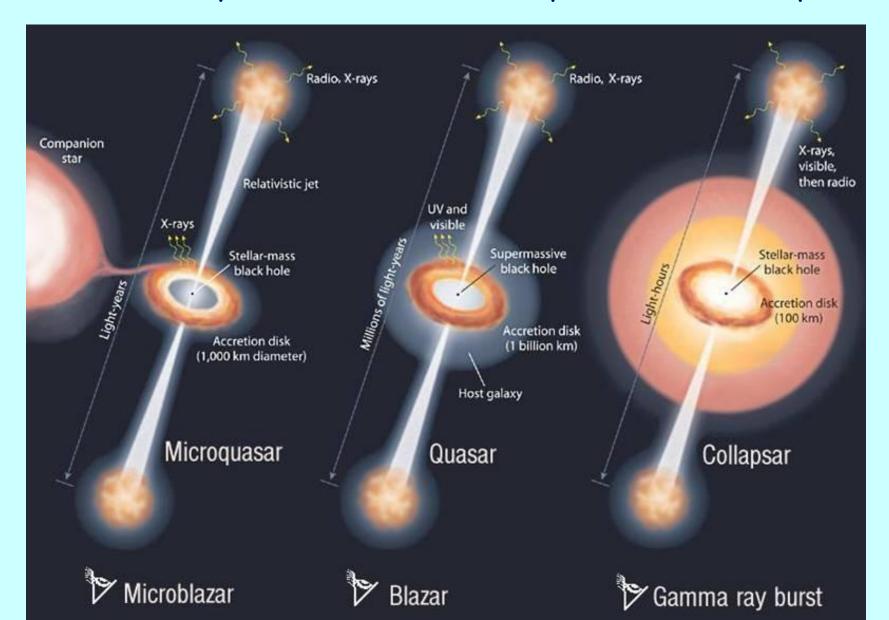
QuickTime™ et un décompresseur codec YUV420 sont requis pour visionner cette image. One finds such supermassive black holes at the centre of most, if not all galaxies.

How did they form?

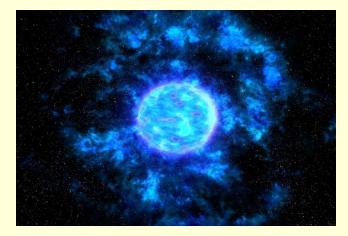
Strongly connected with galaxy formation.

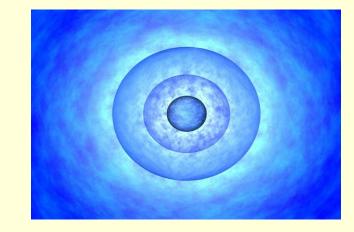


One finds black holes as the building blocks of many astronomical systems where violent phenomena take place

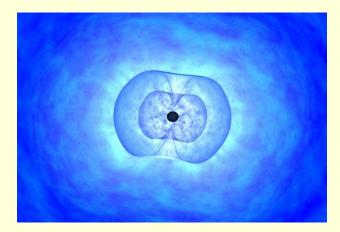


A model for (long) gamma ray bursts





A massive star ends its existence with an explosion Its inner core collapses into a black hole





Collapse is not uniform. There is creation of a jet of particles This jet interacts with the outer layers of the star, which accelerates the particles.

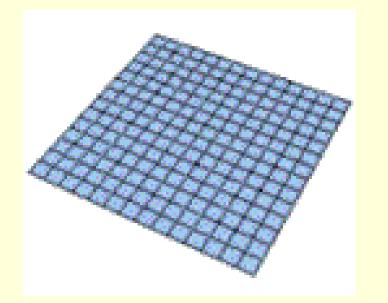


6. Ripples of space-time



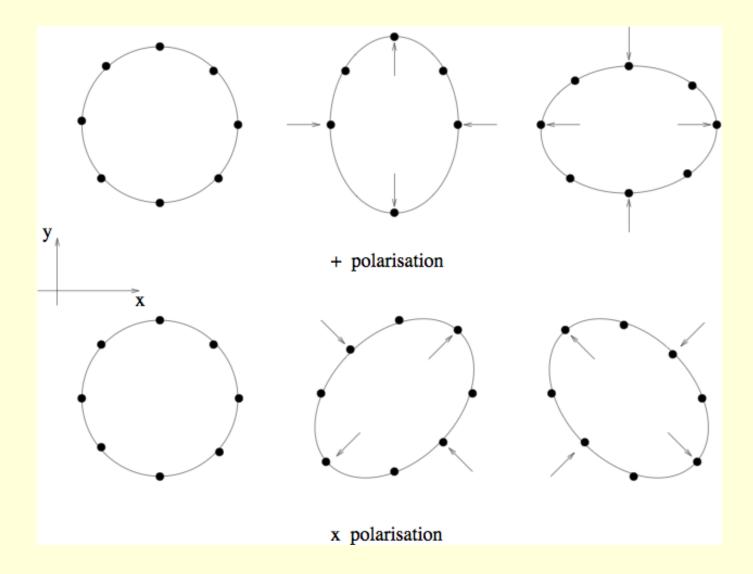
Space-time (4D) is « elastic ».

Any mass or localized form of energy perturbs it and curves it. Just as when you drop a stone in a pond ...



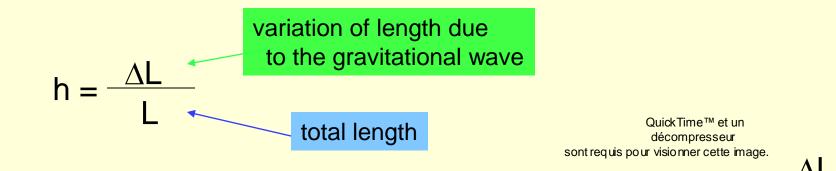


... violent phenomena (sudden motion of bulk of matter) may lead to waves of deformaton of spacetime that will propagate in the Universe.



Two types of polarisation for gravitational waves

One introduces the amplitude of the gravitational wave:



Examples:

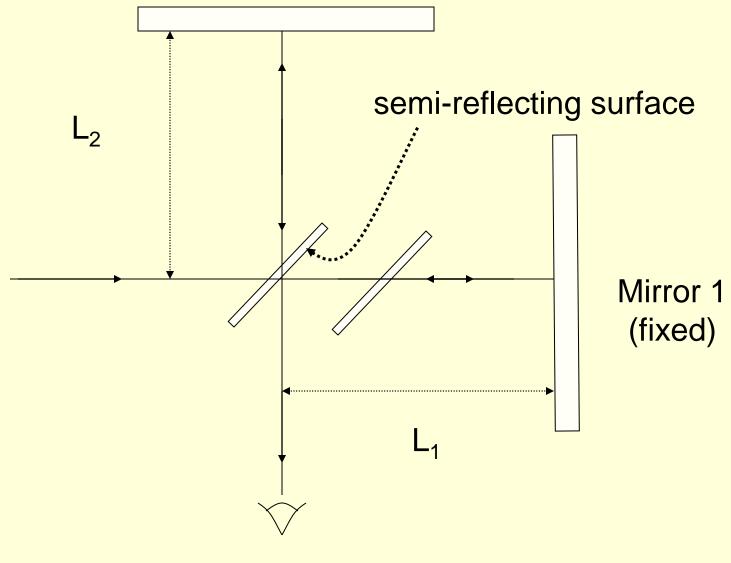
- explosion of a supernova in the Virgo cluster (15Mpc): h=10⁻²¹ à 10⁻²⁴
- binary system of 2 black holes (M=1,4M $_{\odot}$) at 10 Mpc: h=10⁻²² à 10⁻²³

