Science & Technology Facilities Council

Space charge issues In FFAs

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4th ICFA mini-workshop on Space charge 2019

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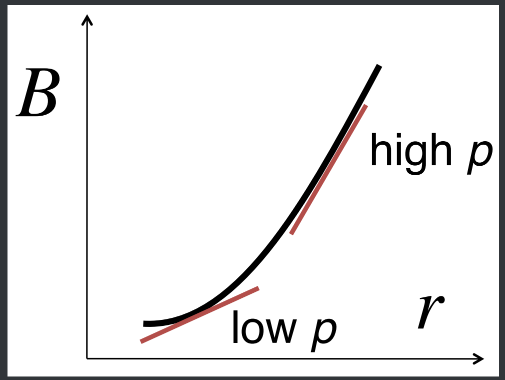
Horizontal FFA

Differences between conventional synchrotrons and zero-chromatic FFAs from beam dynamics point of view:

- Intrinsic non-linearities,



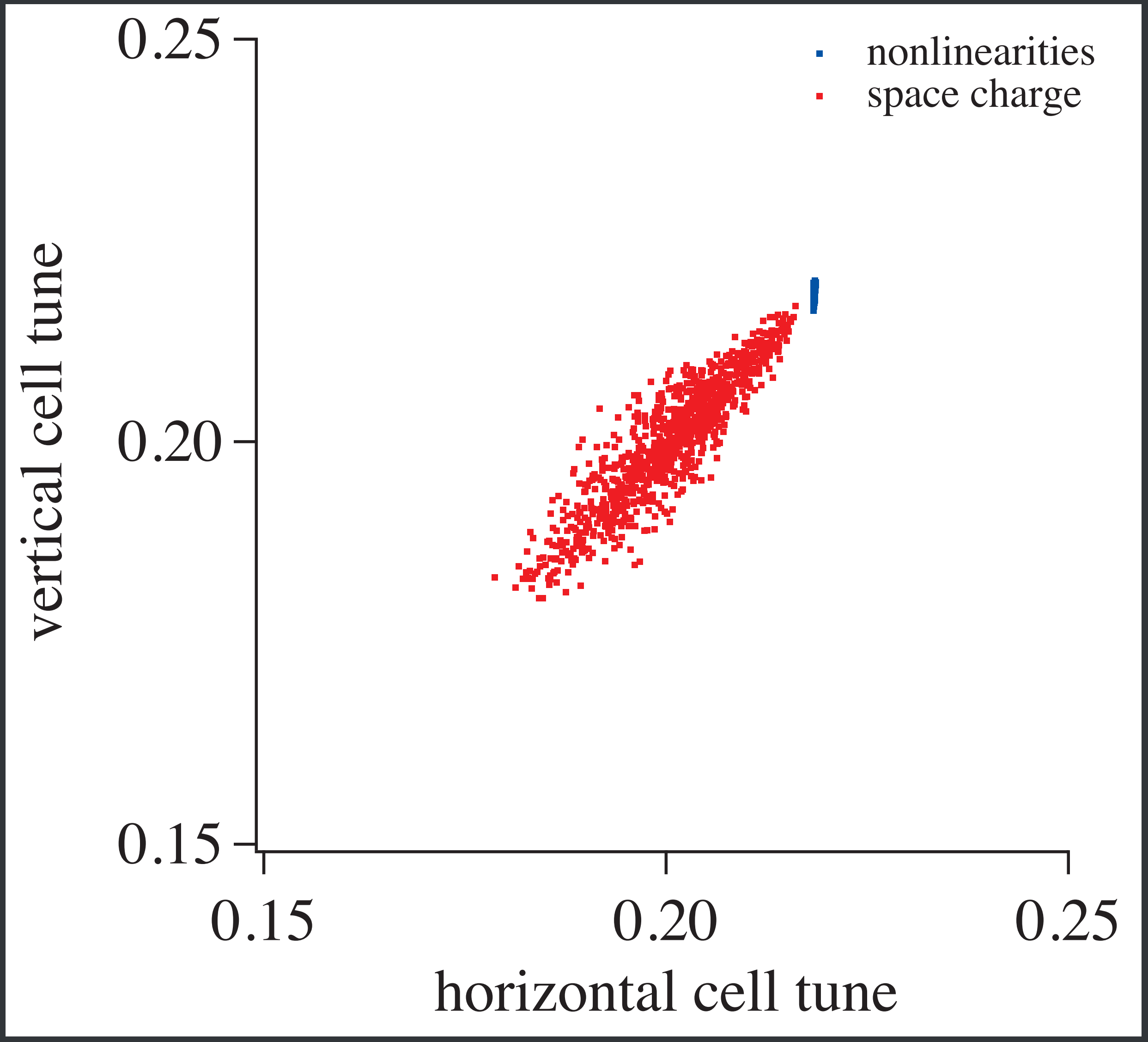
- Big horizontal aperture (with proper tune, large horizontal emittance can be accommodated),
- More flexible longitudinal parameters.

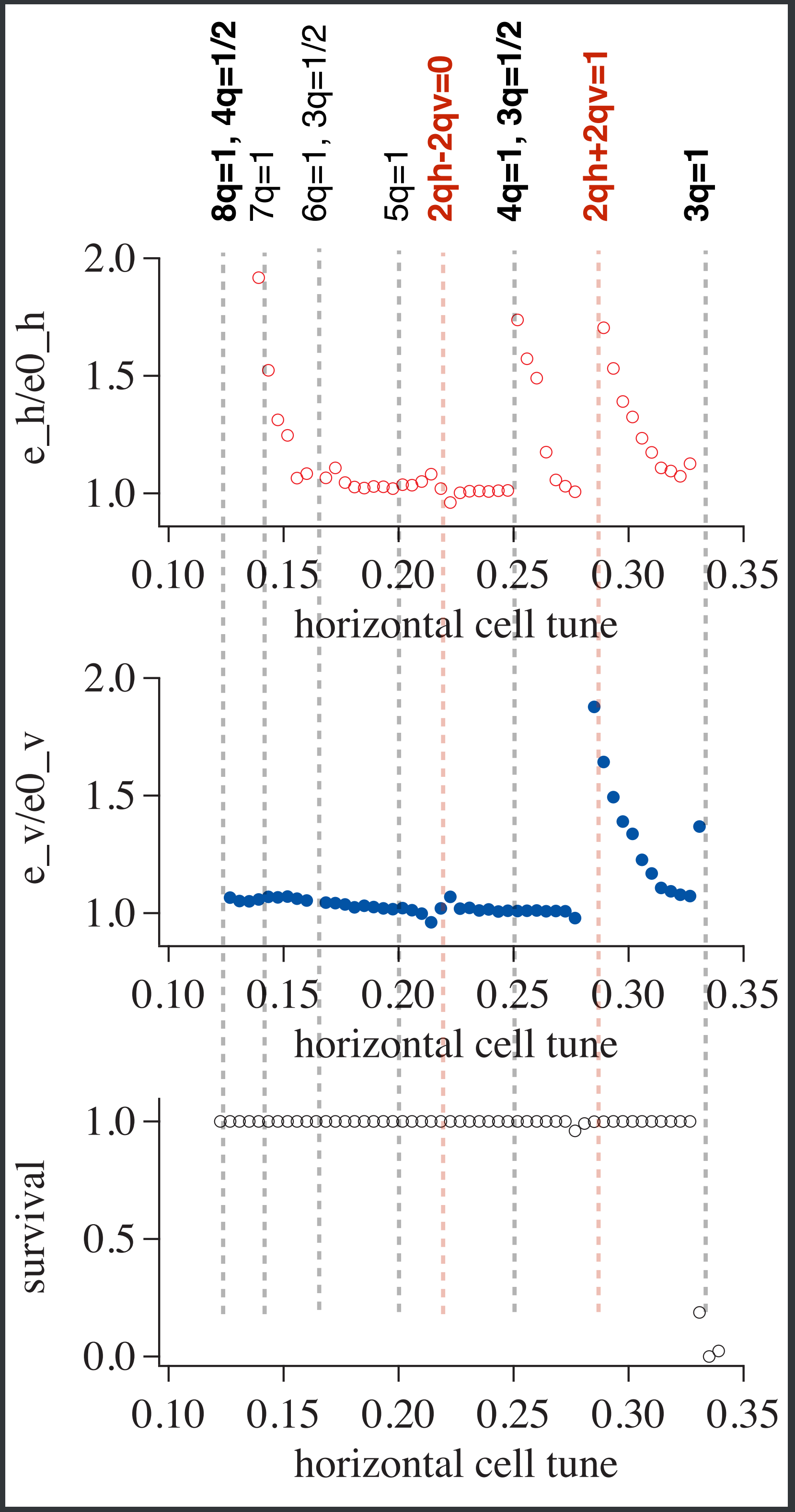
Simulation Parameters

- Lattice: Spiral DF scaling horizontal FFA, 24 cells, k=21, spiral angle=60 deg, average radius=24 m.
- Beam: Gaussian distribution, 4.68 10^{13} protons/bunch, rms emittance=25 π mm.mrad, energy=0.4 - 1.2 GeV.
- RK4 integration in s-dependent magnetic field.
- Frozen space charge (Gaussian potential), with update of beam size after every space charge kick.
- Only transverse space charge kicks, but with longitudinal position dependent line distribution.
- Synchrotron oscillation and acceleration included.

Non-linearities effects

Tune shift due to nonlinearities is much smaller than space charge tune shift in typical horizontal FFAs.





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Vertical excursion FFA

Vertical excursion FFA considered in 1955 as an ”Electron Cyclotron”, rediscovered recently.

Advantages:

- Quasi-isochronicity for relativistic particles,
- Infinite transition energy,
- Orbit radius independent of momentum, like synchrotrons,
- Geometrical arrangement of the lattice footprint independent of the scaling condition, unlike in horizontal scaling FFA,
- Rectangular shape for the main magnets and the coil geometry could be simpler compared to the spiral magnet of horizontal FFA.

Vertical excursion FFA