For masses localized at a distance of one kilometer

$$\Delta L = h L \sim 10^{-22} \cdot 10^3 = 10^{-19} m !$$

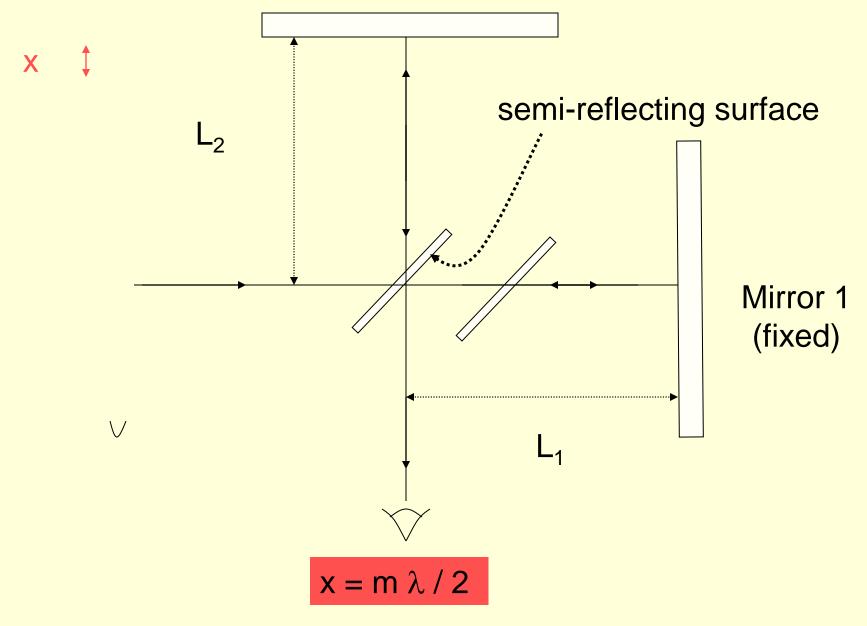
Only known solution : interferometry

Mirrow 2 (mobile)

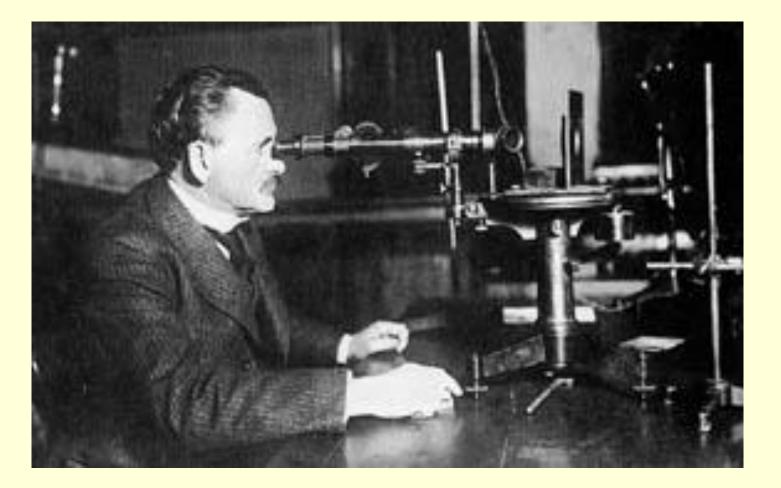


Michelson interferometer

Mirror 2 (mobile)



Albert Michelson counting interference fringes



Sensitivity in 1887: ∆L= 6.10⁻¹⁰ m!

Which size for an interferometer detecting grviational waves?

Size ~ Wavelength of the gravitational wave

~ c / f

Frequency f of gravitational waves $\sim \sqrt{M/R^3}$

(Kepler law for binary systems)

Neutron stars (M ~ 1,4M $_{\odot}$) : f ~ 100 Hz

 \Rightarrow size ~ 3000 km



Supermassive black holes (M ~ $10^6 M_{\odot}$) : f ~ $10^{-4} \text{ à } 10^{-2} \text{ Hz}$

 \Rightarrow size ~ 30 million km

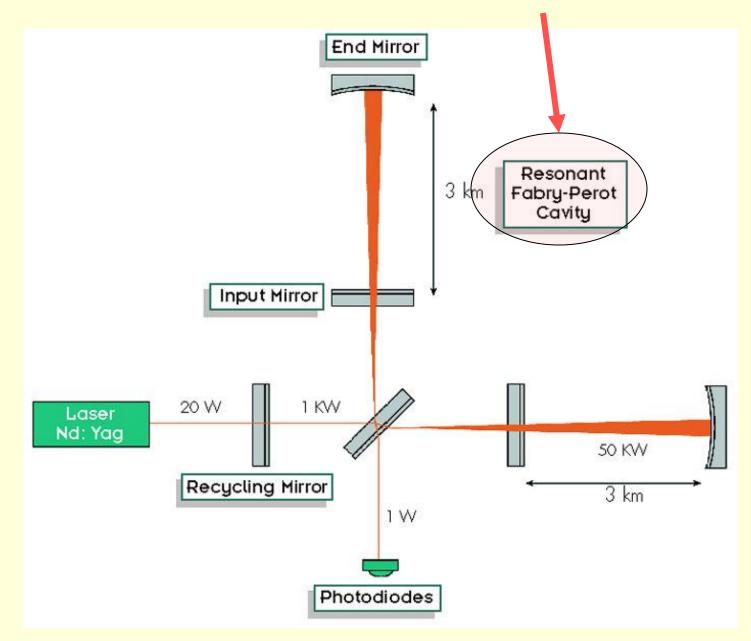




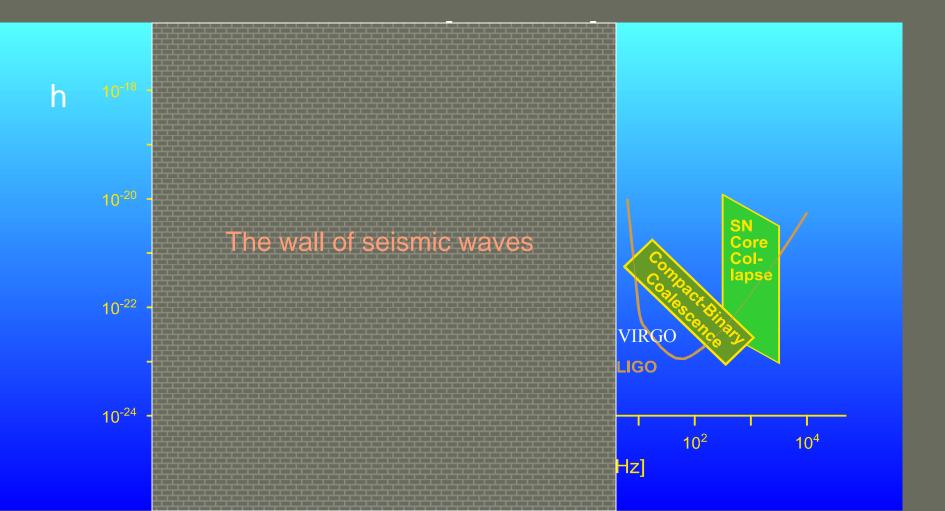
Virgo interferometer near Pisa

Size = 3 km

How to obtain the 3000 km necessary?