To keep the transverse linearised equations of motion independent of momentum, the field must follow

$$B = B_0 e^{m(v-v_0)}$$

with $m = \frac{1}{B}\frac{dB}{dv}$ the vertical normalised field gradient.

⚠️ since m is a vertical gradient, there is coupling between horizontal and vertical plane.

Rectangular Field model

- In the mid-plane ($h = h_0$): Cartesian coordinates (h,v,l)

$$\boxed{B_v(h_0, v, l) = B_0 e^{m(v-v_0)} \mathcal{F}(l)}$$

with m the constant normalised field gradient, and $\mathcal{F}$ the arbitrary fringe field function (*tanh* here).

- From $\left(\overrightarrow{curl}\,\overrightarrow{B}\right)_h = 0$

$$B_l(h_0, v, l) = \int_v \frac{\partial B_v}{\partial l} dv = B_0 \mathcal{F}'(l) \left(\frac{e^{m(v-v_0)}}{m} + g(l) \right)$$

with $g(l)$ an arbitrary function independent of v, must be 0 to keep the invariance of the closed orbits with momentum.

$$\boxed{B_l(h_0, v, l) = \frac{B_0}{m} e^{m(v-v_0)} \mathcal{F}'(l)}$$

- Because of the field symmetry, $\boxed{B_h(h_0, v, l) = 0}$

Rectangular Field model (2)

Cartesian coordinates (*h*,*v*,*l*)

In the mid-plane ($h = h_0$)

$$\begin{cases} B_{h0}(h_0, v, l) = 0 \\ B_{v0}(h_0, v, l) = B_0 e^{m(v-v_0)} \mathcal{F}(l) \\ B_{l0}(h_0, v, l) = \dfrac{B_0}{m} e^{m(v-v_0)} \mathcal{F}'(l) \end{cases}$$

with m the constant normalised field gradient, $\mathcal{F}$ the fringe field function (*tanh* in the models)

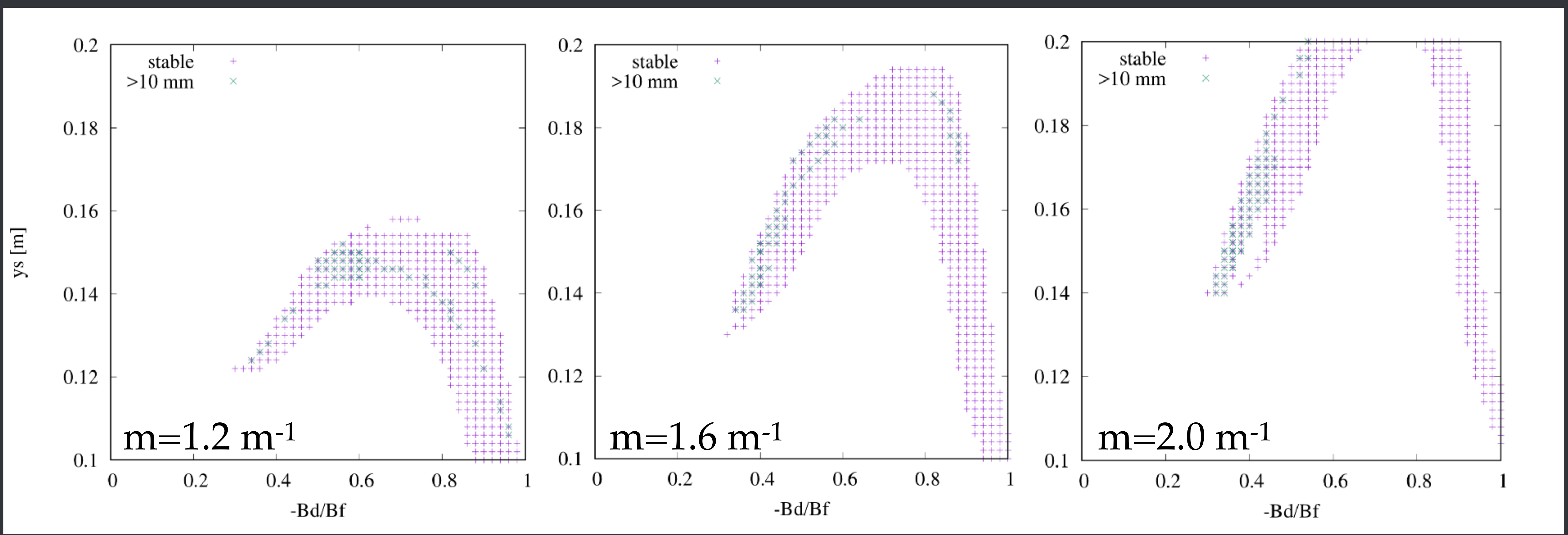
Off mid-plane extrapolation components from Maxwell equations:

$$\begin{cases} B_h(h, v, l) = \dfrac{B_0}{m} e^{m(v-v_0)} \displaystyle\sum_i B_{hi}(l)(h-h_0)^i \\ B_v(h, v, l) = B_0 e^{m(v-v_0)} \displaystyle\sum_i B_{vi}(l)(h-h_0)^i \\ B_l(h, v, l) = \dfrac{B_0}{m} e^{m(v-v_0)} \displaystyle\sum_i B_{li}(l)(h-h_0)^i \end{cases}$$