Sensitivity of ground detectors

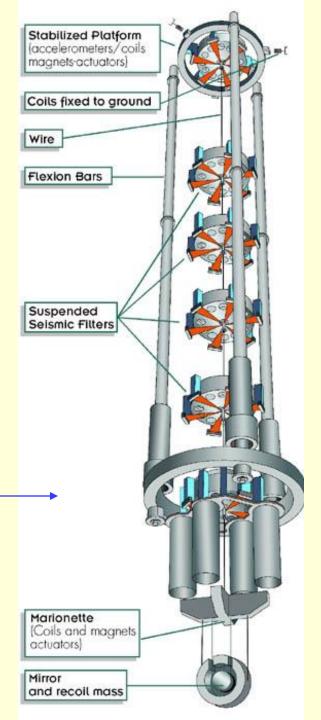


How to ecape as much as possible seismic waves?

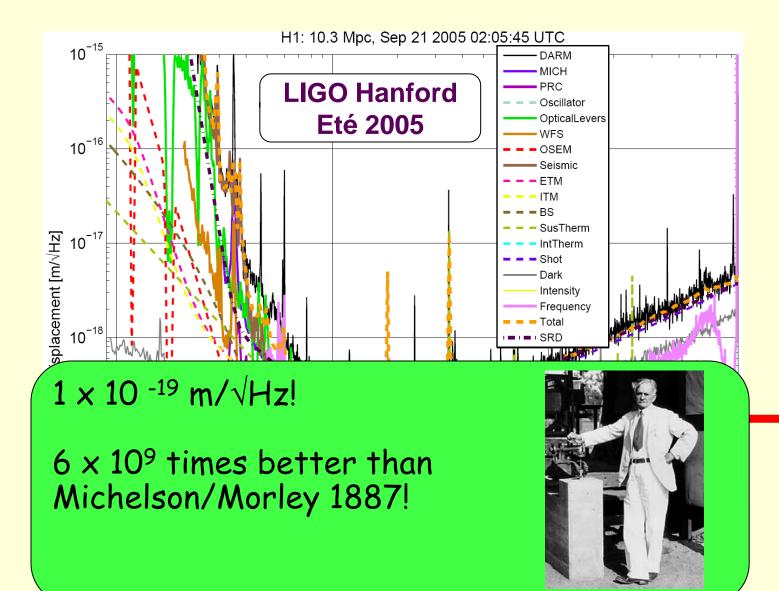
Suspend the interferometer

Virgo suspensions

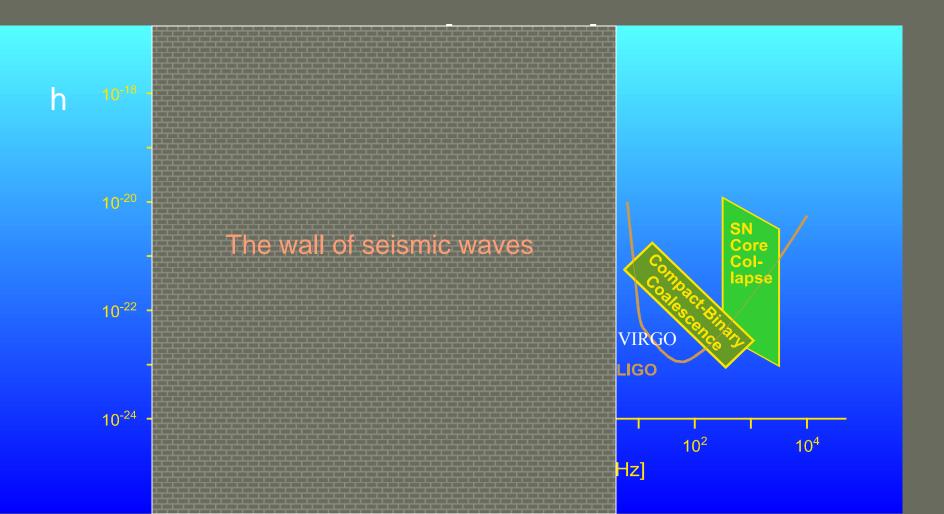
(ou put it underground)



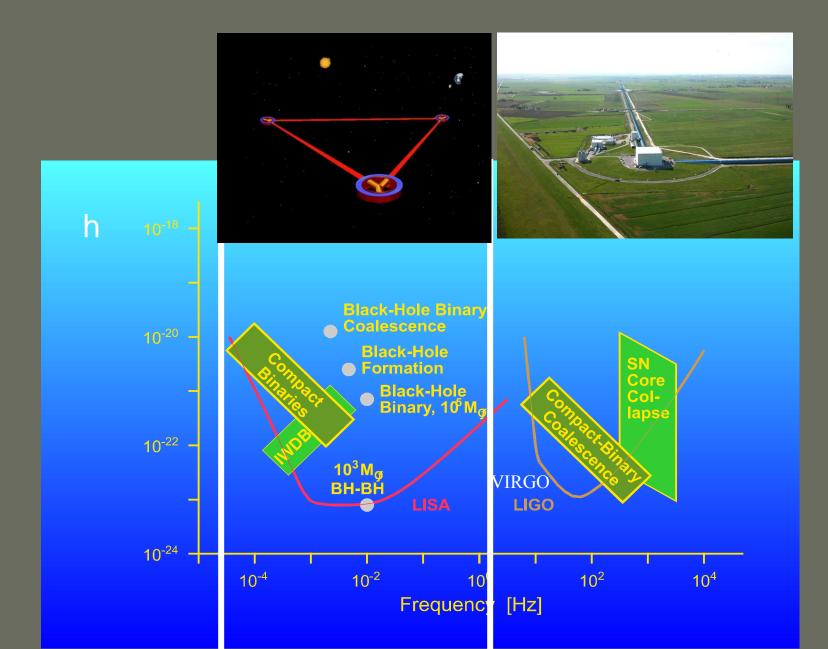
Sensitivity to displacement of ground interférometers