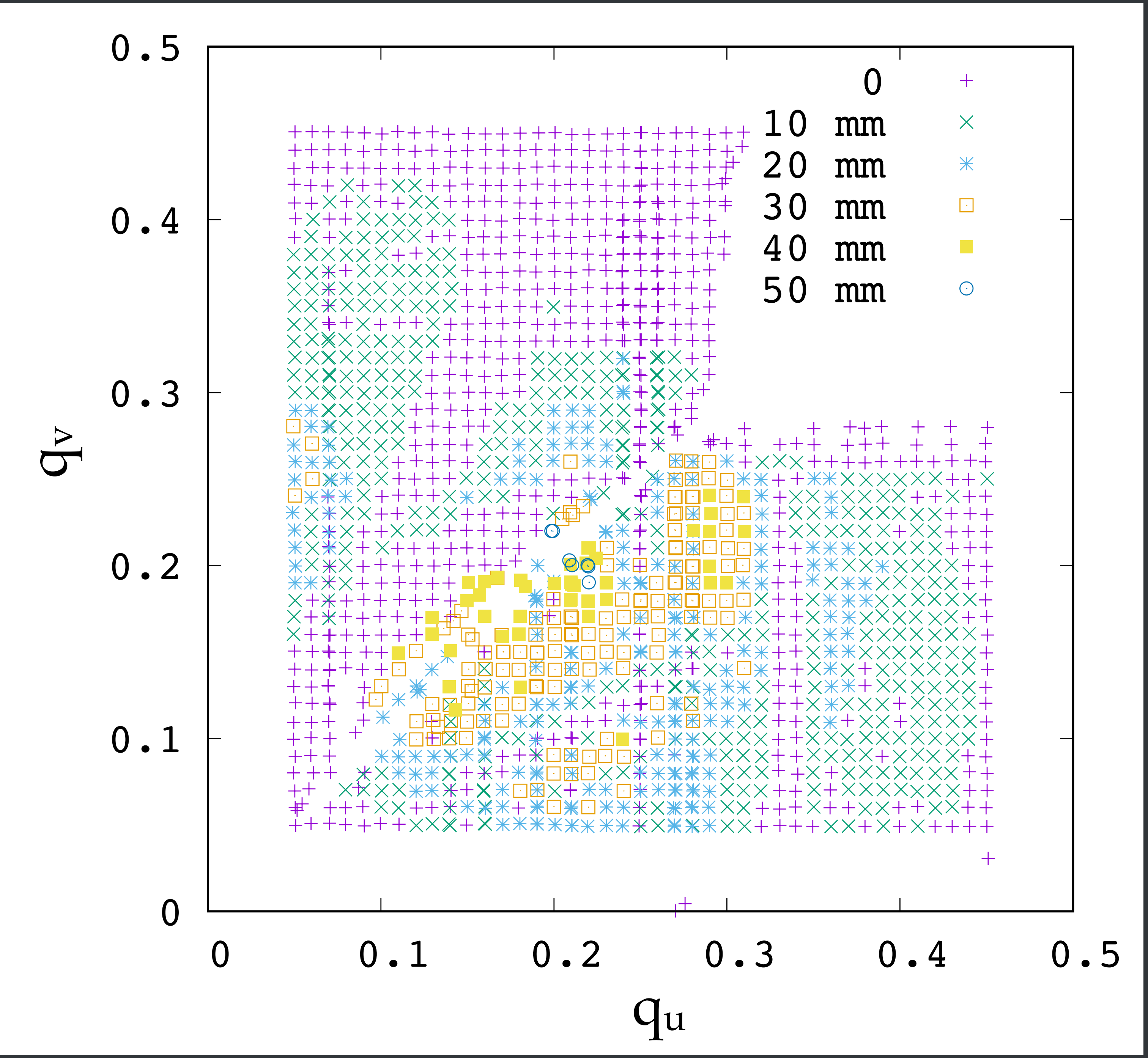
VFFA lattice parameters

3 key parameters in the case of rectangular magnets:

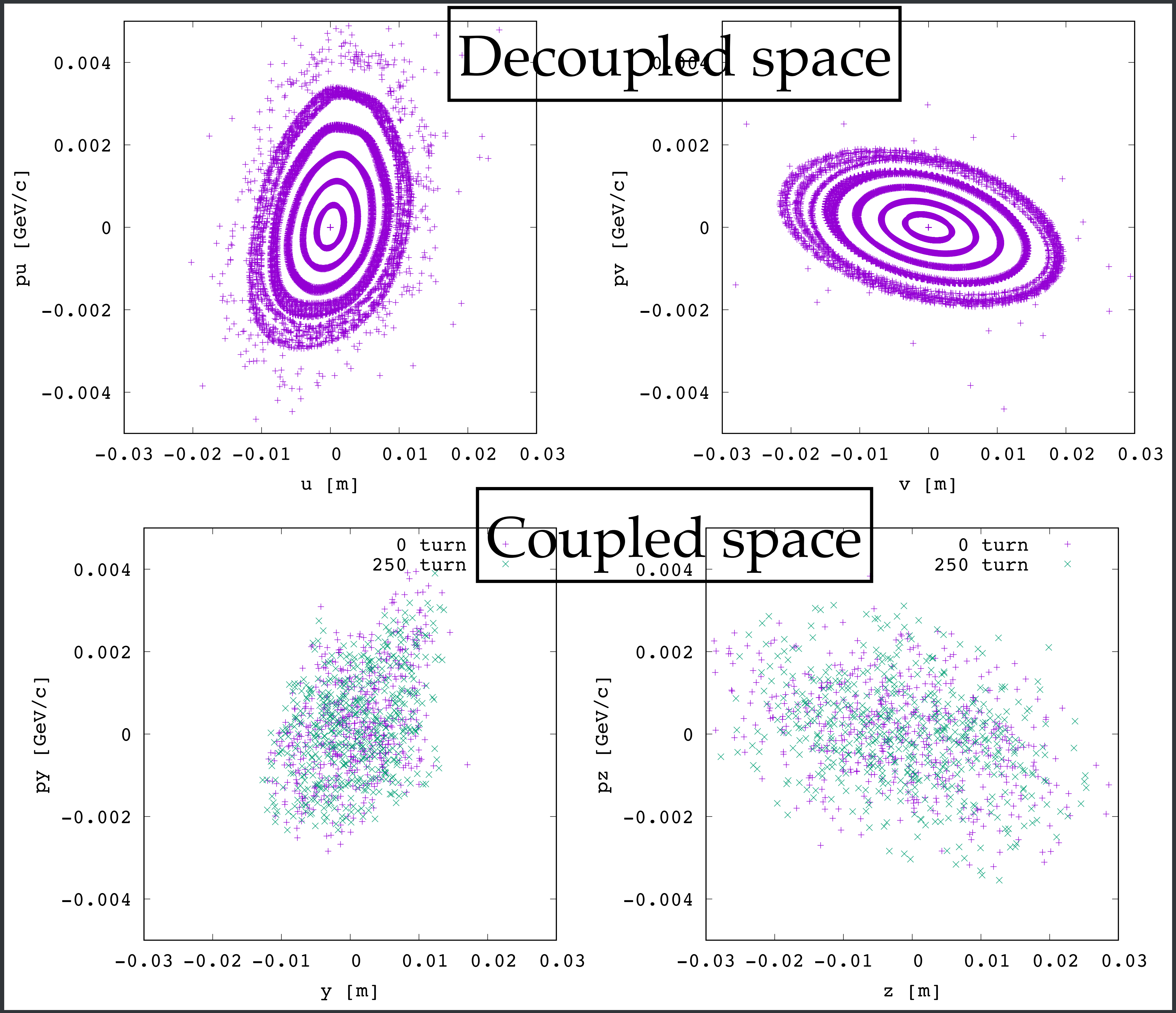
- Normalised field gradient *m*,
- F/D strength ratio,
- Magnet position in the radial direction y_s.