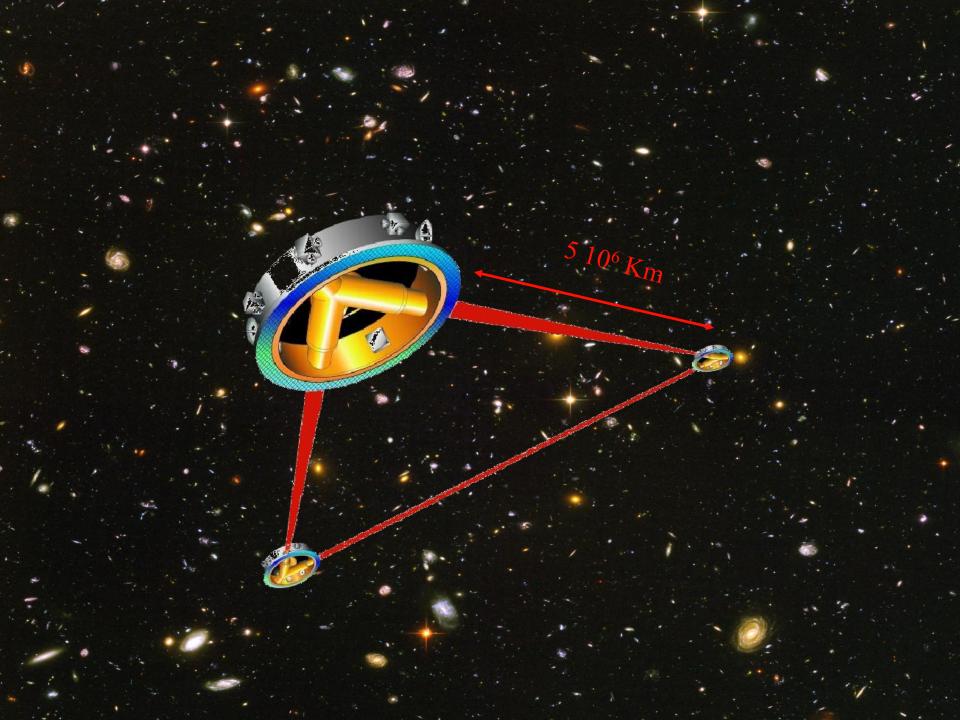


How to completely get away with the seismic wall?



Go into space : LISA interferometer

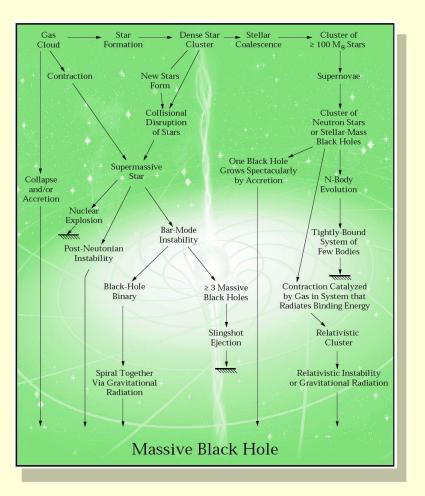




QuickTime™ et un décompresseur Cinepak sont requis pour visionner cette image. One finds such supermassive black holes at the centre of most, if not all galaxies.

How did they form?

Strongly connected with galaxy formation.



QuickTime™ et un décompresseur codec YUV420 sont requis pour visionner cette image.

4. Supernova explosions

Modern theory of supernovae was initiated by Zwicky and Baade in the 30s

Classification of supernovae according to spectroscopy:

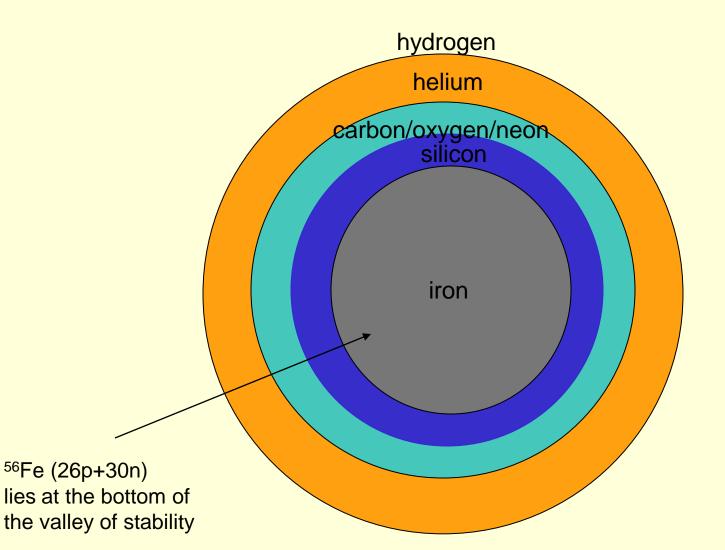
I : Hydrogen lines are absent

- Ia: intermediate mass elements
- Ib: Helium line present
- Ic: Helium lines weak or absent

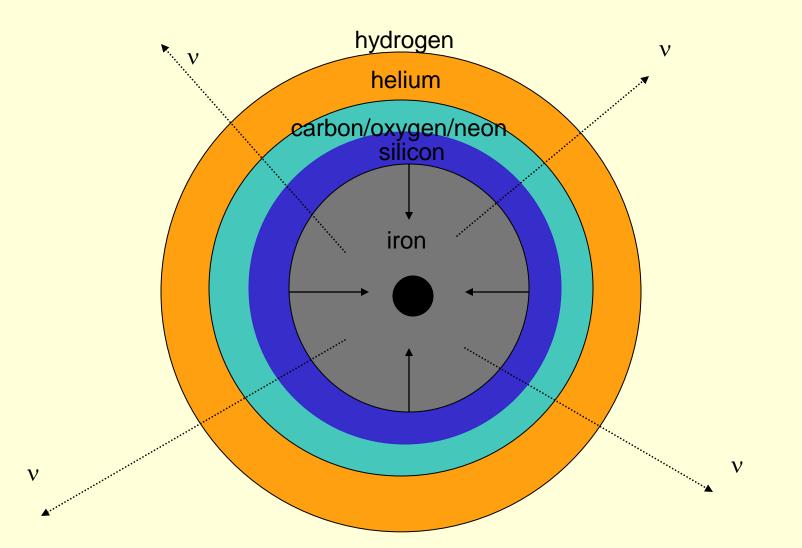
II : Hydrogen lines are present

Supernovae of type II

Pre-supernova stars (M>8 M_{\odot}) have an onion-like structure

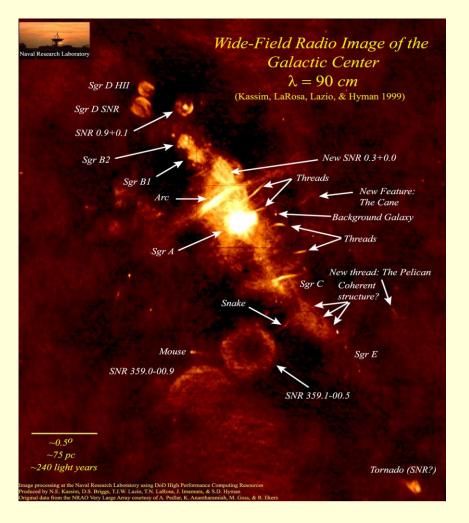


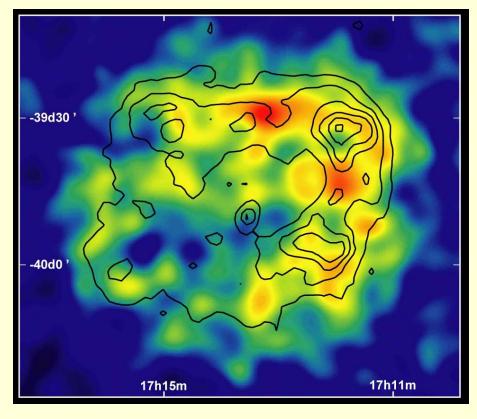
As Si is burned, the mass of the Fe core increases. The density increase turns the electrons relativistic and favours $e+p \rightarrow n+v$. This diminishes the electron degeneracy pressure and leads to a collapse of the core.



QuickTime™ et un décompresseur codec YUV420 sont requis pour visionner cette image.

Radio



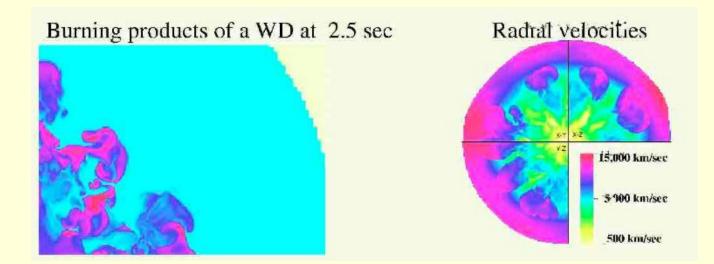


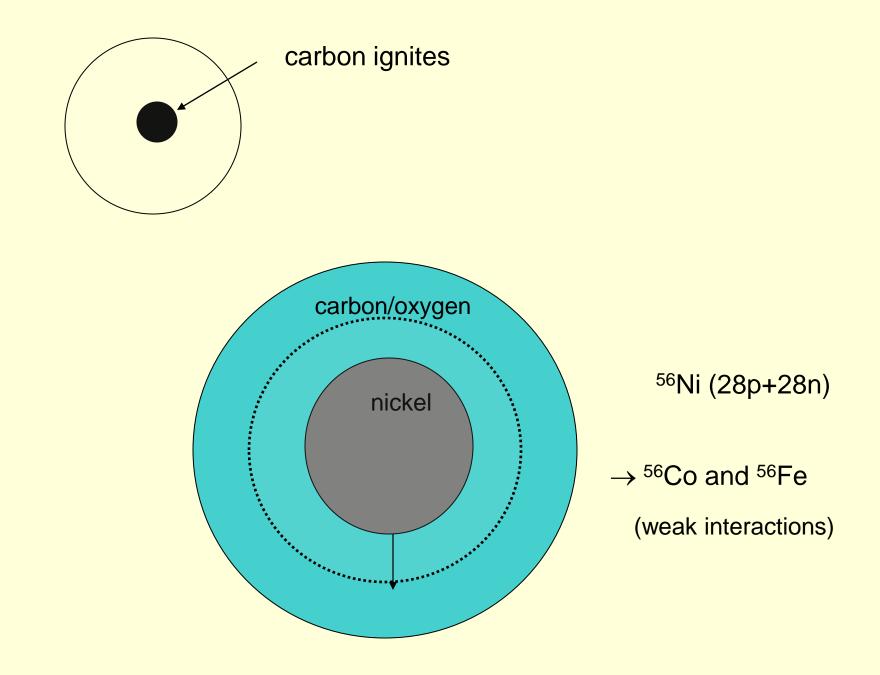
HESS

Supernovae of type la

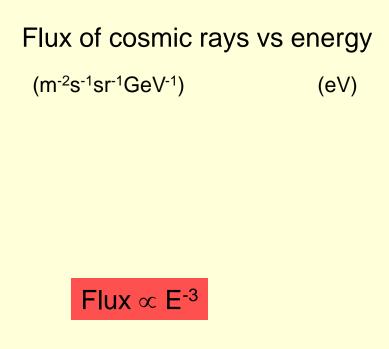
Thermonuclear explosion of white dwarfs:

A carbon-oxygen white dwarf accretes matter (from a companion star) which causes its mass to reach the Chandrasekhar limit: the central core collapses making the carbon burn and causing a wave of combustion that completely disrupts the star.

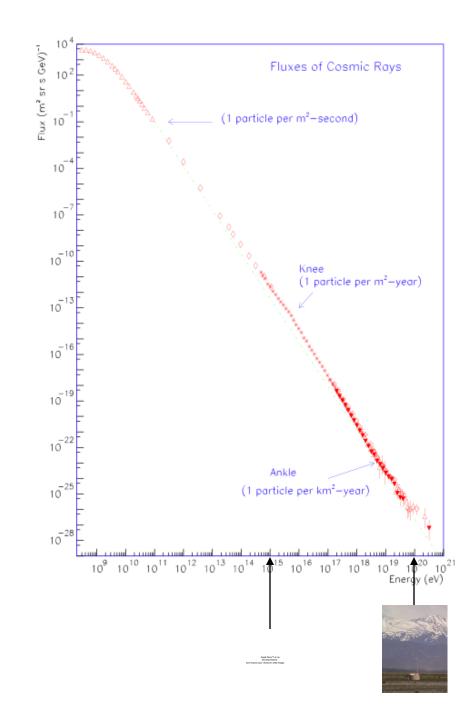




5. Cosmic rays and cosmic acceleration



Single origin for the acceleration?



One easily obtain a power law spectrum $E^{-\gamma}$ if the particles have many encounters where they increase their energy.

One easily obtain a power law spectrum $E^{-\gamma}$ if the particle have many encounters where they increase their energy.

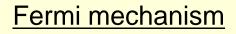
Proof If a test particle of energy E_0 acquires a fraction ξ of its energy at each encounter, then after n encounters:

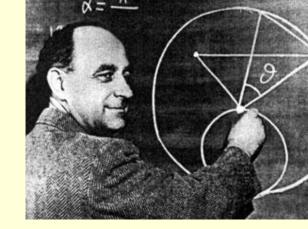
 $E_n = E_0 (1 + \xi)^n$

 ∞

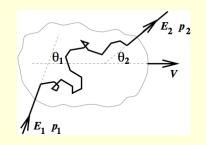
i.e. n encounters necessary to accelerate the particle to energy E: $n = \frac{ln(E/E_0)}{ln(1 + \xi)}$

Define P_{esc} as the probability to escape the acceleration region per encounter, $(1 - P_{esc})^k$ is the probability of remaining in the region after k encounters and the number of particles accelerated beyond energy E is





Fermi (1949) proposes that cosmic rays are accelerated by scattering off magnetized clouds



Consider in the lab frame an ultra-relativistic cosmic ray of energy E_1 and momentum $p_1 = E_1/c$

In the cloud frame

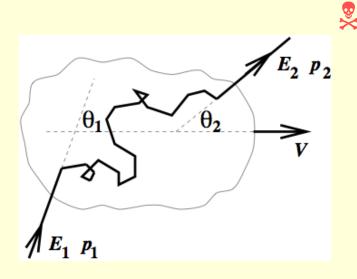
$$B = V/C$$

E'₁ = $\gamma E_1(1 - \beta \cos\theta_1)$
 $\gamma = 1/\sqrt{1-\beta^2}$

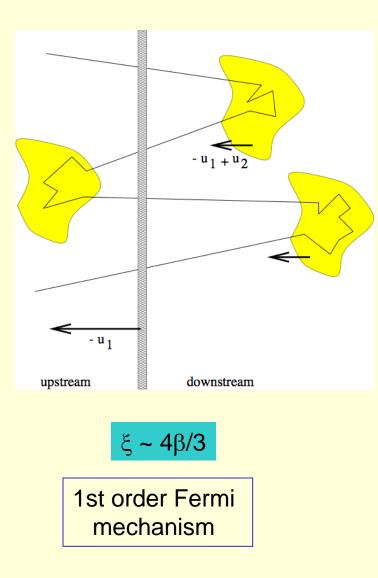
Because scattering is collisionless in the cloud, the final energy E'₂ is equal to E'₁

Back to the lab frame
$$E_2 = \gamma E'_2 (1 + \beta \cos\theta'_2)$$

Then $\xi = \frac{\Delta E}{E} = \frac{E_2 \cdot E_1}{E_1} = \frac{1 - \beta \cos\theta_1 + \beta \cos\theta'_2 - \beta^2 \cos\theta_1 \beta \cos\theta'_2}{1 - \beta^2} - 1$
Since $\frac{dP}{d\cos\theta_1} = \frac{c \cdot V \cos\theta_1}{2c} < \cos\theta_1 > = -\beta/3$
 $\frac{dP}{d\cos\theta'_2} = cst < \cos\theta'_2 > = 0$
2nd order Fermi mechanism



Because the second order is too small, alternate model where the particle multiply crosses a shock front such as induced by supernova explosions.