Cell tune diagram



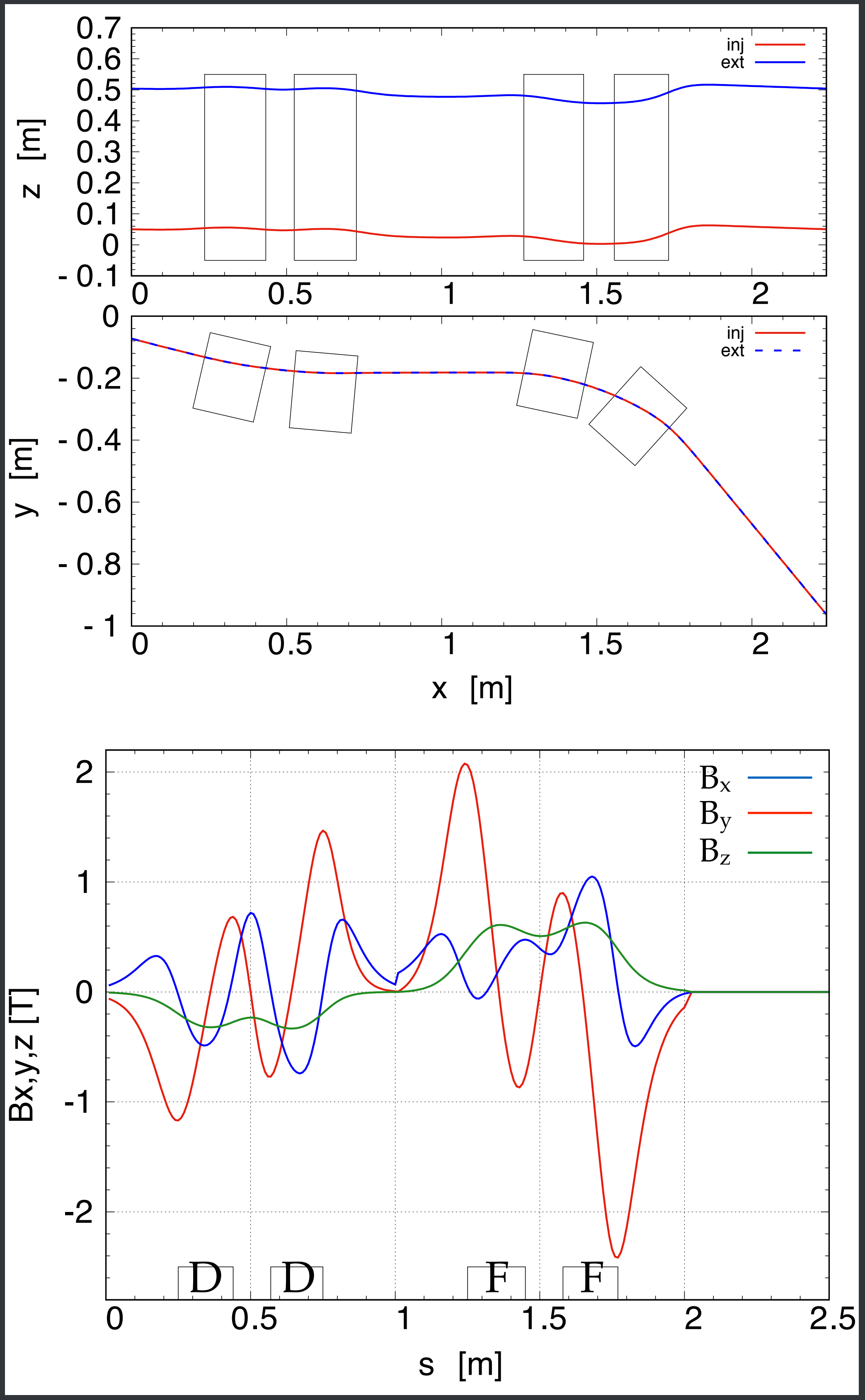
Dynamic aperture (250 turns)



Vertical excursion FFA

Example: ISIS test ring

Kinetic energy	3 - 12 MeV
Reference radius	3.9789 m
Number of cells	10
Packing factor	0.32
Straight section	1.0 m (long), 0.5 m (short)
m-index	1.6 m^{-1}
Ratio Bd/Bf strength	-0.47
Orbit excursion	0.4 m
Cell tune (H, V)	(0.19, 0.16)
Transition gamma	infinite



Questions in VFFA

- What is the definition of a beta-function in a decoupled space?
- Is there any meaning in looking at beta-functions in the decoupled spaces?
- Is it useful to look at the 2D emittance in the decoupled spaces?
- Space charge effects:
  - Does the space charge tune shift depress the tunes in the decoupled space?
  - Since they are 2 modes in the magnets (fringe field=solenoid case, and body=skew quad case), can we learn something by looking at the 2 cases separately (done in previous studies), or should they be combined from the beginning?

Summary

- Space charge in horizontal FFAs are no different than in conventional synchrotrons.

- VFFA is considered now for an upgrade of ISIS synchrotron. The dynamics are highly coupled and under investigation. The high intensity effects are still unknown.