A complementary view of acceleration sites: Hillas diagram

A particle may not stay for ever in an acceleration site

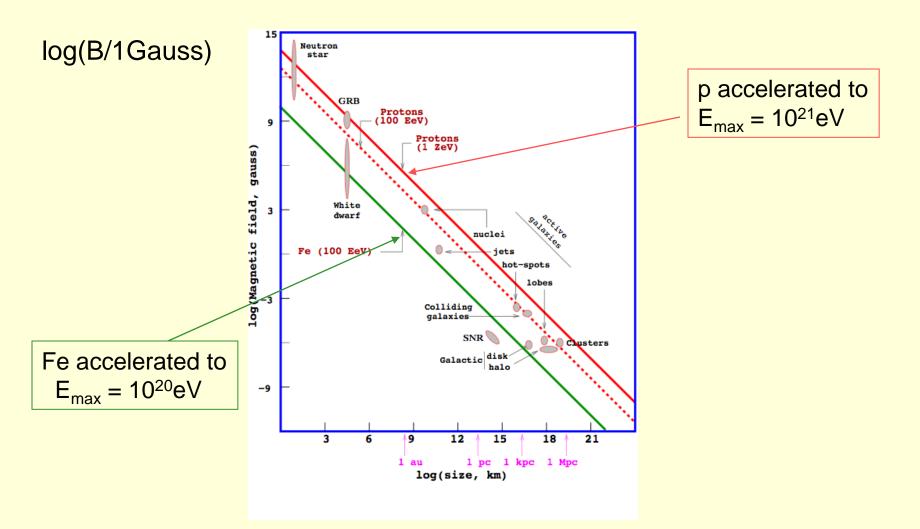
e.g. in a large magnetic field Larmor radius: $r_L = E/(qBc)$ E, v~c

When the energy E increases, r_L may become larger than the size R of the accelerating site.

$$E < E_{max} = qBcR = Z \frac{B}{1\mu G} \frac{R}{1Mpc} 9.3x10^{20}eV$$

q=Ze

Hillas diagram



log(R/1km)

The Greisen-Zatsepin-Kuzmin (GZK) effect

Protons of the highest energy (around 10²⁰ eV) interact with the photons of CMB

 $p\gamma \rightarrow pe^+e^$ $p\gamma \rightarrow \pi N$

The Universe is opaque to such protons.

Protons of the highest energy observed on Earth can only come from its vicinity (sources not further than 100 Mpc).

Observation by Auger of anisotropies for the cosmic rays of the highest energy

 \rightarrow Possibility with more statistics to identify the sources

Astroparticle physics

III - The Universe at large



CERN Summer Student Lecture Programme 2010



- 1. Indirect detection of dark matter
- 2. Looking for standard candles to study dark energy

1. Indirect detection of dark matter

Weakly interacting massive particles remain the best candidate for dark matter (see L. Verde's lectures).

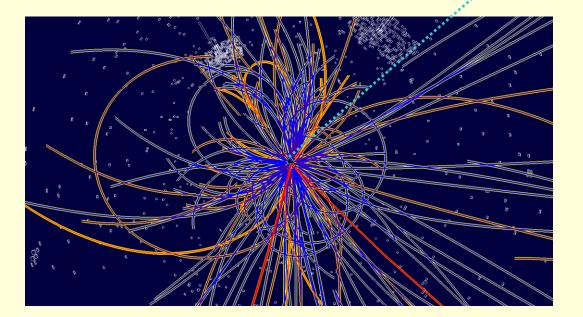
Typically particles with mass around 100 GeV and interactions of the type of the weak interactions of the Standard Model

Appear naturally in many extensions of the Standard Model (in particular Supersymmetry)

They will be searched for at LHC...

These particles are stable and leave the detector unseen while taking away some of the energy :

Signature : missing energy



wimp χ

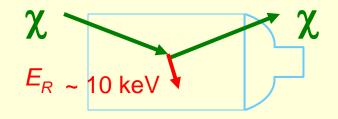
Simulated event in CMS detector

If one discovers at LHC one or several weakly interacting massive stable particles, will this be dark matter?

Not necessarily :

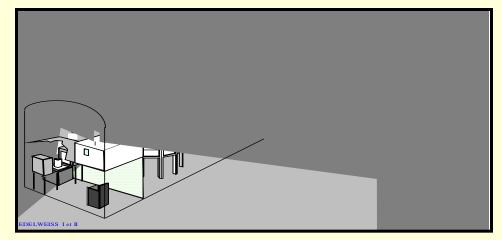
- numerous tests to make to identify their properties: mass, coupling to other particles
- necessary to show that these particles exist in our environment

• direct detection



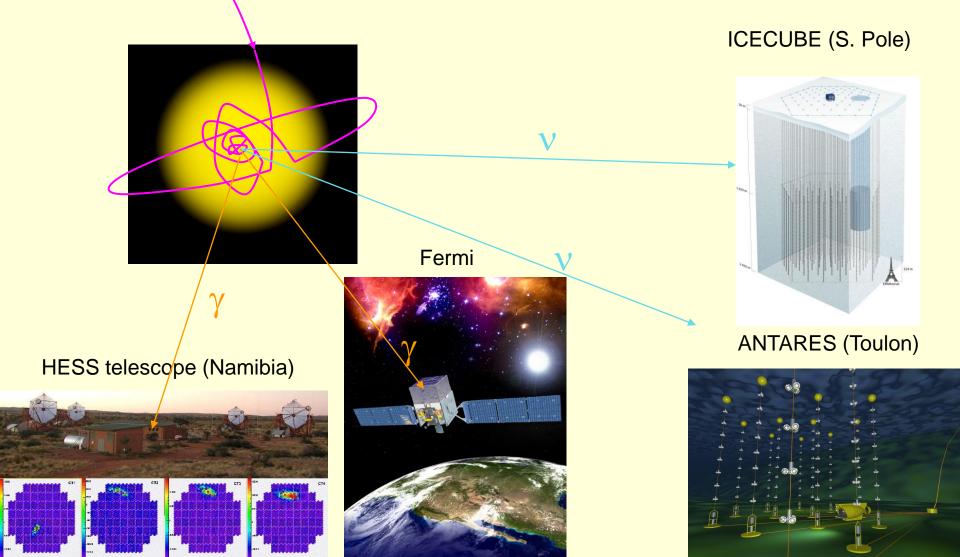
Underground sites (mines, tunnels...)





Modane lab

 indirect detection (wimps accumulate at the centre of the Sun or of the galaxy where they annihilate into energetic neutrinos, gammas, electrons or positrons)



A few surprises!

From PAMELA which detects antiprotons and positrons



antiproton flux

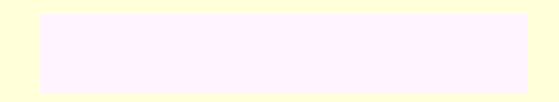
_____ No excess in antiprotons

QuickTime™ et un décompresseur sont requis pour visionner cette image.

Excess in positrons

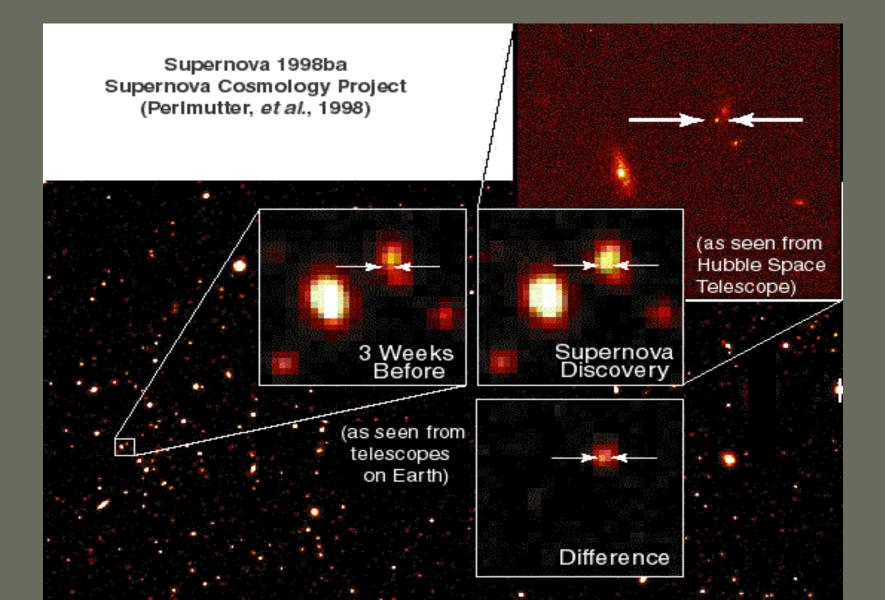
If it is dark matter, it is non-standard because it couples preferentially to leptons.

Astrophysical source?



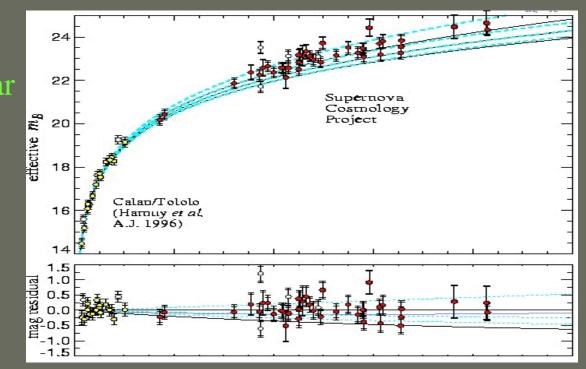
2. Looking for standard candles

Supernovae of type Ia may be used as standard candles to test the geometry of spacetime



Distant supernovae appear less bright than in an expanding universe

 \mathfrak{A} accelerated expansion



 $m_B = 5 \log(H_0 d_L) + M - 5 \log H_0 + 25$

luminosity distance $d_L = l_{H0} z (1 + \frac{1-q_0}{2} z + ...)$

 q_0 deceleration parameter

Why do supernova explosion of type Ia provide standard candles?

Origin: white dwarf where gravitational force is counterbalanced by electron degeneracy pressure (hence independent of the details of the chemical composition)

The star is completely disrupted and all the energy of the explosion goes into the expansion of the products.

But the luminosity depends on the amount of Ni synthesized; for instance, less Ni means lower luminosity, but also lower temperature In the gas and thus less opacity and more rapide energy escape: dimmer supernovae are quicker.

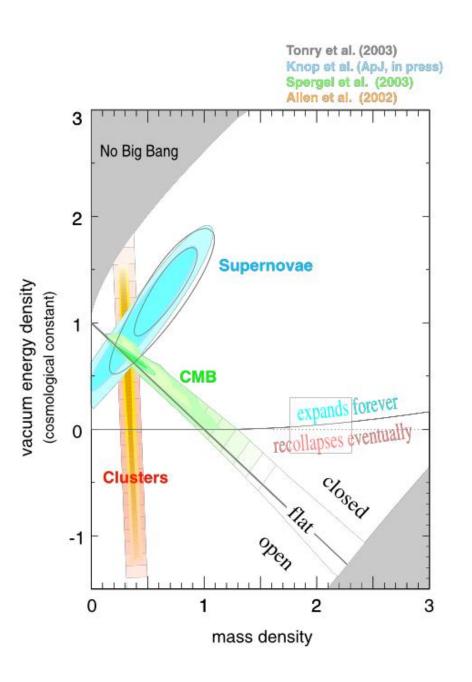
Could this be explained by a cosmological constant ?

Plot $(\Omega_{\Lambda}, \Omega_{M})$:

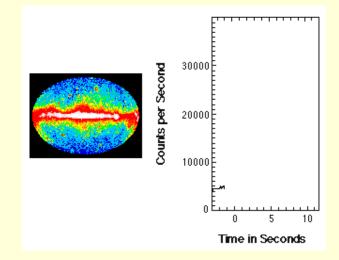
 $\Omega_{\Lambda} \!=\! \rho_{\Lambda} \! / \rho_{c}$, $\Omega_{M} \!=\! \rho_{M} \! / \rho_{c}$

Concordance model

Note: if this is so, the vacuum energy takes the value expected in the context of gravity. Associated energy scale : $\Lambda \sim 10^{-3} \, \text{eV}$

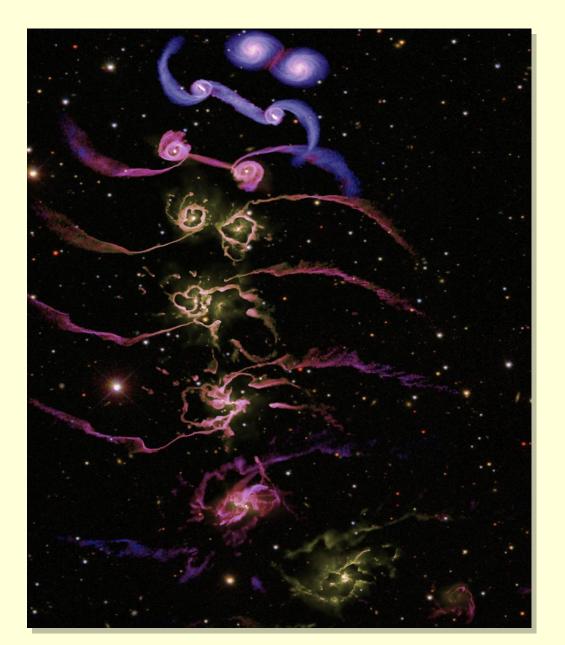


✓ Gamma ray bursts



Determine the luminosity through a relation between the collimation corrected energy E_{γ} and the peak energy

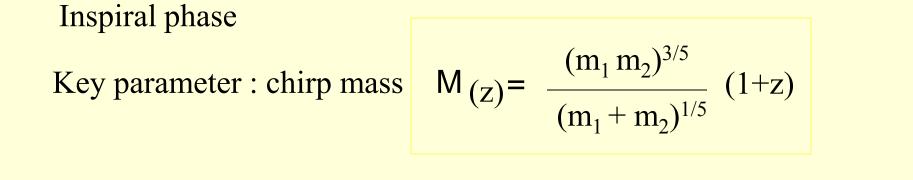
✓ Coalescence of supermassive black holes

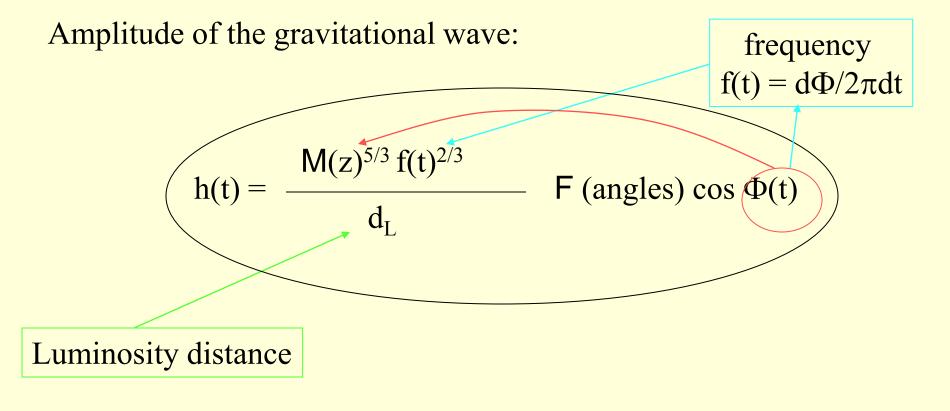


Inspiral phase

Key parameter : chirp mass

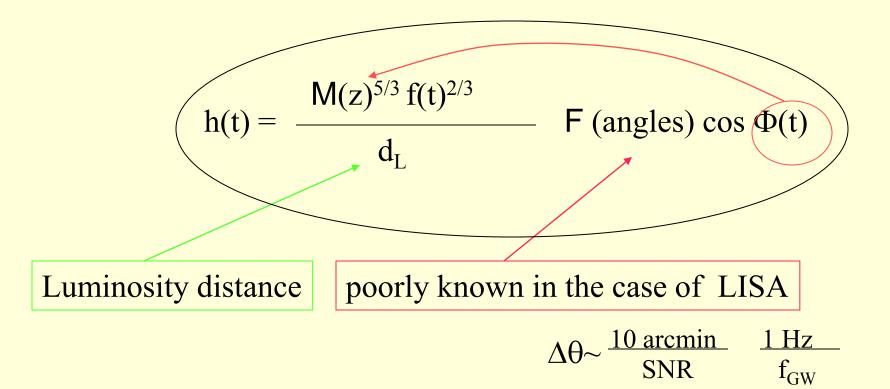
$$M_{(z)} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}} (1+z)$$



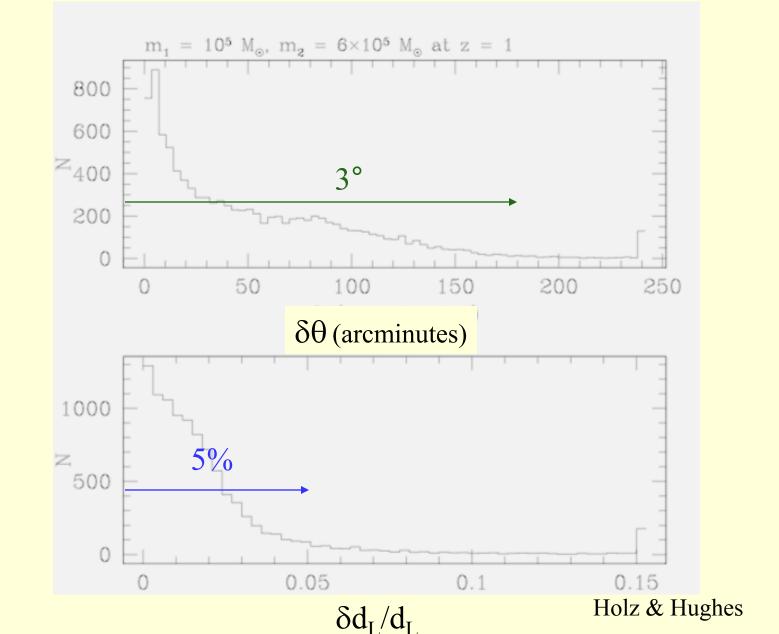


Inspiral phase Key parameter : chirp mass $M_{(Z)} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}} (1+z)$

Amplitude of the gravitational wave:

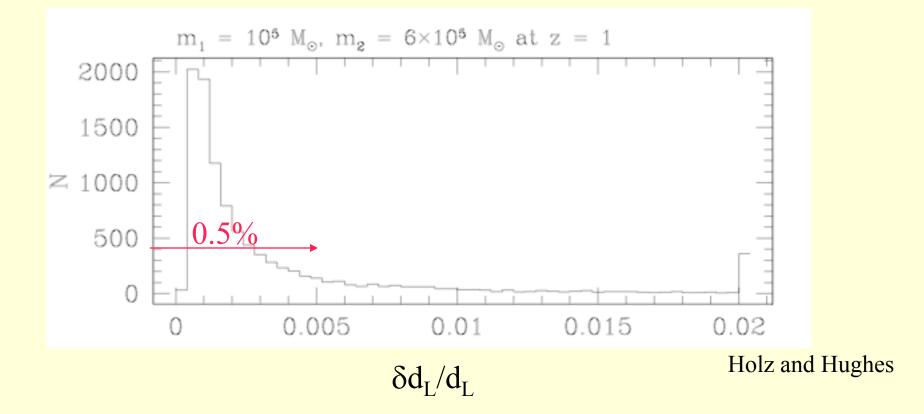


z = 1, $m_1 = 10^5 M_{\odot}$, $m_2 = 6.10^5 M_{\odot}$

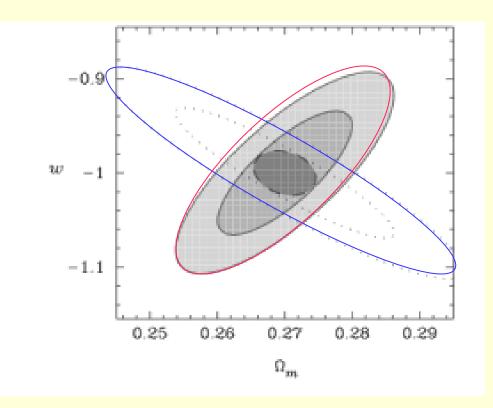


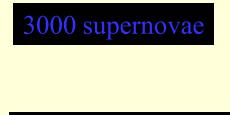
Using the electromagnetic counterpart

Allows both a measure of the direction and of the redshift



Determining the equation of state of dark matter





100 SMBH sources

Dalal et al. astro-ph/0603275

A new window is being opened towards the Universe using the knowledge accumulated over more than 50 years of high energy physics.

Back to where the field started, but this time not to understand the infinitely small, but to grasp the infinitely large.